Neutrino theory & phenomenology

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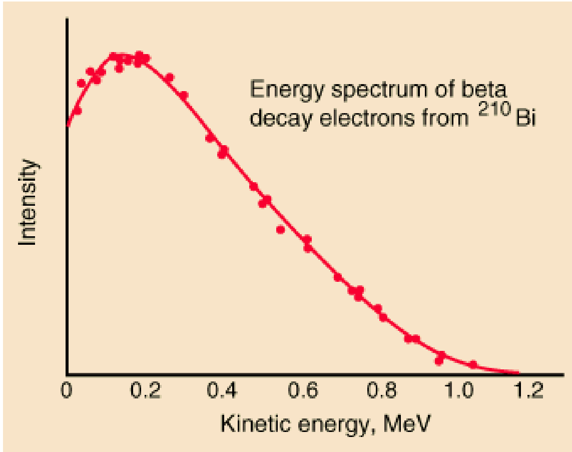
Discovery of neutrinos

- At end of 1800's radioactivity was discovered and three types of particles were identified: α, β, γ. β: an electron coming out of the radioactive nucleus.
- Energy conservation $\Rightarrow e^-$ should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \quad \Rightarrow \quad E_e = M(A, Z+1) - M(A, Z)$$

- But in 1914 James Chadwick showed that the electron energy spectrum is continuous:



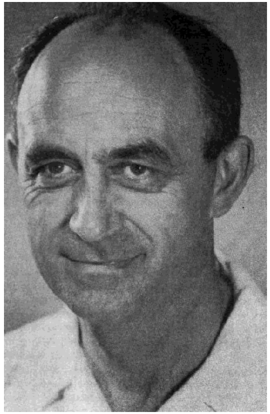
$\Rightarrow$ Do we throw away the energy conservation?

Discovery of neutrinos

- The idea of the neutrino came in 1930, when W. Pauli tried a desperate operation to save the "energy conservation principle".

In his letter addressed to the "Liebe Radioaktive Damen und Herren" (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tübingen, he put forward the hypothesis that a new particle exists as "constituent of nuclei", the "neutron" ν, able to explain the continuous spectrum of nuclear beta decay:

$$(A, Z) \to (A, Z+1) + e^- + \nu$$



- The ν is light (in Pauli's words: "the mass of the ν should be of the same order as the e mass"), neutral and has spin 1/2;
- In order to distinguish them from heavy neutrons, Fermi proposed to name them neutrinos.

Discovery of neutrinos

- Electron (anti)neutrinos produced in **nuclear fission reactions** ($E_\nu \sim$ MeV) were firmly observed at Hanford in 1956 [1] through *inverse beta decay*: $\boxed{\bar{\nu}_e + p \to e^+ + n}$;
  $\Rightarrow$ Nobel prize: Cowan & Reines, 1995
- Muon neutrinos produced in **pion decay** ($E_\nu \sim$ GeV) were detected at Brookhaven in 1962 [2] through *muon appearance*: $\boxed{\nu_\mu + n \to \mu^- + p}$ & $\boxed{\bar{\nu}_\mu + p \to \mu^+ + n}$;
  $\Rightarrow$ Nobel prize: Lederman, Schwartz & Steinberger, 1988
- Tau neutrinos produced in **charmed meson decay** ($E_\nu \sim 100$ GeV) were detected by the DONUT experiment at Fermilab in 2000 [3] through *tau appearance*. Tau tracks were distinguished from muon tracks due to the fast *tau decay*, which induced a "kink" in the track after ~ 2 mm.

[1] C. L. Cowan Jr. *et al.*, Science **124** (1956) 103.
[2] G. Danby *et al.*, Phys. Rev. Lett. **9** (1962) 36.
[3] K. Kodama *et al.* [DONUT Collaboration], Phys. Lett. B **504** (2001) 218 [hep-ex/0012035].

Neutrino properties: interactions

- Already in 1934, Hans Bethe and Rudolf Peierls showed that the cross section between ν and matter is very small:

$$\sigma^{\nu N} \sim 10^{-38}\ \text{cm}^2\ \frac{E_\nu}{\text{GeV}}$$

- Let's consider for example atmospheric $\nu' s$:

$$\Phi_\nu^{\text{ATM}} = 1\ \nu \text{ per cm}^2 \text{ per second} \qquad \text{and} \qquad \langle E_\nu \rangle = 1\ \text{GeV}$$

- How many interact? In a human body:

$$N_{\text{int}} = \Phi_\nu \times \sigma^{\nu N} \times N_{\text{nucleons}}^{\text{human}} \times T_{\text{life}}^{\text{human}} \qquad (M \times T \equiv \text{Exposure})$$

$$\left.\begin{aligned} N_{\text{nucleons}}^{\text{human}} &= \frac{M^{\text{human}} \approx 80\ \text{kg}}{[\text{gr}]} \times N_A = 5 \times 10^{28}\ \text{nucleons} \\ T_{\text{life}}^{\text{human}} &= 80\ \text{years} = 2 \times 10^9\ \text{sec} \end{aligned}\right\} \begin{aligned} &\text{Exposure}_{\text{human}} \\ &\approx 6\ \text{Ton} \times \text{year} \end{aligned}$$

$$N_{\text{int}} = 1 \times (5 \times 10^{28}) \times (2 \times 10^9) \times 10^{-38} \sim \textbf{1 interaction per lifetime}$$

$\Rightarrow$ Need huge detectors with Exposure $\sim$ KTon $\times$ year

Neutrino properties: mass

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay:

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone:

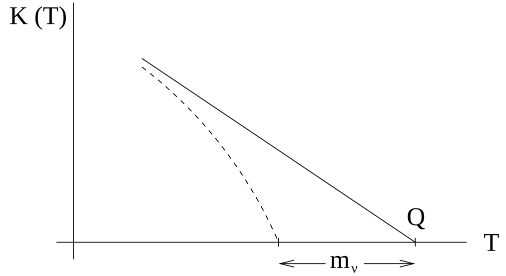
$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{C p_e E_e F(E_e)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}$$

where $T = E_e - m_e$, Q = maximum kinetic energy (for ^3H beta decay $Q = 18.6$ KeV)

- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_\nu = 0 \Rightarrow T_{\text{max}} = Q$$

$$m_\nu \neq 0 \Rightarrow T_{\text{max}} = Q - m_\nu$$



- At present only a bound (Mainz & Troisk experiments): $m_{\nu_e}^{\text{eff}} \equiv \sum m_j |U_{ej}|^2 < 2.2$ eV (at 95% CL)
- Katrin proposes to improve present sensitivity to $m_{\text{eff}}^\beta \sim 0.2$ eV.

Neutrino properties: helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.
- Using the electron capture reaction: (Eu=Europium, Sm=Samarium)

$$e^- + {}^{152}\mathrm{Eu} \to \nu + {}^{152}\mathrm{Sm}^*$$
$$ {}^{152}\mathrm{Sm}^* \to {}^{152}\mathrm{Sm} + \gamma$$

with $J({}^{152}\mathrm{Eu}) = J({}^{152}\mathrm{Sm}) = 0$, $J({}^{152}\mathrm{Sm}^*) = 1$, $L(e^-) = 0$

- Angular momentum conservation $\Rightarrow$ $\begin{cases} J_z(e^-) = J_z(\nu) + J_z(\mathrm{Sm}^*) \\ \qquad\quad\; = J_z(\nu) + J_z(\gamma) \\ \pm 1/2 = \mp 1/2 \quad \pm 1 \end{cases} \Rightarrow \boxed{J_z(\nu) = -\frac{1}{2} J_z(\gamma)}$
- Nuclei are heavy $\Rightarrow$ $\vec{p}({}^{152}\mathrm{Eu}) \simeq \vec{p}({}^{152}\mathrm{Sm}) \simeq \vec{p}({}^{152}\mathrm{Sm}^*) = 0$
  so momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity = γ helicity
- Goldhaber et al. found γ had negative helicity $\Rightarrow$ ν has negative helicity

$\Rightarrow$ **Thus so far ν was a particle with $m_\nu = 0$ and left handed.**
(because for massless fermions helicity≡ chirality...)

Neutrinos in the Standard Model

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- LEP tested this symmetry to 1% precision and the missing particles t, ν_τ were found:

$(1,2)_{-1}$	$(3,2)_{1/3}$	$(1,1)_{-2}$	$(3,1)_{4/3}$	$(3,1)_{-2/3}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Notice there is no ν_R

$\Rightarrow$ Accidental global symmetry:

$B \times L_e \times L_\mu \times L_\tau$

- When SM was invented upper bounds on m_ν:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e)$$

$$m_{\nu_\mu} < 190 \text{ KeV} \qquad (\pi \to \mu + \nu_\mu)$$

$$m_{\nu_\tau} < 18.2 \text{ MeV} \qquad (\tau \to n\pi + \nu_\tau, \text{ with } n > 3)$$

$\Rightarrow$ Neutrinos are conjured to be **massless** and **left-handed**.

Neutrino masses: Dirac or Majorana?

- How to write a mass term for a fermion field? Two possibilities:

Dirac

$$\mathcal{L}^{\text{D}} = -m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R)$$

- can be implemented in the SM via SSB as for up-type quarks:

$$\mathcal{L}^{\text{D}} = -Y^\ell\,\overline{L_L}\,\Phi\,\ell_R - Y^\nu\,\overline{L_L}\,\tilde{\Phi}\,\nu_R + \text{h.c.}$$

- however, it requires **new** field $\nu_R \Rightarrow$ SM extension!

Majorana

$$\mathcal{L}^{\text{M}} = -\frac{1}{2}m\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right)$$

- only ν_L used $\Rightarrow$ no new field required;
- breaks gauge simmetries $\Rightarrow$ unthinkable for **charged** particles (Q is conserved);
- can't be written explicitly in the SM $\Rightarrow$ should be generated *effectively* $\Rightarrow$ SM extension!

- both possibilities are phenomenologically viable $\Rightarrow$ most general case is to use both:

$$\mathcal{L} = -Y^\ell\,\overline{L_L}\,\Phi\,\ell_R - Y^\nu\,\overline{L_L}\,\tilde{\Phi}\,\nu_R - \frac{1}{2}M\,\overline{\nu_R^C}\,\nu_R + \text{h.c.}$$

- ν_R is a singlet under SM symmetries $\Rightarrow$ can have an explicit Majorana mass.

Effects of neutrino masses: oscillations

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \to \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$): $|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}\, |\nu_i\rangle$;
- After a distance L (or time t) it evolves $|\nu(t)\rangle = \sum_{i=1}^{n} U_{\alpha i}\, e^{-i\, E_i t}\, |\nu_i\rangle$;
- it can be detected with flavor β with probability $P_{\alpha\beta} = |\langle\nu_\beta|\nu(t)\rangle|^2$:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j\neq i}^{n} \text{Re}\left[U_{\alpha i}^\star U_{\beta i} U_{\alpha j} U_{\beta j}^\star\right] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2 \sum_{j\neq i}^{n} \text{Im}\left[U_{\alpha i}^\star U_{\beta i} U_{\alpha j} U_{\beta j}^\star\right] \sin\left(\Delta_{ij}\right),$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27\, \frac{(m_i^2 - m_j^2)}{\text{eV}^2}\, \frac{L/E}{\text{Km/GeV}}$$

- $P_{\alpha\beta}$ depends on *Theoretical* Parameters
  - $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles

  and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector
- no information on mass scale nor Dirac/Majorana nature.

Two-neutrino oscillations in vacuum

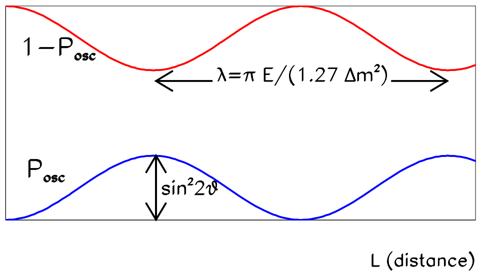
- Equation of motion (**2** parameters): $i\frac{d\vec{\nu}}{dt} = \mathbf{H}\,\vec{\nu}; \qquad \mathbf{H} = \mathbf{U}\cdot\mathbf{H}_0^d\cdot\mathbf{U}^\dagger;$

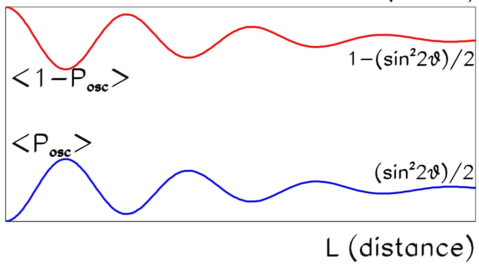
$$\mathbf{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad \mathbf{H}_0^d = \frac{1}{2E_\nu}\begin{pmatrix} -\Delta m^2 & 0 \\ 0 & \Delta m^2 \end{pmatrix}, \qquad \vec{\nu} = \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix};$$

- $P_{\text{osc}} = \sin^2(2\theta)\ \sin^2\left(1.27\,\frac{\Delta m^2 L}{E_\nu}\right)$, $P_{\alpha\alpha} = 1 - P_{\text{osc}}$;
- In real experiments ν's are not monochromatic:

$$\langle P_{\alpha\beta}\rangle = \frac{1}{\mathcal{N}}\int \frac{d\Phi}{dE_\nu}\,\sigma_{CC}(E_\nu)\,\epsilon(E_\nu)\,P_{\alpha\beta}(E_\nu)\,dE_\nu$$

- Maximal sensitivity for $\Delta m^2 \sim E_\nu/L$;
- $\Delta m^2 \ll E_\nu/L \Rightarrow$ No time to oscillate $\Rightarrow \langle P_{\text{osc}}\rangle \simeq 0$;
- $\Delta m^2 \gg E_\nu/L \Rightarrow$ Averaged osc. $\Rightarrow \langle P_{\text{osc}}\rangle \simeq \frac{1}{2}\sin^2(2\theta)$.



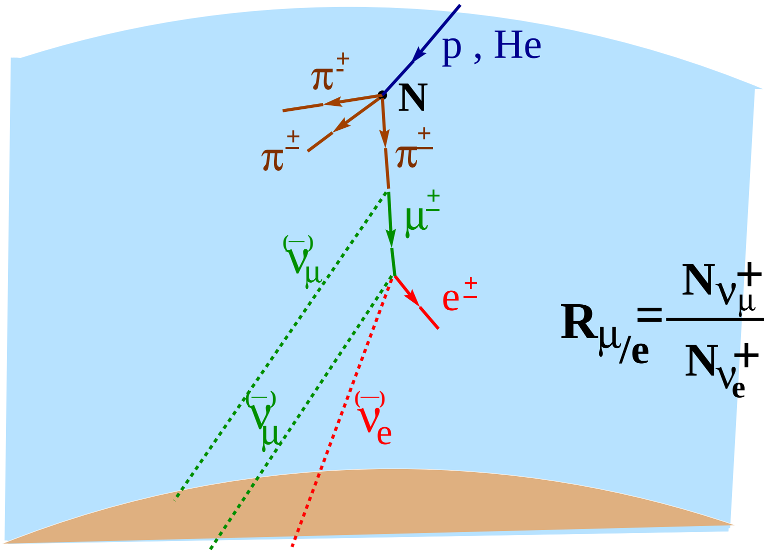


Atmospheric neutrinos

- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, $\dots$) with the Earth's atmosphere:

1	$A_{\text{cr}} + A_{\text{air}} \to \pi^\pm, K^\pm, K^0, \dots$
2	$\pi^\pm \to \mu^\pm + \nu_\mu$,
3	$\mu^\pm \to e^\pm + \nu_e + \nu_\mu$;

- at the detector, some ν interacts and produces a **charged lepton**, which is observed;
- ν_μ and ν_e fluxes have large ($\approx 20\%$) uncertainties;

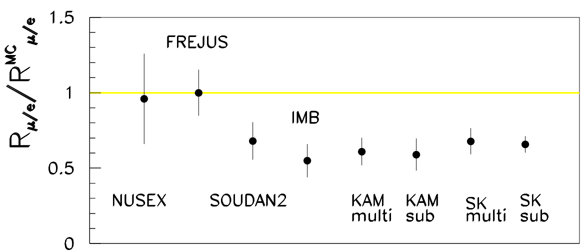


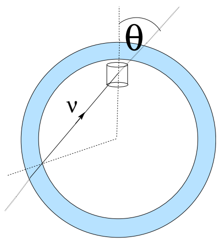
$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar\nu_\mu}}{N_{\nu_e} + N_{\bar\nu_e}} \sim 2$$

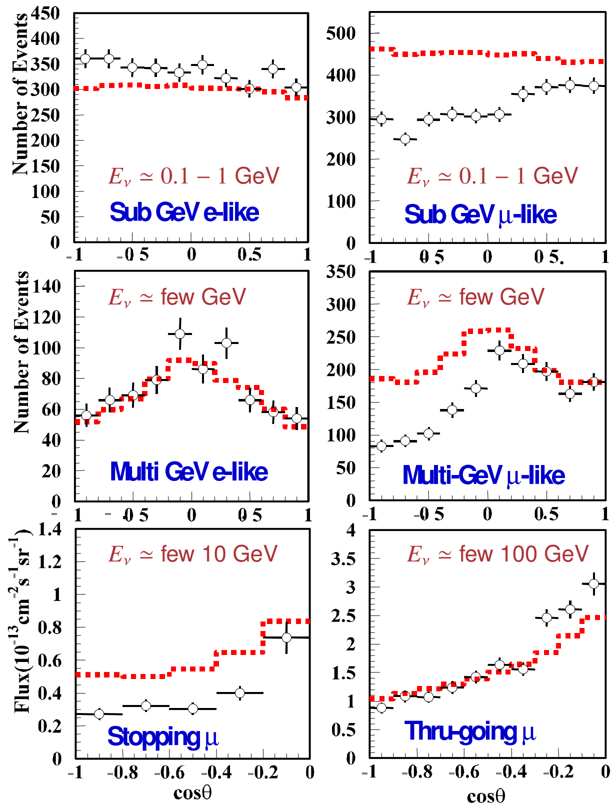
- however, the ν_μ/ν_e ratio is predicted with quite good accuracy ($\approx 5\%$).

Atmospheric neutrinos: experimental status

- historically: { – no deficit in iron calorimeters; – deficit in water Cerenkov;
- possibly a mistake in water Cerenkov?
- ambiguity resolved by Soudan2 and MACRO;
- present data (SK): agreement in ν_e, deficit in ν_μ;
- SK deficit in ν_μ: { – grows with L; – decreases with E_ν;
- deficit cannot be explained by uncertainties;
- solution: $\nu_\mu \to \nu_\tau$ two-neutrino oscillations.





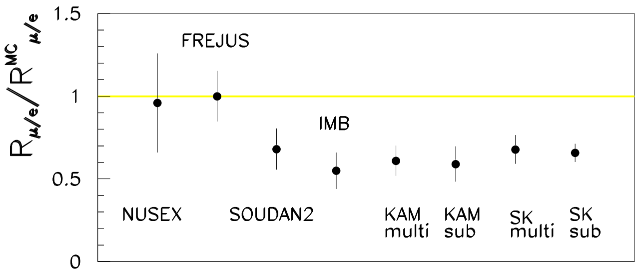


Atmospheric ν oscillations: parameter estimate

- Data: ν_e: good agreement with SM; ν_μ: visible deficit at low energy;

$\Rightarrow$ oscillations in the $\nu_\mu \to \nu_\tau$ channel.

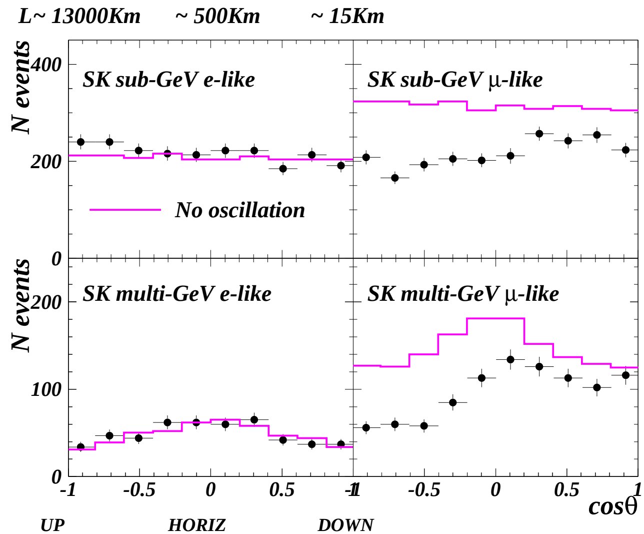
- From total contained event rates:



- $\langle P_{\mu\mu}\rangle = 1 - \sin^2(2\theta_{\text{atm}}) \sin^2\left(1.27 \dfrac{\Delta m^2_{\text{atm}} L}{2E}\right) \sim 0.5 \div 0.7$

$\Rightarrow$ $\sin^2(2\theta_{\text{atm}}) \gtrsim 0.6$;

- From Angular Distribution:



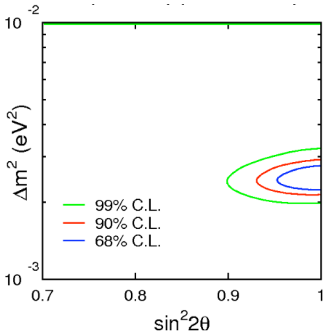
- For $E \sim 1$ GeV: deficit at $L \sim 10^2 \div 10^4$ Km:

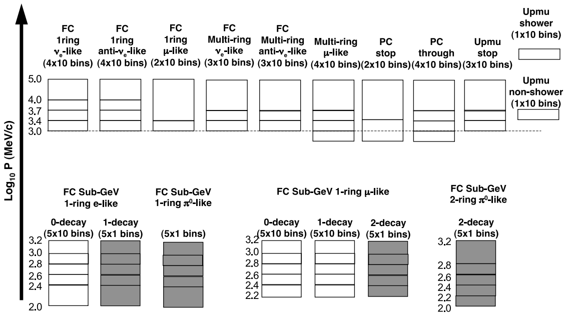
$$\frac{\Delta m^2_{\text{atm}}[\text{eV}^2]\; L[\text{km}]}{2E_\nu[\text{GeV}]} \sim 1$$

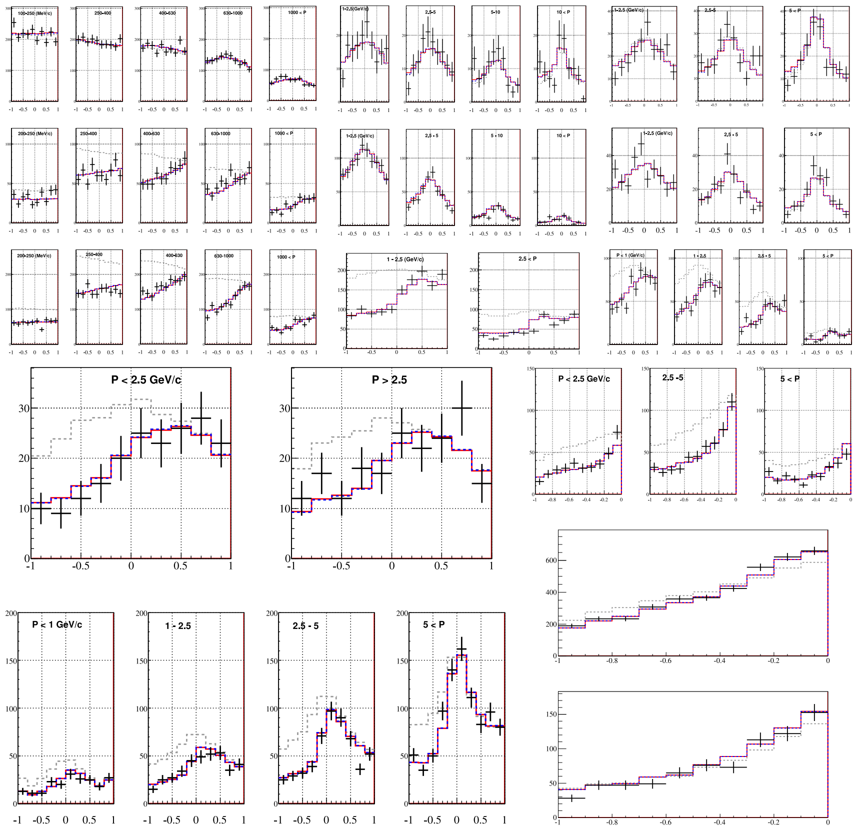
$\Rightarrow$ $\Delta m^2_{\text{atm}} \sim 10^{-4} \div 10^{-2}$ eV^2.

Atmospheric neutrinos: where we are

- SK(1–4) data: 480 bins defined by flavor, charge, topology, momentum, . . . ;
- channel: $\nu_\mu \to \nu_\tau$;
- perfect fit with just 2 params: Δm^2_{atm}, θ_{atm}.





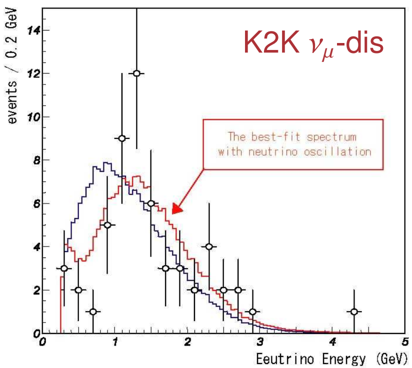


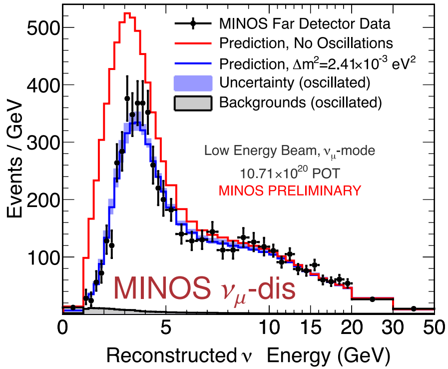
Accelerator neutrino experiments

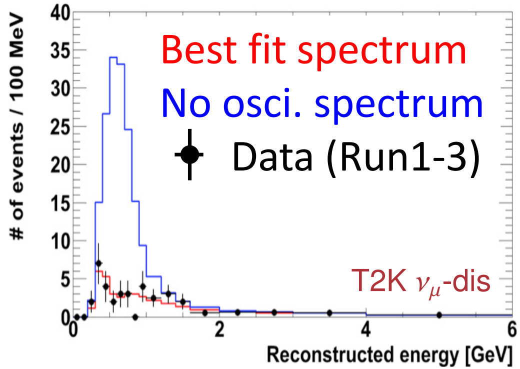
- p + target $\to$ stuff + $\pi^\pm$, then $\pi^\pm \to \mu^\pm + \nu_\mu$ (decay $\mu^\pm \to e^\pm + \nu_e + \nu_\mu$ not exploited);
- detection: focus on ν_μ disappearance and ν_e appearance. For the former:

Exper	Length	Energy	No-osc	Observed	Detector
K2K	250 km	1 GeV	88 (ν_μ)	56 (ν_μ)	single-ring μ-like events in SK
MINOS	735 km	3 GeV	3564 (ν_μ) 464 ($\bar\nu_\mu$)	2894 (ν_μ) 357 ($\bar\nu_\mu$)	dedicated far detector
T2K	295 km	0.6 GeV	196 (ν_μ)	58 (ν_μ)	single-ring μ-like events in SK

- Result: various experiments observed a clear **energy-dependent** ν_μ deficit.

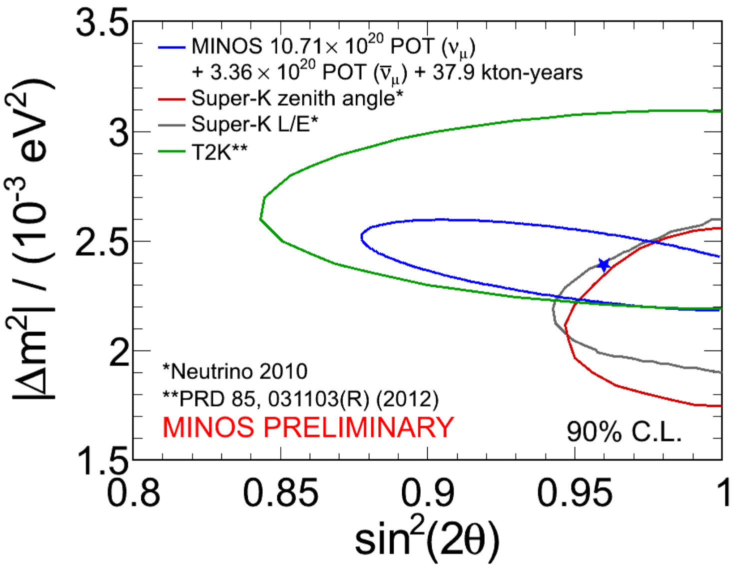


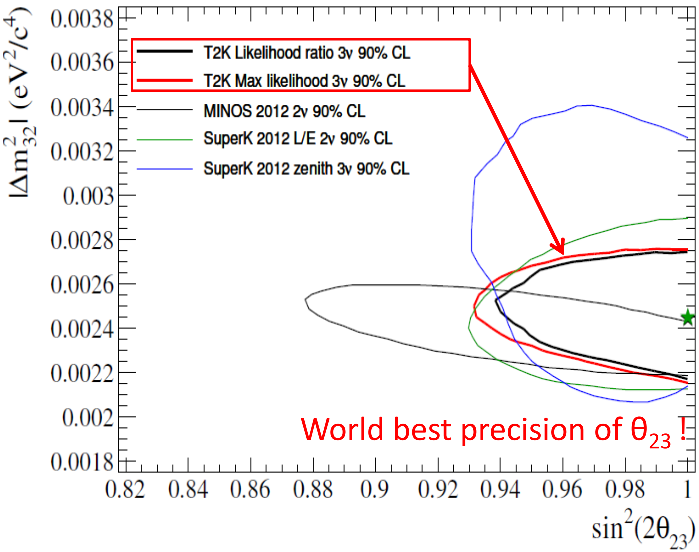




Joint interpretation of atmospheric and accelerator (ν_μ) data

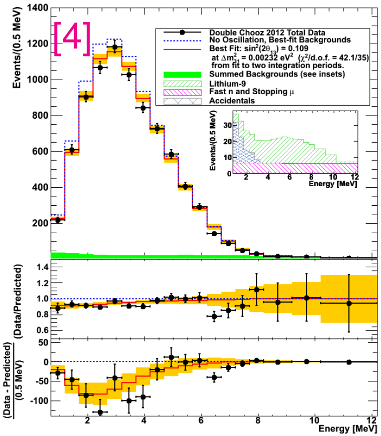
- Hypothesis: $\nu_\mu \to \nu_\tau$ mass-induced oscillations;
- CPT conservation ⇒ same behavior of ν and $\bar{\nu}$ in atmospheric and accelerator data;
- model perfectly explains all the data with only two parameters: $(\Delta m^2_{\text{atm}}, \theta_{\text{atm}})$.

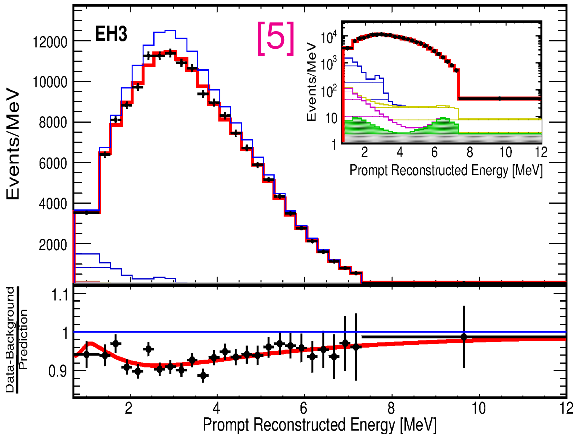


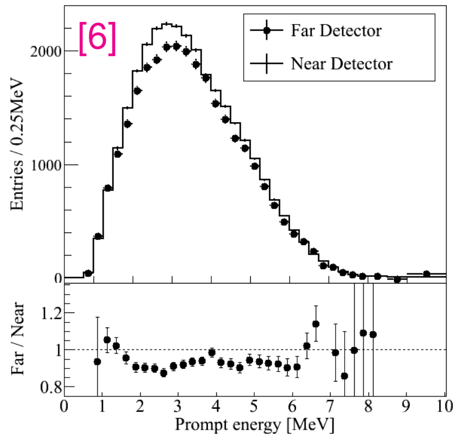


Reactor neutrino experiments

- Electron antineutrinos ($\bar{\nu}_e$) produced by nuclear fission in reactor's core;
- experimental setup: search for $\bar{\nu}_e$ disappearance, $\langle L \rangle \approx 0.1 \to 1$ km;
- early 2012: **positive signal** from DOUBLE-CHOOZ [4], DAYA-BAY [5], RENO [6];
- present status: oscillations established @ 9σ from the combination of all the data.



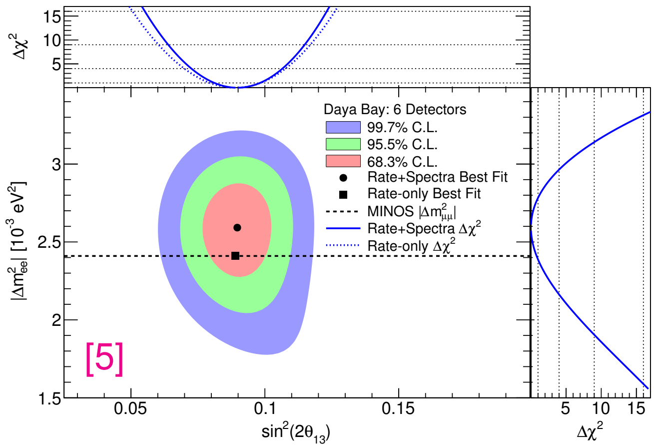


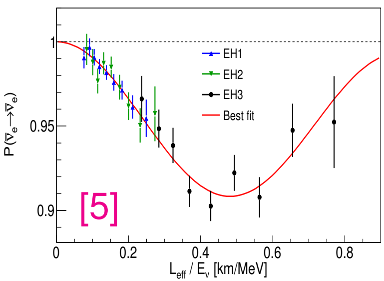


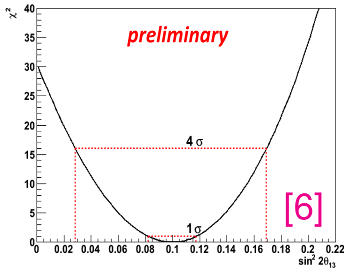
[4] M. Ishitsuka [DOUBLE-CHOOZ], talk presented at Neutrino 2012, Kyoto, Japan, June 3–9, 2012.
[5] F.P. An *et al.* [DAYA-BAY], arXiv:1310.6732, submitted to Phys. Rev. Lett.
[6] S.H. Seo [RENO], talk presented at Neutrino Telescopes 2013, Venice, Italy, March 11–15, 2013.

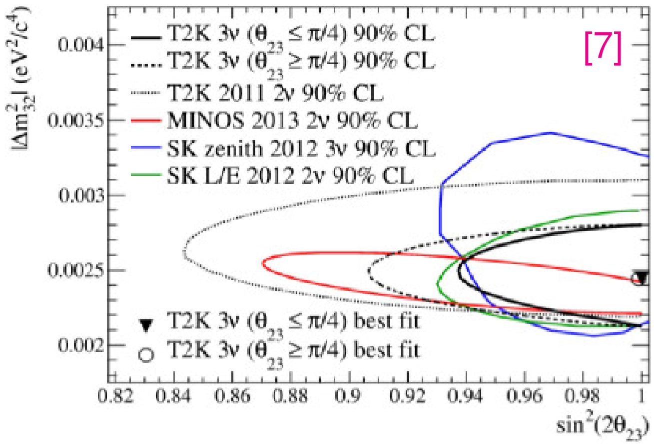
ν oscillations at reactors

- New oscillation channel: $\boxed{\nu_e \to \nu_e}$ $\Rightarrow$ same Δm^2_{atm} as ATM, but different angle θ_{rea};
- sizable deficit at the **far** detector $\Rightarrow$ oscillations $\Rightarrow$ **lower** bound on θ_{rea} and Δm^2_{atm};
- smaller deficit at the **near** detector $\Rightarrow$ not-too-much oscillations $\Rightarrow$ **upper** bound on Δm^2_{atm}.





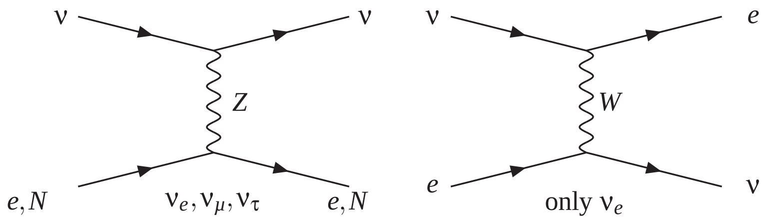




[5] F.P. An *et al.* [DAYA-BAY], `arXiv:1310.6732`.
[6] S.H. Seo [RENO], talk presented at NeuTel 2013.
[7] J. Kameda [T2K], talk presented at TAUP 2013.

Two-neutrino oscillations in matter

- If ν cross matter regions (Sun, Earth...) it interacts *coherently*
- But different flavors have different interactions:



- To include this effect: potential in the evolution equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = \left[\frac{\Delta m^2}{4E_\nu}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix} \pm \begin{pmatrix}V_e & 0\\ 0 & V_X\end{pmatrix}\right]\cdot\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix},$$

$$V_e = \sqrt{2}G_F\left(N_e - \frac{1}{2}N_n\right), \qquad V_\mu = V_\tau = \sqrt{2}G_F\left(-\frac{1}{2}N_n\right), \qquad V_s = 0,$$

$N_{e(n)}$ = electron (neutron) density, sign = + (−) for neutrinos (antineutrinos).

⇒ **Modification of mixing angle and oscillation wavelength.**

Matter effects: effective mass and mixing

- For neutrinos (up to an irrelevant multiple of the identity matrix):

$$\mathbf{H} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix}, \quad \Delta \equiv \frac{\Delta m^2}{4E_\nu}, \quad A = \frac{V_e - V_X}{2};$$

- note that the hamiltonian $\mathbf{H}(x)$ depends on the position along the neutrino trajectory;
- in general, for $x_1 \neq x_2$ we have $[\mathbf{H}(x_1), \mathbf{H}(x_2)] \neq 0$;
- however, for any given x we can diagonalyze $\mathbf{H}(x)$:

$$\mathbf{H} = \begin{pmatrix} \cos\theta_m(x) & \sin\theta_m(x) \\ -\sin\theta_m(x) & \cos\theta_m(x) \end{pmatrix} \cdot \begin{pmatrix} -\Delta_m(x) & 0 \\ 0 & \Delta_m(x) \end{pmatrix} \cdot \begin{pmatrix} \cos\theta_m & -\sin\theta_m(x) \\ \sin\theta_m(x) & \cos\theta_m(x) \end{pmatrix}$$

  and comparing with the previous expression:

$$\left.\begin{aligned} \Delta_m \cos(2\theta_m) &= \Delta\cos(2\theta) - A \\ \Delta_m \sin(2\theta_m) &= \Delta\sin(2\theta) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \Delta_m &= \sqrt{[\Delta\cos(2\theta) - A]^2 + [\Delta\sin(2\theta)]^2} \\ \tan(2\theta_m) &= \frac{\Delta\sin(2\theta)}{\Delta\cos(2\theta) - A} \end{aligned}\right.$$

- for antineutrinos, just replace $A \to -A$.

Matter effects: level crossing and resonant enhancement

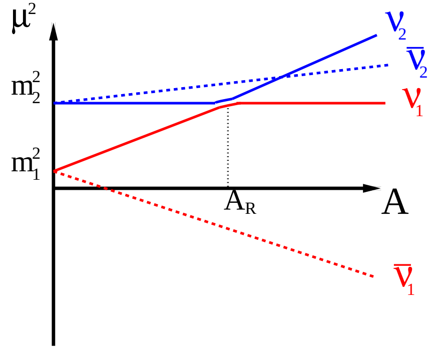
- From the previous transparency:

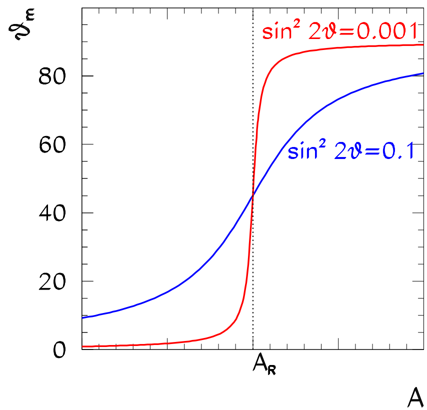
$$\left.\begin{aligned} \Delta_m \cos(2\theta_m) &= \Delta \cos(2\theta) - A \\ \Delta_m \sin(2\theta_m) &= \Delta \sin(2\theta) \end{aligned}\right\} \Rightarrow \quad \tan(2\theta_m) = \frac{\Delta \sin(2\theta)}{\Delta \cos(2\theta) - A};$$

- choosing Δ_m with the same sign as Δ, we see that:
  – $\cos(2\theta_m)$ and $\cos(2\theta)$ have $\frac{\text{the same}}{\text{opposite}}$ sign if $\Delta\cos(2\theta) \frac{>}{<} A$;
  – θ_m is maximal ($45°$) for $\Delta\cos(2\theta) = A$, even if θ is small;
  – the value $A_R = \Delta\cos(2\theta)$ is called *resonant density*.
- for constant matter density, we can define the oscillation lenght in matter as:

$$L_m^{\text{osc}} = L_0^{\text{osc}} \frac{\Delta}{\Delta_m} \quad \text{with} \quad L_0^{\text{osc}} = \frac{\pi}{\Delta};$$

- no level crossing occur if $\Delta\cos(2\theta)$ and A have opposite sign.





Matter effects: the adiabaticity condition

- The evolution equation can be rewritten in the basis of the *instantaneous mass eigen-states in matter*:

$$i\frac{d}{dx}\begin{pmatrix}\nu_1^m \\ \nu_2^m\end{pmatrix} = \begin{pmatrix}-\Delta_m(x) & -i\,\dot\theta_m(x) \\ i\,\dot\theta_m(x) & \Delta_m(x)\end{pmatrix} \cdot \begin{pmatrix}\nu_1^m \\ \nu_2^m\end{pmatrix};$$

- note that, in general, the two mass eigenstates ν_1^m and ν_2^m mix in the evolution, therefore they are NOT energy eigenstates;

⇒ the evolution is called *adiabatic* when the non-diagonal term $i\,\dot\theta_m(x)$ can be neglected, so that the MASS eigenstates are also ENERGY eigenstates;

- from the definition of θ_m we can derive the *adiabaticity condition*:

$$\dot\theta_m = \frac{\Delta\sin^2(2\theta)}{2\Delta_m}\dot A \quad\Rightarrow\quad \Delta_m(x) \gg \frac{\Delta\sin(2\theta)\,A}{2\Delta_m(x)^2}\left|\frac{\dot A}{A}\right|;$$

- the strongest condition is realized when $\Delta_m(x)$ is minimum, *i.e.*, at the resonance.

Defining: $Q \equiv \dfrac{\Delta\sin^2(2\theta)}{h_R\cos(2\theta)}$ with $h_R \equiv \left|\dfrac{\dot A}{A}\right|_R$ ⇒ adiabaticity condition: $Q \gg 1$.

Matter effects: the adiabatic regime

- Survival amplitude of ν_e produced in matter at x_0 and exiting matter at x_1:

$$A_{ee} = \sum_{i,j,m,n} \langle \nu_e(x_1) \,|\, \nu_j(x_1) \rangle \, \langle \nu_j(x_1) \,|\, \nu_n(x_R) \rangle \, \langle \nu_n(x_R) \,|\, \nu_m(x_R) \rangle \, \langle \nu_m(x_R) \,|\, \nu_i(x_0) \rangle \, \langle \nu_i(x_0) \,|\, \nu_e(x_0) \rangle$$

  where x_R is the position at which the resonance occurs.

- We have $\langle \nu_e(x_1) \,|\, \nu_1(x_1) \rangle = \cos\theta$ and $\langle \nu_e(x_1) \,|\, \nu_2(x_1) \rangle = \sin\theta$;
- analogously, $\langle \nu_1(x_0) \,|\, \nu_e(x_0) \rangle = \cos\theta_m$ and $\langle \nu_2(x_0) \,|\, \nu_e(x_0) \rangle = \sin\theta_m$;
- assuming adiabaticity: $\langle \nu_m(x_R) \,|\, \nu_i(x_0) \rangle = \delta_{im} e^{i\phi_i}$ and $\langle \nu_j(x_1) \,|\, \nu_n(x_R) \rangle = \delta_{jn} e^{i\varphi_j}$;
- also, in the adiabatic case $\dot\theta_m(x_R)$ is negligible $\Rightarrow$ $\langle \nu_n(x_R) \,|\, \nu_m(x_R) \rangle = \delta_{mn}$ and:

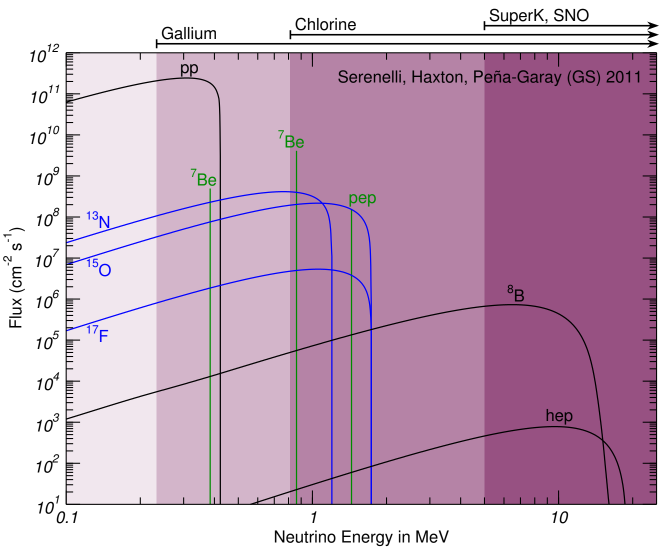
$$P_{ee} = \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta + \frac{1}{2}\sin(2\theta_m)\sin(2\theta)\cos(2\delta)$$

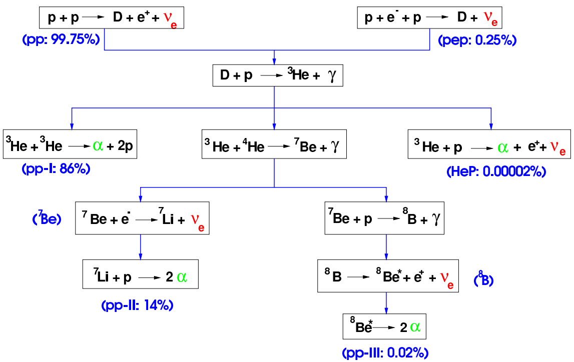
  with $\delta = \int_{x_0}^{x_1} \Delta_m(x) dx = \int_{x_0}^{x_1} \sqrt{[\Delta\cos(2\theta) - A(x)]^2 + [\Delta\sin(2\theta)]^2}\, dx$;

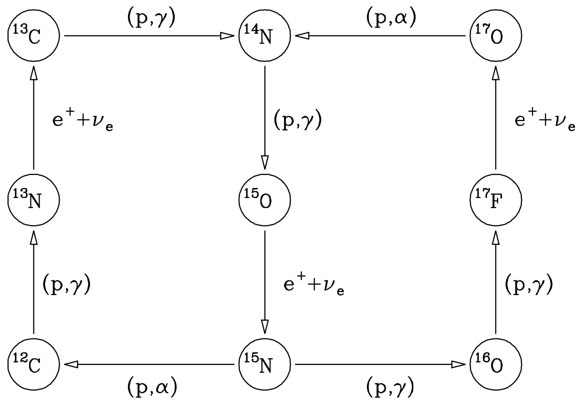
- if $\delta \gg 1 \Rightarrow \cos(2\delta)$ is averaged $\Rightarrow P_{ee} = \frac{1}{2}[1 + \cos(2\theta_m)\cos(2\theta)]$.

Solar neutrinos

- Neutrinos are by *nuclear reactions* in the core of the Sun;
- 2 mechanisms: pp chain and CNO cycle;
- both give $4p \to^4 \text{He} + 2e^+ + 2\nu_e + \gamma$.

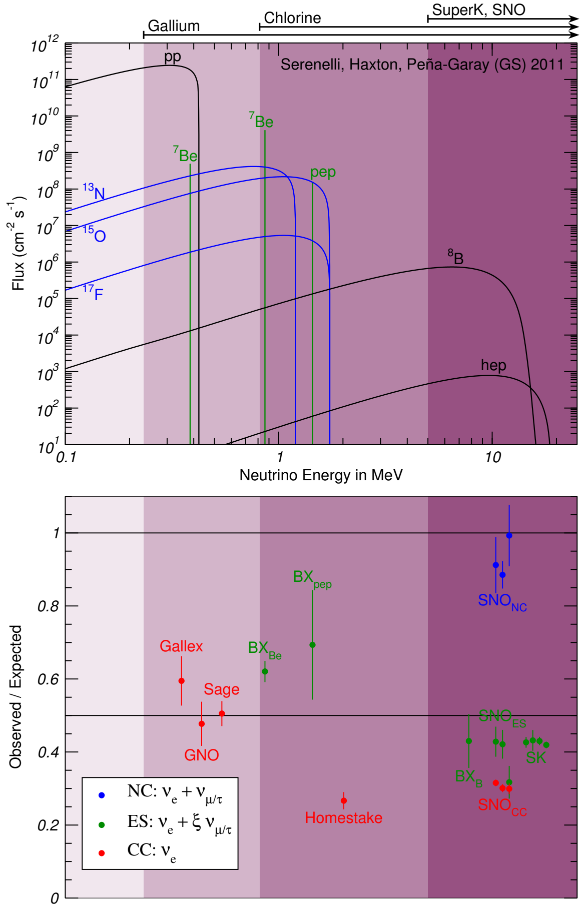






The solar neutrino problem

- Nuclear reactions (**pp-chain** & CNO-cycle) produce *electron neutrinos* of various energies;
- during the last 40 years, a number of underground experiments has measured their flux in different energy windows;
- it is found that ALL the experiments observe a deficit of about $\mathbf{30-60\%}$;
- the deficit is NOT the same for all the experiments, and shows a clear **energy dependence**;
- it is **not possible** to reconcile the data with the Standard Solar Model (SSM) by simply readjusting the parameters of the model;
- the deficit is maximum for CC (ν_e), reduced for ES ($\nu_e + \xi\nu_{\mu/\tau}$), and absent for NC ($\nu_e + \nu_{\mu/\tau}$).

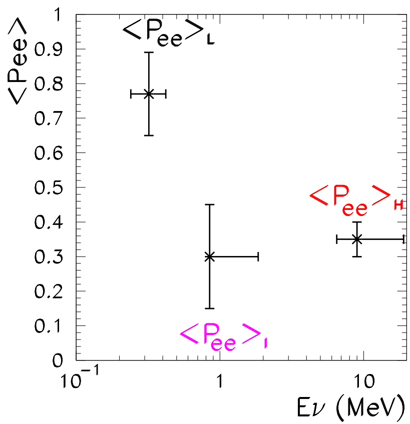


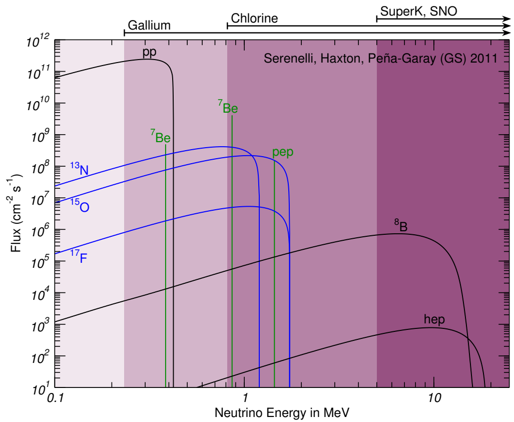
Solar Neutrinos: Flavor Conversion Probabilities

Experiment	E_{th} (MeV)	Type	Detection	R_{th}
Gallium	$E_\nu > 0.233$	CC	$^{71}\text{Ga}(\nu, e^-)^{71}\text{Ge}$	$0.54\,\langle P_{ee}\rangle_L + 0.36\,\langle P_{ee}\rangle_I + 0.10\,f_B\,\langle P_{ee}\rangle_H$
Homestake	$E_\nu > 0.814$	CC	$^{37}\text{Cl}(\nu, e^-)^{37}\text{Ar}$	$0.24\,\langle P_{ee}\rangle_I + 0.76\,f_B\,\langle P_{ee}\rangle_H$
Super-K	$E_e > 5$	ES	$\nu_x e^- \to \nu_x e^-$	$f_B\,[\langle P_{ee}\rangle_H + 0.15\,(1 - \langle P_{ee}\rangle_H)]$
SNO-CC	$T_e > 5$	CC	$\nu_e d \to p p e^-$	$f_B\,\langle P_{ee}\rangle_H$
SNO-NC	$T_\gamma > 5$	NC	$\nu_x d \to \nu_x d$	f_B
SNO-ES	$T_e > 5$	ES	$\nu_x e^- \to \nu_x e^-$	$f_B\,[\langle P_{ee}\rangle_H + 0.15\,(1 - \langle P_{ee}\rangle_H)]$

- Oscillation channel: $\boxed{\nu_e \to \nu_{\mu/\tau}}$;
- data: NC $\to f_B$ and CC, ES $\to \langle P_{ee}\rangle_L, \langle P_{ee}\rangle_I, \langle P_{ee}\rangle_L$;
- the ν_e survival probability:

$$\begin{cases} P_{ee} > 1/2 \text{ at low } E_\nu; \\ P_{ee} < 1/2 \text{ at high } E_\nu. \end{cases}$$





Propagation in the Sun: the MSW effect

- For $R < 0.9 R_\odot$ the solar matter density can be approximated by an exponential:

$$N_e(r) = N_e(0) \exp\left(-\frac{r}{r_0}\right), \qquad r_0 = \frac{R_\odot}{10.54} = 6.6 \times 10^7 \text{ m} = 3.3 \times 10^{-14} \text{ eV}^{-1};$$

- $P_{ee}^\odot$ depends on the relative size of $\Delta \cos(2\theta)$ versus $A_{\text{prod}} = \sqrt{2} G_F N_e(x_{\text{prod}})/2$:

– $\Delta \cos(2\theta) \gg A_{\text{prod}}$: matter effects negligible, propagation occurs as in vacuum:

$$P_{ee}^\odot = 1 - \frac{1}{2} \sin^2(2\theta) \boxed{> \frac{1}{2}};$$

– $\Delta \cos(2\theta) \gtrsim A_{\text{prod}}$: no level crossing. The adiabatic approximation is valid:

$$P_{ee}^\odot = \frac{1}{2} [1 + \cos(2\theta_m) \cos(2\theta)] \boxed{> \frac{1}{2}};$$

– $\Delta \cos(2\theta) < A_{\text{prod}}$ and $Q \gg 1$: level crossing occurs. Adiabatic approximation valid:

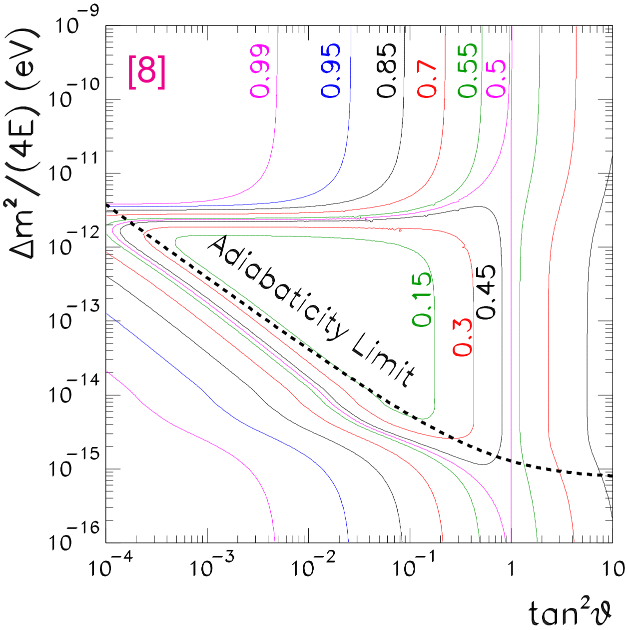
$$P_{ee}^\odot = \frac{1}{2} [1 + \cos(2\theta_m) \cos(2\theta)] \boxed{< \frac{1}{2}};$$

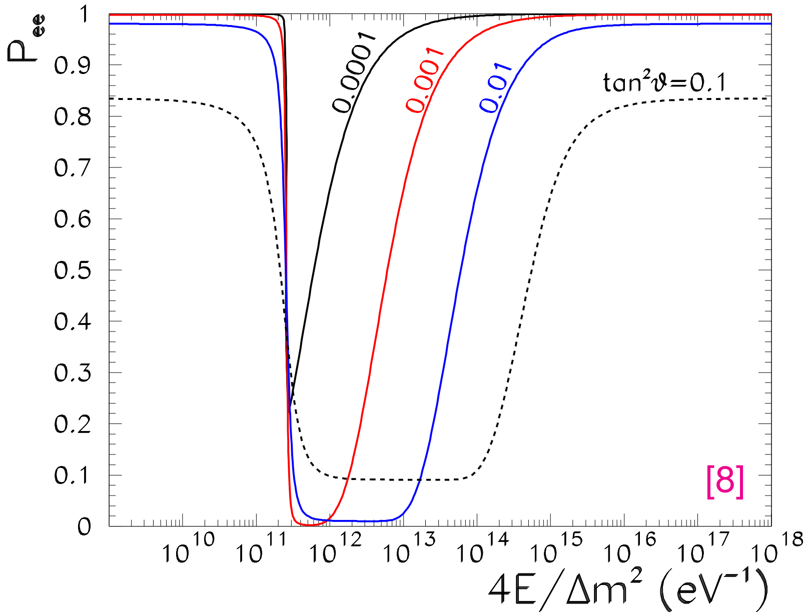
where the last disequality is due to the fact that $\cos(2\theta_m)$ and $\cos(2\theta)$ have now opposite sign. This is known as MSW effect.

– $\Delta \cos(2\theta) < A_{\text{prod}}$ and $Q \lesssim 1$: the adiabatic approximation is no longer valid.

Manifestation of the MSW effect in the Sun

- Full numerical calculations: $P_{ee}^{\odot} < 1/2$ is possible $\Rightarrow$ MSW effect is realized.

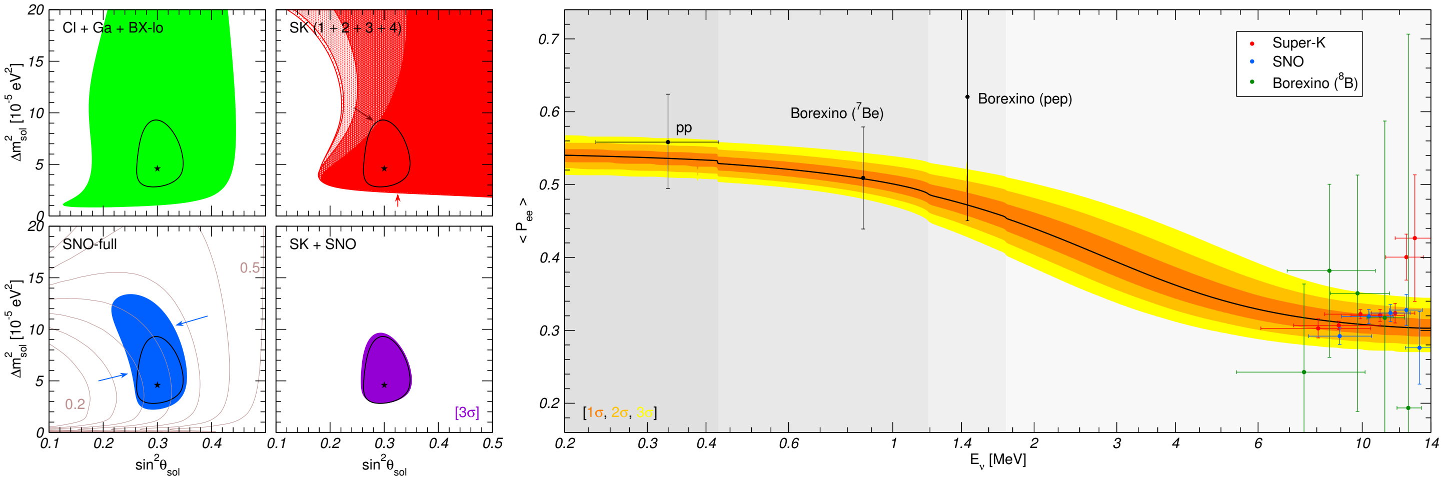




[8] M.C. Gonzalez-Garcia, Y. Nir, Rev. Mod. Phys. **75** (2003) 345 [hep-ph/0202058].

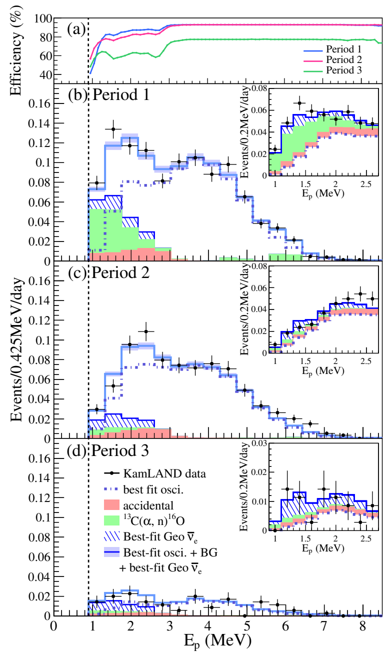
Transition between vacuum and MSW regime in solar data

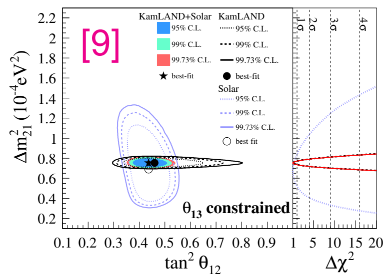
- Evolution: $i\frac{d\vec{\nu}}{dt} = \left[\frac{\Delta m^2_{\text{sol}}}{4E_\nu}\begin{pmatrix}-\cos 2\theta_{\text{sol}} & \sin 2\theta_{\text{sol}} \\ \sin 2\theta_{\text{sol}} & \cos 2\theta_{\text{sol}}\end{pmatrix} \pm \sqrt{2}G_F N_e \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\right]\vec{\nu}, \quad \vec{\nu} = \begin{pmatrix}\nu_e \\ \nu_a\end{pmatrix};$
- limits: $P_{ee} \approx 1 - \frac{1}{2}\sin^2 2\theta_{\text{sol}}$ for low-E (Cl, Ga); $P_{ee} \approx \sin^2\theta_{\text{sol}}$ for high-E (SK, SNO);
- solar region determined by high-E data, low-E contribution marginal;
- SNO-NC measurement confirms the SSM prediction of the ^{8}B flux.



The KamLAND reactor experiment

- Nuclear fission reactions in nuclear power plants produce *electron anti-neutrinos*;
- neutrino flux from many plants in Japan measured by KamLAND (average baseline: ≈ 180 km);
- an energy-dependent deficit of $\bar{\nu}_e$ is observed.
- solution: $\nu_e \to \nu_{\text{active}}$ conversion due to non-zero neutrino masses and flavor mixing;
- CPT conservation $\Rightarrow$ physics of solar (ν) and KamLAND ($\bar{\nu}$) neutrino conversion must be the same;
- only P_{ee} measured $\Rightarrow$ same relevant parameters as solar experiments: θ_{sol} and Δm^2_{sol};
- neutrino oscillation hypothesis provides perfect agreement between solar and KamLAND data.





[9] A. Gando *et al.* [KamLAND collaboration], arXiv:1303.4667 [hep-ex].

Three neutrino oscillations

- Equation of motion: **6** parameters (including CP violating effects):

$$i\frac{d\vec{\nu}}{dt} = H\,\vec{\nu}; \qquad H = U_{\text{vac}} \cdot D_{\text{vac}} \cdot U_{\text{vac}}^{\dagger} \pm V_{\text{mat}};$$

$$U_{\text{vac}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \vec{\nu} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix};$$

$$D_{\text{vac}} = \frac{1}{2E_\nu}\left[\operatorname{diag}(0,\, \Delta m^2_{21},\, \Delta m^2_{31}) + m_1^2 I\right]; \qquad V_{\text{mat}} = \sqrt{2}G_F N_e \operatorname{diag}(1,\, 0,\, 0)\,.$$

Connection with two-neutrino oscillations

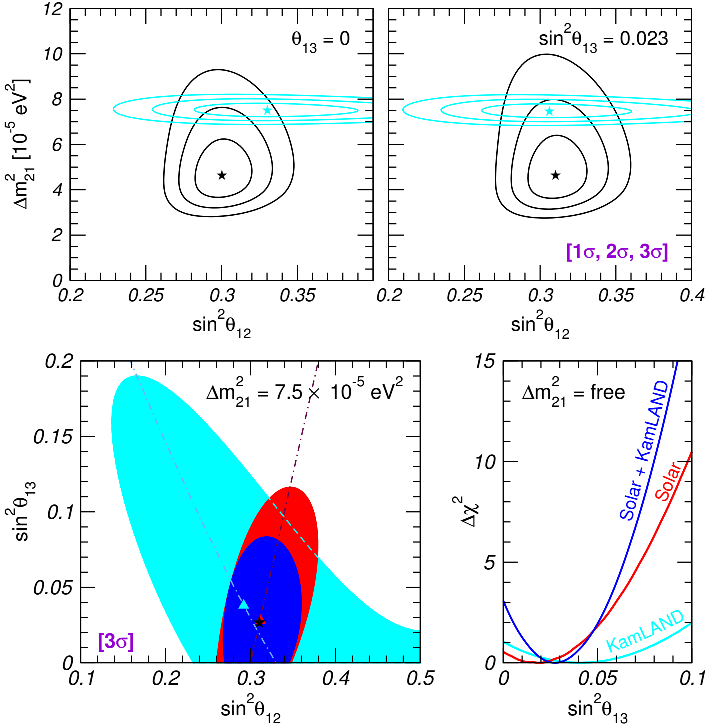
- **Solar** parameters Δm^2_{sol} and θ_{sol} are identified with Δm^2_{21} and θ_{12};
- **atmospheric** parameters Δm^2_{atm} and θ_{atm} are identified with Δm^2_{31} and θ_{23};
- **reactor** angle θ_{rea} involved in reactor experiments corresponds to θ_{13};
- **CP-violating** phase δ_{CP} is a genuine 3ν feature, with no two-neutrino counterpart;
- smallness of θ_{13} and $\Delta m^2_{21}/\Delta m^2_{31}$ implies that solar and atm sectors are decoupled.

Effect of θ_{13} on solar & KamLAND data

- ν_e survival probability:

$$P_{ee} \approx \begin{cases} \text{Kam: } \cos^4\theta_{13}\left(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}\right), \\ \text{low-E: } \cos^4\theta_{13}\left(1 - \frac{1}{2}\sin^2 2\theta_{12}\right), \\ \text{high-E: } \cos^4\theta_{13}\sin^2\theta_{12}; \end{cases}$$

- When θ_{13} increases:
  - KamLAND region shifts to smaller θ_{12};
  - solar region moves to larger θ_{12} (high-E data dominate over low-E ones);
- therefore, a non-zero value of θ_{13} reduces the tension between solar and KamLAND data [10, 11];
- however, a small tension in Δm^2_{21} remains.



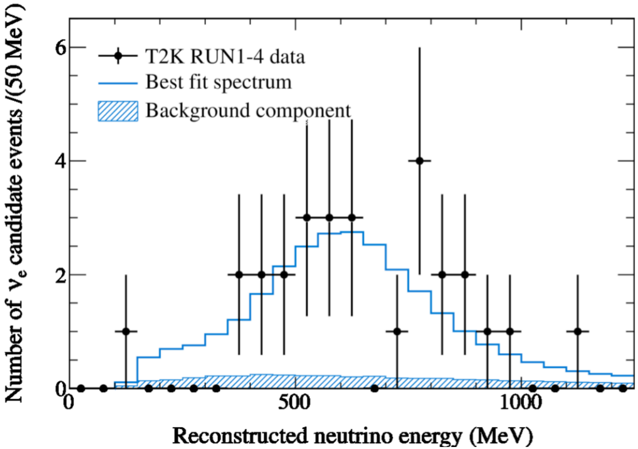
[10] G.L. Fogli *et al.*, Phys. Rev. Lett. **101** (2008) 141801 [arXiv:0806.2649].
[11] T. Schwetz, M.A. Tortola, J.W.F. Valle, New J. Phys. **10** (2008) 113011 [arXiv:0808.2016].

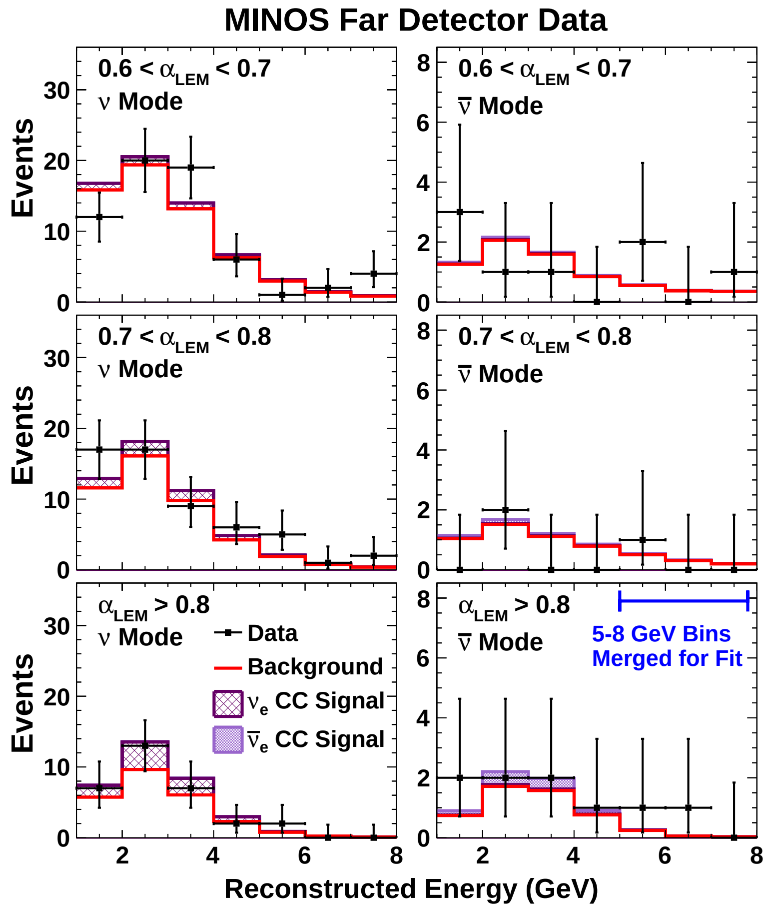
Accelerator experiments: ν_e

- Minos and T2K $\nu_\mu \to \nu_e$ appearance:

Exper	No-osc	Observed
MINOS ($\alpha_{\text{LEM}} > 0.7$)	69.1 (ν_e) 10.5 ($\bar\nu_e$)	88 (ν_e) 12 ($\bar\nu_e$)
T2K	4.6 (ν_e)	28 (ν_e)

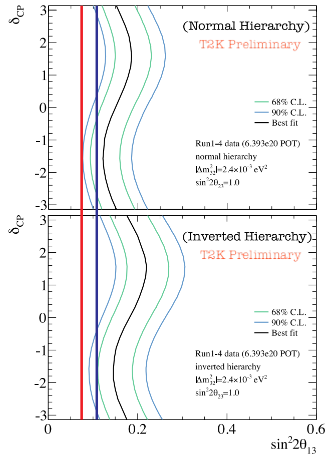
- ν_e excess $\Rightarrow \theta_{13} > 0 \Rightarrow$ reactor data.

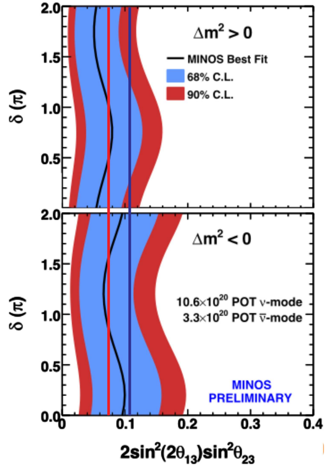


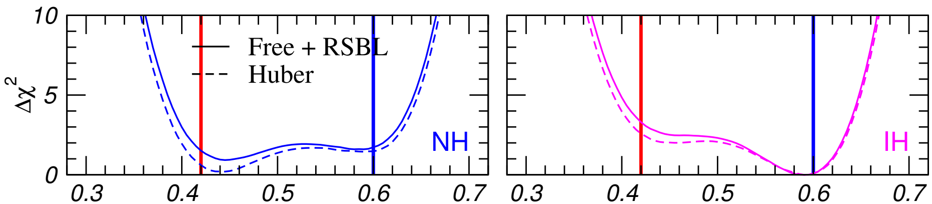


θ_{13}: REACTOR versus LBL-appearance data

- In principle, REA + LBL-APP + LBL-DIS can fix the θ_{23} octant [12]:
  - REACTORS: measure $\sin^2(2\theta_{\text{rea}}) \equiv \sin^2(2\theta_{13})$;
  - LBL-DIS: measure $\sin^2(2\theta_{\text{dis}})$, with $\sin^2\theta_{\text{dis}} \equiv \cos^2\theta_{13}\sin^2\theta_{23}$;
  - LBL-APP: measure $\sin^2(2\theta_{\text{app}}) \equiv \sin^2(2\theta_{13})\, 2\sin^2\theta_{23}$ and δ_{CP};
- in practice, putting explicit numbers:
  - from REACTORS: $\sin^2(2\theta_{13}) \simeq 0.09$;
  - from LBL-DIS: $\sin^2(2\theta_{\text{dis}}) \simeq 0.97$ implies $\sin^2\theta_{23} = 0.42$ or 0.60;
  - hence, REA + LBL-DIS imply $\sin^2(2\theta_{\text{app}}) = 0.076$ or 0.108;
- both values of $\sin^2(2\theta_{\text{app}})$ are in similar agreement with LBL-APP.







[12] G.L. Fogli *et al.*, Phys. Rev. D **86** (2012) 013012 [arXiv:1205.5254].

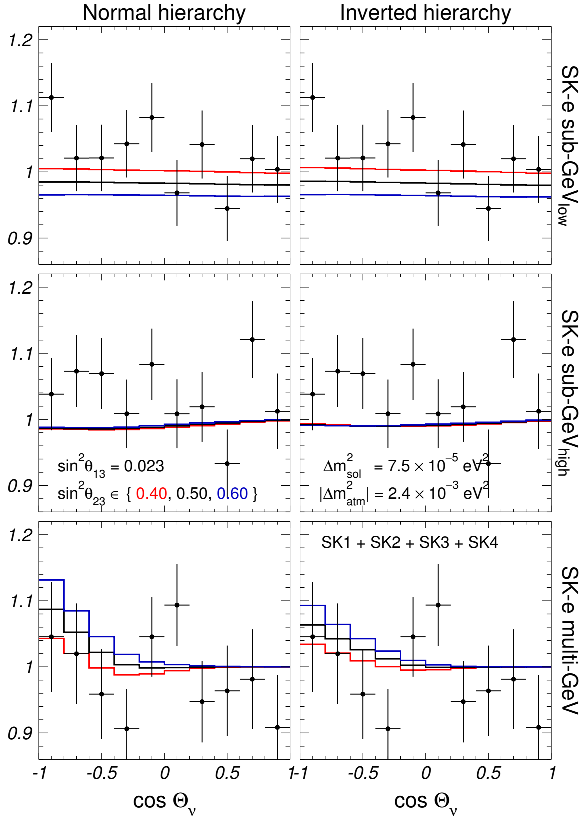
Octant and hierarchy discrimination in atmospheric data

- Excess of e-like events, $\delta_e \equiv N_e/N_e^0 - 1$:

$$\delta_e \simeq (\bar{r}\cos^2\theta_{23} - 1)\, P_{2\nu}(\Delta m^2_{21},\, \theta_{12}) \quad [\Delta m^2_{21} \text{ term}]$$
$$+ (\bar{r}\sin^2\theta_{23} - 1)\, P_{2\nu}(\Delta m^2_{31},\, \theta_{13}) \quad [\theta_{13} \text{ term}]$$
$$- \bar{r}\sin\theta_{13}\sin 2\theta_{23}\ \mathrm{Re}(A^*_{ee} A_{\mu e})\,; \quad [\delta_{\mathrm{CP}} \text{ term}]$$

  with $\bar{r} \equiv \Phi^0_\mu/\Phi^0_e$;

- similar but less pronounced effects also appear in μ-like events (not discussed here);
- resonance in $P_{2\nu}(\Delta m^2_{31},\, \theta_{13}) \Rightarrow$ enhancement of ν ($\bar{\nu}$) oscillations for normal (inverted) hierarchy $\Rightarrow$ **hierarchy discrimination**;
- δ_e distinguishes between light and dark side $\Rightarrow$ **octant discrimination**;
- **present data**: excess in e-like sub-GeV events $\Rightarrow$ preference for light side.



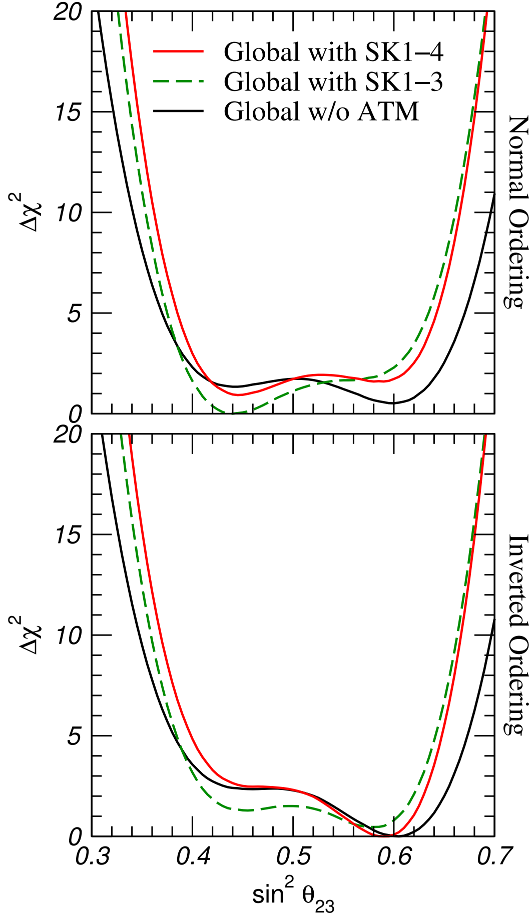
Octant and hierarchy: present status

θ_{23} octant

- Deviation of θ_{23} from maximal mixing **is a physical effect**, which follows from:
  - excess of events in sub-GeV e-like data;
  - zenith distortion in multi-GeV e-like data;
- the effect is not statistically significant, but it is well understood and clearly visible;
- found also by other Fogli *et al.* [12], but **not** by SK.

Mass hierarchy

- Matter effects enhanced for larger θ_{13} $\Rightarrow$ sensitive to specific range of θ_{13};
- no meaningful preference for NH or for IH.



[12] G.L. Fogli *et al.*, PRD **86** (2012) 013012 [arXiv:1205.5254].

Neutrino oscillations: where we are

- Global 6-parameter fit (including δ_{CP}):
  - **Solar**: Cl + Ga + SK(1–4) + SNO-full (I+II+III) + Borexino;
  - **Atmospheric**: SK-1 + SK-2 + SK-3 + SK-4;
  - **Reactor**: KamLAND + Chooz + Palo-Verde + Double-Chooz + Daya-Bay + Reno;
  - **Accelerator**: Minos (DIS+APP) + T2K (DIS+APP);
- best-fit point and 1σ (3σ) ranges:

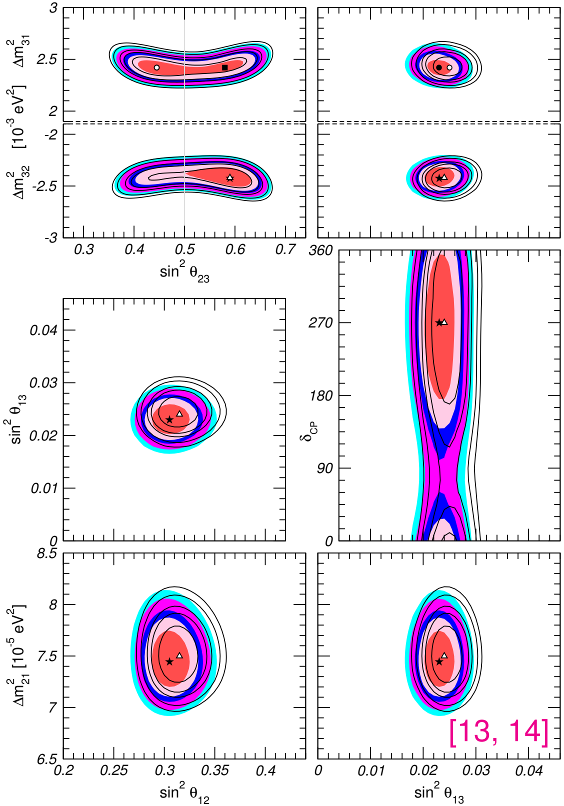
$$\theta_{12} = 33.57\,^{+0.77}_{-0.75}\,\left(^{+2.44}_{-2.20}\right), \qquad \Delta m^2_{21} = 7.45\,^{+0.19}_{-0.16}\,\left(^{+0.60}_{-0.47}\right) \times 10^{-5}\text{ eV}^2,$$

$$\theta_{23} = \begin{cases} 41.9\,^{+0.5}_{-0.4}\,\left(^{+12.6}_{-4.7}\right), \\ 50.3\,^{+1.6}_{-2.5}\,\left(^{+4.2}_{-13.1}\right), \end{cases} \qquad \Delta m^2_{31} = \begin{cases} -2.337\,^{+0.062}_{-0.062}\,\left(^{+0.185}_{-0.191}\right) \times 10^{-3}\text{ eV}^2, \\ +2.417\,^{+0.014}_{-0.014}\,\left(^{+0.206}_{-0.171}\right) \times 10^{-3}\text{ eV}^2, \end{cases}$$

$$\theta_{13} = 8.73\,^{+0.35}_{-0.36}\,\left(^{+1.03}_{-1.17}\right), \qquad \delta_{CP} = 341\,^{+58}_{-46}\text{ (any)};$$

- neutrino mixing matrix:

$$|U|_{3\sigma} = \begin{pmatrix} 0.799 \to 0.844 & 0.515 \to 0.581 & 0.132 \to 0.170 \\ 0.214 \to 0.526 & 0.427 \to 0.706 & 0.598 \to 0.805 \\ 0.234 \to 0.537 & 0.451 \to 0.721 & 0.573 \to 0.787 \end{pmatrix}.$$



[13] M.C. Gonzalez-Garcia *et al.*, JHEP **12** (2012) 123 [arXiv:1209.3023].

[14] M.C. Gonzalez-Garcia *et al.*, NuFIT 1.2 (2013), http://www.nu-fit.org.

What's still missing?

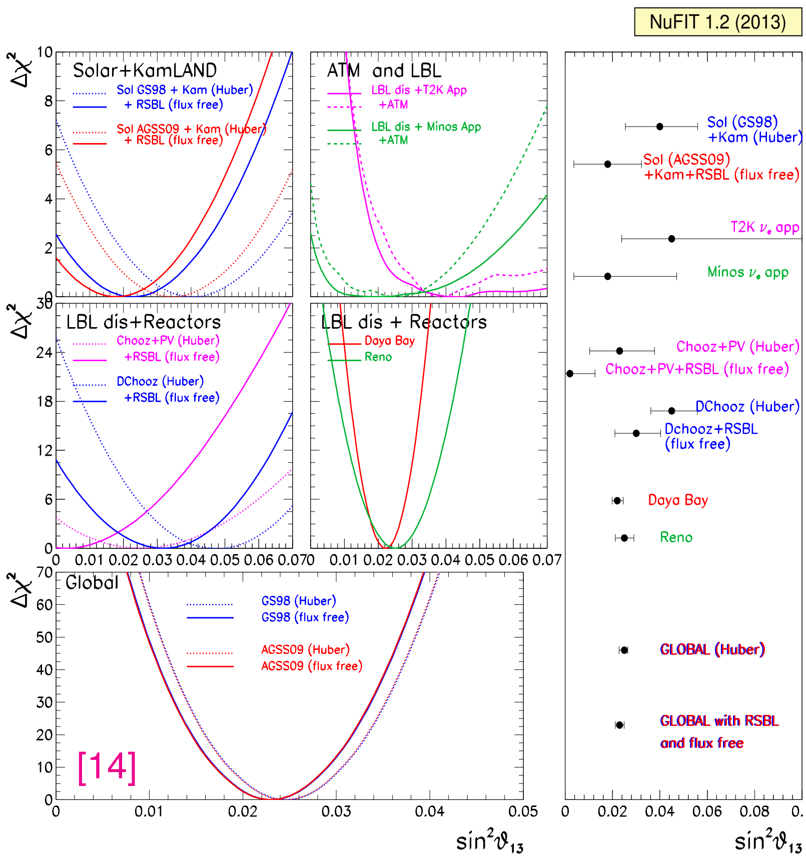
- Neutrino oscillation parameters still to be measured:
  - **value** of δ_{CP}, and whether it differs from 0 and π (CP violation);
  - **size** and **sign** of $\sin^2\theta_{23} - 1/2$ (the θ_{23} octant);
  - **sign** of Δm^2_{31} (neutrino mass hierarchy);
- data that we will almost certainly have (taken from Table 1 of Ref. [15]):

Setup	t_ν [yr]	$t_{\bar\nu}$ [yr]	P_{Th} or P_{Target}	L [km]	Detector technology	m_{Det}
Double Chooz	-	3	8.6 GW	1.05	Liquid scintillator	8.3 t
Daya Bay	-	3	17.4 GW	1.7	Liquid scintillator	80 t
RENO	-	3	16.4 GW	1.4	Liquid scintillator	15.4 t
T2K	5	-	0.75 MW	295	Water Cerenkov	22.5 kt
NOνA	3	3	0.7 MW	810	TASD	15 kt

  plus two atmospheric neutrino detectors: ICECUBE Deep-Core and INO;
- can we answer the remaining questions with this? ⇒ [Lasserre's talk].

[15] P. Huber, M. Lindner, T. Schwetz, W. Winter, JHEP **0911** (2009) 044 [arXiv:0907.1896].

- Most of the present data from solar, atmospheric, reactor and accelerator experiments are well explained by the 3ν oscillation hypothesis. **The three-neutrino scenario is robust**;
- the discovery of **large** θ_{13} is a major breakthrough, and marks the beginning of a new phase in neutrino phenomenology.
- the next step involve searching for CP violation, for non-maximal θ_{23} mixing and for the neutrino mass hierarchy. **With present / approved facilities it may not be easy**.



[14] M.C. Gonzalez-Garcia *et al.*, NuFIT 1.2 (2013), `http://www.nu-fit.org`.