The scientific achievements of Vicent Caselles

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PARTIAL DIFFERENTIAL EQUATIONS, OPTIMAL DESIGN AND NUMERICS
DEDICATED TO THE MEMORY OF VICENT CASELLES
Benasque, August 27 2013

- Vicent (Gata, Alacant, 10-8-1960, Barcelona, 14-8-2013) was an outstanding mathematician, capable of the most rigorous mathematical formalism and, at the same time, eager to find challenging applications for him and his collaborators and to get excellent numerical results.
- As his many collaborators and friends know, he was an excellent person, with an uncommon combination of sincerity, modesty and willingness to help, that made him so charming.
- I hope this little tribute to his memory will serve to illustrate Vicent's personal and scientific achievements.

1. Biography
   - Academic trajectory
   - Awards and prizes
   - Coauthors
2. Some selected topics
   - Gray image denoising
   - Total variation flow
   - Minimizing flow for linear growth functionals
   - Image restoration
   - Color image denoising
   - Geodesic active contours
   - Image inpainting

ACADEMIC TRAJECTORY

- 1977-78 - 1981-82: M.Sc. in Mathematics, Univ. València.
- 1982-83 - 1985: Ph.D. in Mathematics, Univ. València.
- 1985 - 1987: Post-doc at Univ. Tübingen.
- 1987 - 1988: Post-doc at Univ. Franche-Comté, Besançon.
- 1989 - 1990: Research and Teaching Assistant (ATER) at Univ. Franche-Comté, Besançon.
- 1990 - 1999: Univ. Illes Balears, Palma.
- 1999 - 2013: Univ. Pompeu Fabra, Barcelona.

Data taken mainly from Vicent's CV
http://www.dtic.upf.edu/ vcaselles/CV-Vicent.pdf

VICENT'S NUMBERS

- 142 papers in international journals.
- 64 other publications.
- 81 invited conferences.
- 4742 citations (two papers with more than 2000 citations)†
- 74 coauthors (48 with more than one paper)†
- 15 Ph.D. thesis

† Source: ISI-WOK

AWARDS AND PRIZES

- 2003: **Ferran Sunyer i Balaguer** Prize.
- 2006: Invited Conference at **ICM**, Madrid, Spain.
- 2008: **SIAM Outstanding Paper** Prize 2008.
- 2009: **ICREA** Acadèmia (Gen. Cat.) prize for excellence in research.
- 2011: Invited plenary speaker at **ICIAM**, Vancouver, Canada.
- 2011: Test of Time Award at International Conference on Computer Vision (ICCV) for the **ICCV** paper “Geodesic Active Contours” in collaboration with R. Kimmel and G. Sapiro.
- 2012: **SIAM** Activity Group on Imaging Science (SIAG/IS) prize for the PAMI paper “A Perceptually Inspired Variational Framework for Color Enhancement”.
- 2012 : Invited lecture at the **ECM**, Krakow, Poland.
- 2012 : **ERC Advanced Grant** (2.3M euros) “Inpainting Tools for Video Post-production. Variational Theory and Fast Algorithms.”.

COAUTHORS

SAPIRO G (31)
MOREL JM (30)
MAZON JM (20)
ANDREU F (20)
BERTALMIO M (17)
BALLESTER C (16)
NOVAGA M (11)
CHAMBOLLE A (10)
FACCIOLO G (10)
COLL B (8)
ARIAS P (7)
GARRIDO L (7)
VERDERA J (7)
IGUAL L (6)
KIMMEL R (6)
PAPADAKIS N (6)
PROVENZI E (6)
ARANDIGA F (5)
HARO G (5)
LIU YQ (5)
MOLL S (5)
SBERT C (5)
SOLE A (5)
ALMANSA A (4)
BELLETTINI G (4)
BERNOT M (4)
MEINHARDT E (4)
MONASSE P (4)
ROUGE B (4)
ALTER F (3)
BUGEAU A (3)
FERRADANS S (3)
GARGALLO P (3)
SADEK R (3)
TANG B (3)
ALEMAN-FLORES M (2)
ALVAREZ L (2)
BAEZA A (2)
CARDELINO J (2)
CATTE F (2)
CONSTANTINOPOULOS C (2)
DIBOS F (2)
KALMOUN E (2)
LAZCANO V (2)
LISANI JL (2)
MEINHARDT-LLOPIS E (2)
MIRANDA M (2)
RANDALL G (2)
ADALSTEINSSON D (1)
AUJOL JF (1)
CALDERERO F (1)
CAO F (1)
CHUNG DK (1)
D'HONDT O (1)
DIAZ JI (1)
DONAT R (1)
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GONZALEZ M (1)
HERVIEUX A (1)
LUNARDI A (1)
MALLADI R (1)
MARQUES A (1)
MARTINEZ J (1)
PALMA-AMESTOY R (1)
PARDO A (1)
PRECIOZZI J (1)
RANCHIN F (1)
RIZZI A (1)
SANDER O (1)
SETHIAN JA (1)
SOLER J (1)
TANNENBAUM A (1)
VERBENI M (1)
ZACUR E (1)

Ph.D. thesis

Francesc Aràndiga Llaudes, UV, 1992.
Coloma Ballester Nicolau,UIB , 1995. (co-supervisor: J.M. Morel)
Catalina Sbert Juan, UIB, 1995. (J.M. Morel)
Manuel González Hidalgo, UIB, 1995. (J.M. Morel)
Andrés Solé Martínez, UPF, 2002.
Joan Verdera Ribas, UPF, 2004.
José Salvador Moll Cebolla, UV, 2005. (J.M. Mazón, F. Andreu)
Gloria Haro Ortega, UPF, 2005. (R. Donat)
Marc Bernot, ENS Cachan, 2005. (J.M. Morel)
Laura Igual, UPF, 2006. (Luis Garrido)
François Alter, ENS Cachan, 2008. (J.M. Morel)
Gabriele Facciolo Furlan, UPF, 2010.
Enric Meinhardt Llopis, UPF, 2010.
Sira Ferradans Ramonde, UPF, 2011. (M. Bertalmío).
Rida Sadek, UPF , 2012.

PREVIOUS RESEARCH INTERESTS (1982-1990)

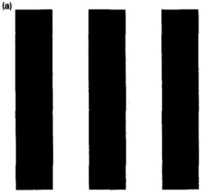
- Functional Analysis: geometry of Banach spaces and operator theory.

RESEARCH INTERESTS (1990-2013)

- Image processing
- Partial Differential Equations and their applications
- Differential geometry and its applications
- Computer vision

IMAGE DENOISING

- Image (gray): $u\colon \Omega := (0,1)^2 \to \mathbb{R}$ ($\mathbb{R}^3 \equiv {\color{red}R} \times {\color{green}G} \times {\color{blue}B}$ for color), $u(x,y) \equiv$ gray level at (x,y):



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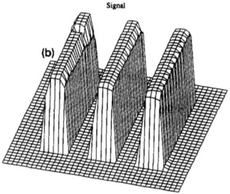
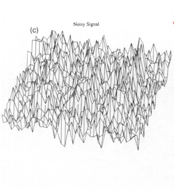


- Image acquisition introduces noise: $z = u + n$, z recorded image, n noise (unknown, up to some statistics):

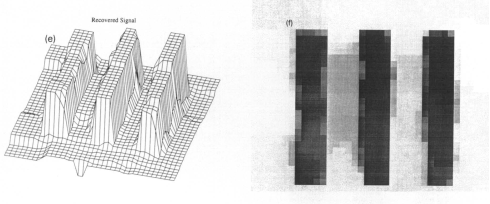


z

Pictures taken from [Rudin, Osher, Fatemi, 92]

IMAGE DENOISING

- Goal: approximate u, preserving discontinuities (edges)



- Assume we know $\|n\|_2 := \left(\int_\Omega n^2\right)^{\frac{1}{2}} = \sigma$; then solve

$$\min F(u), \quad \text{subject to} \|\overbrace{z-u}^{n}\|_2 = \sigma$$

where $F\colon \mathcal{X} \to \mathbb{R}$ measures the regularity of u in some sense.

- This is equivalent to

$$\min F(u) + \frac{\lambda}{2}\|z-u\|_2^2$$

for suitable Lagrange multiplier λ, with Euler-Lagrange equation:

$$F'(u) + \lambda(u-z) = 0.$$

- If

$$F(u) = \int_\Omega |\nabla u|^2 dxdy, \quad |\nabla u| = \sqrt{u_x^2 + u_y^2},$$

for $u \in \mathcal{X} = H^1(\Omega)$, then the E-L equation reads:

$$-\Delta u + \lambda(u - z) = 0$$

a linear elliptic equation with continuous solutions (no edge recovery).

- [Rudin, Osher and Fatemi, 1992] use

$$F(u) = TV(u) := \int_\Omega |\nabla u| dxdy \tag{1}$$

(**Total Variation** of u), $u \in \mathrm{BV}(\Omega)$, for then u can be discontinuous along curves. Of course, (1) is only valid for differentiable functions, a weak formulation applies otherwise.

IMAGE DENOISING

- The (formal) E-L equation is now:

$$-\text{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda(u - z) = 0$$

and the associated parabolic equation reads

$$u_t = \text{div}\left(\frac{\nabla u}{|\nabla u|}\right) - \lambda(u - z)$$

- These equation do not make sense for non-differentiable functions or with extrema (where the denominator $|\nabla u|$ vanishes).

TOTAL VARIATION FLOW

- Of course, somebody had to give sense to all these equations. Vicent, together with F. Andreu and J.M. Mazón, studied the **total variation flow** in a series of papers starting at 2000:

$$u_t = \text{div}\left(\frac{\nabla u}{|\nabla u|}\right), \tag{2}$$

  giving correct sense to all the terms involved in these formulation.
- This equation corresponds to the minimization of the total variation and can be used to denoise without prior knowledge of σ:

$$\min \int_\Omega |\nabla u|. \tag{3}$$

- Related to anisotropic diffusion ([Perona, Malik, 1987], [Catte, Coll, Lions, Morel, 1992]).

TOTAL VARIATION FLOW

1. Minimizing total variation flow, Andreu, F; Ballester, C; Caselles, V; Mazon, JM, C.R. Acad. Sci. 2000.
2. The Dirichlet problem for the total variation flow, Andreu, F; Ballester, C; Caselles, V; Mazon, JM J., Functional Analysis, 2001.
3. The total variation flow in $\mathbb{R}^N$, Bellettini, G; Caselles, V; Novaga, M, J. Diff. Eq., 2002.
4. Some qualitative properties for the total variation flow, Andreu, F; Caselles, V; Diaz, JI; Mazon, JM, J. Functional Analysis, 2002.
5. The minimizing total variation flow with measure initial conditions, Andreu, F; Mazon, JM; Moll, JS; Caselles, V, Comm. Contemp. Math., 2004.
6. Evolution of characteristic functions of convex sets in the plane by the minimizing total variation flow, Alter, F; Caselles, V; Chambolle, A, Interfaces and Free Boundaries, 2005.
7. Explicit solutions of the eigenvalue problem $-\text{div}(\nabla u/|\nabla u|) = u$ in $\mathbb{R}^2$, Bellettini, G; Caselles, V; Novaga, M, SIAM J. Math. Analysis, 2005.
8. On the jump set of solutions of the Total Variation flow, Caselles V, Jalalzai K, Novaga M, Sem. Matematico della Università di Padova, 2013.

MINIMIZING FLOW FOR LINEAR GROWTH FUNCTIONALS

- Vicent and coauthors then study generalizations of (2) and (3):

$$\min_u \int_\Omega f(x, \nabla u(x))dx, \quad \Omega \text{ open subset of } \mathbb{R}^N$$

for suitable convex functions $f(x, \xi)$ with linear growth (on $\xi \equiv \nabla u$), which give gradient flows:

$$u_t(x,t) = \text{div}\left[a(x, \nabla u(x,t))\right], \quad a(x,\xi) = \nabla_\xi f(x,\xi).$$

($f(x,\xi) = |\xi|$ gives the minimizing total variation flow)

MINIMIZING FLOW FOR LINEAR GROWTH FUNCTIONALS

1. Existence and uniqueness of a solution for a parabolic quasilinear problem for linear growth functionals with L^1 data, Andreu, F; Caselles, V; Mazon, JM, Mathematische Annalen, 2002.
2. A parabolic quasilinear problem for linear growth functionals, Andreu, F; Caselles, V; Mazon, JM, Rev. Matem. Iberoamericana, 2002
3. The Cauchy problem for linear growth functionals, Andreu, F; Caselles, V; Mazon, JM, J. Evol. Equat., 2003.
4. Evolution problems associated to linear growth functionals: The Dirichlet problem, Andreu, F; Caselles, V; Mazon, JM, Evolution equations: Applications to physics, industry, life sciences and economics, 2003.

IMAGE RESTORATION

- Image acquisition introduces blur and noise: $z = Ku + n$, z recorded image, n noise and $Ku = k * u$, for **known** kernel k, with the goal of getting $\hat{u} \approx u$ (preserving edges)

u

Image restoration

- Image acquisition introduces blur and noise: $z = Ku + n$, z recorded image, n noise and $Ku = k * u$, for **known** kernel k, with the goal of getting $\hat{u} \approx u$ (preserving edges)

z

IMAGE RESTORATION

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$\hat{u}$

Image restoration

- Assume that we know that $n = z - Ku$ is a Gaussian white noise with variance σ^2 (and 0 mean) uncorrelated to Ku; then

$$\Pr\left(\fint_{\Omega'} n^2 \leq \sigma^2\right) = 1$$

for any open $\Omega' \subseteq \Omega = (0,1)^2$.

- If we impose **globally** $\int_\Omega n^2 \leq \sigma^2$, then variational problem reads as:

$$\min TV(u), \quad \text{subject to } \frac{1}{2}(\int_\Omega (Ku - z)^2 - \sigma^2) \leq 0 \tag{4}$$

- The Euler-Lagrange equation is now:

$$-\text{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda K^*(Ku - z) = 0 \quad (K^* \equiv \text{adjoint of } K)$$

where λ is the Lagrange multiplier for the (**global**) constraint.

IMAGE RESTORATION

- It is observed that the solution $\hat{u}$ of (4) yields an approximated noise $\hat{n} = z - K\hat{u}$ which is correlated with $K\hat{u}$:

$\hat{n}$

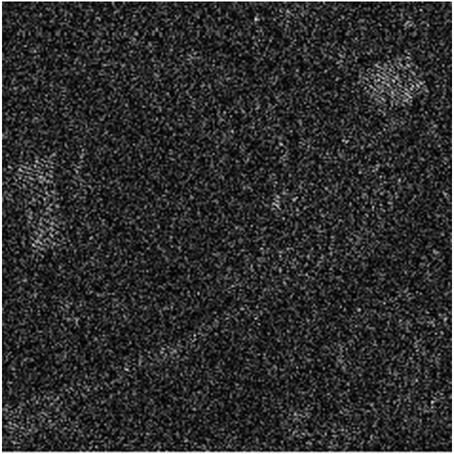


IMAGE RESTORATION

- It is also observed that the approximated noise $\hat{n}$ does not verify

$$\fint_{\Omega'} \hat{n}^2 \leq \sigma^2, \quad \forall \text{ open } \Omega' \subseteq \Omega$$

- In [Almansa, Ballester, Caselles, Haro 2008] they propose a suitable discretization of the problem (with **local constraints**)

$$\begin{aligned} &\min TV(u), \\ &\text{subject to } \fint_{B_\delta(x,y)} (Ku - z)^2 \leq \sigma^2, \forall (x,y) \in \Omega \end{aligned}$$

  for some $\delta > 0$.
- They use a variant of Uzawa's algorithm for the numerical solution.

Image restoration: results

$\hat{n}$ global constraint

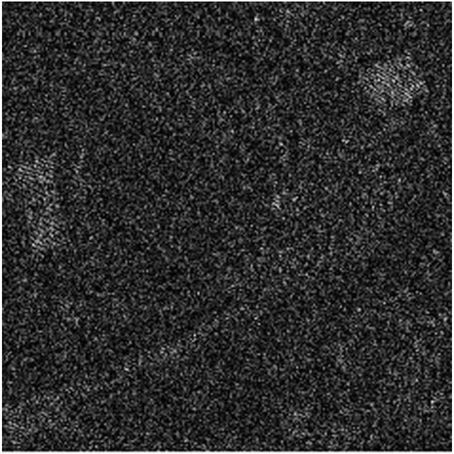


IMAGE RESTORATION: RESULTS

$\hat{n}$ local constraints

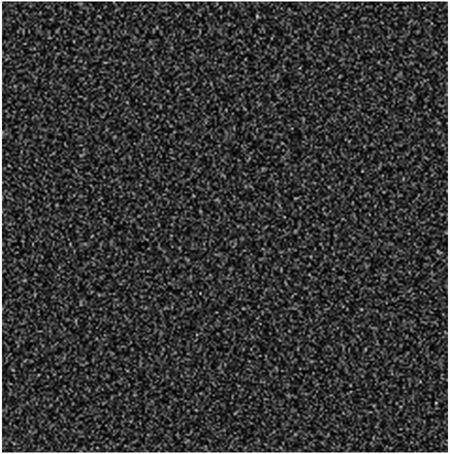


Image restoration: Results

$\hat{u}$ global constraint

IMAGE RESTORATION: RESULTS

$\hat{u}$ local constraints

IMAGE RESTORATION

1. TV based image restoration with local constraints, Bertalmio, M; Caselles, V; Rouge, B; Sole, A, J. Sci. Comput., 2003.
2. Restoration and zoom of irregularly sampled, blurred, and noisy images by accurate total variation minimization with local constraints, Almansa, A; Caselles, V; Haro, G; Rouge, B, Multiscale Modeling & Simulation, 2006.
3. A TV based restoration model with local constraints, Almansa, A.; Ballester, C.; Caselles, V.; Haro, G., J. Sci. Comput., 2008.

COLOR IMAGE DENOISING

- Color image: $u\colon \Omega := (0,1)^2 \to \mathbb{R}^3$ ((R, G, B)-components)
- Typical strategy for color image processing: apply gray-level procedure to each component $\Rightarrow$ false colors.
- In [Tang, Sapiro, Caselles, 2001], u is decomposed in a different way:

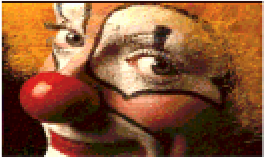
$$
\begin{array}{lll}
M = |u| & : \Omega \to \mathbb{R} & \text{brightness (magnitude)} \\
C = u/|u| & : \Omega \to S^2 & \text{chromaticity (direction)}
\end{array}
$$

- M is processed by a gray-level technique
- C is processed by solving variational problem:

$$
\min \int_\Omega \left(|C_x|^2 + |C_y|^2\right)^{p/2}, \quad 1 < p < 2
$$
$$
\text{subject to } |C(x,y)|^2 = 1, \quad \forall (x,y) \in \Omega
$$

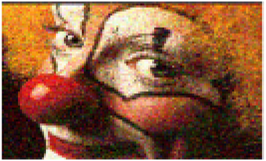
COLOR IMAGE DENOISING

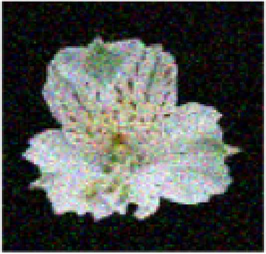
original



COLOR IMAGE DENOISING

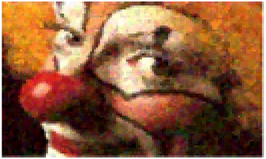
noisy

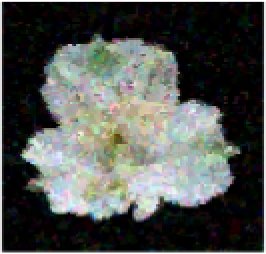




Color image denoising

vector directional filter ([Karakos, Trahanias, 1997])





COLOR IMAGE DENOISING

Vicent's model

Geodesic active contours

- Goal: detect **object boundaries** in image.
- Classical approach [Kass et. al., 1988] based on deforming an initial contour C_0 towards boundary of the object to be detected.
- The deformation is obtained by trying to minimize a functional designed so that its (local) minimum is obtained at the boundary of the object.
- Let $I\colon \Omega := (0,1)^2 \to \mathbb{R}$ be the (given) image and $g\colon [0,\infty) \to (0,\infty)$ be strictly decreasing and $g(r) \to 0$, when $r \to \infty$, define energy of a closed curve $\mathcal{C} : [0,1] \to \Omega$ by:

$$E(\mathcal{C}) = \int_0^1 |\mathcal{C}'(q)|^2\, dq + \beta \int_0^1 g(|\nabla I(\mathcal{C}(q))|)dq$$

Geodesic active contours

- Classical **active contours** (**snakes**):

$$\min_{\mathcal{C}} \underbrace{\int_0^1 |\mathcal{C}'(q)|^2 \, dq}_{\text{internal energy}} + \underbrace{\beta \int_0^1 g(|\nabla I(\mathcal{C}(q))|) dq}_{\text{external energy}}$$

- Internal energy forces $\mathcal{C}$ to be regular (i.e. $|\mathcal{C}'| \downarrow 0$).
- External energy pushes $\mathcal{C}$ to discontinuity of I ($|\nabla I(\mathcal{C}(q))| \to \infty \Rightarrow g(|\nabla I(\mathcal{C}(q))|) \to 0$).
- But functional is not intrinsic (energy changes with reparametrization of the curves), depends on parameter β and cannot deal with topological changes of the curve.

GEODESIC ACTIVE CONTOURS

- **Geodesic active contours:** [Caselles, Kimmel, Sapiro, 1997] define

$$E(\mathcal{C}) = \int_0^1 g(|\nabla I(\mathcal{C}(q))|)|\mathcal{C}'(q)|dq.$$

- E is independent of parametrizations.
- In fact, $E(\mathcal{C})$ is the length of $\mathcal{C}$ when considering the Riemannian metric given by the first fundamental form $g_{i,j} = g(|\nabla I|)\delta_{i,j}$.
- Therefore

$$\min_{\mathcal{C}} E(\mathcal{C})$$

is equivalent to finding a **geodesic** for the new metric.

- Minimizing flow for $\mathcal{C} = \mathcal{C}(t, q)$:

$$\mathcal{C}_t = (\tilde{g}(\mathcal{C})\kappa(\mathcal{C}) - \nabla\tilde{g}(\mathcal{C}) \cdot \mathcal{N})\mathcal{N}, \quad \tilde{g}(x) = g(|\nabla I(x)|), \tag{5}$$

where $\kappa(\mathcal{C})$ is the curvature of $\mathcal{C}$ and $\mathcal{N}$ is the inward unit normal to $\mathcal{C}$.

Geodesic active contours

- To deal with topological changes in $\mathcal{C}$ a **level set** formulation can be applied: embed $\mathcal{C}$ as 0-level set of unknown $u(x,t)$, so (5) is equivalent to:

$$u_t = \left(\tilde{g}\,\text{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \nabla\tilde{g}\cdot\frac{\nabla u}{|\nabla u|}\right)|\nabla u|$$

$$u_t = \text{div}\left(g(|\nabla I|)\frac{\nabla u}{|\nabla u|}\right)|\nabla u|,$$

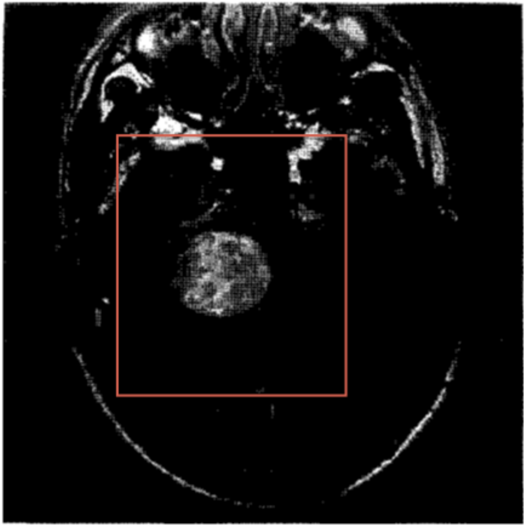
where we have taken into account that

$$\kappa(\text{level set of } u) = \text{div}\left(\frac{\nabla u}{|\nabla u|}\right)$$

$$\mathcal{N}(\text{level set of } u) = -\frac{\nabla u}{|\nabla u|}$$

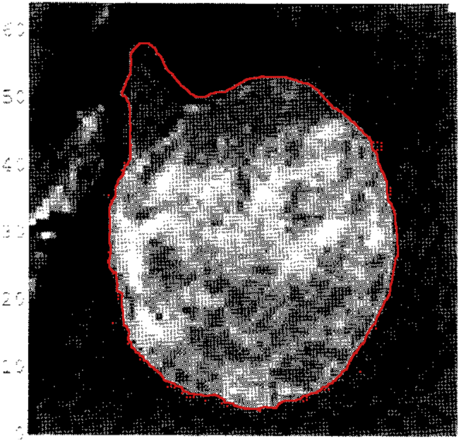
GEODESIC ACTIVE CONTOURS

Example of tumor detection in MRI



GEODESIC ACTIVE CONTOURS

Example of tumor detection in MRI



GEODESIC ACTIVE CONTOURS

1. A geometric model for active contours in image-processing, CASELLES, V; CATTE, F; COLL, T; DIBOS, F, Numerische Mathematik, 1993.
2. Geodesic active contours, CASELLES, V; KIMMEL, R; SAPIRO, G, Fifth ICCV, Proceedings, 1995.
3. Geometric models for active contours, Caselles, V, International Conference on Image Processing - Proceedings, vols I-III, 1995.
4. Geodesic active contours, Caselles, V; Kimmel, R; Sapiro, G, International J. Comput. Vision, 1997.
5. Texture-oriented anisotropic filtering and geodesic active contours in breast tumor ultrasound segmentation, Aleman-Flores, M; Alvarez, L; Caselles, V, J. Math. Imag. Vis., 2007.
6. Breast nodule ultrasound segmentation through texture-based active contours, Aleman-Flores, M; Alvarez, L; Caselles, V, Progress in Industrial Mathematics at ECMI 2006, 2008.

Image inpainting

- Filling-in missing data (**inpainting** in art restoration) in digital images has a number of fundamental applications:
  - Removal of scratches in old photographs and films,
  - Removal of superimposed text like dates, subtitles, or publicity from a photograph,
  - Recovery of pixel blocks corrupted during binary transmission.

IMAGE INPAINTING

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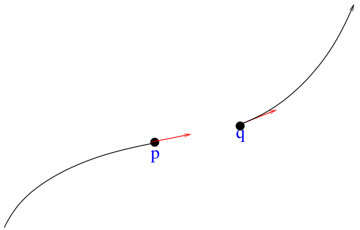
IMAGE INPAINTING

- Inpainting may be tackled by interpolatory techniques, e.g.

$$\begin{aligned} \Delta u = 0 & \quad \text{harmonic extension} \\ \nabla^2 u(\nabla u, \nabla u) = 0 & \quad \text{Absolute Minimal Lipschitz Extension} \end{aligned}$$

  with $u|_{\partial\widetilde{\Omega}} = u_0|_{\partial\widetilde{\Omega}}$, where:
  - $\widetilde{\Omega} \subset\subset \Omega \subset\subset (0,1)^2$ is the **hole**,
  - $B = \Omega \setminus \widetilde{\Omega}$ is the **band** surrounding the hole $\widetilde{\Omega}$
  - $u_0 \colon (0,1)^2 \setminus \widetilde{\Omega} \to \mathbb{R}$ is known.
- This may work well for small $\widetilde{\Omega}$, specially AMLE (studied by Vicent and coauthors in a series of papers), but need more sophisticated techniques for larger $\widetilde{\Omega}$.
- Idea: try to continue the level sets of u_0 affected by occlusion.

- **Euler's elastica**: to continue a partially occluded curve, knowing end points p, q and tangent vectors τ_p, τ_q at p, q



solve for given parameters $\alpha, \beta > 0$

$$\min_{\mathcal{C}} \int_{\mathcal{C}} (\alpha + \beta\kappa^2) ds,$$

subject to $\mathcal{C}(0) = p, \mathcal{C}(1) = q, \mathcal{C}'(0) = \tau_p, \mathcal{C}'(1) = \tau_q$

for ds the arc length measure and κ the curvature.

IMAGE INPAINTING

- [Ambrosio, Masnou, 2001] extend Euler's elastica to join level sets of u_0 (defined on $(0,1)^2 \setminus \widetilde{\Omega}$):

$$\min_u \int_\Omega |\nabla u|(\alpha + \beta|\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)|^p)ds, \quad p \geq 1$$
$$u|_B = u_0|_B.$$

Image inpainting

- [Ballester, Bertalmio, Caselles, Sapiro, Verdera, 2001], [Ballester, Caselles, Verdera, 2003] relax a similar problem, introducing an auxiliary variable θ that should be in the limit $\frac{\nabla u}{|\nabla u|}$, i.e., the (outward) normal to the level set:

$$\min_{u,\theta} \int_\Omega |\mathrm{div}(\theta)|^p(\gamma + \beta|\nabla k * u|)$$
$$|\theta| \leq 1, \nabla u - \theta|\nabla u| = 0$$
$$u|_B = u_0|_B$$
$$(\theta - \theta_0) \cdot \nu|_{\partial\Omega} = 0$$

  where u_0 is the image known in $(0,1)^2 \setminus \widetilde{\Omega} \supseteq B$ and θ_0 is any vector field in B such that $(\nabla u_0 - \theta_0|\nabla u_0|)|_B = 0$ and ν is the unit normal to $\partial\Omega$.
- Convolution by kernel k necessary for proving well-posedness.
- Recent (and future!) work on video inpainting and stereo video inpainting ("3D" video) (got **ERC advanced grant** with these topics).

Image inpainting

Image inpainting

IMAGE INPAINTING

Image inpainting

- Image inpainting, Bertalmio, M; Sapiro, G; Caselles, V; Ballester, C, SIGGRAPH 2000 Conference Proceedings, 2000.
- Inpainting surface holes, Verdera, J; Caselles, V; Bertalmio, M; Sapiro, G, 2003 International Conference on Image Processing, vol 2, Proceedings, 2003.
- A variational model for disocclusion, Ballester, C; Caselles, V; Verdera, J, 2003 International Conference on Image Processing, vol 3, Proceedings, 2003.
- An inpainting-based deinterlacing method, Ballester, C; Bertalmio, M; Caselles, V; Garrido, L; Marques, A; Ranchin, F, IEEE Trans. Imag. Proc., 2007.
- On geometric variational models for inpainting surface holes, Caselles, V.; Haro, G.; Sapiro, G.; Verdera, J., Computer Vision and Image Understanding, 2008.
- A Variational Framework for Non-local Image Inpainting, Arias, P; Caselles, V; Sapiro, G, Energy Minimization Methods in Computer Vision and Pattern Recognition, Proceedings, 2009.

IMAGE INPAINTING

- A Comprehensive Framework for Image Inpainting, Bugeau, A; Bertalmio, M; Caselles, V; Sapiro, G, IEEE Trans. Imag. Proc., 2010.
- A Variational Framework for Exemplar-Based Image Inpainting, Arias, P; Facciolo, G; Caselles, V; Sapiro, G, INTERN. J. Computer Vision, 2011.
- Stereoscopic Image Inpainting Using Scene Geometry, Hervieux, A.; Papadakis, N.; Bugeau, A.; Gargallo, P.; Caselles, V., 2011 IEEE International Conference on Multimedia and Expo (ICME), 2011.
- Analysis of a Variational Framework for Exemplar-based Image Inpainting, Arias, P.; Caselles, V.; Facciolo, G. Multiscale Modeling & Simulation, 2012.
- Nonlocal Variational Models for Inpainting and Interpolation, Arias, P; Caselles, V; Facciolo, G; Lazcano, V; Sadek, R, Mathematical Models & Methods in Applied Sciences, 2012.
- Exemplar-Based Image Inpainting Using Multiscale Graph Cuts, Liu, Y; Caselles, V, IEEE Trans. Imag. Proc, 2013.

OTHER TOPICS

- Image histogram equalization / contrast enhancement.
- Irrigation / transport problems.
- Image compression
- Flux limited equations / “relativistic” heat equation
- Optical flow
- Video editing / camera replay simulation
- and many more . . .