Relativistic effects in large-scale structure

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Outline

♦ How do relativistic effects distort our observables?
  ➔ Effect on: the galaxy number counts $\Delta$
  the convergence $\kappa$ (or magnification)

♦ How can we measure relativistic effects?
  ➔ We can isolate these effects by looking at anti-symmetries in the correlation function.

♦ How can we use them to test gravity?
Galaxy survey

- We want to measure **fluctuations** in the distribution of galaxies.
- We pixelise the map.
- We count the number of **galaxies** per **pixel**: \[ \Delta = \frac{N - \bar{N}}{\bar{N}} \]
- Question: what are the effects contributing to \( \Delta \)?
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Question: what are the effects contributing to \( \Delta \)?
Galaxy number counts

We observe
\[
\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{N(z)}
\]

\[
N(z, \mathbf{n}) = \rho(z, \mathbf{n}) V(z, \mathbf{n}) \quad \text{and} \quad \bar{N} = \bar{\rho}(z) \bar{V}(z)
\]

\[
\Delta = \frac{\rho(z, \mathbf{n}) \cdot V(z, \mathbf{n}) - \bar{\rho}(z) \cdot \bar{V}(z)}{\bar{\rho}(z) \cdot \bar{V}(z)}
\]

\[
\Delta(z, \mathbf{n}) = b \cdot \delta(z, \mathbf{n}) + \frac{\delta V(z, \mathbf{n})}{V} - 3 \frac{\delta z}{1 + z}
\]
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\[ \rho = \bar{\rho} + \delta\rho \quad \text{and} \quad V = \bar{V} + \delta V \quad z = \bar{z} + \delta z \]

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Fluctuations

Perturbed Friedmann universe:

\[ ds^2 = -a^2 \left[ (1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right] \]

We follow the propagation of photons from the galaxies to the observer and calculate:

- Changes in energy
- Changes in direction
Result

\[ \Delta(z, n) = b \cdot D - \frac{1}{H} \partial_r (V \cdot n) \]

- \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega (\Phi + \Psi)

+ \left( 1 - \frac{\mathcal{H}}{H^2} - \frac{2}{rH} \right) V \cdot n + \frac{1}{H} \dot{V} \cdot n + \frac{1}{H} \partial_r \Psi

+ \Psi - 2\Phi + \frac{1}{H} \dot{\Phi} - \frac{3}{k} \dot{V} + \frac{k}{2} \int_0^r dr' (\Phi + \Psi)

+ \left( \frac{\mathcal{H}}{H^2} + \frac{2}{rH} \right) \left[ \Psi + \int_0^r dr' (\Phi + \Psi) \right]

\rightarrow \text{potential}
Result

standard expression

\[ \Delta(z, n) = b \cdot D - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n}) \]

\[ - \int_0^r dr' \frac{r - r'}{rr'} \Delta_{\Omega}(\Phi + \Psi) \]

\[ + \left( 1 - \frac{\mathcal{H}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi \]

\[ + \Psi - 2\Phi + \frac{1}{\mathcal{H}} \dot{\Phi} - \frac{3}{k} \mathcal{H} V + \frac{2}{r} \int_0^r dr' (\Phi + \Psi) \]

\[ + \left( \frac{\mathcal{H}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) \left[ \Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right] \]

\[ \frac{\mathcal{H}}{k} D \]

\[ \left( \frac{\mathcal{H}}{k} \right)^2 D \]

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)
Convergence

- Galaxy surveys observe also the shape and the luminosity of galaxies → measure of the convergence.
- The convergence $\kappa$ measures distortions in the size.
- The shear $\gamma$ measures distortions in the shape.
- Relativistic distortions affect the convergence at linear order.

We solve Sachs equation

$$\frac{D^2 \delta x^\alpha(\lambda)}{D \lambda^2} = R^\alpha_{\beta\mu\nu} k^\beta k^\mu \delta x^\nu$$
Convergence

Gravitational lensing

\[ \kappa = \frac{1}{2r} \int_0^r dr' \frac{r - r'}{r'} \Delta \Omega (\Phi + \Psi) + \left( \frac{1}{rH} - 1 \right) \mathbf{V} \cdot \mathbf{n} \]

- \[ \frac{1}{r} \int_0^r dr' (\Phi + \Psi) + \left( 1 - \frac{1}{rH} \right) \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \]

\[ + \left( 1 - \frac{1}{rH} \right) \Psi + \Phi \rightarrow \text{Sachs Wolfe} \]

Doppler lensing

The moving galaxy is further away → it looks smaller, i.e. demagnified

Integrated terms

CB (2008)
Bolejko et al (2013)
Bacon et al (2014)
Observations

♦ Due to relativistic effects, $\Delta$ and $\kappa$ contain additional information.

$\delta, V, \Phi, \Psi \quad (\Phi + \Psi), V, \Phi, \Psi$

♦ This can help testing gravity by probing the relation between density, velocity and gravitational potentials.

♦ Two difficulties:
  • The relativistic effects are small: we need to go to large scales.
  • We always measure the sum of all the effects.

♦ We need a way of isolating relativistic effects
  $\rightarrow$ look for anti-symmetries in the correlation function.
Density

The density contribution \( \Delta = b \cdot \delta \), generates an isotropic correlation function.

\[
\xi(s) = \langle \Delta(x)\Delta(x') \rangle \text{ depends only on the separation } s = |x - x'| 
\]

\[
\xi(s) = \frac{A b^2 D_1^2}{2 \pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s - 1} T_\delta^2(k) j_0(k \cdot s) 
\]
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\]
Redshift distortions break the isotropy of the correlation function.

$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

**Quadrupole**  Hamilton (1992)

$$\xi_2 = -D_1^2 \left( \frac{4fb}{3} + \frac{4f^2}{7} \right) \mu_2(s) P_2(\cos \beta)$$

$$\mu_2(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s-1} T_2^2(k) j_2(k \cdot s)$$

$$P_2(\cos \beta) = \frac{3}{2} \cos^2 \beta - \frac{1}{2}$$
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Redshift distortions break the isotropy of the correlation function.

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Hexadecapole Hamilton (1992)

\[ \xi_4 = D_1^2 \frac{8 f^2}{35} \mu_4(s) P_4(\cos \beta) \]

\[ \mu_4(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s - 1} T_\delta^2(k) j_4(k \cdot s) \]

\[ P_4(\cos \beta) = \frac{1}{8} \left[ 35 \cos^4 \beta - 30 \cos^2 \beta + 3 \right] \]
Redshift distortions

\[ \xi_0(s) = \frac{1}{2} \int_{-1}^{1} d\mu \xi(s, \mu) \]

\[ \xi_2(s) = \frac{5}{2} \int_{-1}^{1} d\mu \xi(s, \mu) P_2(\mu) \]

\[ \xi_4(s) = \frac{9}{2} \int_{-1}^{1} d\mu \xi(s, \mu) P_4(\mu) \]

\[ \mu = \cos \beta \]

\[ \mu_4(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s-1} T^2_\delta(k) j_4(k \cdot s) \]

\[ f = \frac{d \ln D_1}{d \ln a} \]

Measure separately:

\[ b \cdot \sigma_8 \quad \text{and} \quad f \cdot \sigma_8 \]
Relativistic effects

The relativistic effects break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.

This differs from the breaking of **isotropy**, due to redshift distortions, which is symmetric.

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.
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Relativistic effects

The relativistic effects break the \textit{symmetry} of the correlation function.

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Cross-correlation

The following terms **break** the symmetry:

\[ \Delta_{\text{rel}} = \frac{1}{\mathcal{H}} \partial_r \Psi + \left( 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r \mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} \]
Dipole in the correlation function

CB, Hui and Gaztanaga (2013)

\[ \xi(s, \beta) = D_1^2 f \frac{\dot{H}}{H_0} \left( \frac{\ddot{H}}{H^2} + \frac{2}{rH} \right) (b_B - b_F) \nu_1(s) \cdot \cos(\beta) \]

\[ \nu_1(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s-1} T_\delta(k) T_\Psi(k) j_1(k \cdot s) \]

By fitting for a dipole in the correlation function, we can measure relativistic effects, and separate them from the density and redshift space distortions.

\[ \xi_1(s) = \frac{3}{2} \int_{-1}^{1} d\mu \, \xi(s, \mu) \cdot \mu \quad \mu = \cos \beta \]
$z = 0.25$

Multipoles

Monopole

Quadrupole

Hexadecapole

Dipole
Convergence

\[ \kappa_g = \frac{1}{2r} \int_0^r dr' \frac{r - r'}{r'} \Delta \Omega (\Phi + \Psi) \quad \kappa_v = \left( \frac{1}{r \mathcal{H}} - 1 \right) \mathbf{V} \cdot \mathbf{n} \]

We can isolate the Doppler lensing by looking for anti-symmetries in \( \langle \Delta \kappa \rangle \)
The dipole due to gravitational lensing is completely subdominant.
Testing Euler equation

The monopole and quadrupole in $\Delta$ allow to measure $V$

The dipole allows to measure:

$$\Delta_{\text{rel}} = \frac{1}{\mathcal{H}} \partial_r \Psi + \left( 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) V \cdot n + \frac{1}{\mathcal{H}} \dot{V} \cdot n$$

If Euler equation is valid: $\dot{V} \cdot n + \mathcal{H} V \cdot n + \partial_r \Psi = 0$

$$\Delta_{\text{rel}} = - \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) V \cdot n$$

→ With the dipole, we can test Euler equation.
Measuring the anisotropic stress

♦ The dipole in the convergence is sensitive to:

\[
\kappa_v = \left( \frac{1}{r \mathcal{H}} - 1 \right) \mathbf{V} \cdot \mathbf{n}
\]

♦ The standard part \( \kappa_g = \frac{1}{2r} \int_0^r dr' \frac{r - r'}{r'} \Delta_\Omega (\Phi + \Psi) \)

can be measure through \( \langle \kappa \kappa \rangle \) and \( \langle \gamma \gamma \rangle \)

♦ Assuming Euler equation, we can test the relation between the two metric potentials \( \Phi \) and \( \Psi \).
Conclusion

- Our **observables** are affected by relativistic effects.

- These effects have a different **signature** in the **correlation** function: they induce anti-symmetries.

- By measuring these anti-symmetries we can isolate the relativistic effects and use them to **test** the **relations** between the density, velocity and gravitational potential.
Contamination

The density and velocity evolve with time: the density of the faint galaxies in front of the bright is larger than the density behind. This also induces a dipole in the correlation function.

\[
\begin{align*}
\text{observer} & \quad \uparrow \quad \mathbf{n} \\
F & \quad \rightarrow \quad \text{smaller density} \\
\text{Larger redshift} & \\
B & \quad \text{relativistic evolution}
\end{align*}
\]

\[
\begin{align*}
\xi_1^2 & \quad \text{total} \\
\xi_1 & \quad \text{evolution} \\
0 & \quad \text{relativistic}
\end{align*}
\]
Dipole in the correlation function

\[ \xi(s, \beta) = D_1^2 \int \frac{\mathcal{H}}{\mathcal{H}_0} \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r \mathcal{H}} \right) (b_B - b_F) \nu_1(s) \cdot \cos(\beta) \]

\[ \nu_1(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s - 1} T_\delta(k) T_\Psi(k) j_1(k \cdot s) \]

\[ \mathcal{H} = \frac{\dot{H}}{H^2} \]

\[ \beta \]

\[ \xi(s) \]

\[ n [\text{Mpc}/h] \]

\[ s [\text{Mpc}/h] \]

\[ z = 0.25 \]

\[ z = 0.5 \]

\[ z = 1 \]

\[ b_B - b_F \approx 0.5 \]

CB, Hui and Gaztanaga (2013)