From Mainz 2013 to Benasque 2013:

Tensor Networks Reach Quantum Simulation!

October 1st, 2013

Workshop on "Quantum Simulations", Benasque, Spain




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Reference: arXiv:1309.4075

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Outline

- Introduction and motivation: *cavity-lattice quantum simulation*
- Tensor networks and the way they can gift quantum simulation
  - **Case study:** Tensor-network simulation of kagome (photonic) quantum simulator
- New horizons for photonic quantum simulation

Photon lattices for quantum simulation

- **Quantum simulation:** employing *well-controlled* quantum systems to simulate *complex* quantum matter

- **Physical implementation:** using photons as particles in a "photon lattice" quantum simulator (an array of circuit QED elements)

  → Flexible lithographic fabrication and easily attainable strong coupling

- **"On-chip" many-body physics:** superfluid–Mott-insulator transition, macroscopic quantum self-trapping, and fractional quantum Hall physics etc

- Well-controlled quantum systems with "**circuit excitations**" rather than **physical particles**

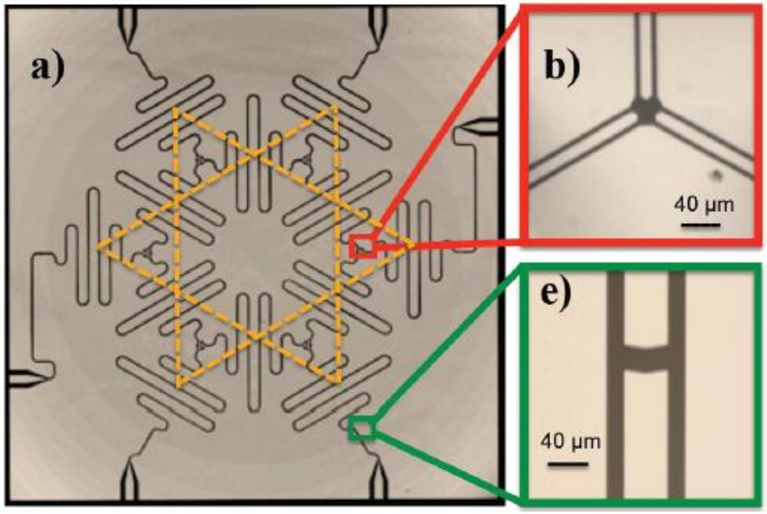
Andrew A. Houck, Hakan E. Türeci, and Jens Koch, Nature Phys. **8**, 292 (2012).

Kagome cavity lattices

- Arrays of on-chip microwave resonators in a kagome geometry described by the Jaynes-Cummings-Hubbard (JCH) Hamiltonian:

$$H = \sum_i H_i^{\mathrm{JC}} - \kappa \sum_{i,j} (a_i^\dagger a_j + \mathrm{H.c.})$$

$$H_i^{\mathrm{JC}} = \omega_d a_i^\dagger a_i + \epsilon \sigma_i^+ \sigma_i^- + g(a_i \sigma_i^+ + a_i^\dagger \sigma_i^-)$$



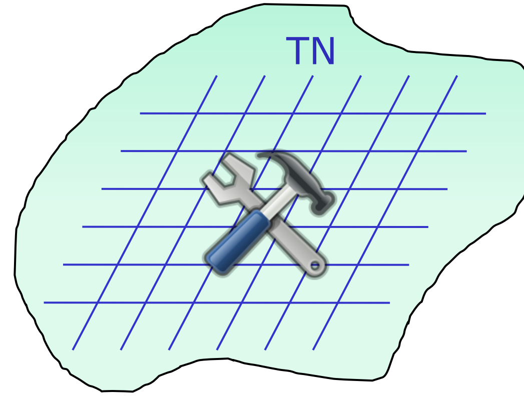
- Needing sophisticated and efficient numerical techniques that can capture many-polariton correlations

→ address larger arrays and collective phenomena and possible ***phase transitions of light***

D. L. Underwood, W. E. Shanks, J. Koch, and A. A. Houck, Phys. Rev. A **86**, 023837 (2012)

Tensor networks (TN)

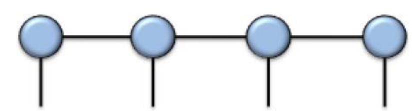
- Powerful tools for *classical* simulation of quantum many-body systems by representing the state of a system as an *efficiently*-contractible network of multi-index tensors optimized *variationally*



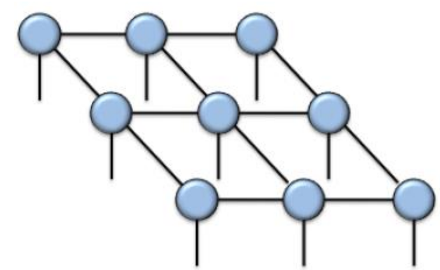
Many-body Hilbert space

- **1D:** Matrix-product states (**MPS**)

$$|\Psi_{\mathrm{G}}\rangle = \sum_{i_1, i_2, \cdots, i_N = 1}^{d} \left(\prod_{k=1}^{N} \mathcal{A}_{[k]}^{i_k} \right) \bigotimes_{k=1}^{N} |i_k\rangle$$



- **2D:** Projected-entangled-pair states (**PEPS**)



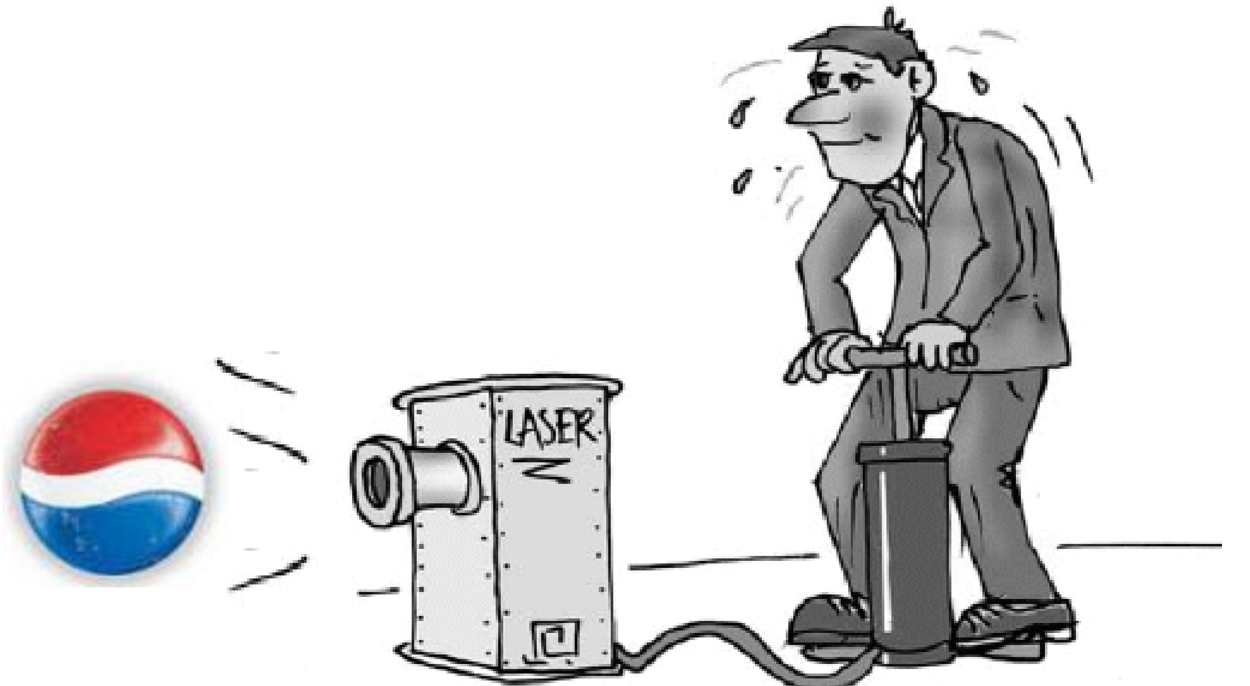
$$|\Psi_{\mathrm{G}}\rangle = \sum_{i_1, i_2, \cdots, i_N = 1}^{d} \mathcal{C}(\mathcal{A}_{[1]}^{i_1}, \mathcal{A}_{[2]}^{i_2}, \cdots, \mathcal{A}_{[N]}^{i_N}) \bigotimes_{k=1}^{N} |i_k\rangle$$

F. Verstraete, V. Murg, and J. Cirac, Adv. Phys. **57**, 143 (2008); R. Orus, arXiv:1306.2164

Myths and facts about tensor networks!

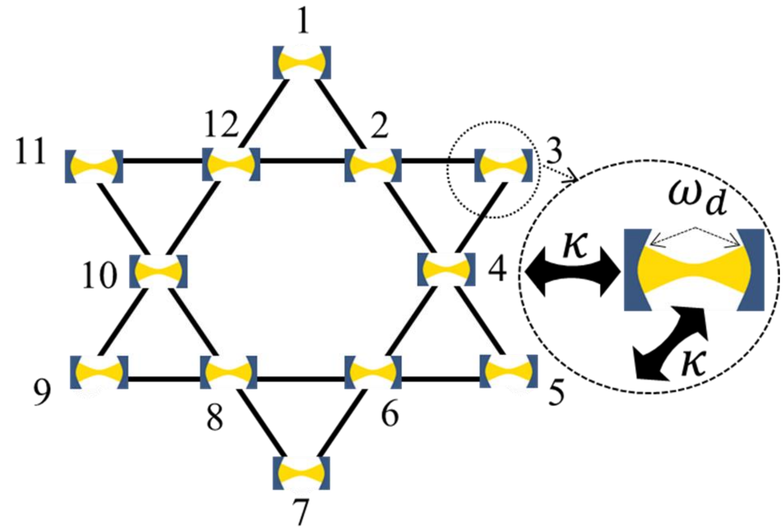
Myths	Facts
TNs do not produce any physics!	TNs do produce physics addressing (efficiently) otherwise intractable problems such as multi-channel Kondo model etc
TNs outperform quantum monte carlo all the time!	Not suffering minus sign but rather tricky contraction schemes beyond 1D
…	

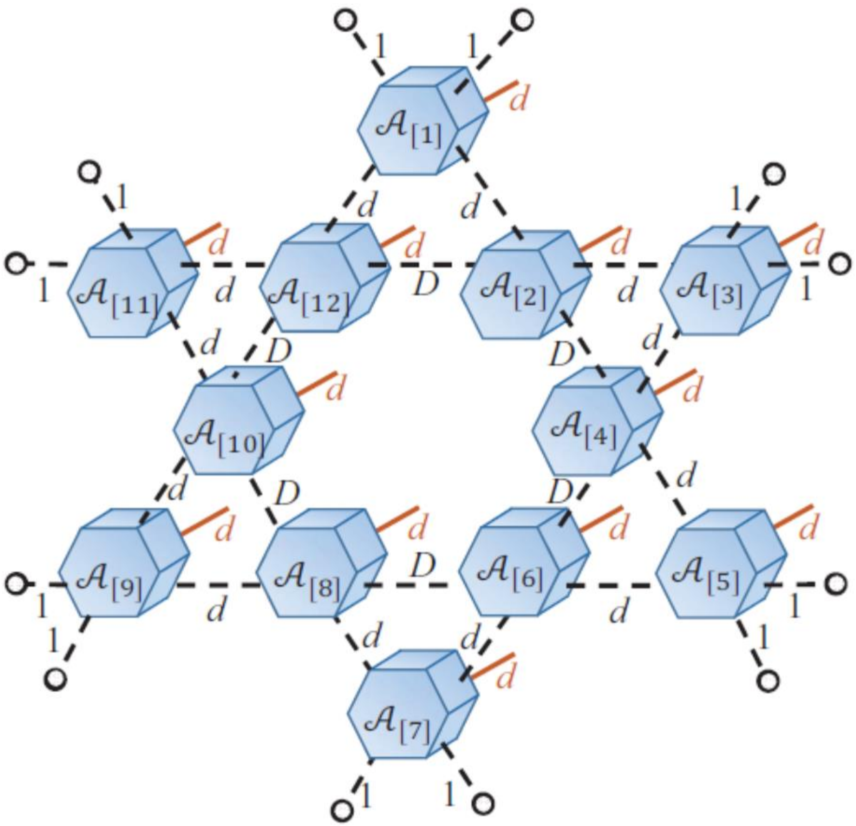
Photonic PEPS



Kagome PEPS

$$\hat{\mathcal{H}} = \hbar\omega_d \sum_{k=1}^{12} \hat{a}_k^\dagger \hat{a}_k - \kappa \sum_{\langle k,k' \rangle} (\hat{a}_k^\dagger \hat{a}_{k'} + \mathrm{H.c.})$$





- PEPS ansatz to the many-photon ground state:

$$|\Psi_{\mathrm{G}}\rangle = \sum_{i_1,\cdots,i_{12}=1}^{d} \mathcal{C}(\mathcal{A}_{[1]}^{i_1}, \cdots, \mathcal{A}_{[12]}^{i_{12}}) \bigotimes_{k=1}^{12} |i_k\rangle$$

$$d = N + 1$$

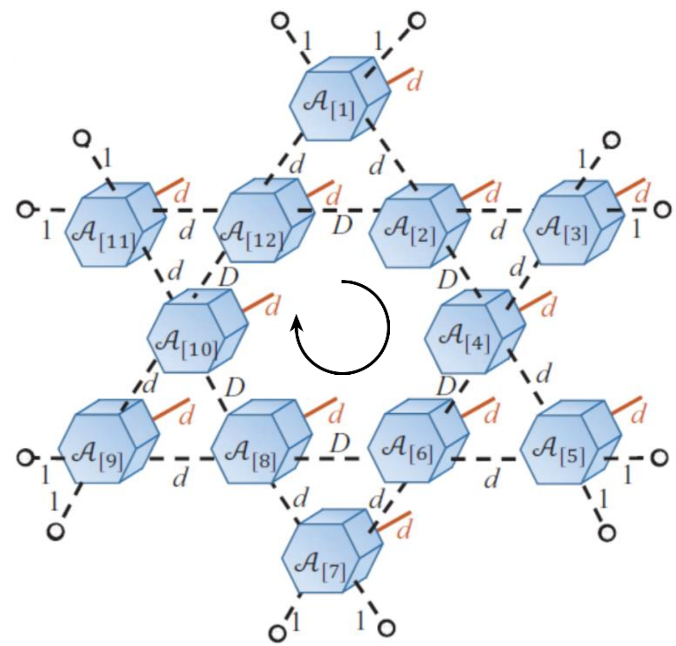
# of photons

Capturing equilibrium properties

- Target the ground-state energy $|\Psi_{\mathrm{G}}\rangle$:

$$\min_{|\Psi\rangle\in\{\mathrm{PEPS}\}} \frac{\langle\Psi|\hat{\mathcal{H}}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

- Variational “sweeping procedure”:



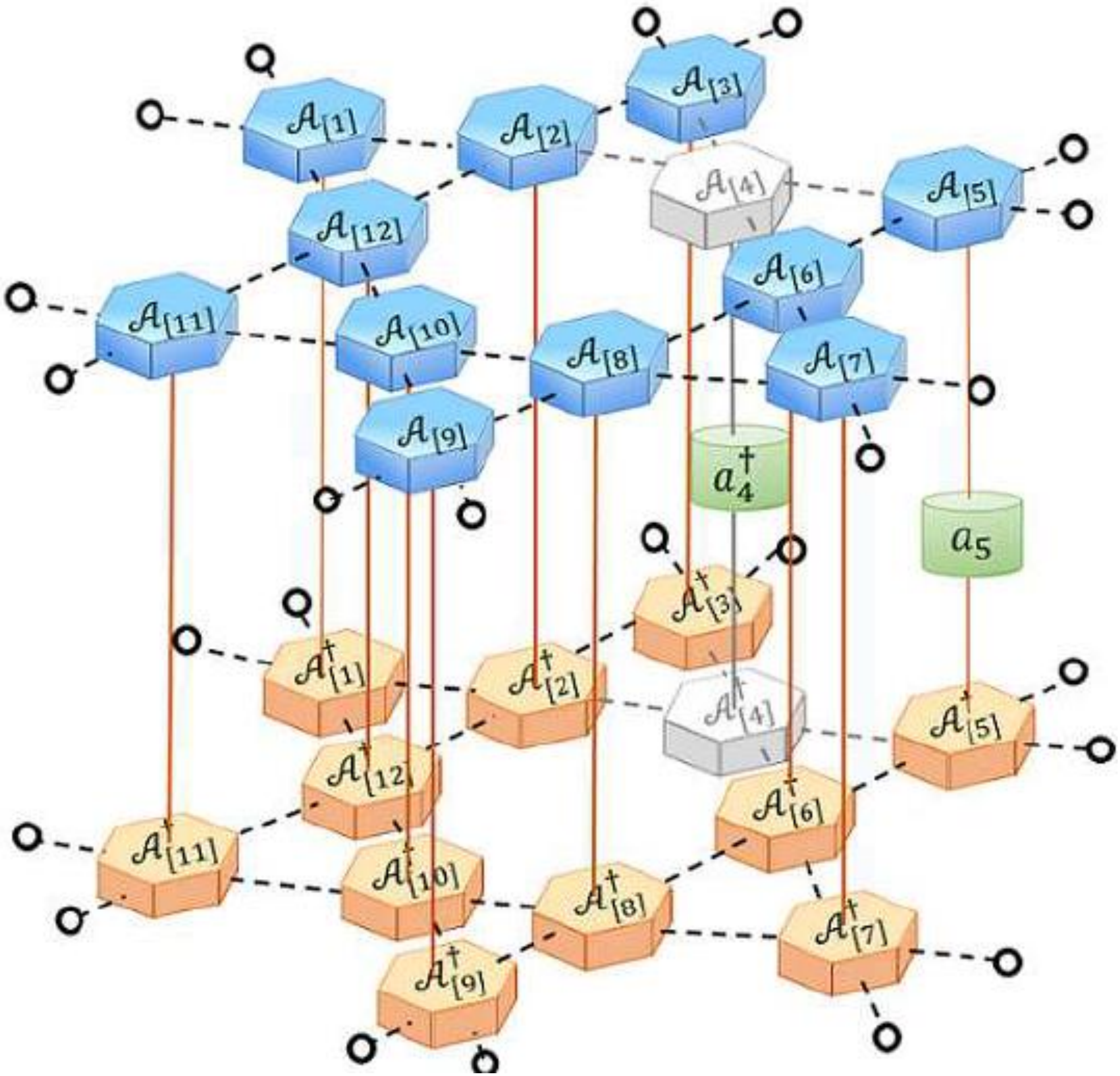
$$\langle\Psi|\hat{\mathcal{H}}|\Psi\rangle = \boldsymbol{A}_k^{\dagger}\mathcal{H}_k^{\mathrm{eff.}}\boldsymbol{A}_k$$
$$\langle\Psi|\Psi\rangle = \boldsymbol{A}_k^{\dagger}\mathcal{N}_k^{\mathrm{eff.}}\boldsymbol{A}_k$$

- Generalized eigenvalue problem:

$$\mathcal{H}_k^{\mathrm{eff.}}\boldsymbol{A}_k = \xi_k\mathcal{N}_k^{\mathrm{eff.}}\boldsymbol{A}_k$$

$$E_{\mathrm{G}} \leq \min_j \xi_k^j \equiv \xi_k^{\min}$$

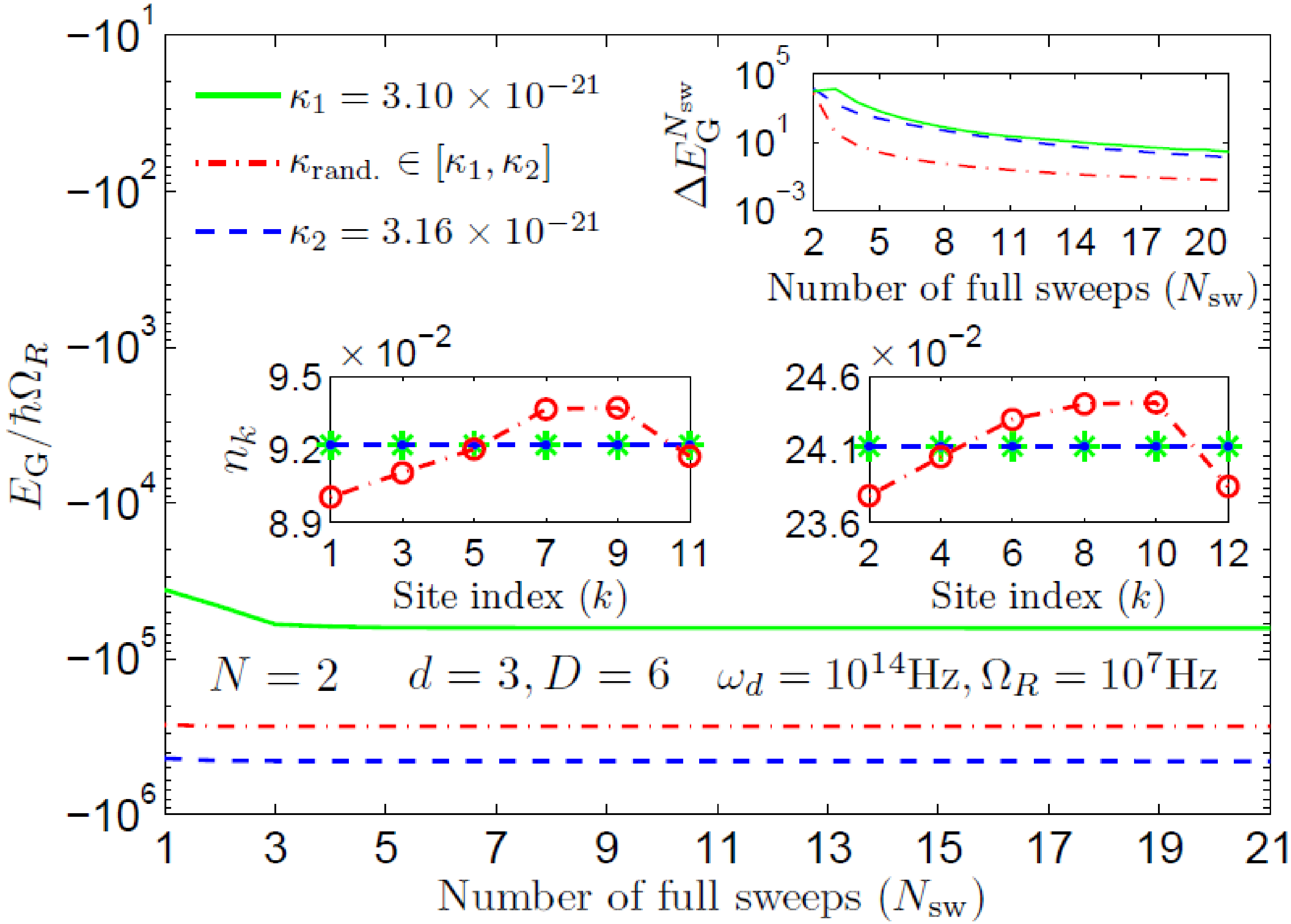
Effective Hamiltonian



$\Rightarrow \quad \mathcal{H}_k^{\text{eff.}}$

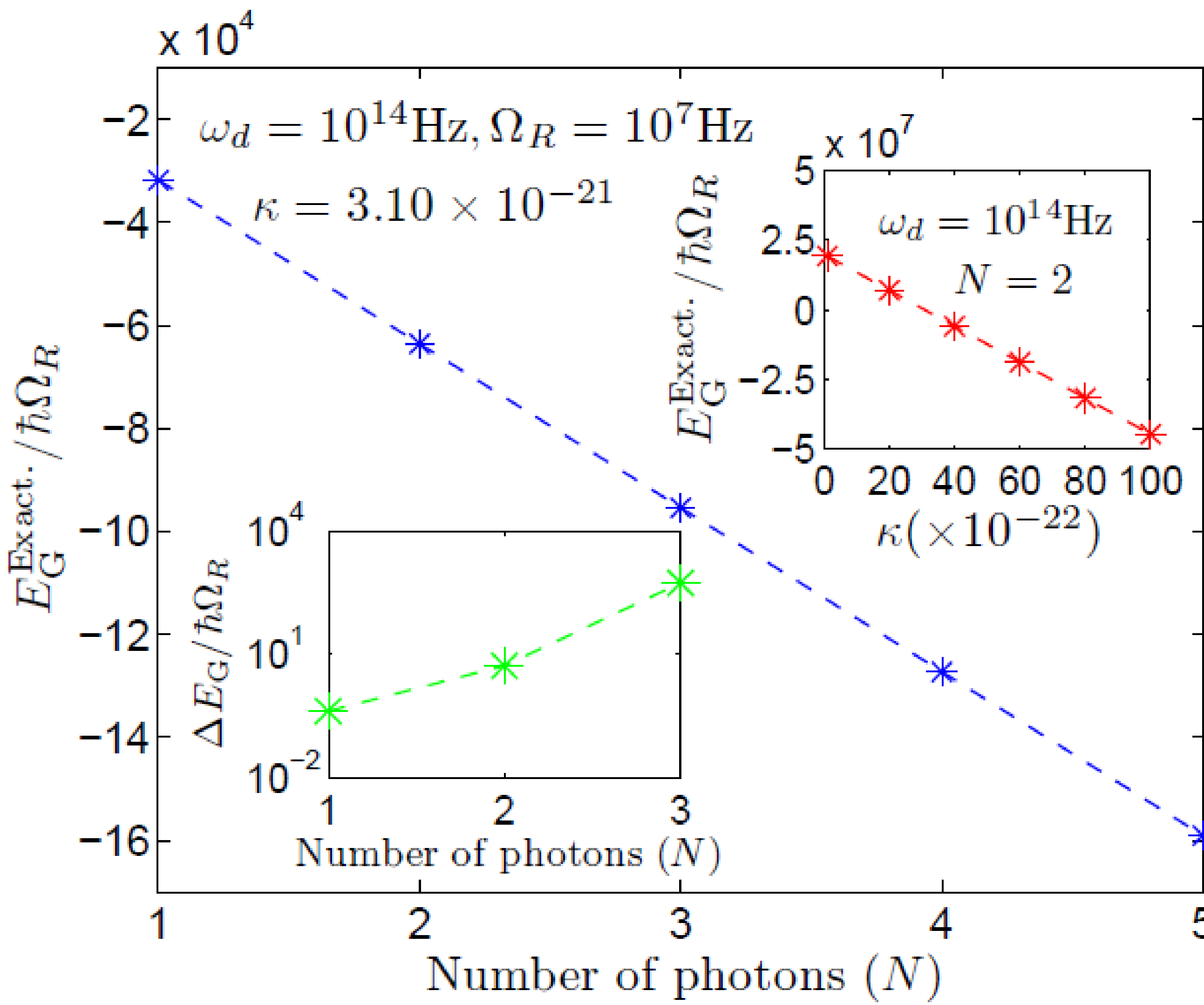
$$\mathcal{H}_k^{\text{eff.}} \boldsymbol{A}_k = \xi_k \mathcal{N}_k^{\text{eff.}} \boldsymbol{A}_k$$

Equilibrium properties



$$n_k = \langle \Psi_G | \hat{a}_k^\dagger \hat{a}_k | \Psi_G \rangle \qquad N = 6(n_{\text{inner}} + n_{\text{outer}})$$

Equilibrium properties



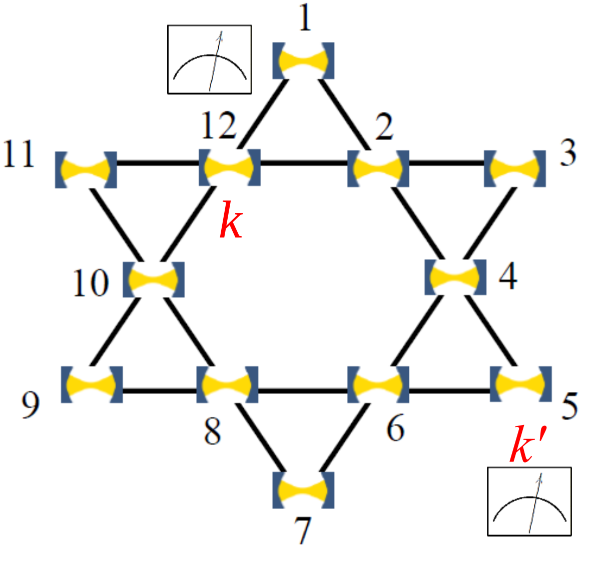
Dynamical properties

- Two-point correlation functions associated with the propagation of two-photonic excitations:

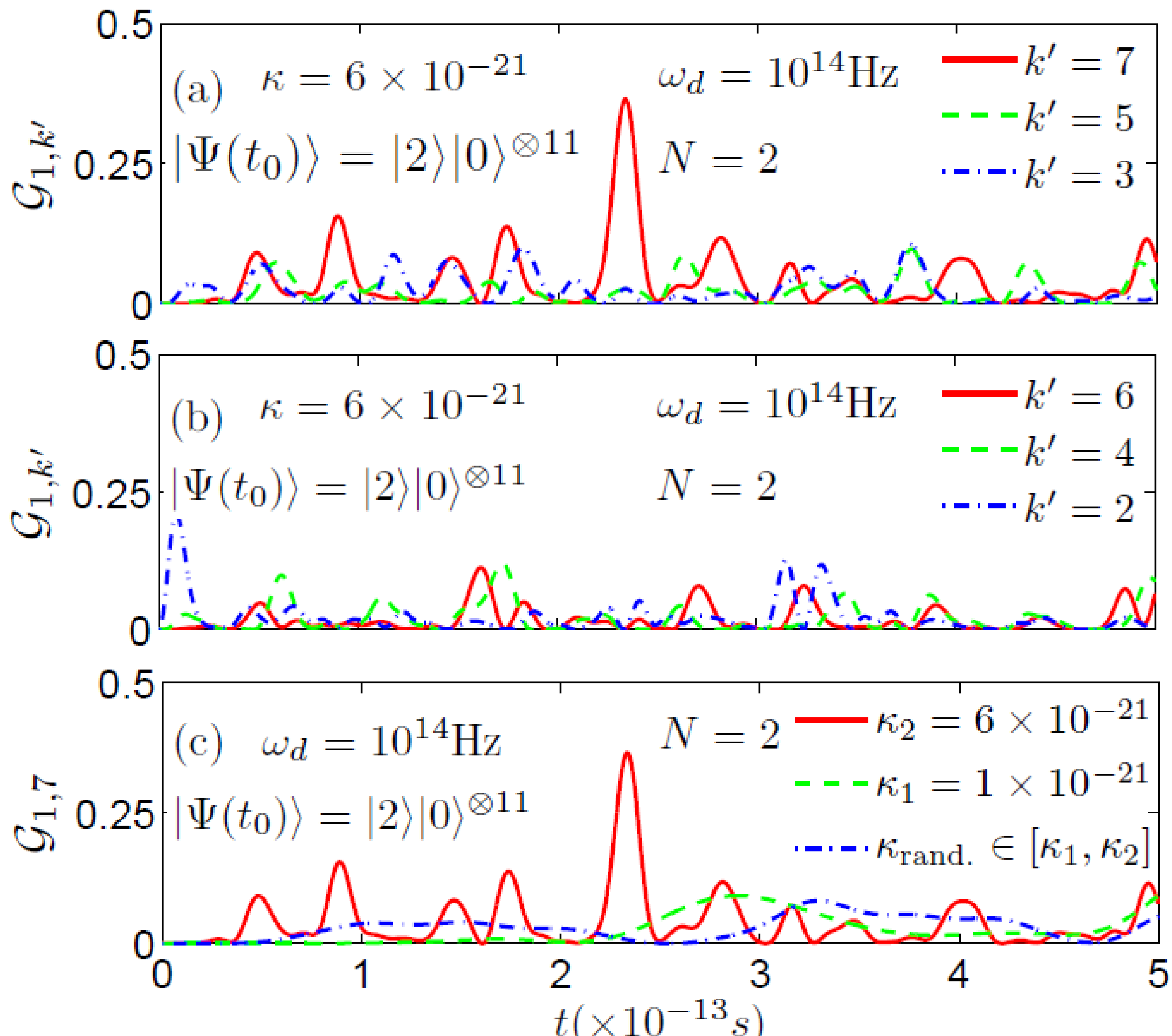
$$\mathcal{G}_{k,k'}(t) = \langle \Psi(t) | \hat{n}_k \hat{n}_{k'} | \Psi(t) \rangle$$

$$|\Psi(t)\rangle = e^{-i\hat{\mathcal{H}}t/\hbar} |\Psi(t_0)\rangle$$

> The average result of a joint photon-number measurement performed on cavities k and k'



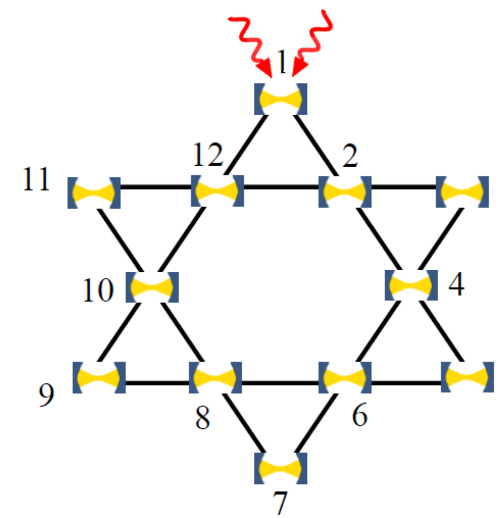
Propagation of *localized* excitations



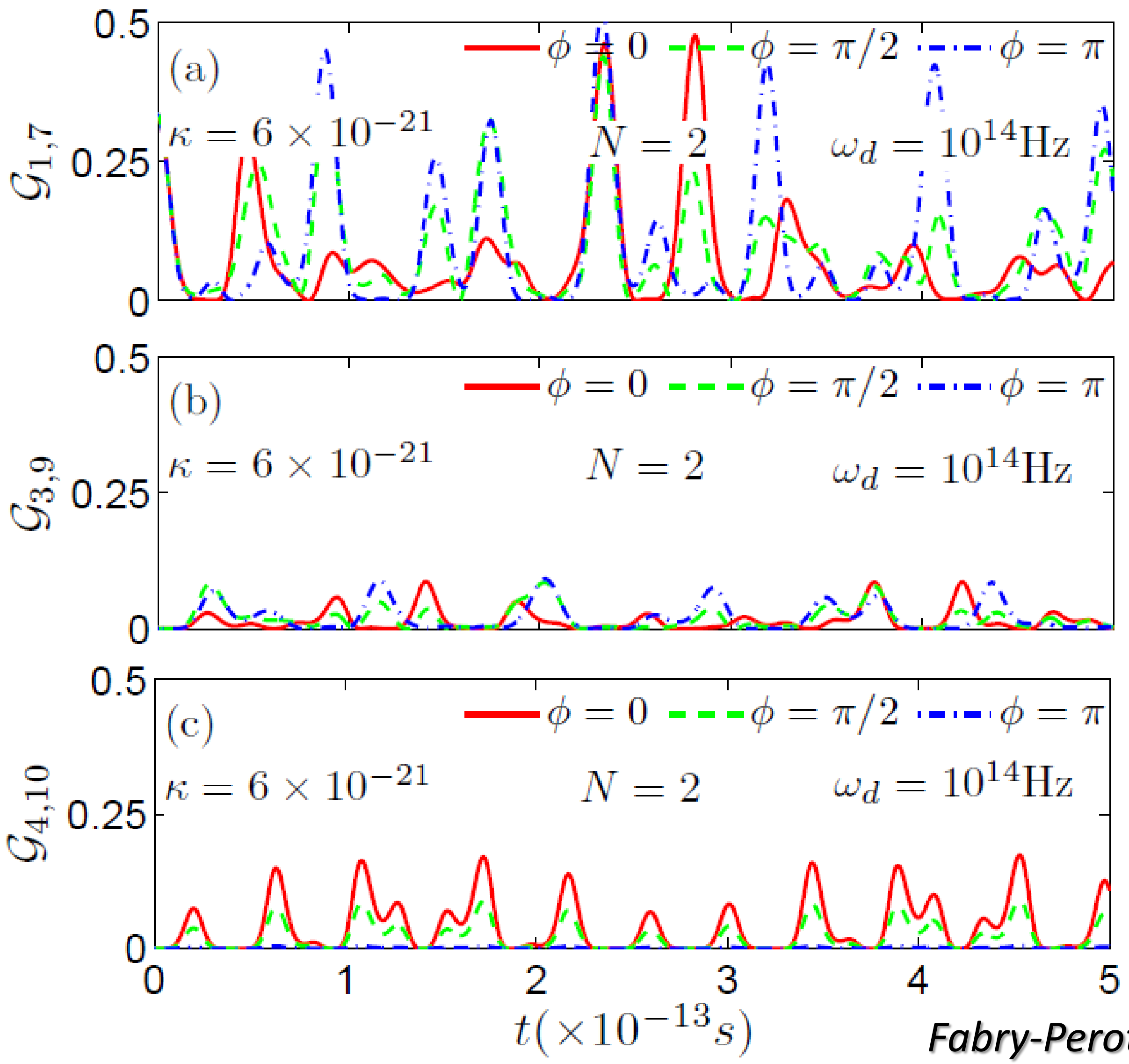
Reference cavity:

$k = 1$

$|\Psi(t_0)\rangle = |2\rangle|0\rangle^{\otimes 11}$

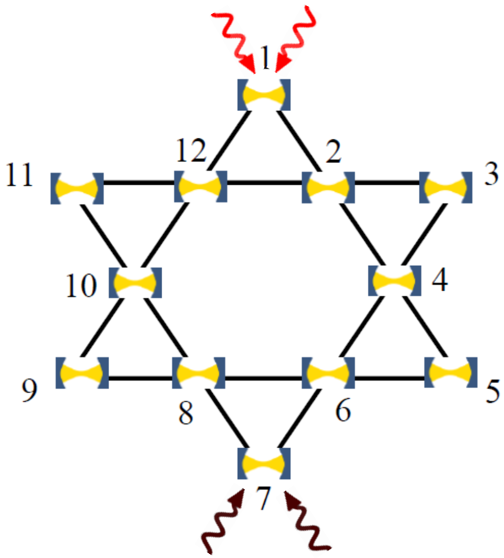


Propagation of *delocalized* superposition



$$|\Psi(t_0)\rangle = \frac{1}{\sqrt{3}}(|2\rangle|0\rangle^{\otimes 11} + |0\rangle^{\otimes 6}|2\rangle|0\rangle^{\otimes 5} + e^{i\phi}|1\rangle|0\rangle^{\otimes 5}|1\rangle|0\rangle^{\otimes 5}) ,$$

Relative phase



Fabry-Perot-type resonances

New horizons?

- Tensor networks offer efficient *classical* simulation of photonic *quantum* simulators
- Proposed a flexible numerical framework for unraveling ***exotic phases of light*** on a kagome geometry
- Simulation of the kagome lattice in the ***ultrastrong coupling regime*** of light-matter interaction T. Niemczyk *et al*, Nature Phys. **6**, 772 (2010).
- Paving the way for studying a variety of thrilling strongly correlated many-photon phenomena such as possible fermionization of photons, anomalous Hall effects etc upon a systematic extension of the proposed numerical framework to larger kagome arrays

Andrew A. Houck, Hakan E. Türeci, and Jens Koch, Nature Phys. **8**, 292 (2012).

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Thank you

either for your patience **or** **for your attention!**

From the cover of the March 1998 issue of the “Physics World”

q.e.d.