Mildly non-linear effects in the large scale structure: resummation vs effective approaches

Diego Blas

w/ M. Garny and T. Konstandin

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Precision cosmology: power spectrum

\[ \delta_n \equiv \frac{\rho_n(x, t)}{\bar{\rho}_n(t)} - 1 \]

\[ \langle \delta(k, t) \delta(k', t) \rangle = P(k, t) \delta^{(3)}(k + k') \]

**DES, Euclid, LSST,...: 1% level at different redshifts! (also higher n-point correlation functions)**
Matter power spectrum at decoupling

gaussian initial scale invariant PS + radiation-matter transition + BAO imprint

\[ P_k \sim \frac{k}{(1 + k^2/k_0^2)^2} \]

Small quantity for PT: \( \delta_k(z = 1000) \)
Gravity makes matter clump: \( \delta_k \sim a(t) \)
perturbations grow! PT will break down

what can we learn from PT?
Non-linearity in the Universe

\[ \Delta^2 \equiv \frac{k^3 P(k)}{(2\pi)^3} \]

\[ M \text{ [M}_\odot\text{]} \]

\[ k \text{ [Mpc}^{-1}\text{]} \]

\[ \Delta^2(k) \]

Cosmic Cluster Galactic

CDM

Unknown small scale behavior

non-linear (simulation)
linear (analytic)

Baryon Acoustic Oscillations

ADM

WDM(8keV)
Theoretical framework

\[ 8\pi G T^m_{\mu\nu} = G_{\mu\nu} \]

Matter Non-relativistic and small \( \phi \)

Micro: ‘Particles’ (sampling \( \delta_{DM} \)) interacting through gravity

\[ p^i_A \equiv a m_A v^i_A , \quad \frac{dp^i_A}{dt} = -a m_A \partial_i \phi \]

\[ \frac{df(x, p, t)}{dt} = \frac{\partial f}{\partial t} + \frac{p^i_A}{a m_A} \partial_i f - a m_A \partial_i \phi \frac{\partial f}{\partial p^i_A} = 0 \]

Linear: free streaming particles

Fully non-linear: N-body

Mildly non-linear PT?
Fluid description

Taking moments:

Particles per volume
\[ \int d^3p f(x, p, t) \equiv \rho(x, t) , \quad \delta_n \equiv \frac{\rho_n(x, t)}{\bar{\rho}_n(t)} - 1 \]

Velocity field
\[ \int d^3p \frac{p^i}{am} f(x, p, t) \equiv \rho(x, t) v^i(x, t) \]

\[ \frac{df(x, p, t)}{dt} = \frac{\partial f}{\partial t} + \frac{p_A^i}{am_A} \partial_i f - am_A \partial_i \phi \frac{\partial f}{\partial p_A^i} = 0 , \quad \Delta \phi = \frac{3}{2} \mathcal{H}^2 \sum \Omega_n \delta_n \]

\[ \dot{\delta} + \partial_i ([1 + \delta] v^i) = 0 \]
\[ \dot{v}^i + \mathcal{H} v^i + v^j \partial_j v^i = -\partial_i \phi - \frac{1}{\rho} \partial_j \left[ \int d^3p \frac{p^i p^j}{(am)^2} f - \rho v^i v^j \right] \]

Pressureless perfect fluid interacting through gravity

Deviation from single flow Suppressed by
\[ k v_p \mathcal{H}^{-1} \]
Dealing with short scales

\[ \dot{v}^i + \mathcal{H} v^i + v^j \partial_j v^i = -\partial_i \phi - \frac{1}{\rho} \partial_j \left[ \int d^3 p \frac{p^i p^j}{(am)^2} f - \rho v^i v^j \right] \]

- Short scales: non-linear and no-fluid before.
- Hard to really ‘integrate-out’ modes

To restrict to long ‘quasi-linear’ scales, can the short scales be cut off (e.g. mean field) and get the right predictions?

\[ \dot{v}_L^i + \mathcal{H} v_L^i + v_L^j \partial_j v_L^i = -\partial_i \phi_L + O_L O_S + O_S + O_{mf} \]

EFT approach: encapsulate these effects in operators of \( L \)

\[ \dot{v}_L^i + \mathcal{H} v_L^i + v_L^j \partial_j v_L^i = -\partial_i \phi_L + c_L; s(t)^2 \partial_i \delta_L + c_L; bv(t)^2 \partial_i \partial_j v_L^j + \ldots \]

Pietroni et al 11, Carrasco et al 12
‘Effective’ coefficients

\[
\dot{v}_L^i + \mathcal{H}v_L^i + v_L^j \partial_j v_L^i = -\partial_i \phi_L + c_{L,s}(t)^2 \partial_i \delta_L + c_{L,bv}(t)^2 \partial_i \partial_j v_L^j + \ldots
\]

- Related to non-linear scales: not clear power-counting
- Functions of time
- Some effects are known (decoupling of virialized structures, \(k^2\) tail, measured non-linearities, …)

**measurements**

- Shell-crossing
  \[
  \dot{\theta}(k, t) + \mathcal{H}\theta(k, t) + \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(k, t) = q_\theta(k, t)
  \]
  \[\theta \equiv \partial_i v^i\]

- EFT matching
  \[O(k^2)\) small effect: can only be measured at mildly non-linear scales

Pueblas, Scoccimarro 09

Carrasco et al 13
The linearized equations of motion, Eqs. (35-37), the of Eq. (39) for redshifts

In it, we show the measured left and right hand sides in all cases is very good, improving, as expected, for
corrections to the PPF approximation due to the orbit-
spectrum reads

illustrative.

At redshifts

velocity divergence (three top lines) and density power
tutions are computed in linear theory, Eqs. (45) and (48),
all cases, we plot their absolute values. These correc-
(dotted). Note that the actual correction is negative in

We are interested in estimating the large-scale cor-
Pueblas, Scoccimarro 09

\[
\begin{align*}
\dot{\theta}(k, t) + & \mathcal{H}\theta(k, t) \\
+ & \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(k, t) = q\theta(k, t)
\end{align*}
\]

FIG. 9: Correction to the PPF approximation for the

Thus extrapolation well beyond

and matching,

After many approximations

Effects of effective coefficients

Negligible!

Crucial!
\[ \Omega_m = 1 \]

\[ \delta(k, t) = \sum_n a(t)^n F_n(k_1, \ldots, k_n) \delta(k_1, t_0) \ldots \delta(k_n, t_0) \delta^3(k - \sum k_i) \]

**Power spectrum**

\[ \langle \delta(k, t) \delta(k', t) \rangle = P(k, t) \delta^3(k + k') \]

NLO (1L): \( \sim \delta(k, t_0)^4 \)

\[ P_{13} = 3P_0(k) \int d^3q F_3^s(k, q, -q) P_0(q) \]

\[ P_{22} = 2 \int d^3q [F_2^s(q, k - q)]^2 P_0(q) P_0(|k - q|) \]

NNLO (2L): \( \sim \delta(k, t_0)^6 \)

\[ P(k, t) = a^2 P_0 + a^4(2P_{13} + P_{22}) + a^6(2P_{15} + 2P_{24} + P_{33}) + \ldots \]
3 Loop ‘disaster’

$z = 0.375$

- $N$-body
  - Horizon Run 2, Kim et al. 11

$P(k)/P_{\text{nowiggle}}(k)$

$k$ [h/Mpc]

$N_{\text{NNNLO}}$

$N_{\text{NLO}}$

$N_{\text{Linear}}$
Figure 1: One, two and three-loop contributions to the equal-time power spectrum obtained from a numerical Monte Carlo integration within standard perturbation theory at \( z = 0 \). The linearpowerspectrum from CAMB [20] using the \( \Lambda \)CDM model with WMAP5 parameters.

For the three-loop order, the error bars show an estimate for the numerical error obtained by multiplying the error output of the CUBA routine Suave by a factor of two. The relative error is \(<0.002\) for \( k \leq 0.55\) h/Mpc. The black diamonds and grey crosses correspond to two different parametrization soft hand looploophotonicfactors (see App. A).

For even larger momentum \( k \), one observes that each loop contribution features the expected behavior (3.2) with a logarithmic enhancement compared to the linear spectrum. But also in this regime, the loop expansion appears to be divergent.

The picture might change if one goes to larger redshift \( z \), where the expansion parameter can be efficiently suppressed since \( \sigma^2_l \propto D + (z)^2 \sim (1 + z)^{-2} \). In Figs. 2 and 3 we show some comparisons between our three-loop SPT results (black lines and diamonds) and N-body simulations (red dots, Horizon Run 2 [27]) for various redshifts (see App. C for further details). For large redshift (\( z \gtrsim 1.75 \)) the three-loop contribution may lead to an improved agreement with the N-body data, while it clearly degrades the agreement compared to the two-loop at lower redshifts. The same happens for the two-

NNNLO (3 Loop)
A resummation: Padé integrands

\[ P_{\text{low-k}} = -\frac{244\pi}{315} k^2 P_{\text{lin}}(k, z) \int dq P_{\text{lin}}(k, z) \sum L C_L \left[ 4\pi \int_0^q dp p^2 P_{\text{lin}}(p, z) \right]^{2(L-1)} \]

\[ K(x) = \sum C_L x^{L-1} \quad \Rightarrow \quad K(x)^{\text{Padé}} = \frac{1 + \sum_{i=1}^n a_i x^i}{1 + \sum_{j=1}^m b_j x^j} \]

The resummation **damps** the UV dependence! May made the series **convergent**! (1% target attainable)
Padé results: perturbation theory

\[ P(k, t) = P_l + P_{1\text{-loop}} + \ldots = P_l + P^{\text{Padé}}_{\text{low}-k} + \Delta P_{1\text{-loop}} \ldots \]

\[ \Delta P_{L\text{-loop}} \equiv P_{L\text{-loop}} - P_{L\text{-loop}}^{\text{small}-k}, \quad P^{\text{Padé}}_{\text{low}-k} \equiv \sum P_{L\text{-loop}}^{\text{small}-k} \]

\( z = 0.375 \)

\( N\text{-body} \)

Horizon Run 2, Kim et al. 11

NNNLO(P)

NLO(P)

NNNLO(SPT)

Linear
Padé results: redshift dependence

\[ z = 0 \]

\[ z = 0.375 \]

\[ z = 0.833 \]

\[ z = 1.75 \]
Further checks

Correction to $P(k,z)$ rel. to one–loop

$P(k,z)/P_{-\text{loop}}(k,z)$

redshift $z$

$P_{\text{1\ loop}}$, $P_{\text{2\ loop}}$, $P_{\text{3\ loop}}$

$K_{01}^{\text{pade}}$, $K_{02}^{\text{pade}}$, $K_{11}^{\text{pade}}$

Nbody HR2

DB, Chan, Garny, Konstandin Unpublished
Conclusions

- Future surveys will test cosmological expansion and structure formation to **percent level**.
- At this precision, the Universe at large scales behaves **almost** as pressureless perfect fluid.
- On-going discussion on the importance of short modes. **Necessary/irrelevant** for convergence at semi-linear $k$.
- PT series is **not convergent**! Reminds asymptotic series (result at 3 loop).
- **Padé ansatz**: parameter free resummation. Much better convergence properties and agreement with $N$-body. (percent accuracy at BAO scales and $z = 0$ reachable)
For the future

- More analytical understanding, Borel-Padé. UV sens. Other basis...

- Other observables ($P_{\theta \theta}$, bispectrum, ...), other IC (NG).

- Predictions for observations: results in redshift space, parametrization of BAOs, bias...

- Putting all together? **EFTofLSS + resummations**

- Including neutrinos...

- Lagrangian space. Work in phase space.