Numerical Investigation
Aeroelasticity
Ice accretion

PDE’s, optimal design and numerics
Benasque 2013

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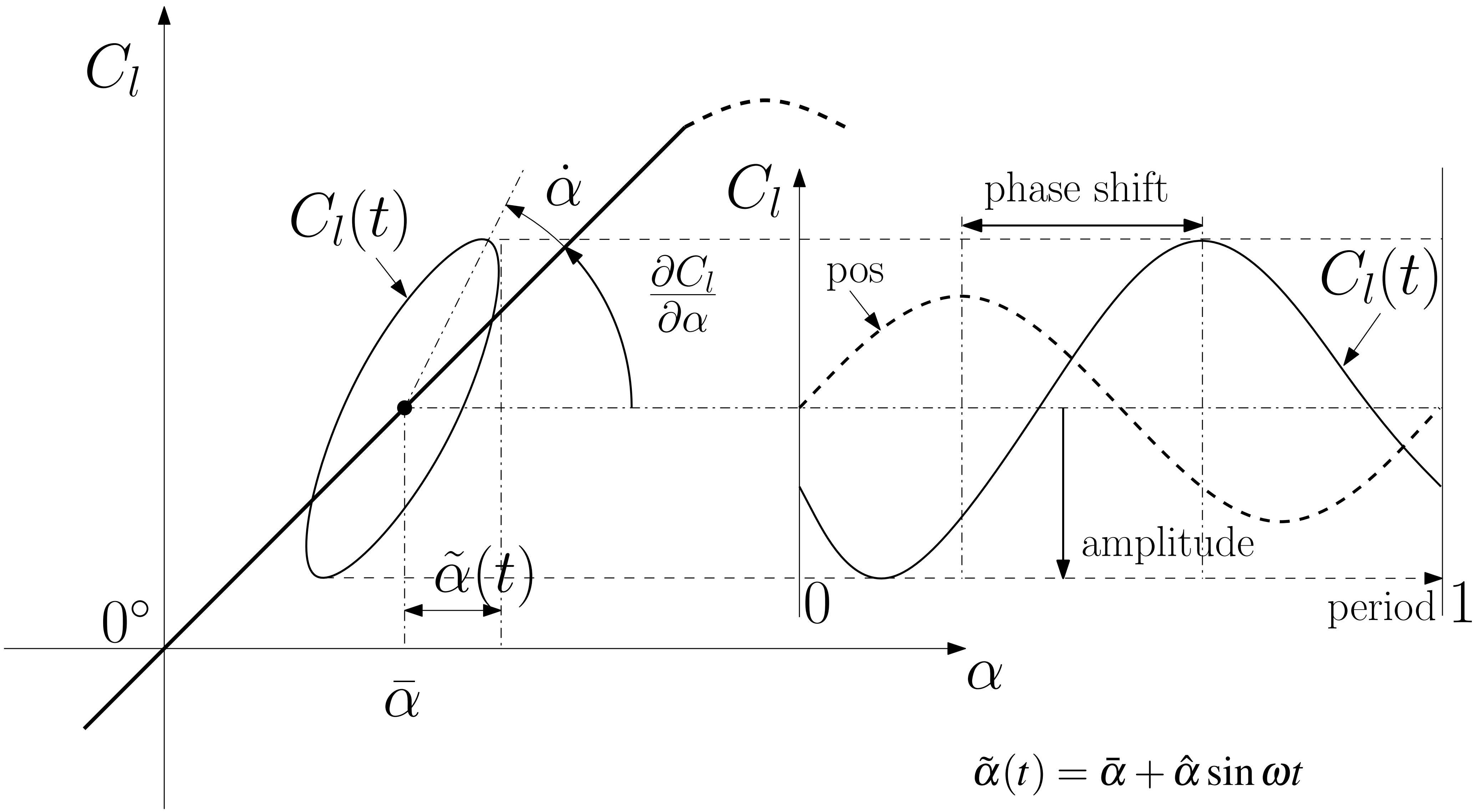
DLR

Unsteady harmonic motions



$$\tilde{\alpha}(t) = \bar{\alpha} + \hat{\alpha}\sin\omega t$$

Linearization of Unsteady Reynolds-Averaged Navier-Stokes (URANS) equations

time-dependent coordinate: $\mathbf{x}(t) = [x(t), y(t), z(t)]^T$

velocity field: $\mathbf{U}(\mathbf{x}(t), t) = [u(\mathbf{x}, t), v(\mathbf{x}, t), w(\mathbf{x}, t)]^T$

nondimensionalized integral strong conservation form:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega(t)} \mathbf{W} \, \mathrm{d}|\Omega| + \int_{\partial\Omega(t)} (\mathbf{f}_c \cdot \mathbf{n} - \mathbf{f}_v \cdot \mathbf{n} - \mathbf{W} \dot{\mathbf{x}} \cdot \mathbf{n}) \, \mathrm{d}|\partial\Omega| = \int_{\Omega(t)} \mathbf{Q} \, \mathrm{d}|\Omega|, \quad \Omega(t) \subset \mathbb{R}^3$$

with $\mathbf{W} := [\rho, \rho u, \rho v, \rho w, \rho E, \rho \check{\nu}]^T$

isolating the time derivative

$$\mathbf{R}(\mathbf{W}, \mathbf{x}, \dot{\mathbf{x}}) := \int_{\partial\Omega(t)} (\mathbf{f}_c \cdot \mathbf{n} - \mathbf{f}_v \cdot \mathbf{n} - \mathbf{W} \dot{\mathbf{x}} \cdot \mathbf{n}) \, \mathrm{d}|\partial\Omega| - \int_{\Omega(t)} \mathbf{Q} \, \mathrm{d}|\Omega|$$

Linearization of URANS equations

semi-discrete form (a):

$$\frac{\mathrm{d}(\mathbf{M}\mathbf{W})}{\mathrm{d}t} + \mathbf{R}(\mathbf{W},\mathbf{x},\dot{\mathbf{x}}) = 0$$

M being the mass matrix $\mathbf{M} := \mathbf{M}(\mathbf{x}) := diag(|\Omega_i(t)|), \;\; i = 1,\dots,N$

linearization around steady state (Taylor-series) - (b)

$$\begin{aligned}
\mathbf{W}(\mathbf{x},\dot{\mathbf{x}}) &\approx \overline{\mathbf{W}}(\bar{\mathbf{x}}) + \widetilde{\mathbf{W}}(\mathbf{x},\tilde{\mathbf{x}},,\dot{\tilde{\mathbf{x}}}) + \dots, \quad \|\widetilde{\mathbf{W}}\| \ll \|\overline{\mathbf{W}}\| \\
\mathbf{x}(t) &\approx \bar{\mathbf{x}} + \tilde{\mathbf{x}}(t) + \dots \\
\dot{\mathbf{x}}(t) &\approx \dot{\bar{\mathbf{x}}} + \dot{\tilde{\mathbf{x}}}(t) + \dots, \quad \dot{\bar{\mathbf{x}}} = 0 \\
\mathbf{M}(\mathbf{x}) &\approx \overline{\mathbf{M}}(\bar{\mathbf{x}}) + \left.\frac{\partial \mathbf{M}}{\partial \mathbf{x}}\right|_{\bar{\mathbf{x}}} \tilde{\mathbf{x}} = \overline{\mathbf{M}}(\bar{\mathbf{x}}) + \widetilde{\mathbf{M}}(\bar{\mathbf{x}},\tilde{\mathbf{x}}) + \dots, \\
\mathbf{R}(\mathbf{W},\mathbf{x},\dot{\mathbf{x}}) &\approx \mathbf{R}(\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}) + \left.\frac{\partial \mathbf{R}}{\partial \mathbf{W}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} \widetilde{\mathbf{W}} + \left.\frac{\partial \mathbf{R}}{\partial \mathbf{x}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} \tilde{\mathbf{x}} + \left.\frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} \dot{\tilde{\mathbf{x}}} + \dots = \\
&= \overline{\mathbf{R}}(\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}) + \widetilde{\mathbf{R}}(\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}},\widetilde{\mathbf{W}},\tilde{\mathbf{x}},\dot{\tilde{\mathbf{x}}}) + \dots
\end{aligned}$$

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Linearization of URANS equations

after (b) into (a) we obtain:

$$\left(\overline{\mathbf{M}}(\bar{\mathbf{x}})+\widetilde{\mathbf{M}}(\bar{\mathbf{x}},\tilde{\mathbf{x}})\right)\frac{\mathrm{d}}{\mathrm{d}t}\left(\overline{\mathbf{W}}(\bar{\mathbf{x}})+\widetilde{\mathbf{W}}(\bar{\mathbf{x}},\tilde{\mathbf{x}})\right)+$$
$$+\left(\overline{\mathbf{W}}(\bar{\mathbf{x}})+\widetilde{\mathbf{W}}(\bar{\mathbf{x}},\tilde{\mathbf{x}})\right)\left.\frac{\partial \mathbf{M}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\bar{\mathbf{x}}}\dot{\tilde{\mathbf{x}}}+$$
$$+\overline{\mathbf{R}}\left(\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}\right)+\widetilde{\mathbf{R}}\left(\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}},\widetilde{\mathbf{W}},\tilde{\mathbf{x}},\dot{\tilde{\mathbf{x}}}\right)=0$$

using discrete Fourier-series (c) for harmonic excitations - transformation into frequency domain

$$\widetilde{\mathbf{W}}:=\sum_{n=1}^{\infty}\widehat{\mathbf{W}}_n e^{in_h\omega t},\quad \widehat{\mathbf{W}}_n\in\mathbb{C}$$

$$\tilde{\mathbf{x}}=\sum_{n=1}^{\infty}\hat{\mathbf{x}}_n e^{in_h\omega t},\quad \hat{\mathbf{x}}_n\in\mathbb{C}$$

x is known which is a mode (bending, torsion, …)

Linearization of URANS equations

after (c) into (b) we obtain:

$$\left[i\omega \overline{\mathbf{M}}(\bar{\mathbf{x}}) + \left.\frac{\partial \mathbf{R}}{\partial \mathbf{W}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} \right] \widehat{\mathbf{W}} e^{i\omega t} +$$

$$+ \left[\left.\frac{\partial \mathbf{R}}{\partial \mathbf{x}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} + i\omega \left(\left.\frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} + \overline{\mathbf{W}}(\bar{\mathbf{x}}) \left.\frac{\partial \mathbf{M}}{\partial \mathbf{x}}\right|_{\bar{\mathbf{x}}} \right) \right] \hat{\mathbf{x}} e^{i\omega t} +$$

$$+ \left.\frac{\partial \mathbf{M}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\bar{\mathbf{x}}} \left(-\omega^2 + i\omega \right) \hat{\mathbf{x}} \widehat{\mathbf{W}} e^{i2\omega t} = 0$$

Implementation! - neglect higher order term - solve for hat W

$$\left[i\omega \overline{\mathbf{M}}(\bar{\mathbf{x}}) + \left.\frac{\partial \mathbf{R}}{\partial \mathbf{W}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} \right] \widehat{\mathbf{W}} = - \left[\left.\frac{\partial \mathbf{R}}{\partial \mathbf{x}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} + i\omega \left(\left.\frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}}\right|_{\overline{\mathbf{W}},\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}}} + \overline{\mathbf{W}}(\bar{\mathbf{x}}) \left.\frac{\partial \mathbf{M}}{\partial \mathbf{x}}\right|_{\bar{\mathbf{x}}} \right) \right] \hat{\mathbf{x}}$$

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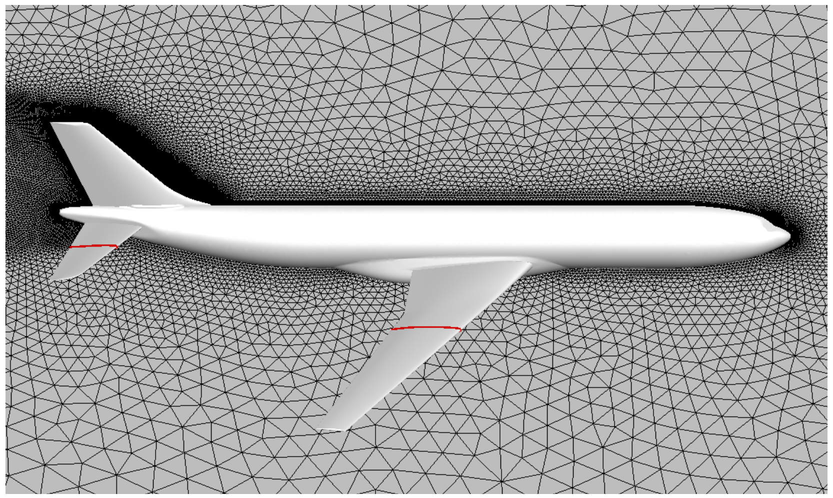
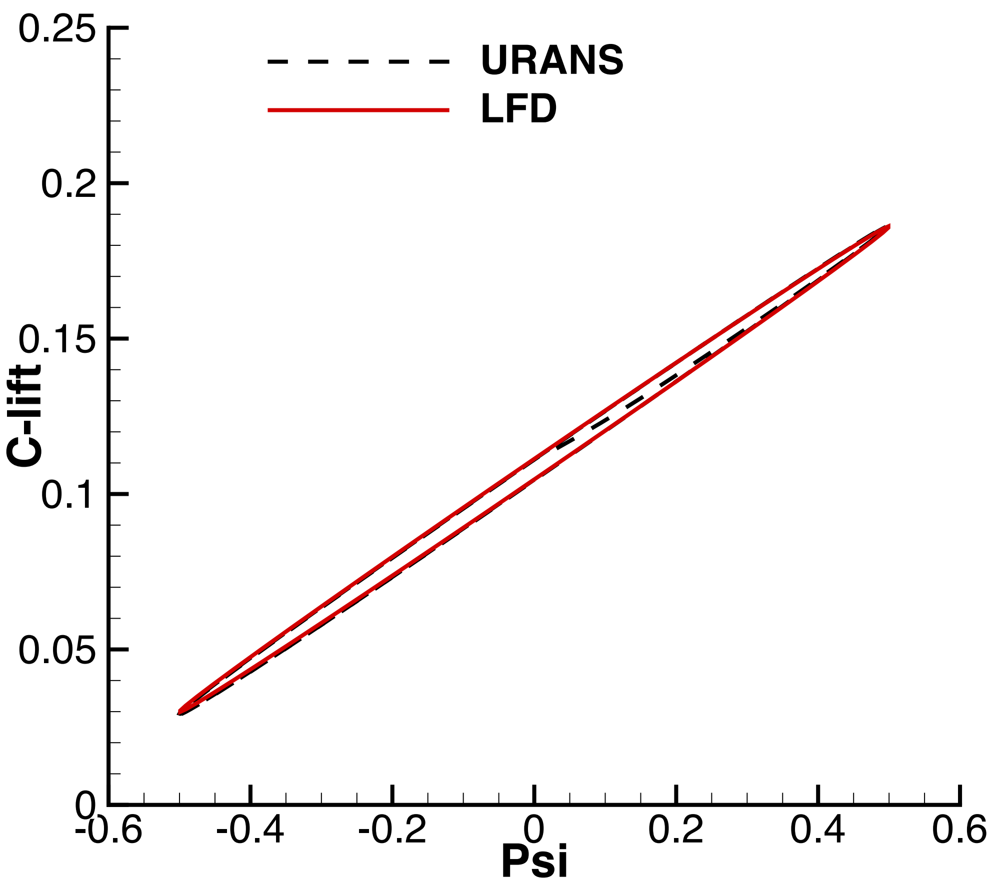
DLR F12 - RANS

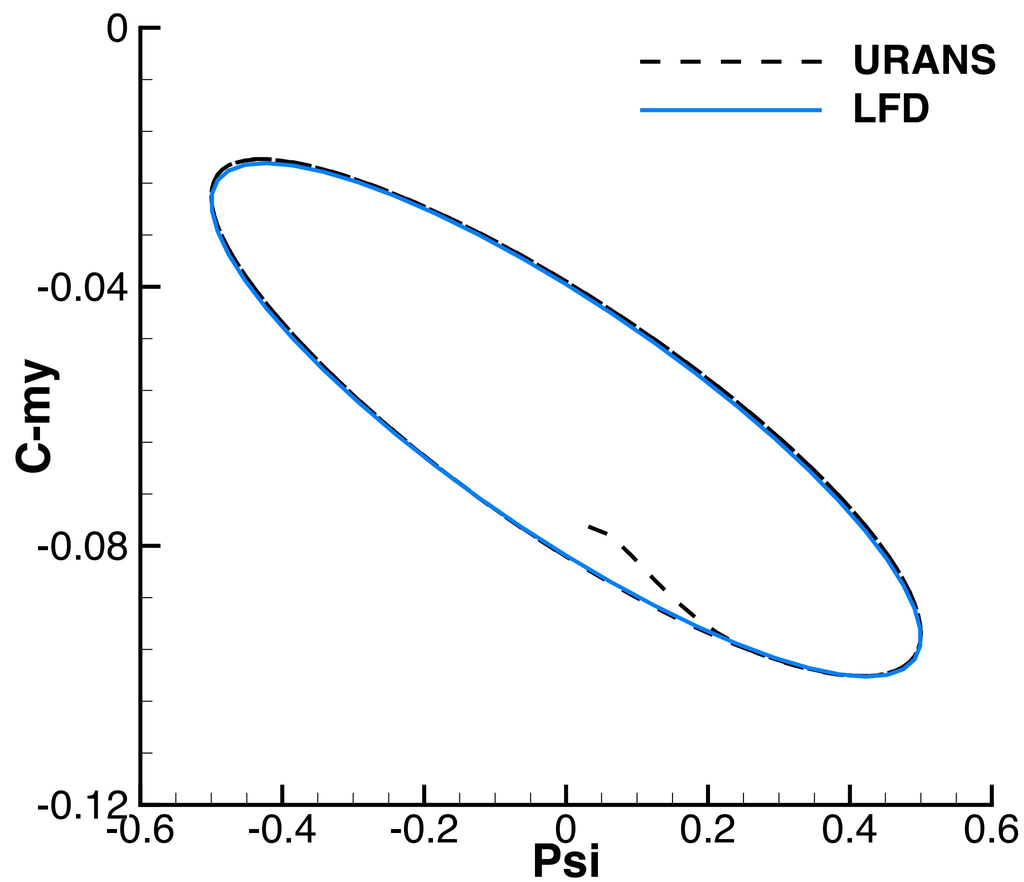
Ma = 0.85, Re = 1.28 mill., α = 0.0°, amp = 0.5°, red. f = 0.068

Dynamic response



Simulation	Magnitude [rad]	Phase [rad/s]
URANS	8.9966	-4.20E-02
LFD	8.9519	-4.24E-02
URANS	4.5619	-2.580
LFD	4.5436	-2.585





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Linearization of URANS equations

for higher harmonics nh we obtain:

$$
\begin{aligned}
\left[\left(in_1\omega\overline{\mathbf{M}}+\frac{\partial\mathbf{R}}{\partial\mathbf{W}}\right)\widehat{\mathbf{W}}_1+\left[\frac{\partial\mathbf{R}}{\partial\mathbf{x}}+in_1\omega\left(\frac{\partial\mathbf{R}}{\partial\dot{\mathbf{x}}}+\overline{\mathbf{W}}\frac{\partial\mathbf{M}}{\partial\mathbf{x}}\right)\right]\hat{\mathbf{x}}_1\right]e^{in_1\omega t}&+\\
+\ldots&+\\
+\left[\left(in_h\omega\overline{\mathbf{M}}+\frac{\partial\mathbf{R}}{\partial\mathbf{W}}\right)\widehat{\mathbf{W}}_{n_h}+\left[\frac{\partial\mathbf{R}}{\partial\mathbf{x}}+in_h\omega\left(\frac{\partial\mathbf{R}}{\partial\dot{\mathbf{x}}}+\overline{\mathbf{W}}\frac{\partial\mathbf{M}}{\partial\mathbf{x}}\right)\right]\hat{\mathbf{x}}_{n_h}\right]e^{in_h\omega t}&+\\
+\frac{\partial\mathbf{M}}{\partial\mathbf{x}}\left((-\omega^2+i\omega)n_1\hat{\mathbf{x}}_1\widehat{\mathbf{W}}_1\right)e^{in_2\omega t}&+\\
+\ldots&+\\
+\frac{\partial\mathbf{M}}{\partial\mathbf{x}}\left(-n_h^2\omega^2+n_hi\omega\right)\hat{\mathbf{x}}_{n_h}\widehat{\mathbf{W}}_{n_h}e^{i(2n_h)\omega t}&=0
\end{aligned}
$$

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