



# Probing electron excitations with inelastic x-ray scattering spectroscopies

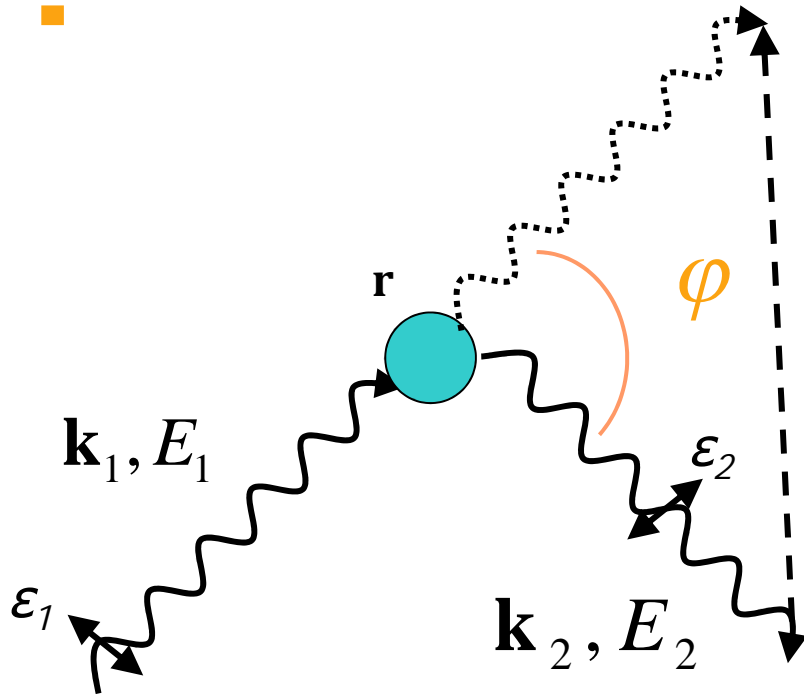
**Simo Huotari**

University of Helsinki, Finland

Benasque TDDFT workshop January 2012



# Inelastic x-ray scattering



$$\mathbf{Q} = \mathbf{k}_1 - \mathbf{k}_2$$

Dynamic structure factor  $S(\mathbf{Q}, E)$

$$S(\mathbf{Q}, E) \propto \sum_f |\langle \Psi_f | e^{i \mathbf{Q} \cdot \mathbf{r}} | \Psi_i \rangle|^2 \delta(E_i - E_f + E)$$

Momentum transfer  $\mathbf{Q} = \mathbf{k}_1 - \mathbf{k}_2$

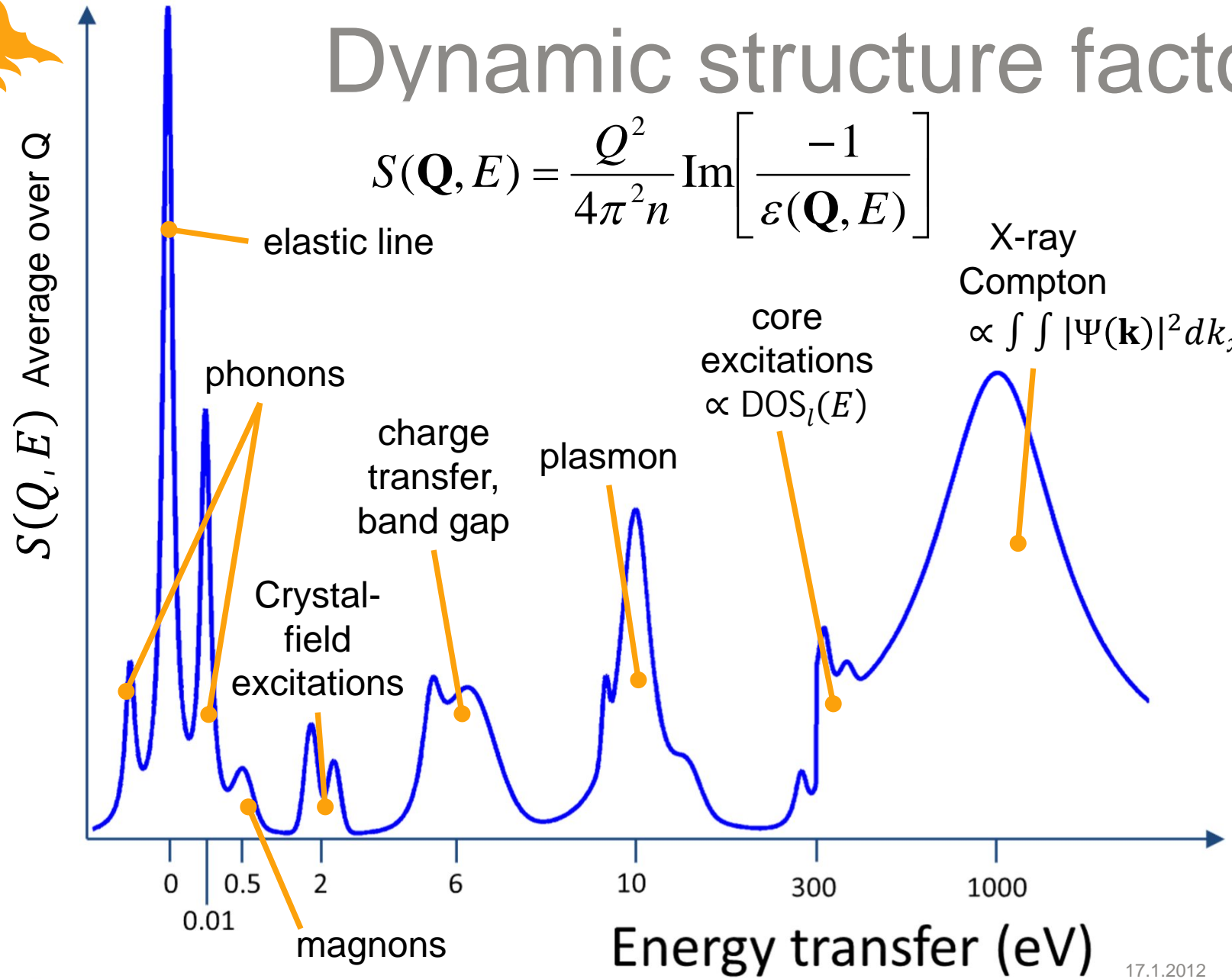
Energy transfer  $E = E_1 - E_2$

$$S(\mathbf{Q}, E) = \frac{Q^2}{4\pi^2 n} \text{Im} \left[ \frac{-1}{\varepsilon(\mathbf{Q}, E)} \right]$$



# Dynamic structure factor

$$S(\mathbf{Q}, E) = \frac{Q^2}{4\pi^2 n} \text{Im} \left[ \frac{-1}{\epsilon(\mathbf{Q}, E)} \right]$$





# Outline

**Part 1 Metal-to-insulator transition**

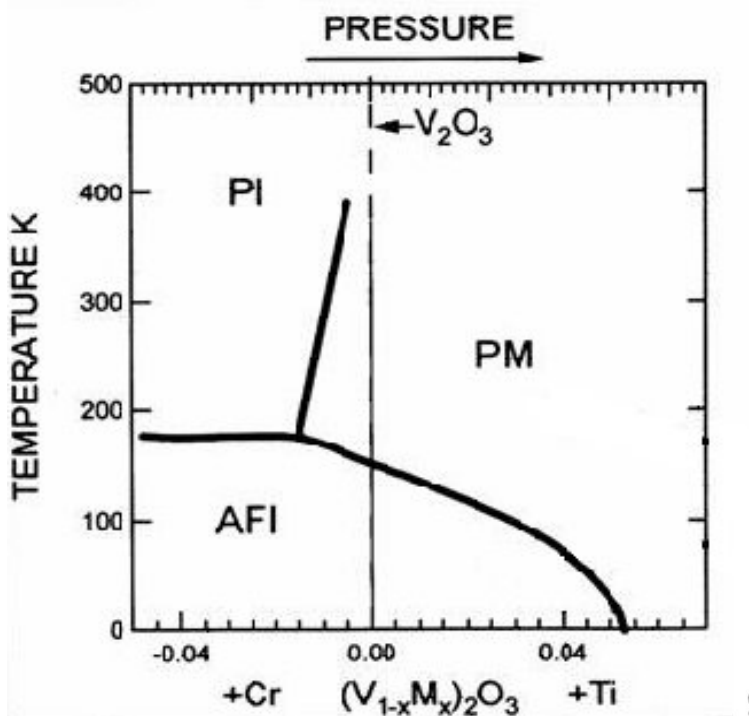
Part 2 Plasmons and e-h continuum

Part 3 Double plasmons

Part 4 Quasiparticle renormalisation

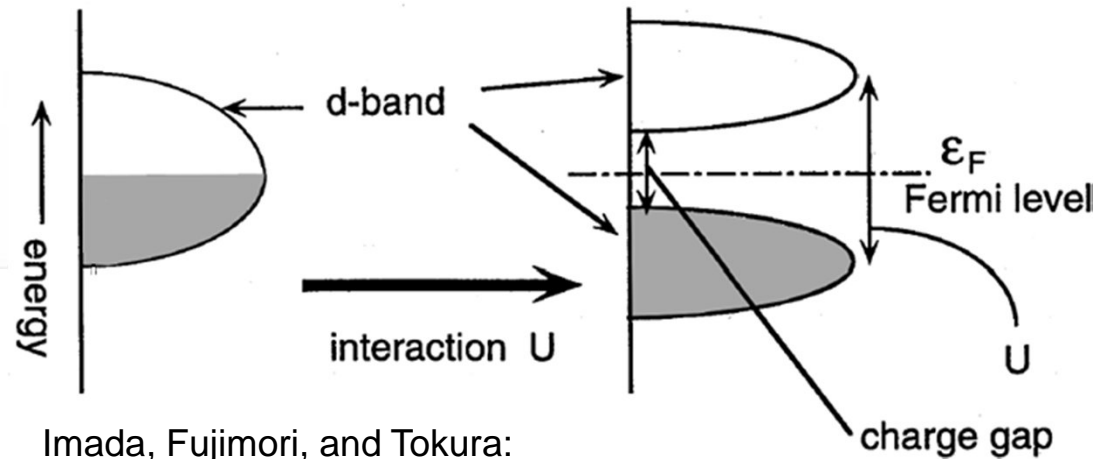


# $V_2O_3$ and the Mott transition

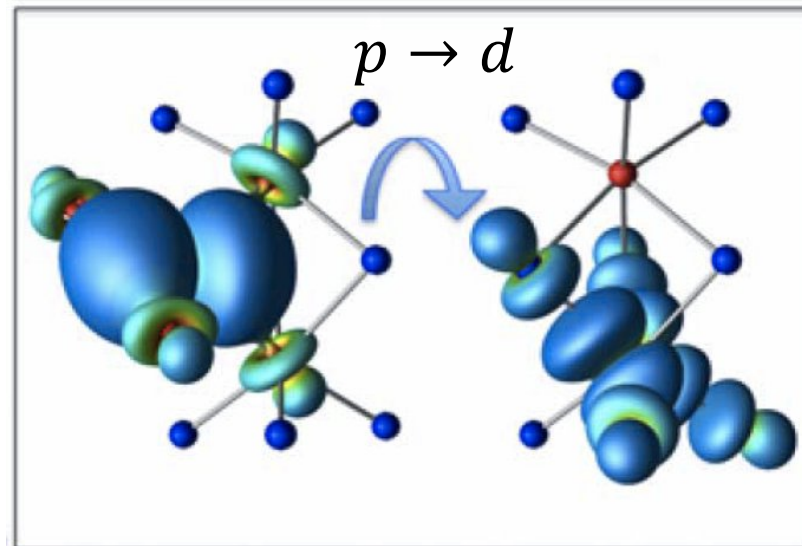
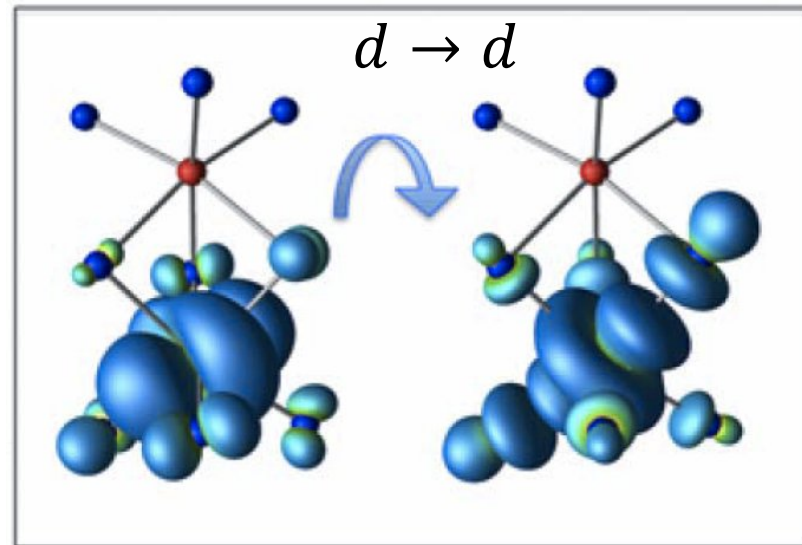
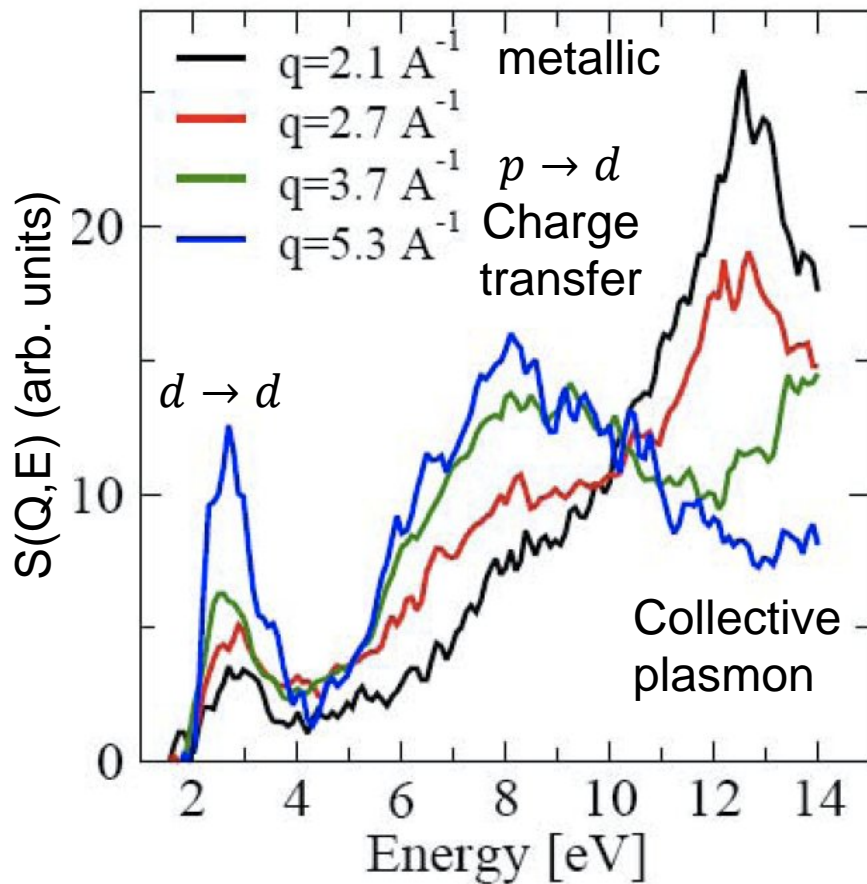


McWhan et al., PRL 27 (1971)

- $V_2O_3$  is one of the canonical model systems for a Mott-Hubbard insulator
- The Mott transition is usually explained within the Hubbard model with  $U$  = on-site Coulomb repulsion

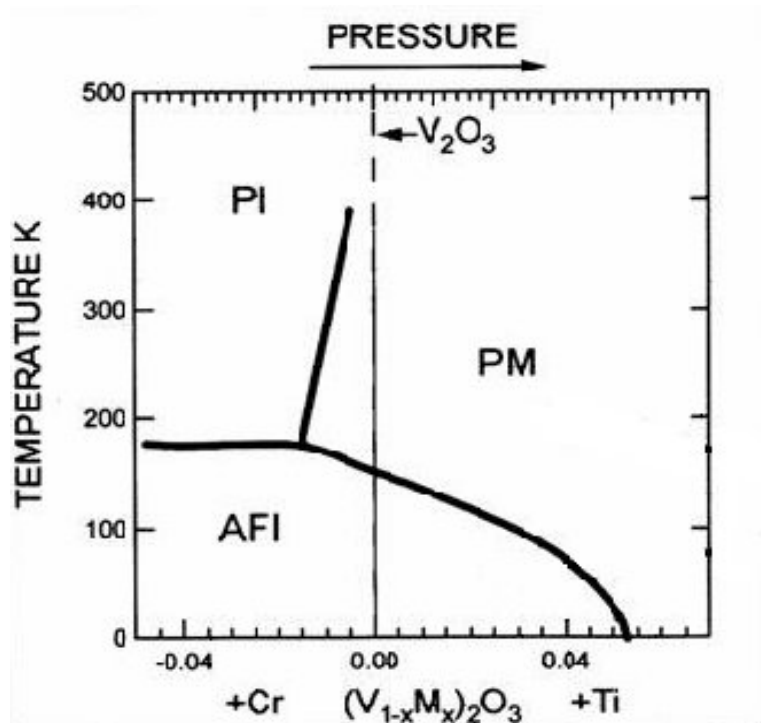


Imada, Fujimori, and Tokura:  
Metal-insulator transitions, RMP 70 1039 (1998)



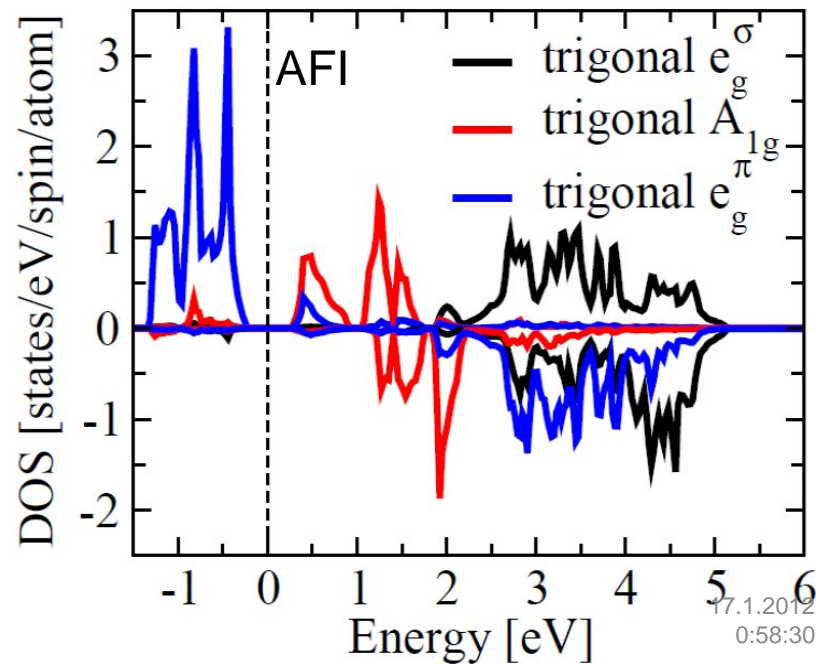
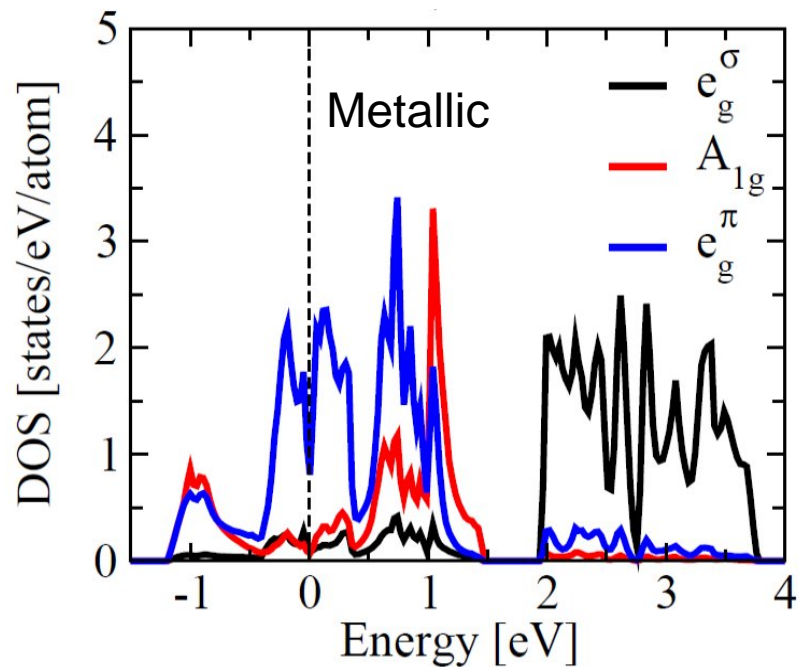
$$e^{i\mathbf{Q}\cdot\mathbf{r}} = 1 + i\mathbf{Q}\cdot\mathbf{r} - (\mathbf{Q}\cdot\mathbf{r})^2/2 + \dots$$

A.V.Kozhevnikov et al., in preparation

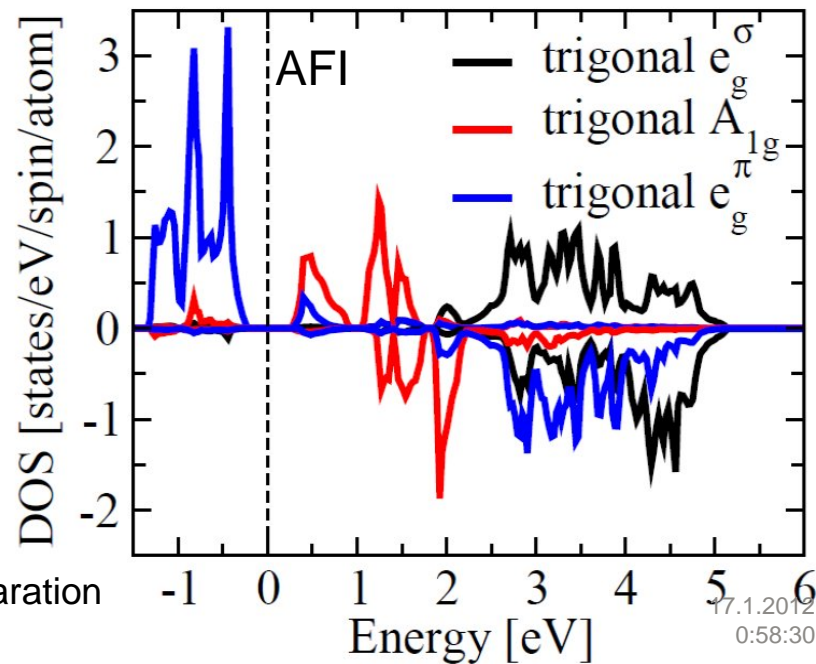
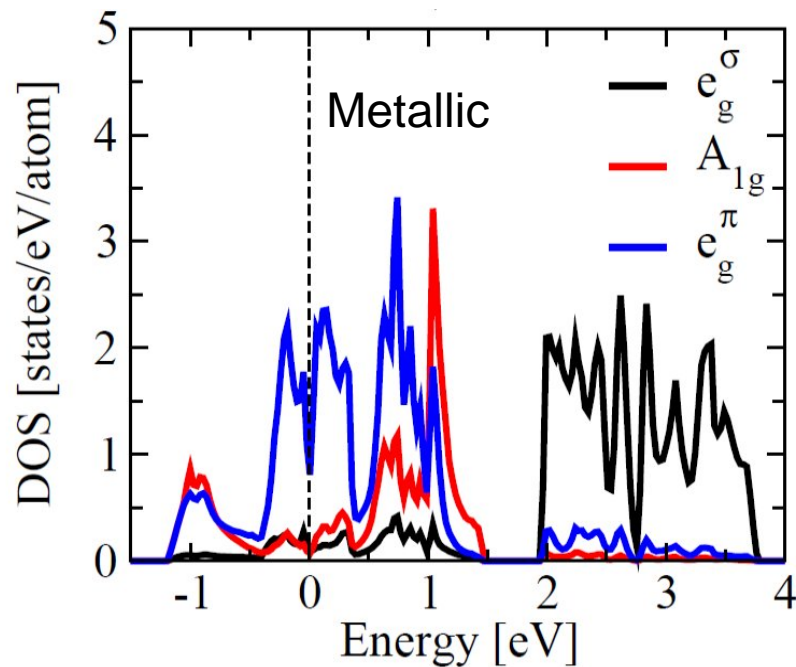
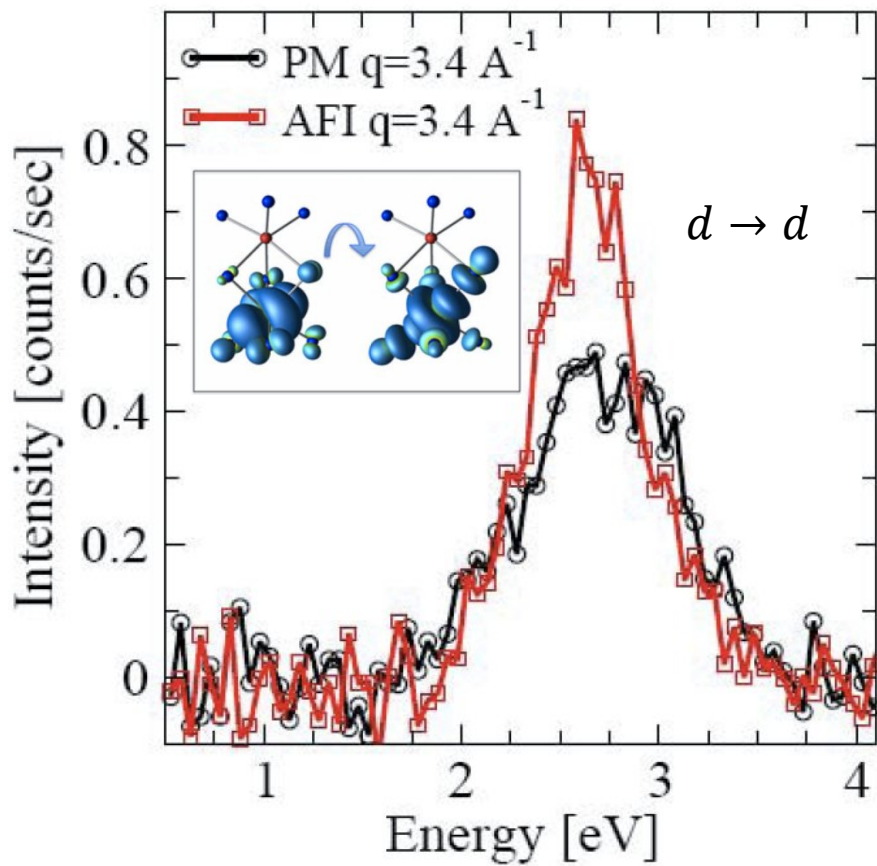


McWhan et al., PRL 27 (1971)

A.V.Kozhevnikov et al., in preparation







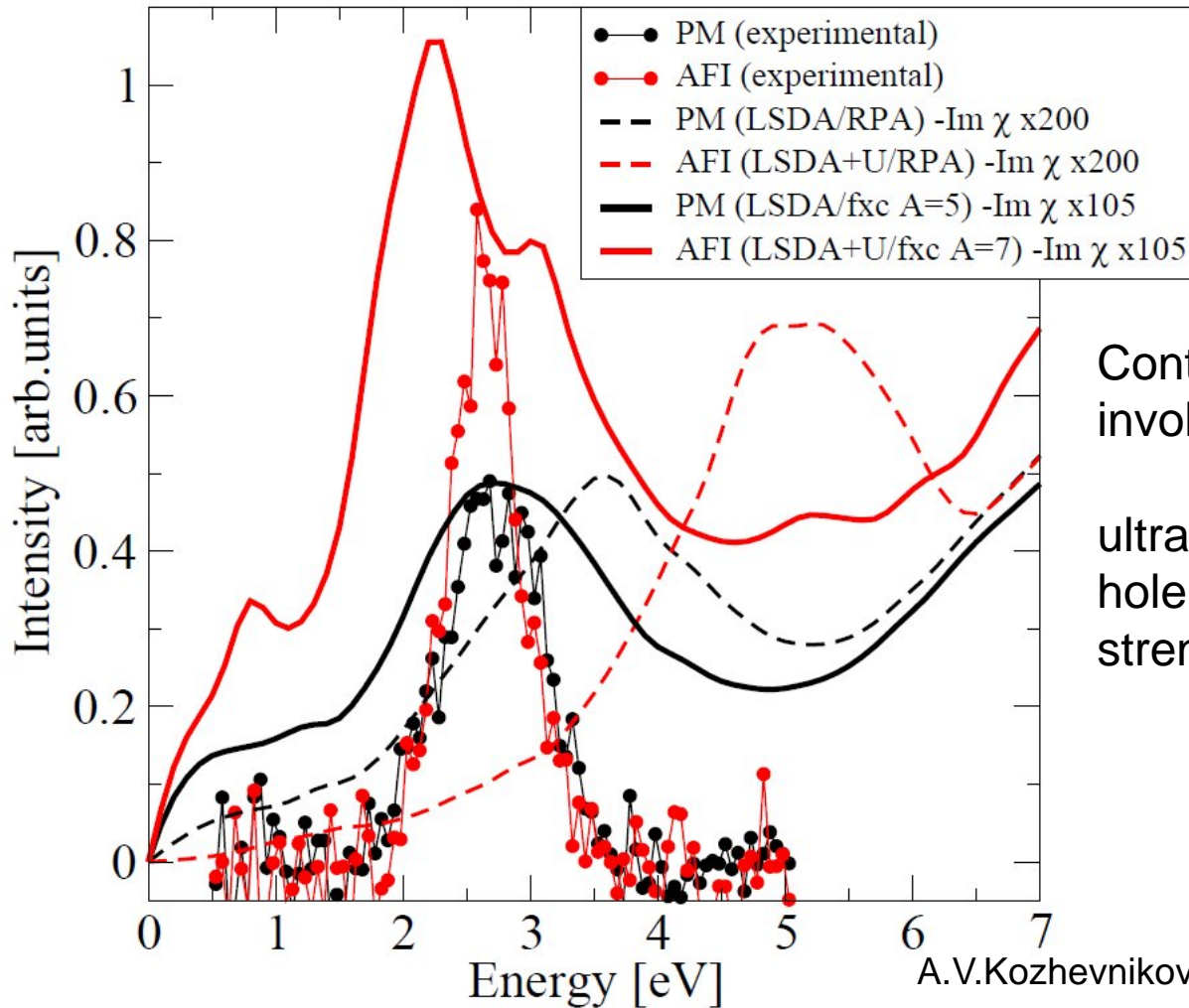
A.V.Kozhevnikov et al., in preparation

Simo Huotari, Benasque TDDFT workshop 2012





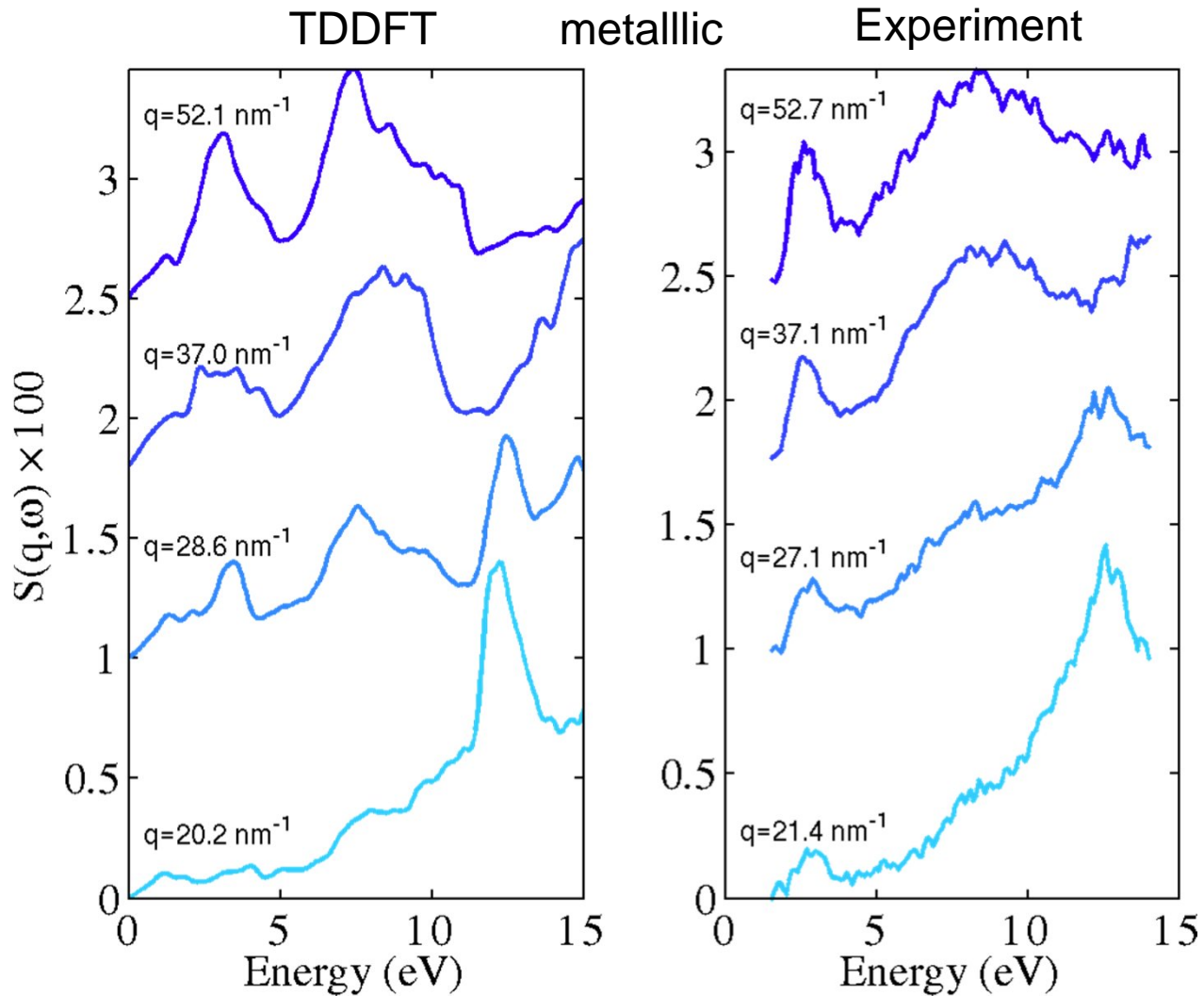
# Metal-to-insulator transition in $V_2O_3$



Contact exciton model invoked:

ultrashort-range electron-hole attraction, tunable strength with parameter A

A.V.Kozhevnikov et al., in preparation



A.V.Kozhevnikov et al., in preparation

Simo Huotari, Benasque TDDFT workshop 2012



# Metal-to-insulator transition in $V_2O_3$

## Conclusions for metal-insulator transition of $V_2O_3$ :

- ❑ d-d excitation changes surprisingly little upon going to insulating phase
- ❑ RPA would predict much larger change
- ❑ Must imply a switching-on of a excitonic interaction upon the MIT?

A.V. Kozhevnikov, M.C. Tropicovsky, T. C. Schulthess, A.G. Eguiluz, T. Pylkkänen, L. Simonelli, G. Monaco, and S. Huotari, under preparation



# Outline

Part 1 Metal-to-insulator transition

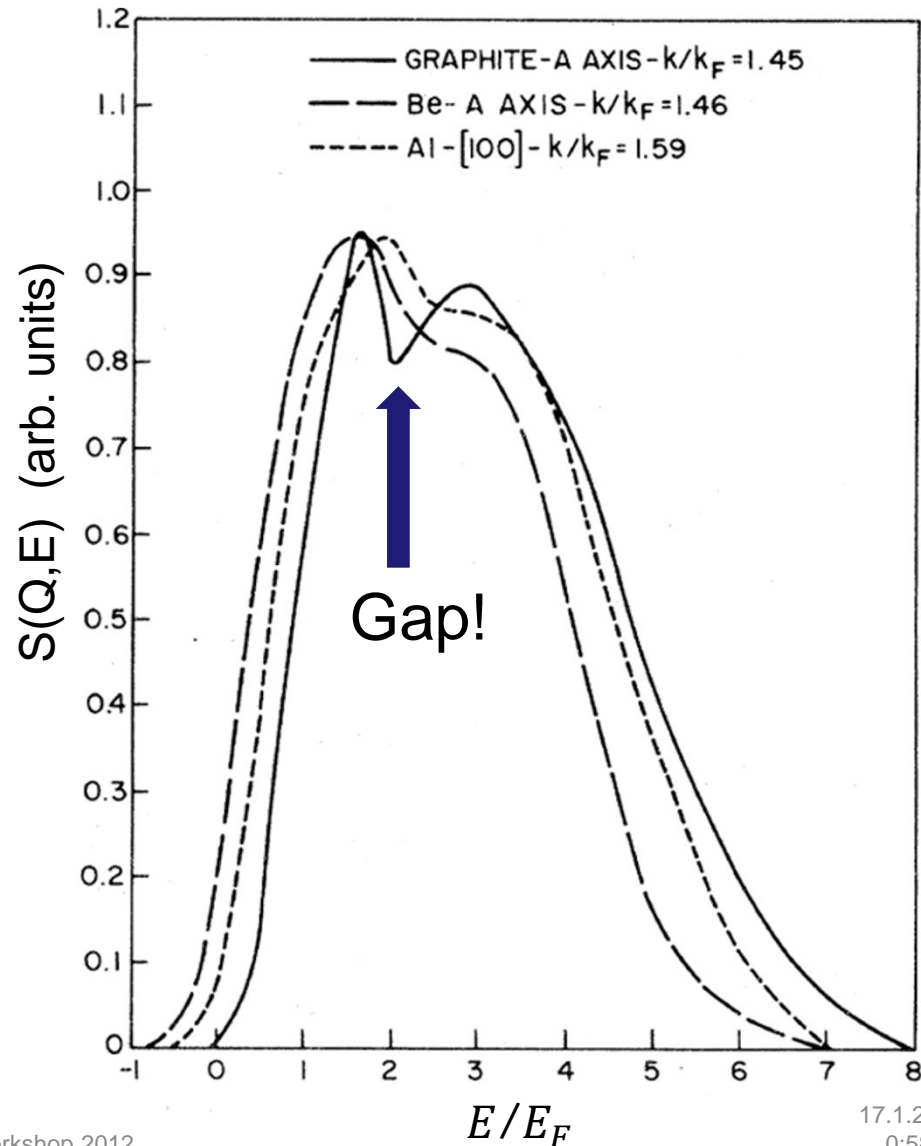
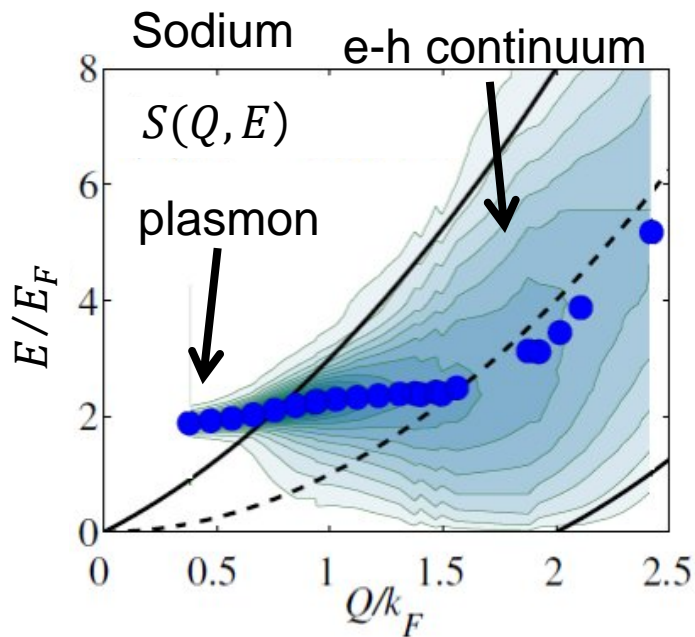
Part 2 Plasmons

Part 3 Double plasmons

Part 4 Quasiparticle renormalisation



# Electron gas response



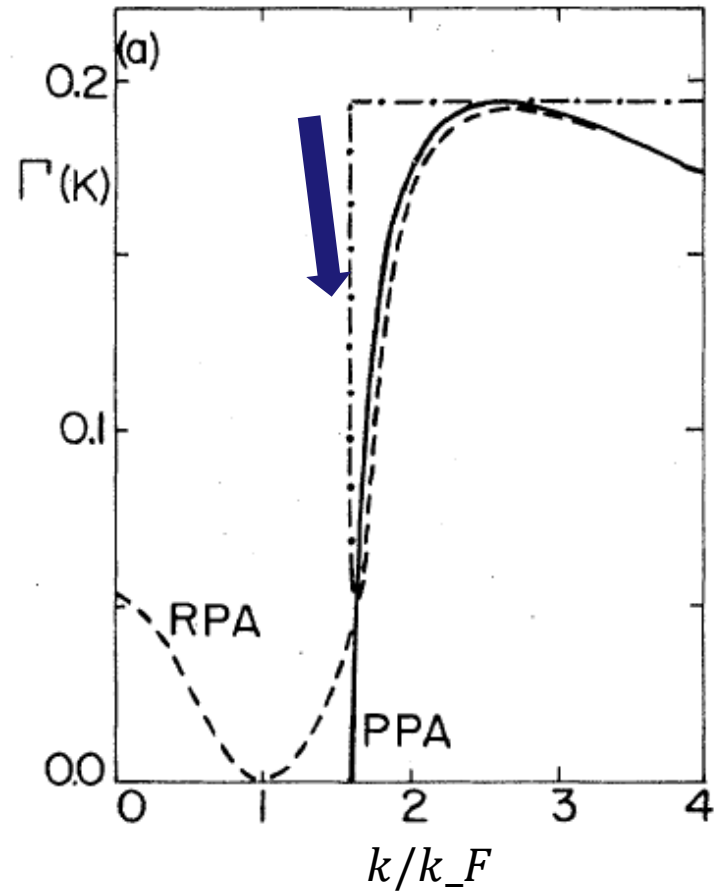
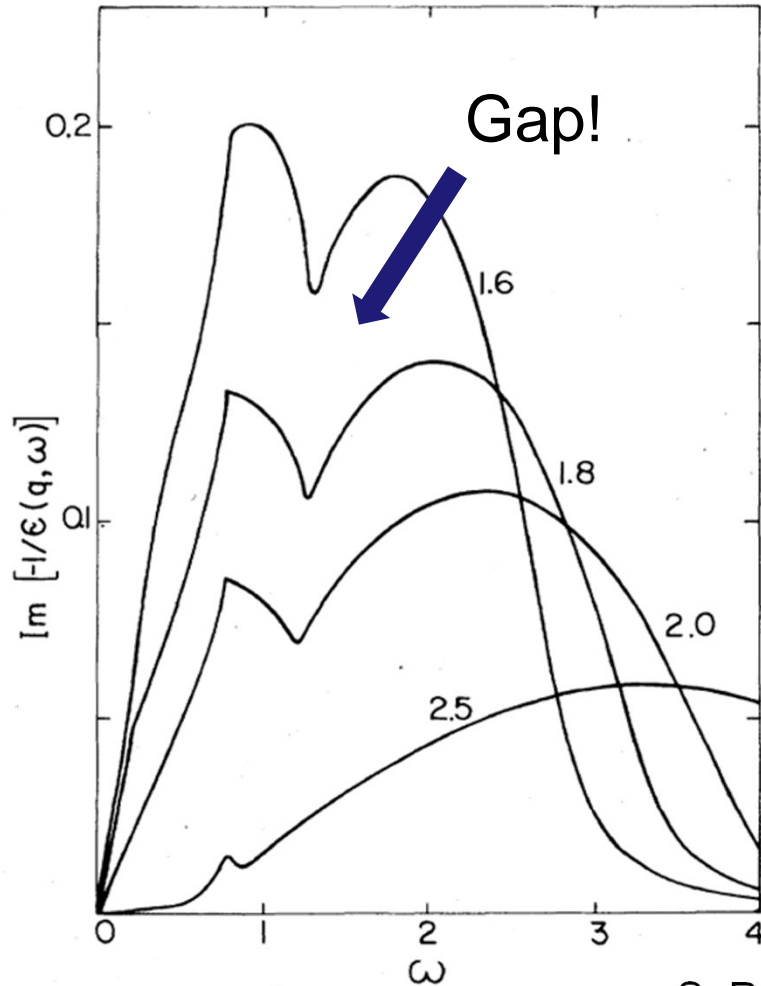
In 1974,  $S(Q, E)$  for many materials seemed to suggest a "universal shape" of the electron gas response function

P.M.Platzman and P.Eisenberger,  
PRL 33, 152 (1974)



# Theory for electron gas

Gap structure appears when taking into account quasiparticle lifetimes

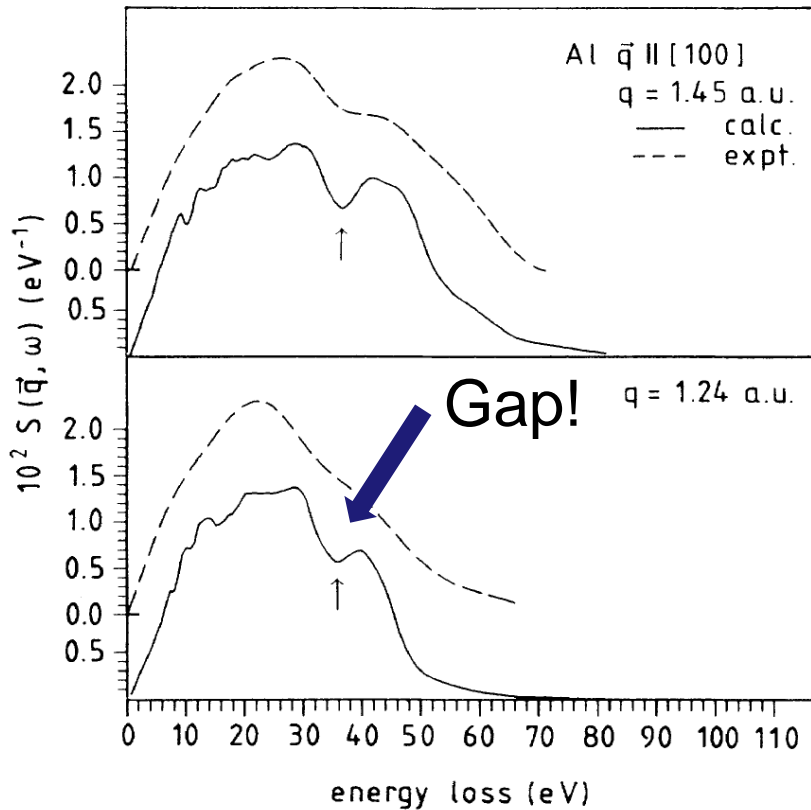


S. Rahman and G. Vignale, PRB 30, 6951 (1984)



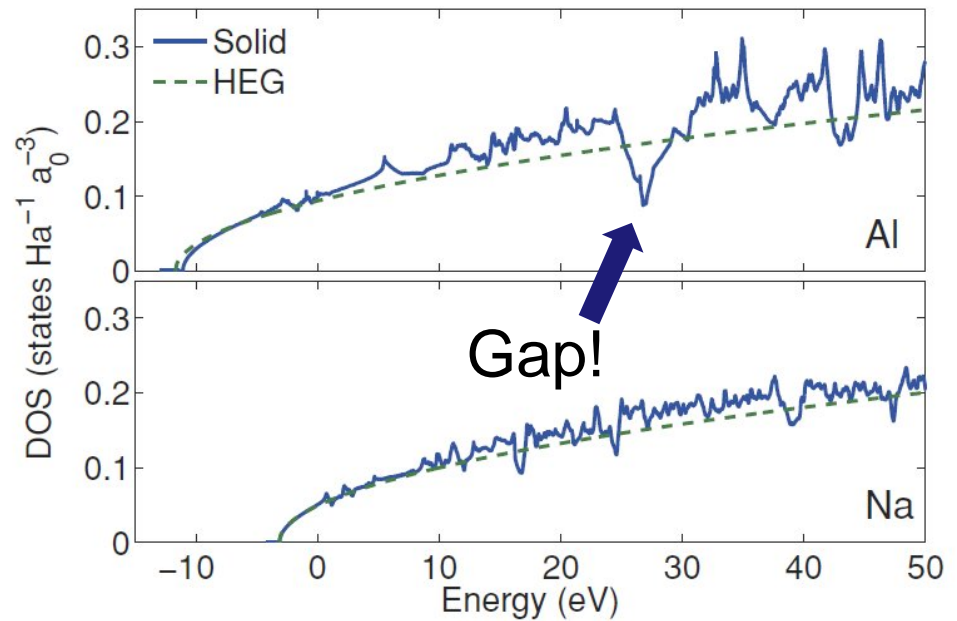


# Gap-structure in Al



Winfried Schülke et al. PRB 47, 12426 (1993)

Explained to be a band-structure effect: a gap in the DOS

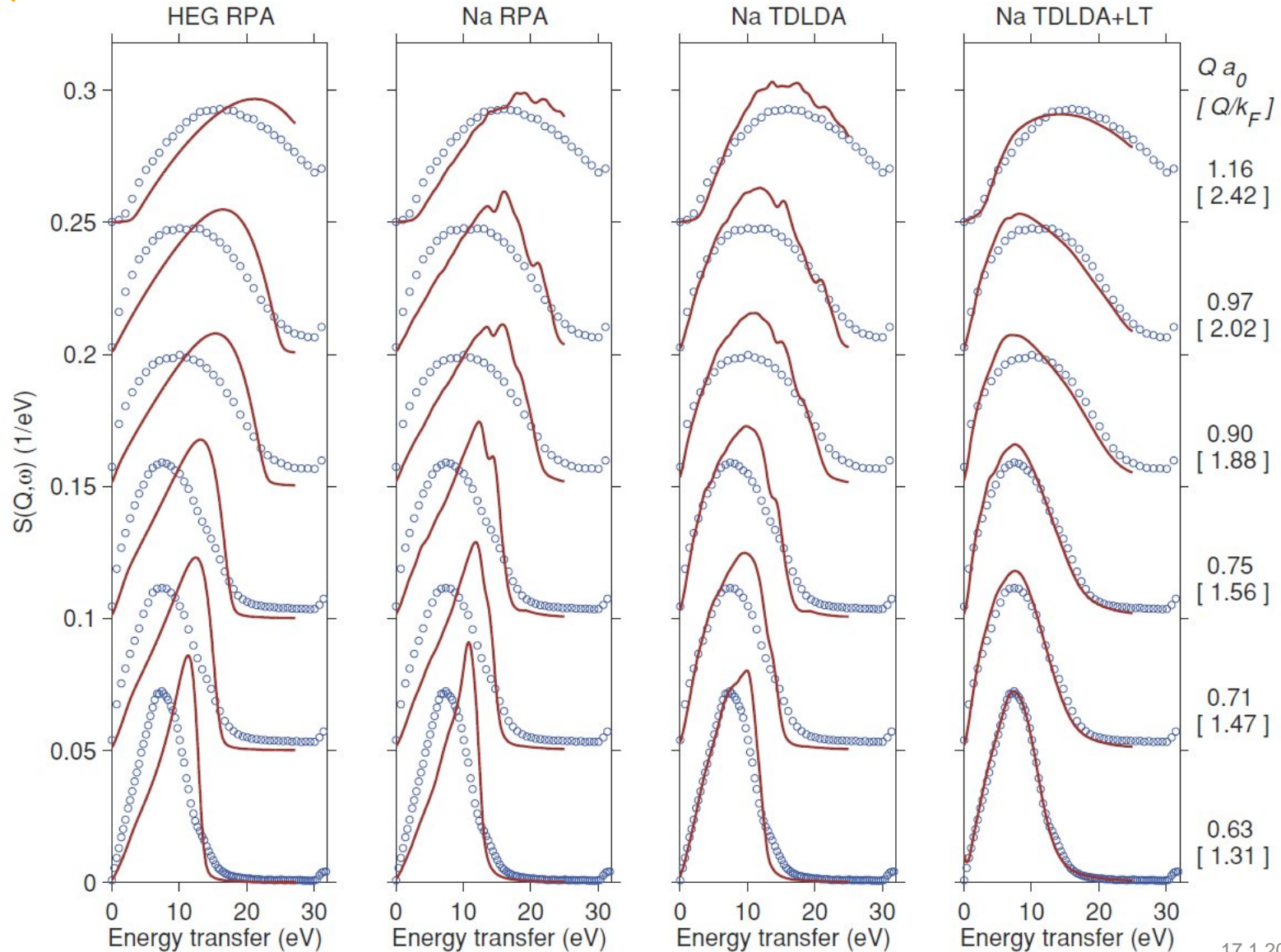


S. Huotari, M. Cazzaniga et al., PRB 84, 075108 (2011)



# Results for sodium

S. Huotari et al., PRB  
**84**, 075108 (2011)



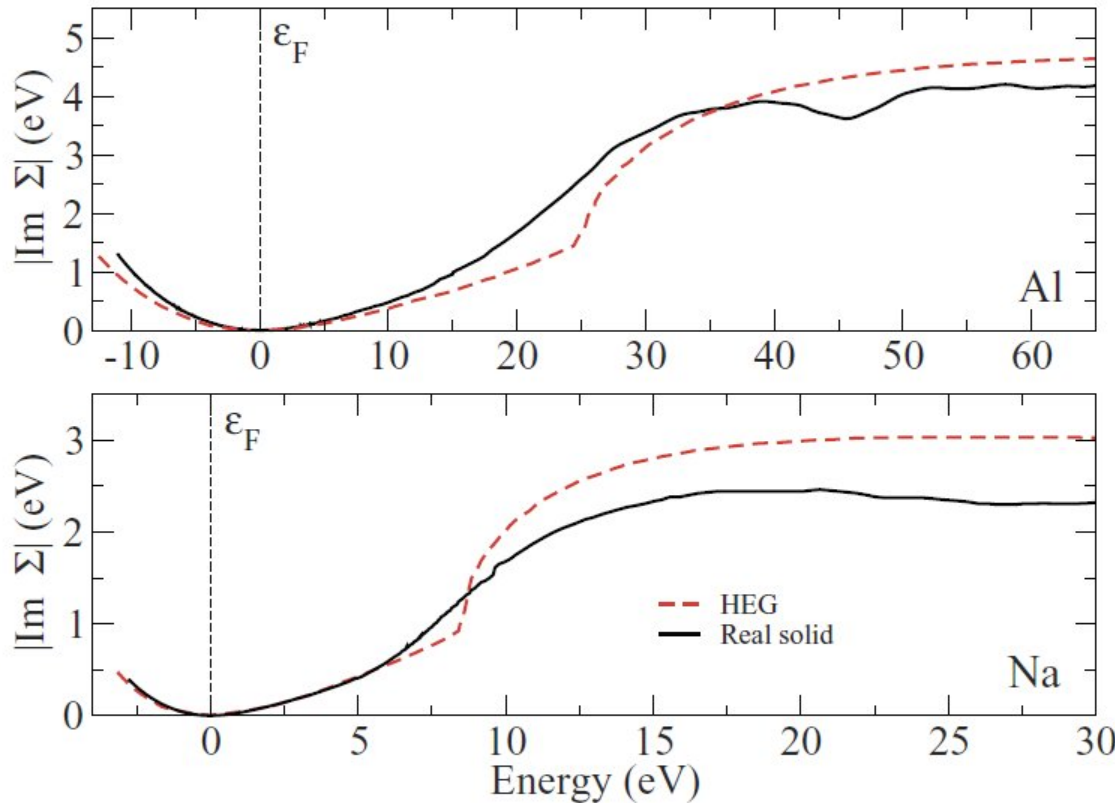


# TDLDA+LifeTimes (LT)

H.-C. Weissker et al., Phys. Rev. Lett. 97, 237602 (2006)

M. Cazzaniga et al., Phys. Rev. B 84, 075109 (2011)

$$\chi_{G,G'}^0(\mathbf{q}, \omega) = -\frac{1}{V_{\text{BZ}}} \sum_{j,j'} \int_{\text{BZ}} d^3k [f(\epsilon_{j'}(\mathbf{k} + \mathbf{q})) - f(\epsilon_j(\mathbf{k}))] \frac{\langle \mathbf{k}, j | e^{-i(\mathbf{q}+\mathbf{G})\cdot\hat{\mathbf{r}}} | \mathbf{k} + \mathbf{q}, j' \rangle \langle \mathbf{k} + \mathbf{q}, j' | e^{i(\mathbf{q}+\mathbf{G}')\cdot\hat{\mathbf{r}}} | \mathbf{k}, j \rangle}{\omega - [\epsilon_{j'}(\mathbf{k} + \mathbf{q}) - \epsilon_j(\mathbf{k})] + i\eta}$$



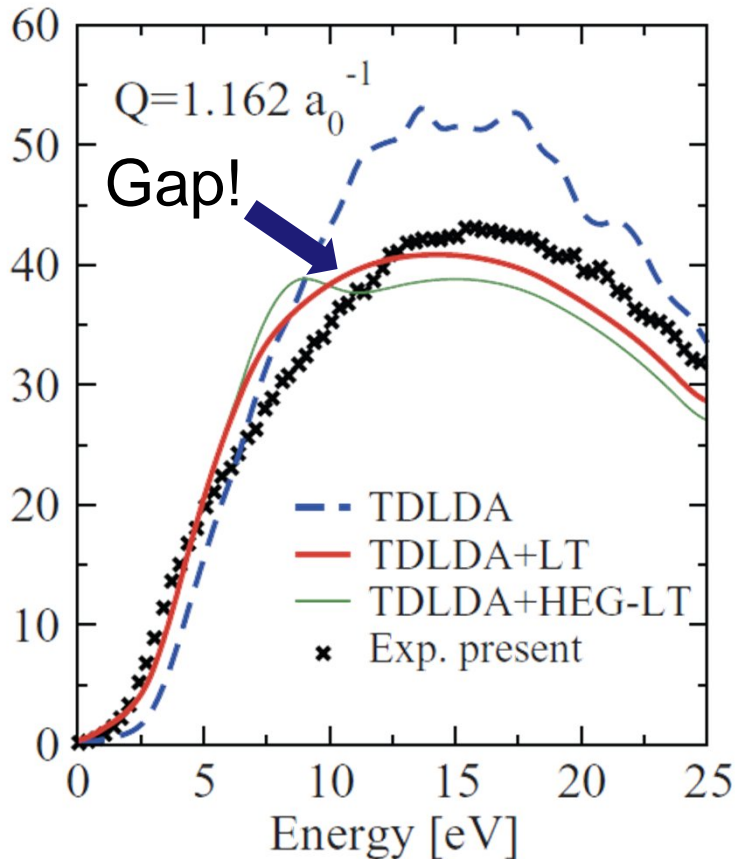
$$i|\text{Im } \Sigma_{j'}(\mathbf{k} + \mathbf{q})| + i|\text{Im } \Sigma_j(\mathbf{k})|$$

Inclusion of lifetimes  
via a modified  
independent particle  
polarizability  $\chi_0^{LT}$



# Results for sodium

M. Cazzaniga et al., PRB **84**, 075109 (2011)



Using HEG lifetimes to HEG or real-metal Na, does a gap structure in the  $S(Q,E)$ .

The gap is washed out if real-metal lifetimes are used.

Fictitious HEG does in fact keep the gap structure but Na is not close enough to the HEG!



# Plasmons

## Conclusions about plasmons in Na:

- ❑ Correlation effects beyond RPA surprisingly large in Na
- ❑ Quasiparticle lifetimes have to be taken into account properly
- ❑ Fine structure to the  $S(Q,E)$  expected in HEG; washed out in Na

PHYSICAL REVIEW B **84**, 075108 (2011)

### **Dynamical response function in sodium studied by inelastic x-ray scattering spectroscopy**

Simo Huotari,<sup>1,2,\*</sup> Marco Cazzaniga,<sup>3,4</sup> Hans-Christian Weissker,<sup>4,5,6</sup> Tuomas Pylkkänen,<sup>1,2</sup> Harald Müller,<sup>2</sup>  
Lucia Reining,<sup>4,5</sup> Giovanni Onida,<sup>3,4</sup> and Giulio Monaco<sup>2</sup>

PHYSICAL REVIEW B **84**, 075109 (2011)

### **Dynamical response function in sodium and aluminum from time-dependent density-functional theory**

Marco Cazzaniga,<sup>1,2,\*</sup> Hans-Christian Weissker,<sup>2,3,4</sup> Simo Huotari,<sup>5,6</sup> Tuomas Pylkkänen,<sup>5,6</sup> Paolo Salvestrini,<sup>1,7</sup>  
Giulio Monaco,<sup>5</sup> Giovanni Onida,<sup>1,2</sup> and Lucia Reining<sup>2,3</sup>



# Outline

Part 1 Metal-to-insulator transition

Part 2 Plasmons

Part 3 Double plasmons

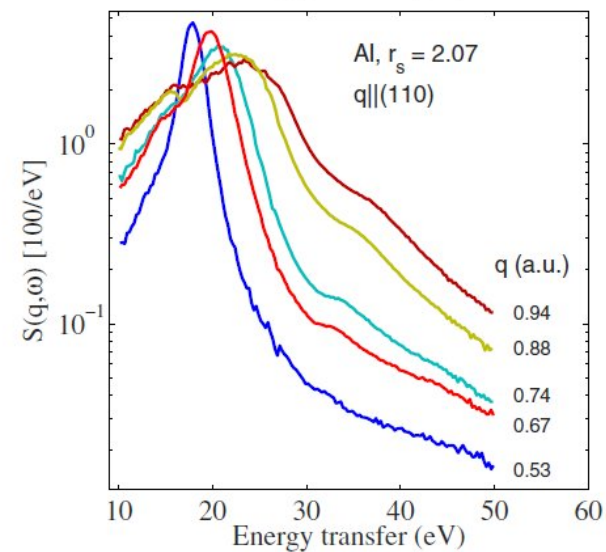
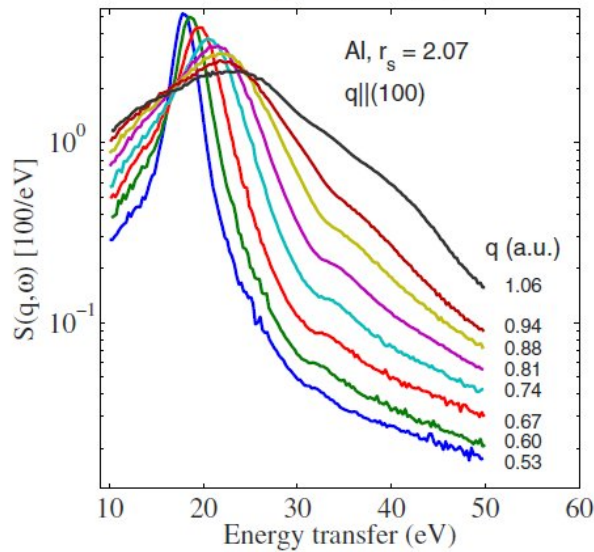
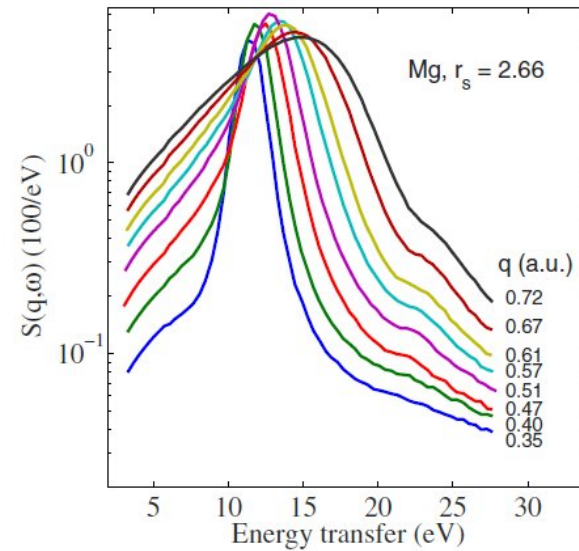
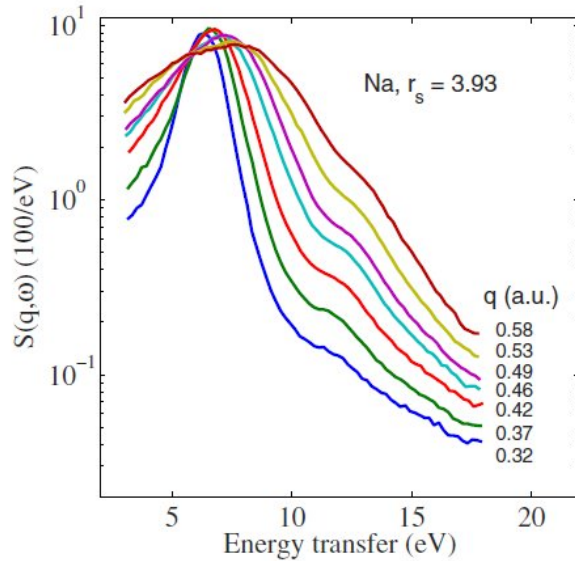
Part 4 Quasiparticle renormalisation





# Double plasmons

S. Huotari et al.,  
PRB 77, 195125 (2008)

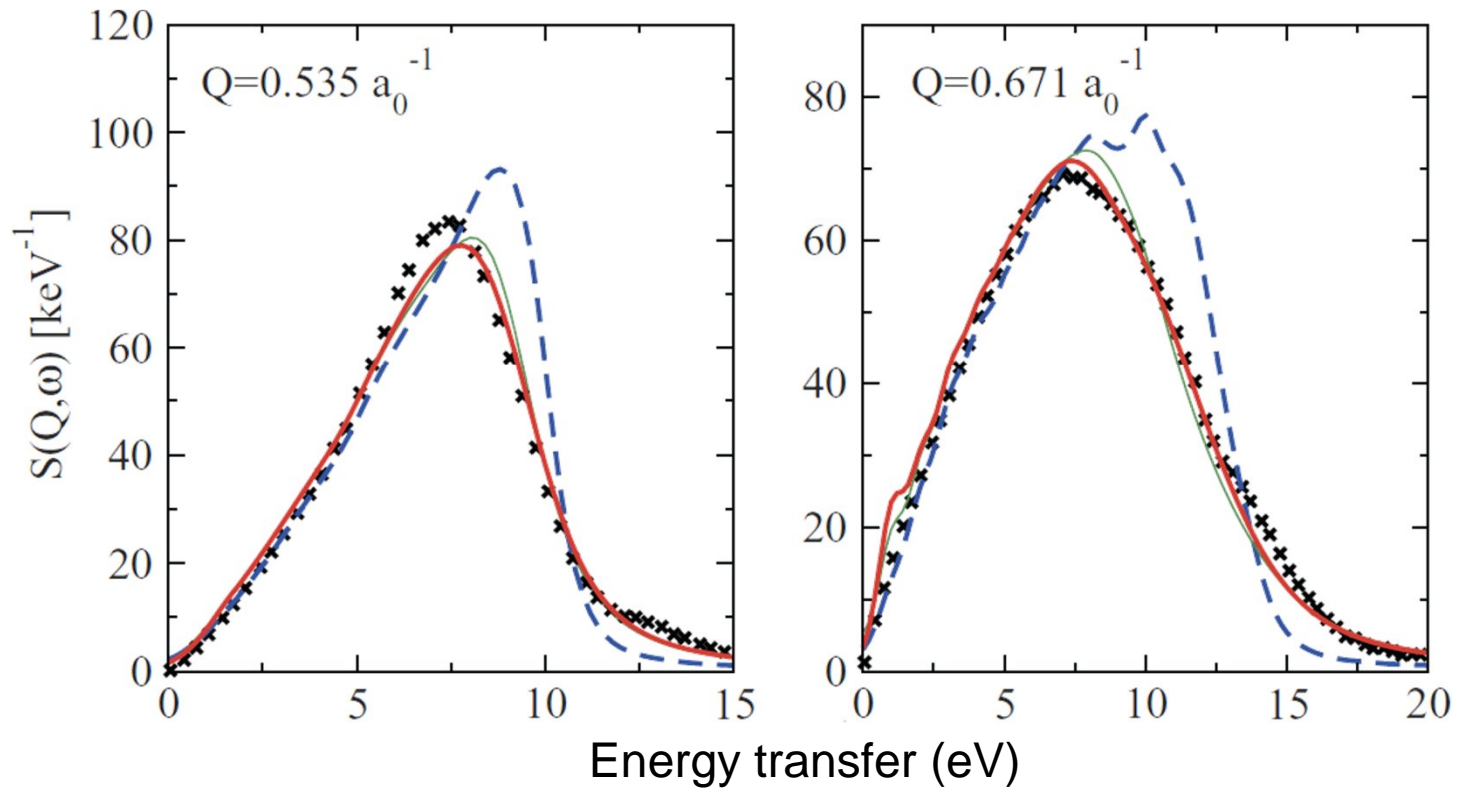




# Double plasmons

Not visible in TDDFT results!

- - TDLDA
- TDLDA+LT
- TDLDA+HEG-LT
- × Exp. present



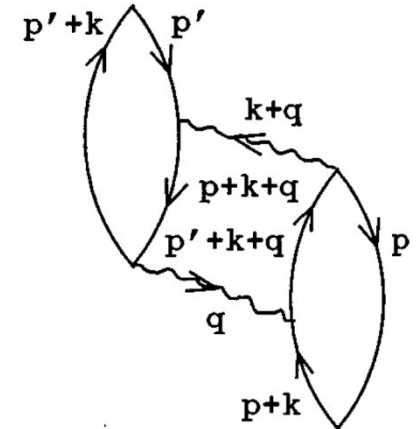
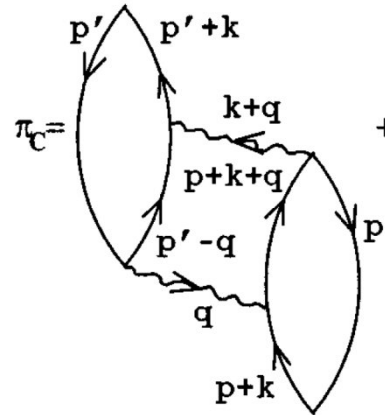
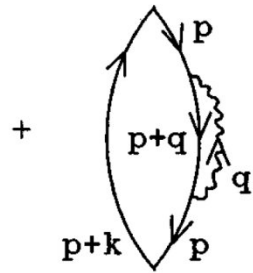
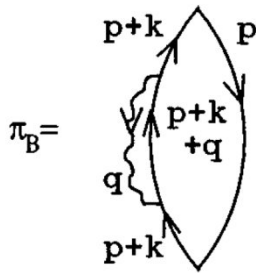
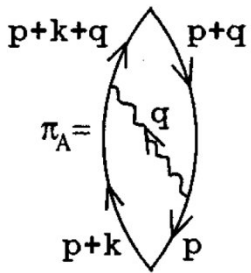
M. Cazzaniga et al., PRB **84**, 075109 (2011)



# Double plasmons

Need to go many-body!

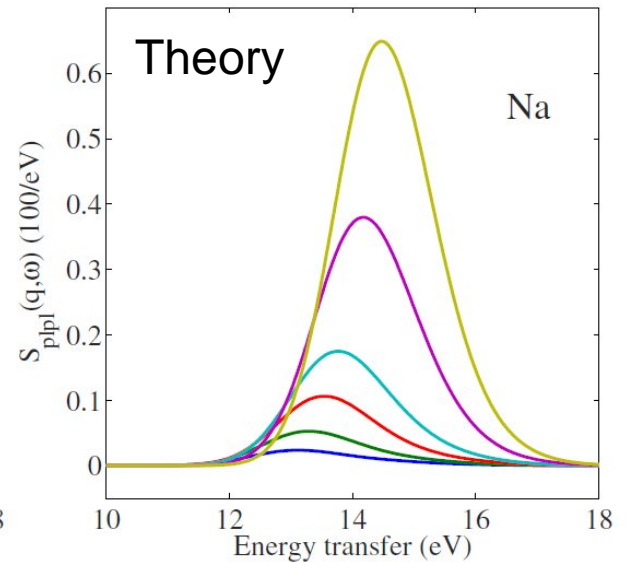
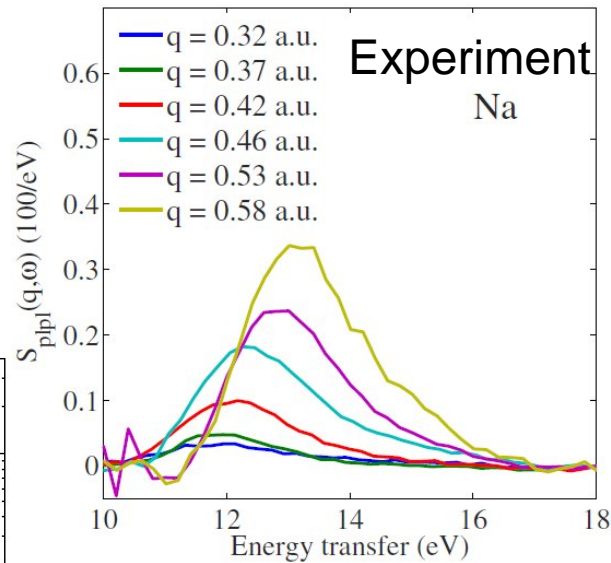
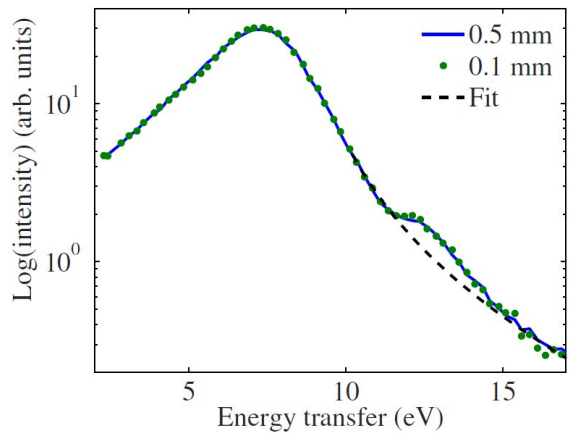
$$S(\mathbf{q}, \omega) = \frac{q^2}{4\pi^2 n_0} \text{Im} \left( \frac{-1}{\epsilon(\mathbf{q}, \omega)} \right) \approx \frac{1}{\pi n_0} \times \frac{\text{Im}[\pi_0(\mathbf{q}, \omega) + \pi_A(\mathbf{q}, \omega) + \pi_B(\mathbf{q}, \omega) + \pi_C(\mathbf{q}, \omega)]}{\epsilon_L^2(\mathbf{q}, \omega)}$$



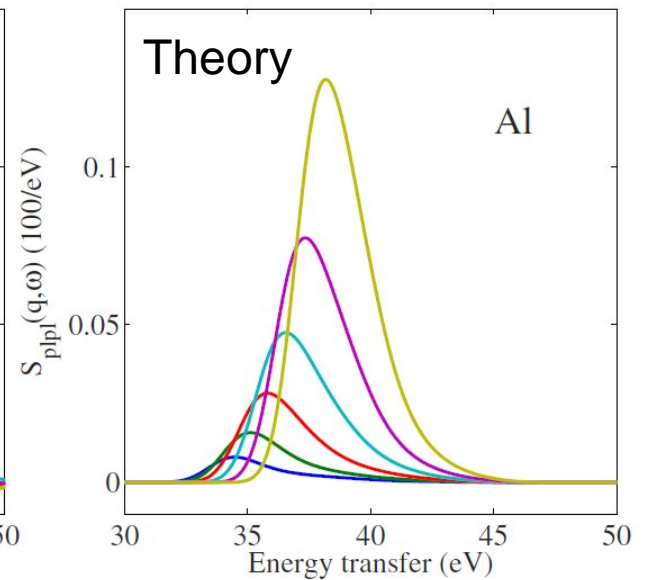
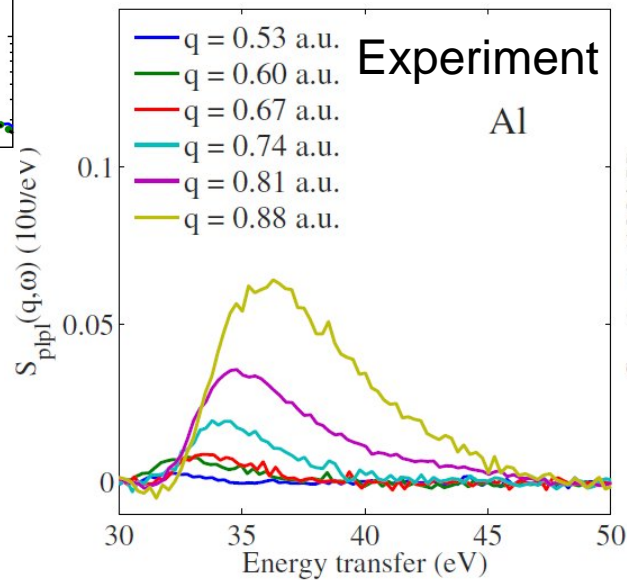
K. Sturm and A. Gusarov, PRB 62, 16474 (2000)



# Double plasmons



S. Huotari et al.,  
PRB 77, 195125 (2008)





# Double plasmons

## Conclusions about double plasmons in Al,Na and Mg:

- ❑ Double plasmons observed in  $S(Q,E)$ , not explained by TDDFT
- ❑ Many-body effects explain the spectra to  $\sim 2$  eV and factor of 2 in intensity
- ❑ Should one try to incorporate the appropriate interaction in  $f_{xc}$  in TDDFT?

PRL **95**, 157401 (2005)

PHYSICAL REVIEW LETTERS

week ending  
7 OCTOBER 2005

### **Correlation-Induced Double-Plasmon Excitation in Simple Metals Studied by Inelastic X-Ray Scattering**

C. Sternemann,<sup>1</sup> S. Huotari,<sup>2</sup> G. Vankó,<sup>2</sup> M. Volmer,<sup>1</sup> G. Monaco,<sup>2</sup> A. Gusarov,<sup>3,4</sup> H. Lustfeld,<sup>3</sup>  
K. Sturm,<sup>3</sup> and W. Schülke<sup>1</sup>

PHYSICAL REVIEW B **77**, 195125 (2008)

### **Electron-density dependence of double-plasmon excitations in simple metals**

S. Huotari,<sup>1</sup> C. Sternemann,<sup>2</sup> W. Schülke,<sup>2</sup> K. Sturm,<sup>3</sup> H. Lustfeld,<sup>3</sup>  
H. Sternemann,<sup>2</sup> M. Volmer,<sup>2</sup> A. Gusarov,<sup>4</sup> H. Müller,<sup>1</sup> and G. Monaco<sup>1</sup>



# Outline

Part 1 Metal-to-insulator transition

Part 2 Plasmons

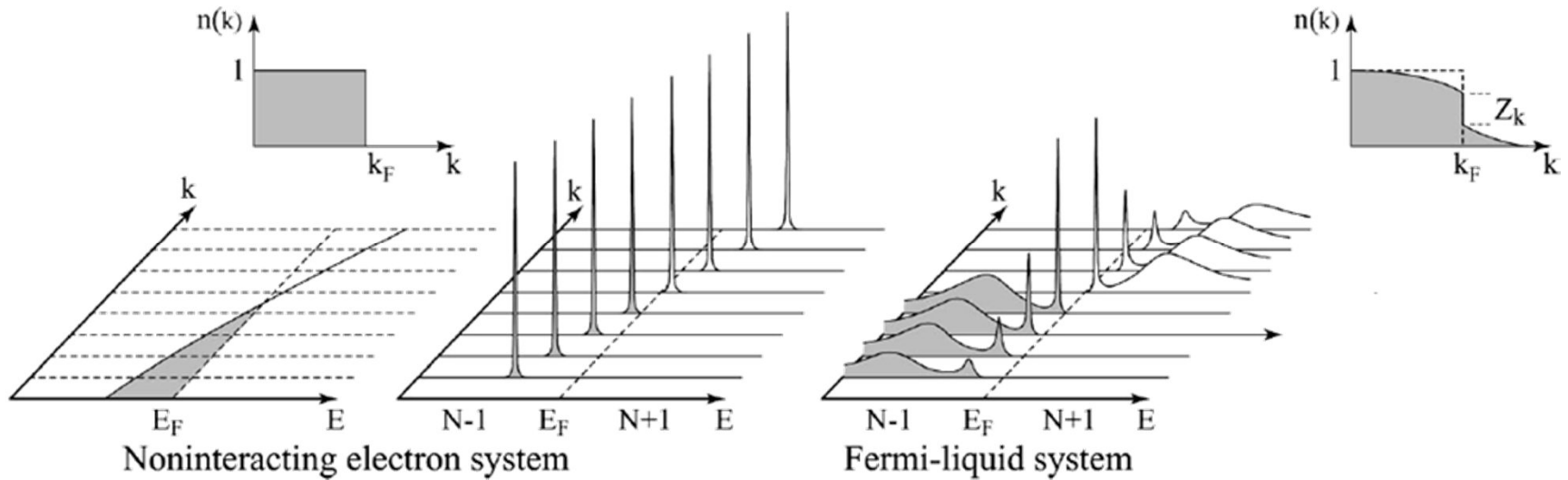
Part 3 Double plasmons

Part 4 Quasiparticle renormalisation





# Quasiparticle renormalisation

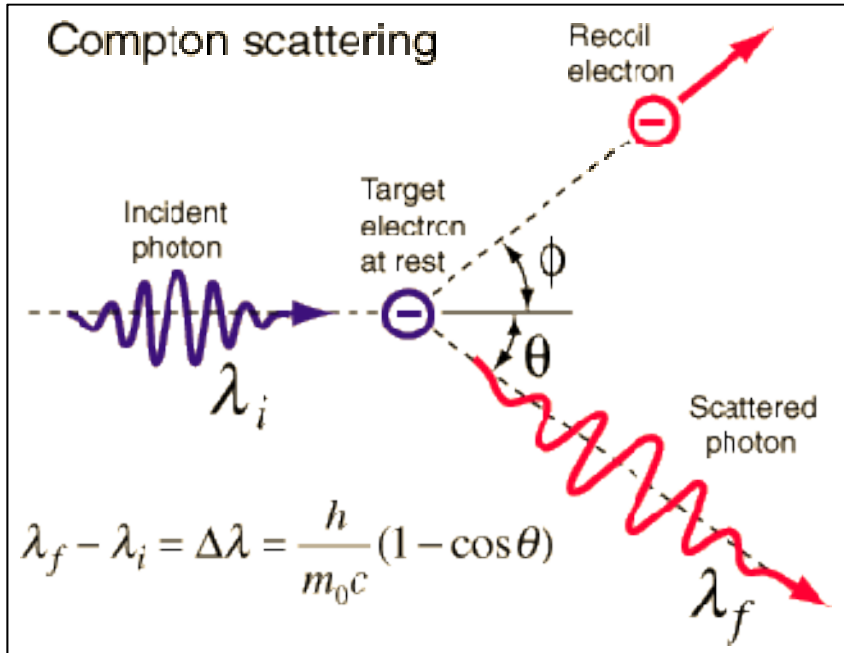


~~$$\text{PES} \propto \sum_{f,i} |M_{f,i}^{\text{P}}|^2 A(\mathbf{p}, E) \delta(E_{\mathbf{k}} + E_{\mathbf{m}}^{N-1} - E_i^N - \omega_1)$$~~

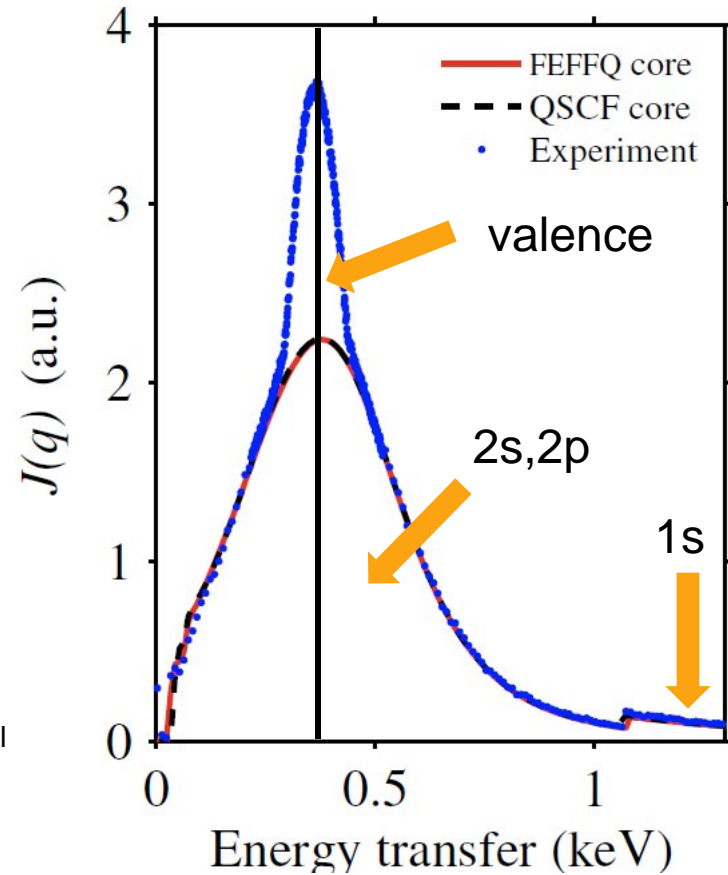
A. Damascelli et al.,  
 Rev. Mod. Phys 75, 473 (2003)



# Compton spectroscopy



<http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/compton.html>



Compton profile  $J(q)$

$$J(q) = \frac{3}{8\pi p_F^3} \int_{4\pi} d\Omega \int_{|q|}^{\infty} pn(\mathbf{p}) dp$$

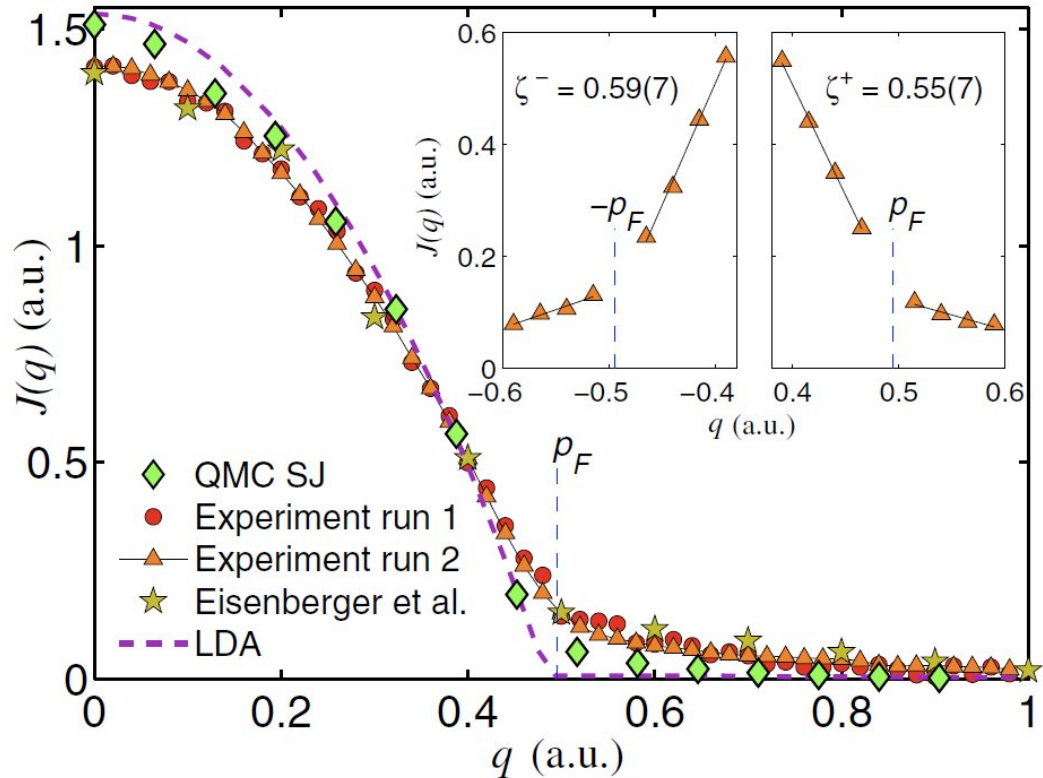
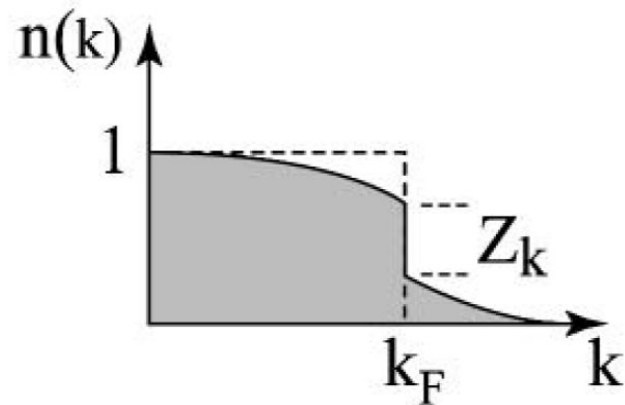
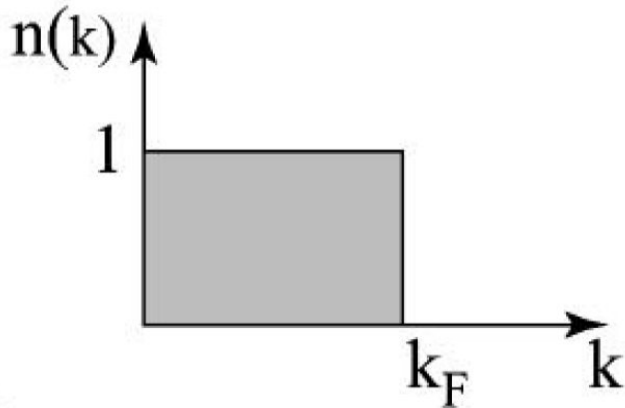
S. Huotari et al.,  
PRL 105, 086403 (2010)



# Compton spectroscopy

Independent electrons:

$$J(q) \propto (p_F^2 - q^2) \text{ for } |q| < p_F$$



A. Damascelli et al.,  
Rev. Mod. Phys 75, 473 (2003)

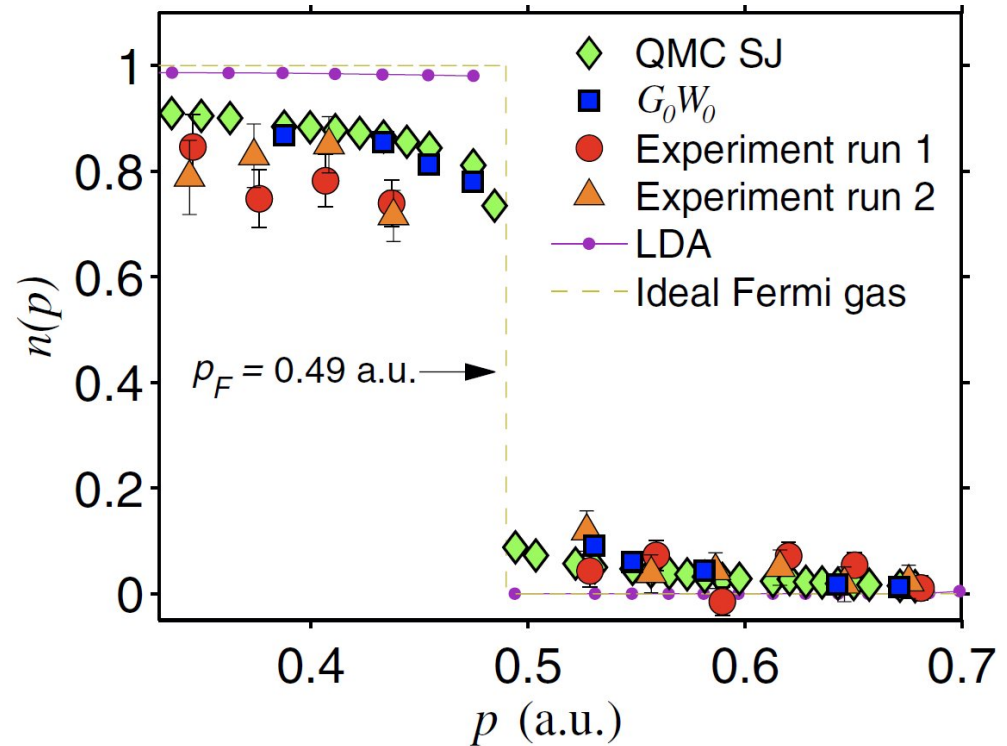
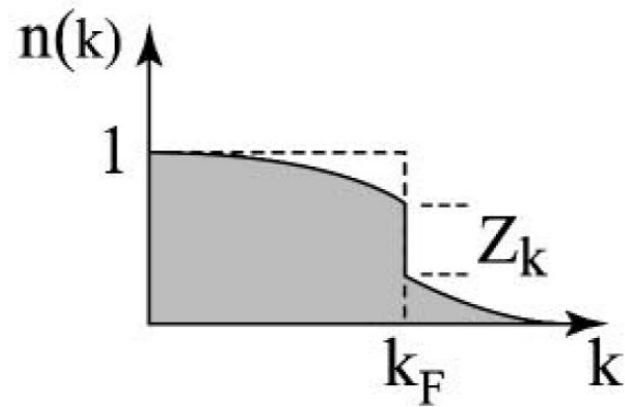
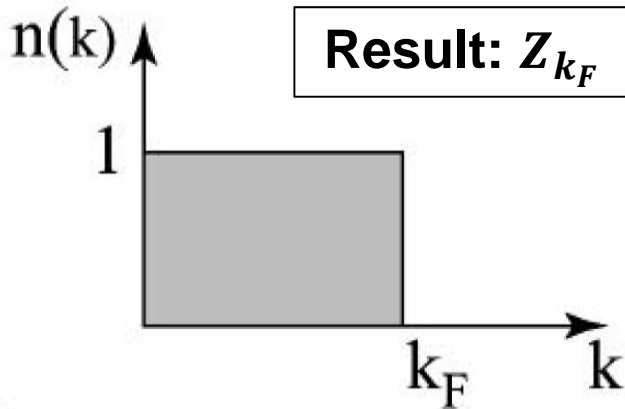
S. Huotari et al.,  
PRL 105, 086403 (2010)



# Momentum density of the electron gas

**Result:  $Z_{k_F} = 0.58 \pm 0.07$  for Na**

$$n(p) = -\frac{2p_F^3}{3p} \frac{dJ(q)}{dq} \Big|_{q=p}$$



S. Huotari et al.,  
PRL 105, 086403 (2010)



# Quasiparticle renormalisation

## Conclusions about the QP renormalisation factor:

- ❑ Is in principle the area under the coherent peak in photoemission
- ❑ Can be independently measured by momentum-density experiment
- ❑ Good results and agreement with theory obtained for Na

PRL **105**, 086403 (2010)

PHYSICAL REVIEW LETTERS

week ending  
20 AUGUST 2010

## Momentum Distribution and Renormalization Factor in Sodium and the Electron Gas

Simo Huotari,<sup>1,2</sup> J. Aleksi Soininen,<sup>2</sup> Tuomas Pylkkänen,<sup>1,2</sup> Keijo Hämäläinen,<sup>2</sup> Arezki Issolah,<sup>3</sup> Andrey Titov,<sup>4</sup> Jeremy McMinis,<sup>5</sup> Jeongnim Kim,<sup>5</sup> Ken Esler,<sup>5</sup> David M. Ceperley,<sup>5</sup> Markus Holzmann,<sup>6</sup> and Valerio Olevano<sup>4</sup>

<sup>1</sup>European Synchrotron Radiation Facility, B.P. 220, F-38043 Grenoble, France

<sup>2</sup>Department of Physics, P.O. Box 64, FI-00014, University of Helsinki, Finland

<sup>3</sup>Université de Tizi-Ouzou, Campus de Hasnaoua, 15000 Tizi-Ouzou, Algeria

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<sup>5</sup>Department of Physics and NCSA, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

<sup>6</sup>LPTMC, UPMC-CNRS, Paris, France, and LPMCM, UJF-CNRS, F-38042 Grenoble, France

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J. Aleksi Soininen (Univ. Helsinki)

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V. Olevano, M. Holtzmann (CNRS, Grenoble) (+everybody from QMC)

A. Eguiluz, A.V.Kozhevnikov, M.C.Troparevsky, T.C.Schulthess et al.