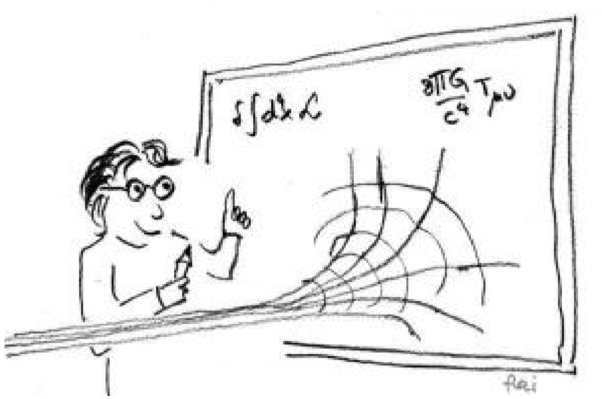
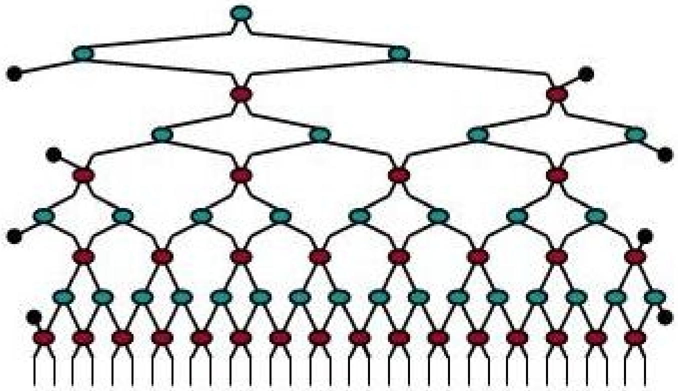
Holography, Tensor Networks and correlations between disjoint regions at criticality





ArXiv: 1108.1277 JMV. Pasquale Sodano. JHEP10(2011)011

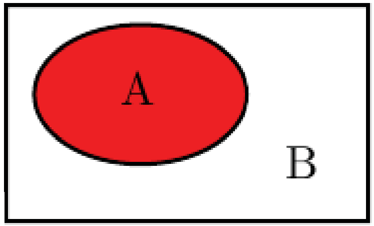
Entanglement Entropy in CFT

Entanglement Entropy

- Quantum system ($\mathcal{H}$) in the ground state $|\Psi\rangle$
  Density matrix $\rho = |\Psi\rangle\langle\Psi| \quad \Longrightarrow \quad \mathrm{Tr}\rho^n = 1$

- Two observers: each one measures only a subset of a complete set of cummuting observables

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



- A's reduced density matrix $\boxed{\rho_A = \mathrm{Tr}_B \rho}$

- Entanglement entropy $\equiv$ Von Neumann entropy of ρ_A

$$\boxed{S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A)}$$

It measures the amount of information shared by A and B

- Notoriously difficult to compute in QFT

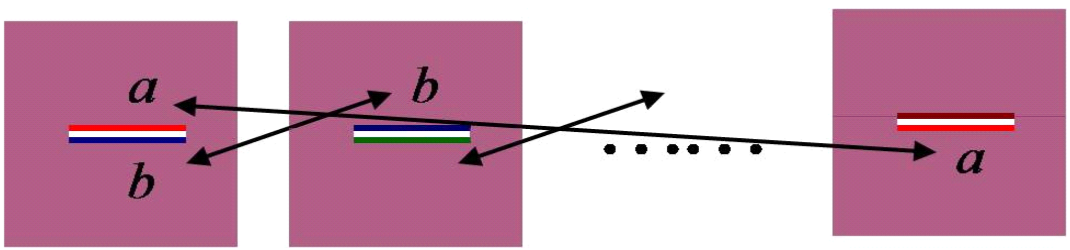
Entanglement Entropy in CFT

EE in CFT. The Replica Trick

[Holzhey, Larsen, Wilczek, NPB (1994)]
[Calabrese, Cardy, JSTAT (2004)]

- i.e, sewing different replicas of $[\rho_A]$

$\mathbf{Tr}(\rho_A)^n =$

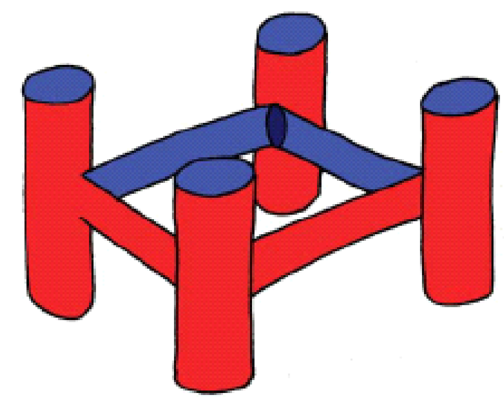


$=$ a path integral over n - sheeted Riemann surface $\sum_{n,1}$

- The trick

$$S_A = -\partial_n Tr(\rho_A)^n \,|_{n=1} = \frac{c}{3}\log\left(\frac{L}{\varepsilon}\right) + c_1'$$

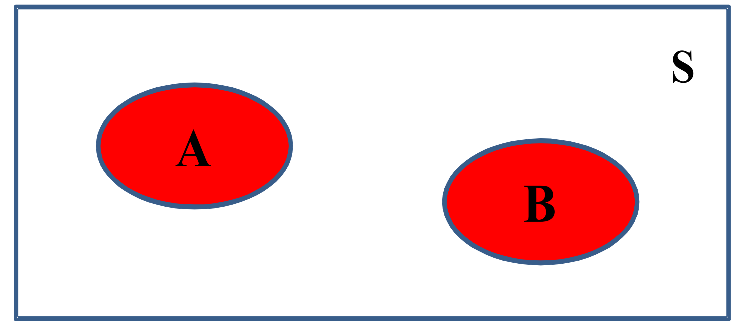
Ex: $\Sigma_{4,1}$



Mutual Information between non complementary regions in CFT

- Mutual Information (MI) between two disjoint regions is UV - finite quantity

$$I_{(A:B)} = S_A + S_B - S_{A\cup B}$$



- In general, MI bound correlators between operators in A & B

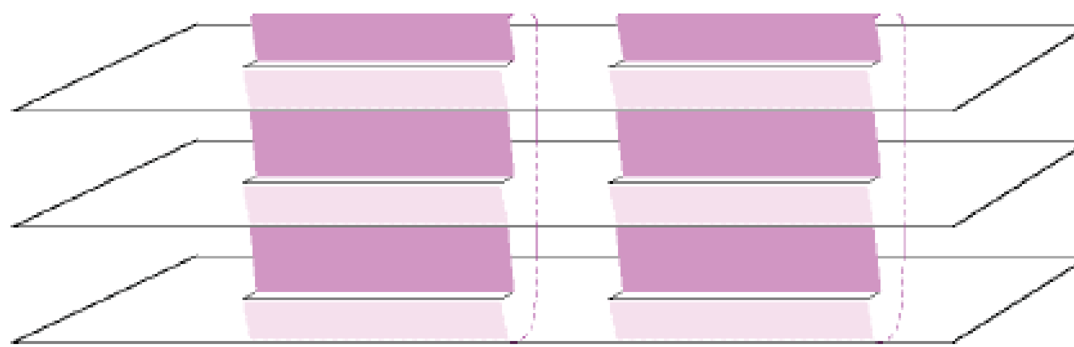
$$I_{(A:B)} \geq \left(\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle\right)^2$$

Wolf, Verstraete, Cirac, 2007

- S_{AUB} : Need to compute EE of 2 disjoint intervals: much harder to compute from ***replica trick*** than 1 interval; now depends on the full operator content of the theory, not just c (central charge)

$$A = A_1 \cup A_2 = [u_1, v_1] \cup [u_2, v_2]$$

Ej: Riemann surface $\Sigma_{3,2}$



Mutual Information between non complementary regions in CFT

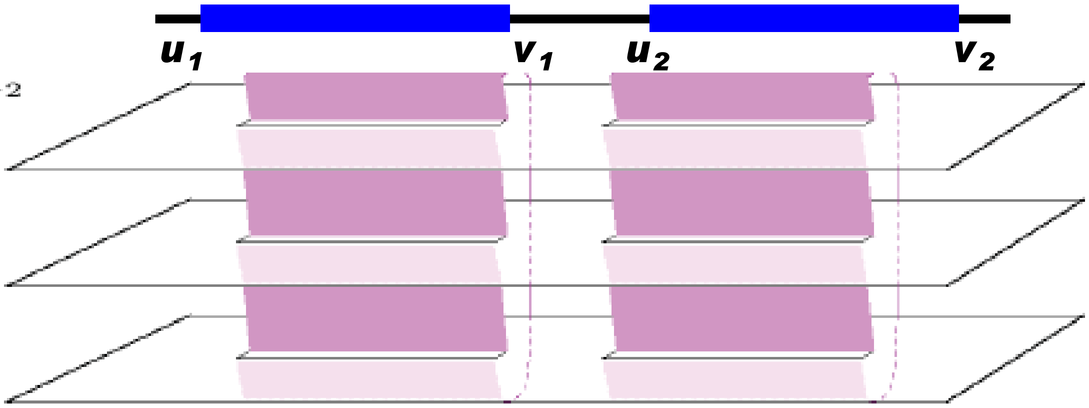
- $$\mathrm{Tr}\rho_A^n \equiv Z_{\mathcal{R}_{n,2}} = \underbrace{c_n^2 \left(\frac{|u_1 - u_2||v_1 - v_2|}{|u_1 - v_1||u_2 - v_2||u_1 - v_2||u_2 - v_1|} \right)^{\frac{c}{6}(n-1/n)}}_{Z^W_{\mathcal{R}_{n,2}}} \mathcal{F}_n(x)$$

$$x = \frac{(u_1 - v_1)(u_2 - v_2)}{(u_1 - u_2)(v_1 - v_2)}$$

[Calabrese, Cardy, JSTAT (2004)]

- Analytical continuation for $n \to 1$ of $\mathcal{F}_n(x)$
  $\Longrightarrow$ Mutual information for any value of the parameters

- Holographic computation of $I_{A_1:A_2}$



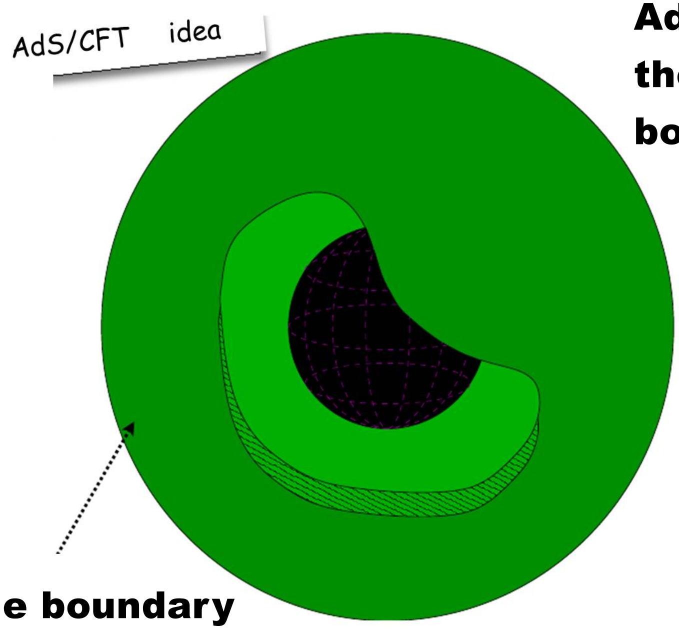
$$A = A_1 \cup A_2 = [u_1, v_1] \cup [u_2, v_2]$$

AdS / CFT correspondence

AdS_{d+2}/CFT_{d+1} correspondence

Maldacena, Gubser, Klebanov, Polyakov, Witten

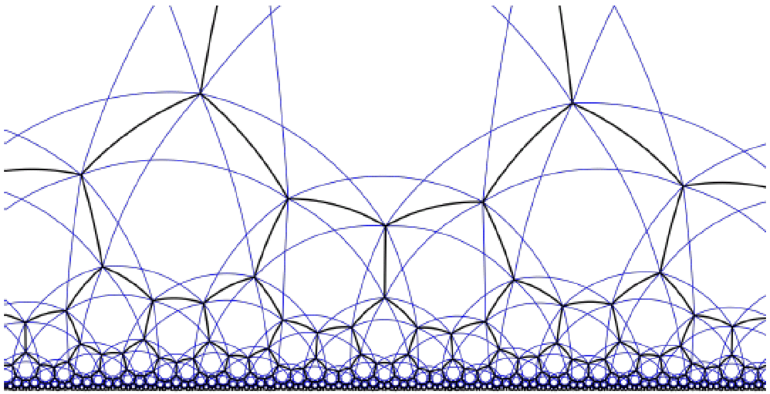
$$\mathcal{Z}_{\mathrm{SUGRA}}(\mathcal{M},\varphi)=\mathcal{Z}_{\mathrm{CFT}}(\partial\mathcal{M},\mathcal{O}) \qquad \mathcal{M}=\mathrm{AdS}_{d+2}$$



AdS_3 black hole = thermal CFT_2 at the boundary

CFT_2 at the boundary quantum criticality in 1 + 1 dimensions

- AdS space has a boundary
- Isometry of AdS space : SO(2, d+1) = conformal symmetry in d+1 dimensions



AdS / CFT correspondence: The bulk extra dimension

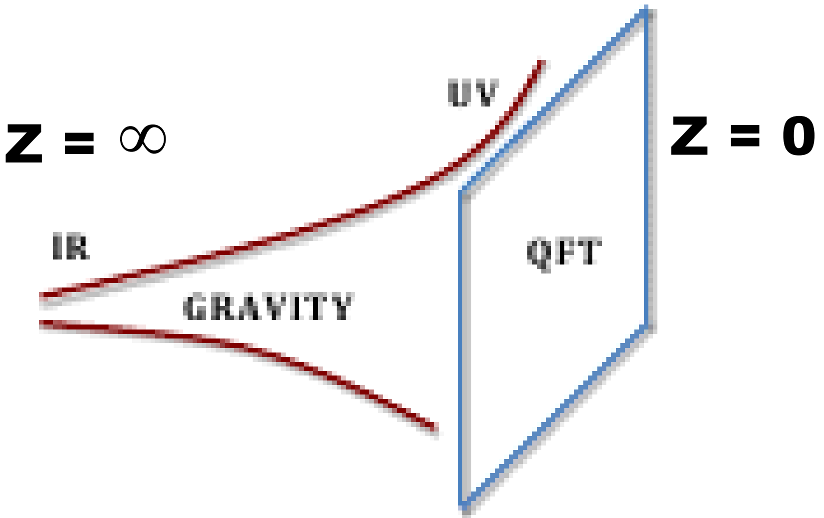
The duality is holographic in nature: the dual gravitational system has at least one extra dimension z and much of the field theory properties can be extracted by working on the boundary.

The extra dimension z should be interpreted as an energy scale. It represents the renormalization group flow of the quantum field theory defined on the boundary.

In this sense the AdS/CFT correspondence "geometrizes" the field theory energy scale.

geometrization: in the dual bulk gravitational description the energy scale is treated geometrically on an equal footing to the spatial directions of the boundary field theory

AdS / CFT correspondence: The bulk extra dimension



Einstein-Hilbert action

$$S_G = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{|g|}\,(R-2\Lambda)$$

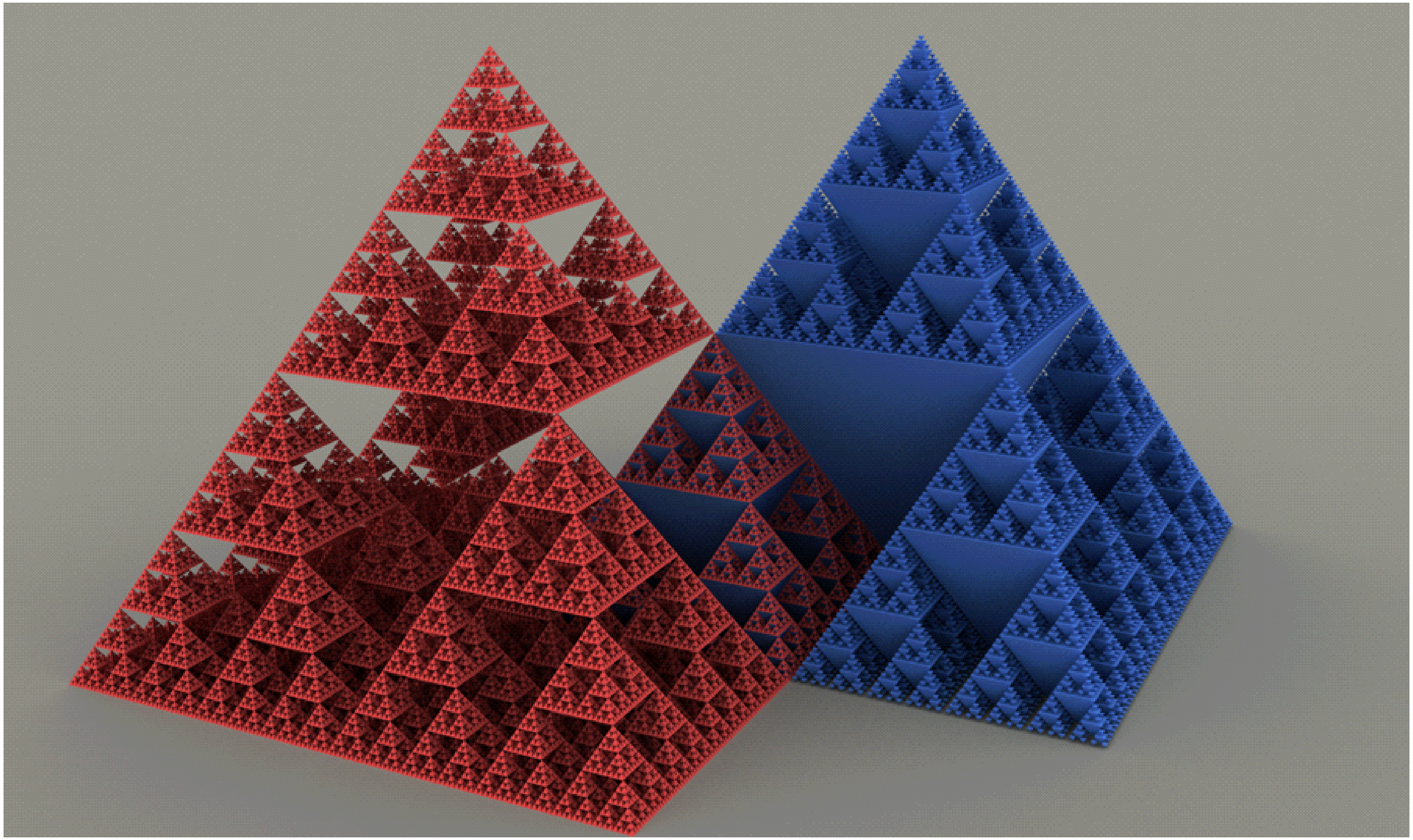
- g: the determinant of the metric $g_{\mu\nu}$
- d spatial dimensions $r, x^1, \ldots, x^{d-1}$ + 1 time
- κ^2: $\propto$ the Newton constant
- R: the Ricci scalar
- Λ: the cosmological constant

Sol: AdS (in Poincaré coord)

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^i dx^i + dz^2\right)$$

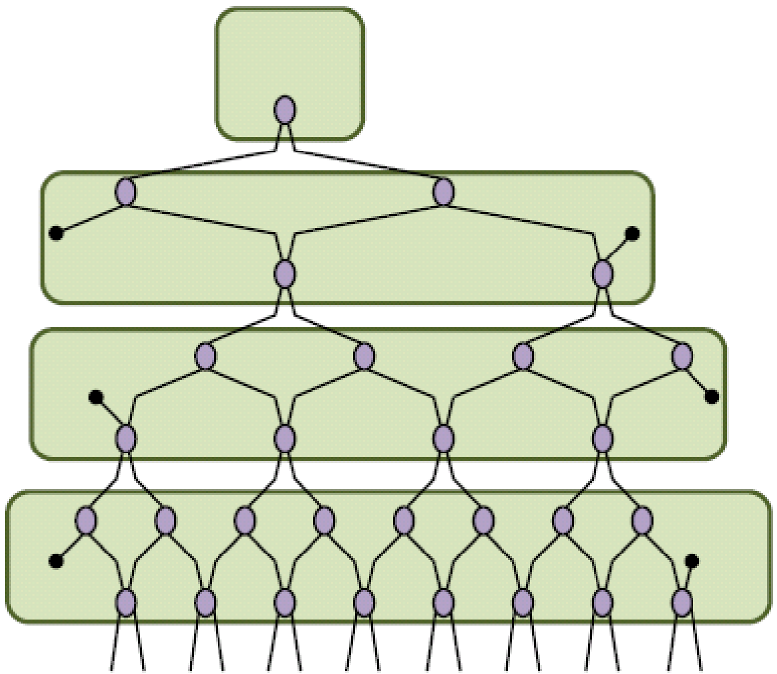
- (t, x^i): coordinates in the boundary
- z: the extra radial coordinate running from
  - $z = 0$: the boundary
  - $z = \infty$: the origin

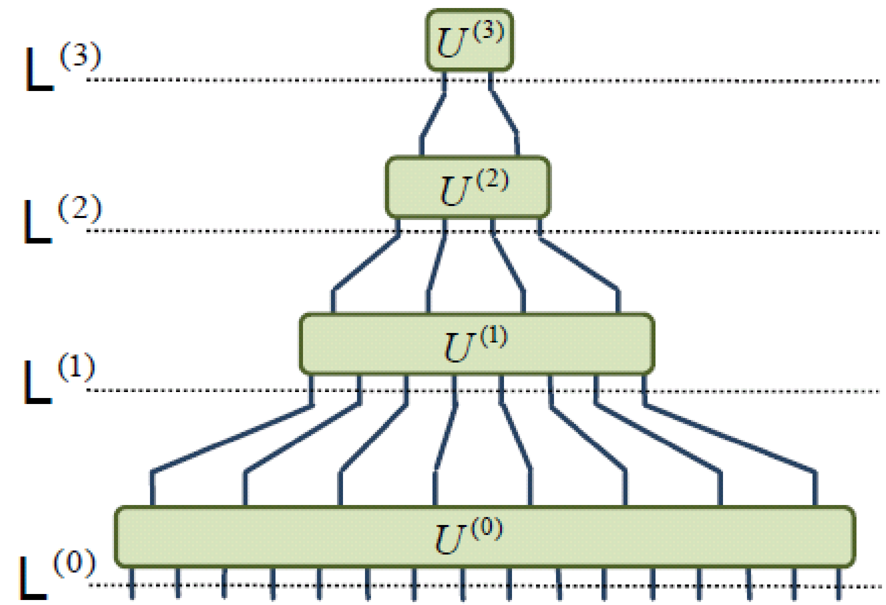
Entanglement Renormalization Tensor Networks

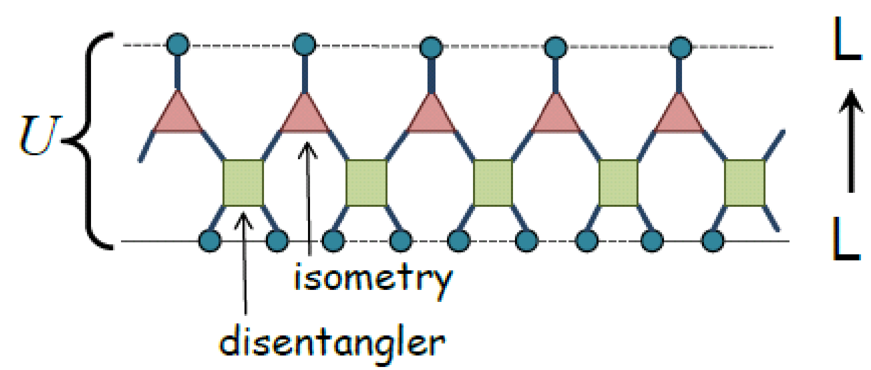


Simplified representation of MERA

$$\mathcal{L}_0 \rightarrow \mathcal{L}_1 \rightarrow \cdots \rightarrow \mathcal{L}_{w-1} \rightarrow \mathcal{L}_w \rightarrow$$

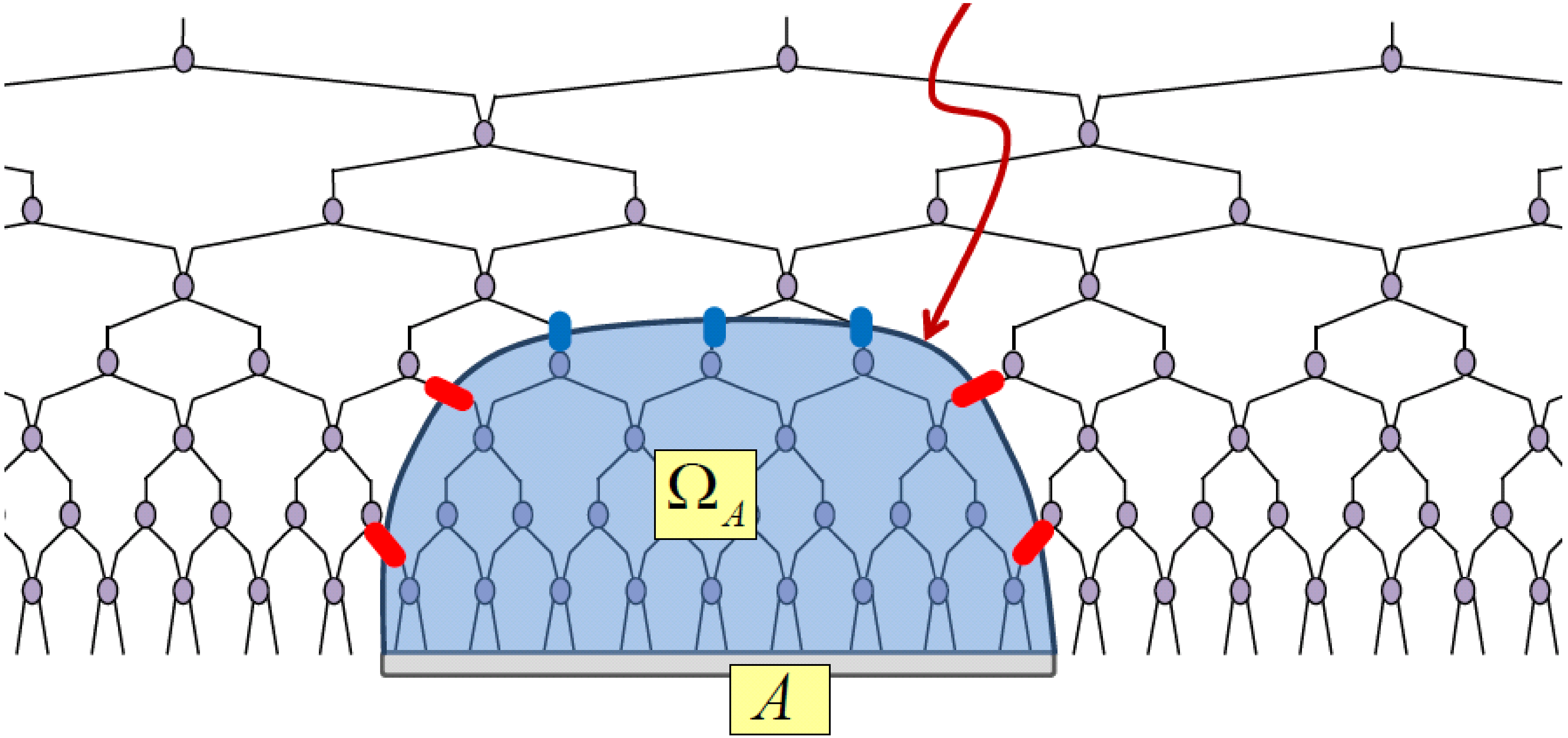






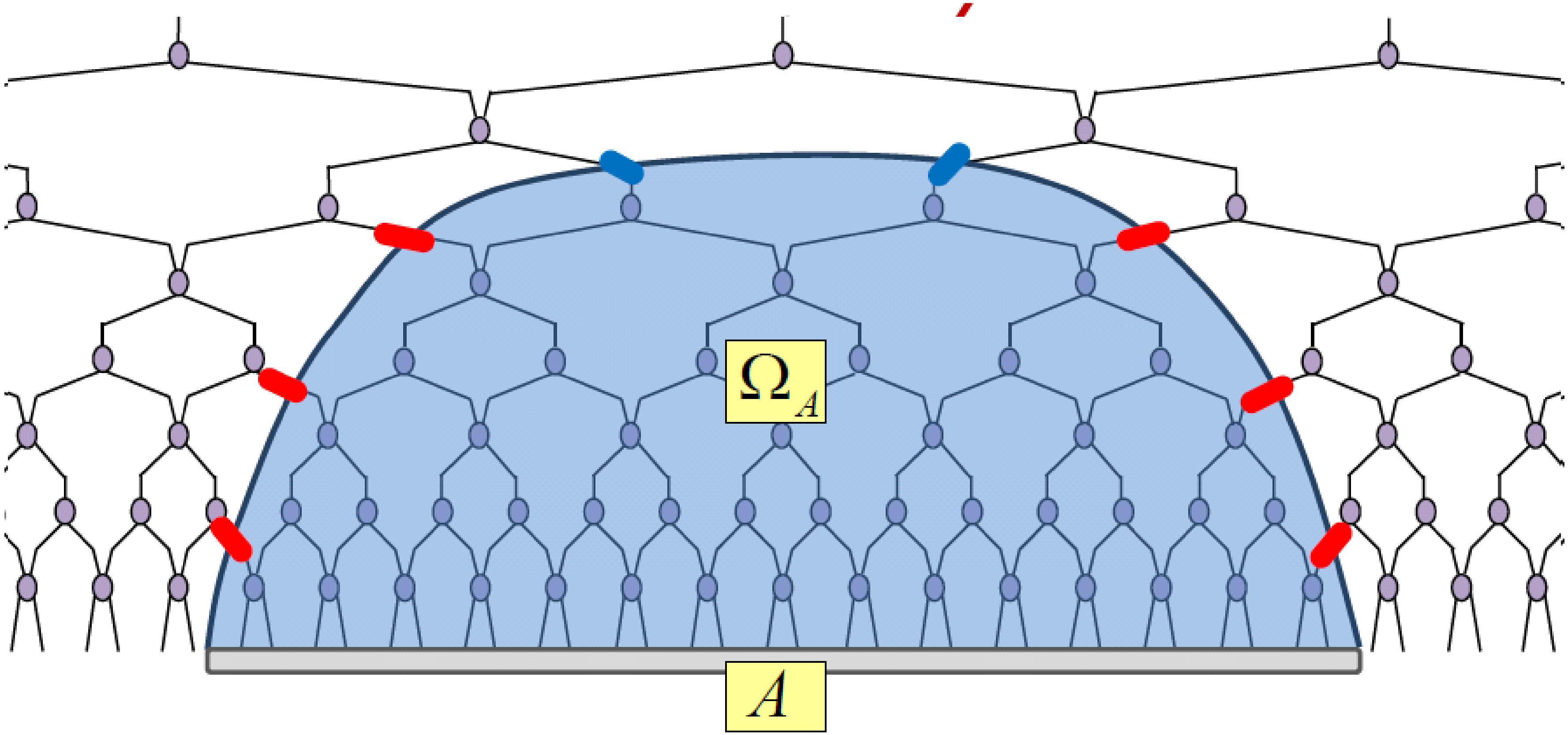
Vidal, Phys Rev Lett. 99, 220405 (2007)

Computing Entanglement Entropy in MERA



$$S_A \sim |\partial\Omega_A|$$

Computing Entanglement Entropy in MERA

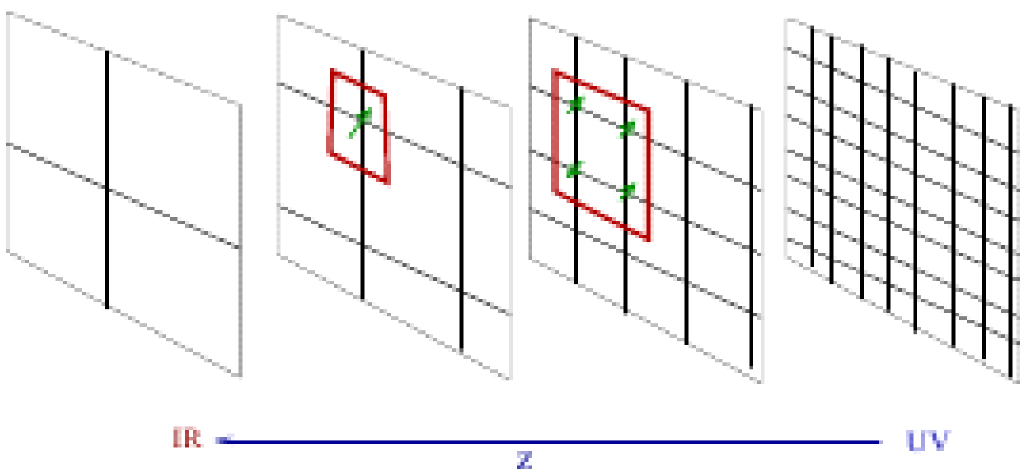


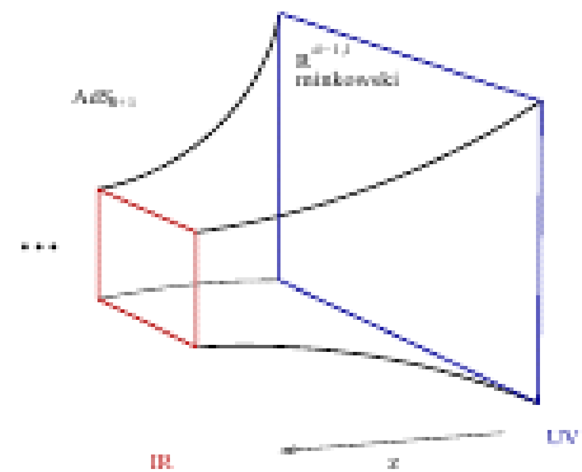
$$S_A \sim |\partial\Omega_A|$$

Entanglement Renormalization and Holography

MERA Tensor Network implements a discrete version of Anti de Sitter (AdS) Space

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^i dx^i + dz^2\right)$$





Emergence of the gravity dual picture is intimately related to the quantum entanglement of degrees of freedom in the quantum system located at MERA level w =0.

Swingle: arXiv: 0905.1317
Evenbly & Vidal arXiv : 1106.1082

Entanglement Renormalization and Holography

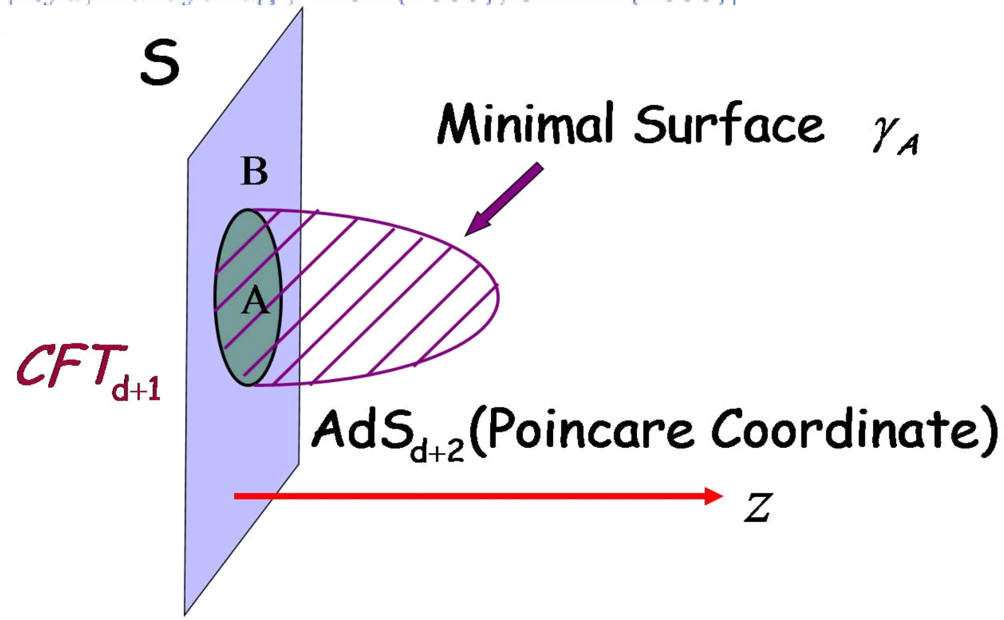
Holographic Entanglement Entropy

[Ryu, Takayanagi, PRL (2006), JHEP (2006)]

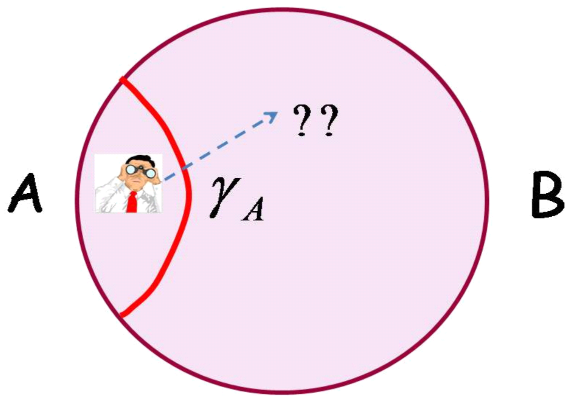
- Prescription: in regularized AdS_{d+2}

  Find the *minimal area* surface γ_A s.t. $\partial\gamma_A = \partial A$

$$\boxed{S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{(d+2)}}}$$



- $d=1$ formula $S_A = (c/3)\log(l/a)$
- Apply also to disconnected regions
- Motivation for the prescription



Holographic Mutual Information

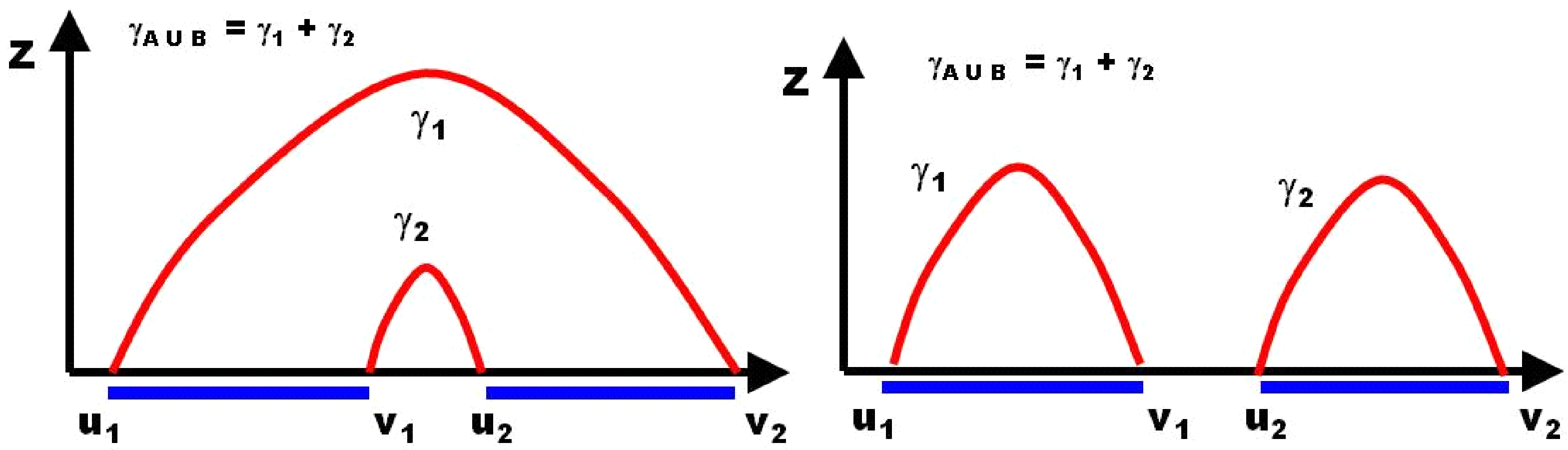
[Headrick: 1006.0047]

- Applying the RT prescription. Prediction: *Phase transition in holographic mutual information*

$$I_{(A:B)} = \frac{1}{4G_N}\left\{Length(\gamma_A) + Length(\gamma_B) - \min\left[Length(\gamma^{con}_{A\cup B}), Length(\gamma^{dis}_{A\cup B})\right]\right\}$$

Brown & Henneaux 86

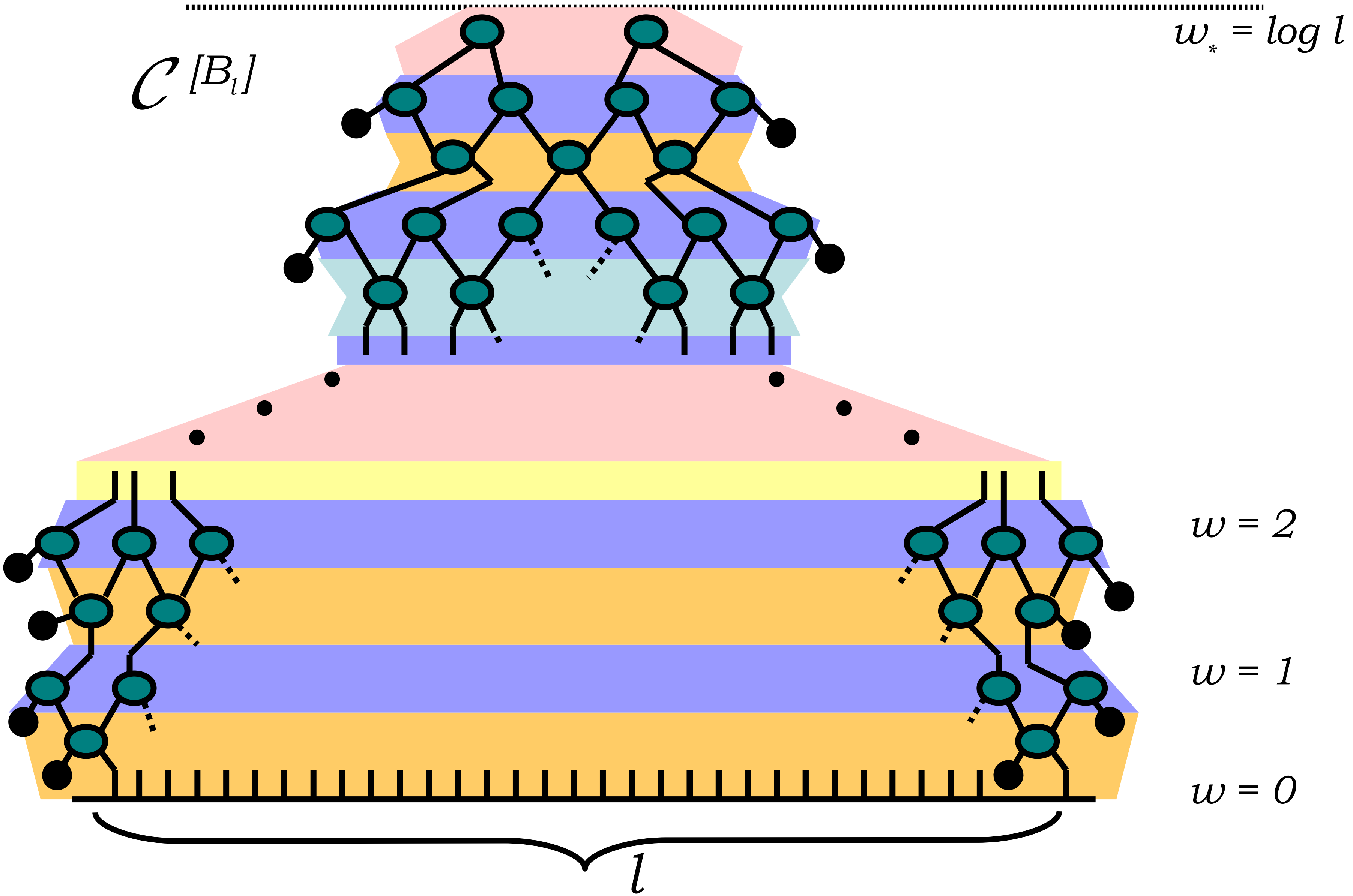
$$c = \frac{3L}{2G_N}$$



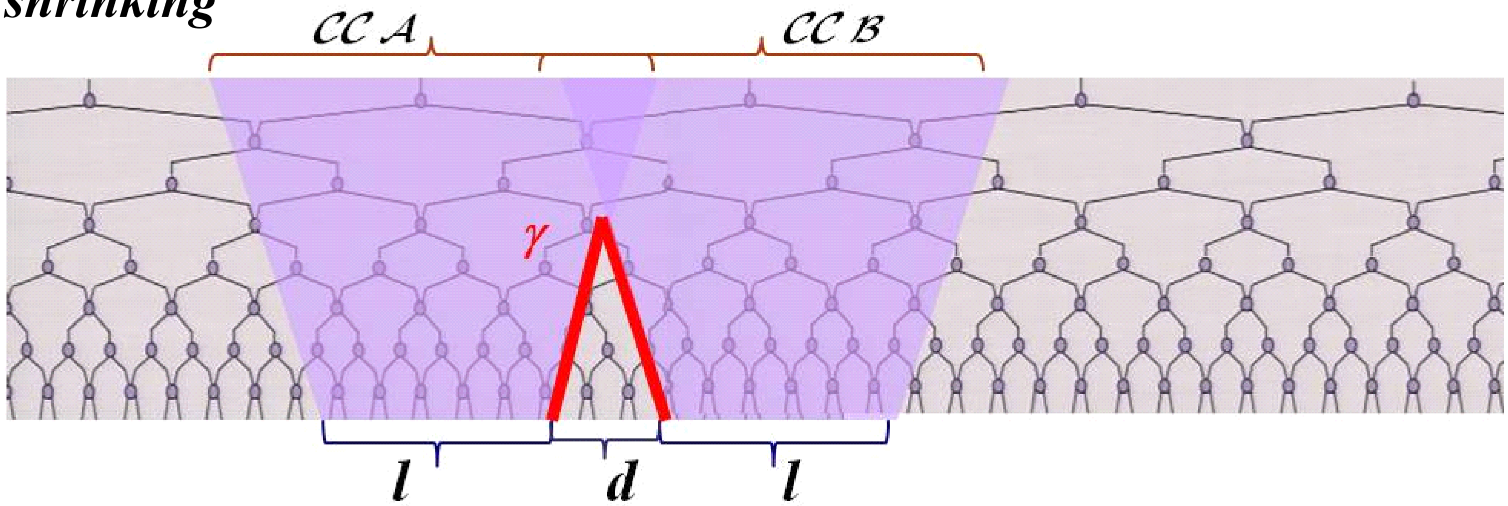
$$I_{(A:B)}(x) = \frac{c}{3}\log\left(\frac{x}{(1-x)}\right) \qquad x < 1/2$$

$$I_{(A:B)}(x) = 0 \qquad x \geq 1/2$$

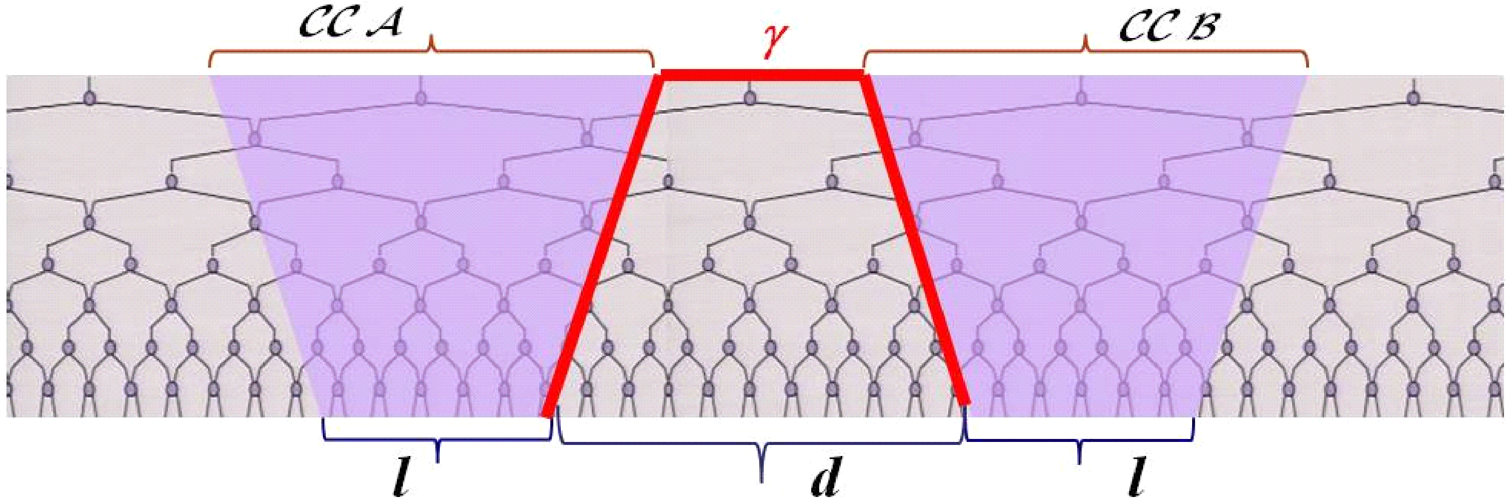
- Holographic prescription provides $Z^W_{\mathcal{R}_{n,N}}$
- *How do* $\mathcal{F}_n(x)$ *functions appear in AdS/CFT?*


$\mathcal{C}^{[B_l]}$
$w_* = \log l$
$w = 2$
$w = 1$
$w = 0$
l

- *CC overlap before shrinking*



- *CC do not overlap before shrinking*



End of the MERA geometry for the intervals ?

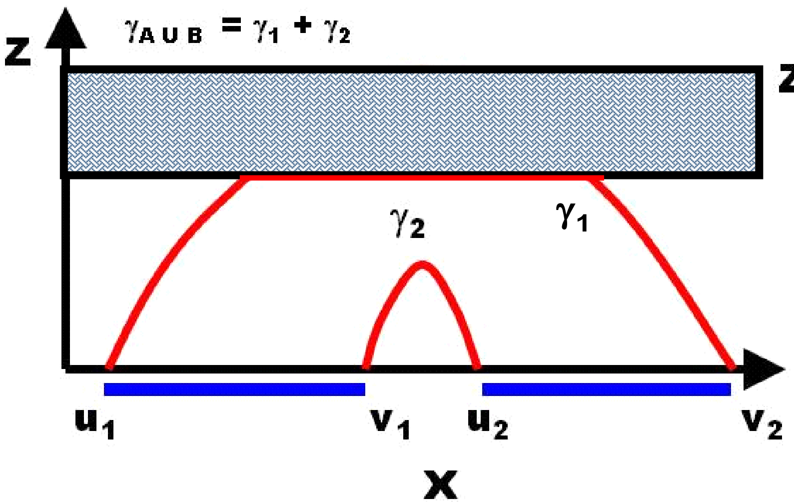
- **Each iteration of the ER coarse-graining of a block of l lattice sites**
  (i) disentangling across the boundaries of the block, followed by
  (ii) coarse-graining of sites within the block.

- **After $\log(l)$ iterations, when the block has been coarse-grained into a single site, it is no longer valid to continue this interpretation (in the next step one loses track of the sites that correspond to the original block)**

- **Inspired by AdS/ MERA we hypothesized a dual effective geometry for suitably computing the holographic MI between disjoint intervals consisting in an AdS Black Hole (AdS geometry with an horizon)**

$$z_H = l \sim 1/T \qquad ds^2_{BH} = \frac{L^2}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2\right) \qquad f(z) = 1 - \left(\frac{z}{z_H}\right)^2$$

Holographic Mutual Information 2.0



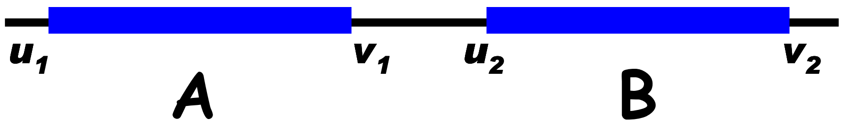
$$I_{A:B} = \frac{c}{3}\log\left(\frac{sinh(\pi T|u_1 - u_2|)sinh(\pi T|v_1 - v_2|)}{sinh(\pi T|u_2 - v_1|)sinh(\pi T|u_1 - v_2|)}\right)$$

Finite Size systems

$$\lim_{NT\to\infty}\left[\frac{\theta\nu(wT \mid iNT)\partial_z\theta_1(0 \mid iNT)}{\theta\nu(0 \mid iNT)\theta_1(wT \mid iNT)}\right] = \left[\frac{\pi T}{4sinh(\pi T\rho)}\left[1 \pm e^{-\pi NT}\cosh 2\pi T\rho + \ldots\right]\right]$$

$$w = i\rho$$

Holographic Mutual Information 2.0



- $$I_{(A:B)} = \frac{c}{3}\log\left(\frac{x}{(1-x)}\right) + \frac{c}{3}\log\left(\frac{\theta_\nu(iT|u_1-v_2||\tau)\theta_\nu(iT|u_2-v_1||\tau)}{\theta_\nu(iT|u_1-u_2||\tau)\theta_\nu(iT|v_1-v_2||\tau)}\right)$$

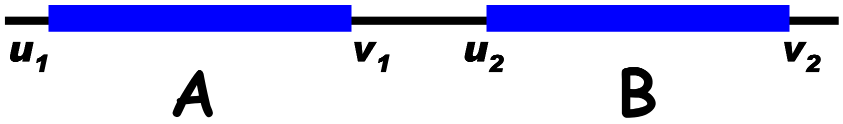
- $$I_{A:B} = \frac{c}{3}\log\left(\frac{x}{(1-x)}F_\nu(x,\tau)\right)$$

- $$F_\nu(x,\tau) = \frac{\theta_\nu(iT|u_1-v_2||\tau)\theta_\nu(iT|u_2-v_1||\tau)}{\theta_\nu(iT|u_1-u_2||\tau)\theta_\nu(iT|v_1-v_2||\tau)}$$

$$\lim_{x\to 1} F_\nu(x,\tau) = 1$$

$$\tau = iNT \qquad x = \frac{|u_1-v_1||u_2-v_2|}{|u_1-u_2||v_1-v_2|} = \frac{l^2}{(l+d)^2}$$

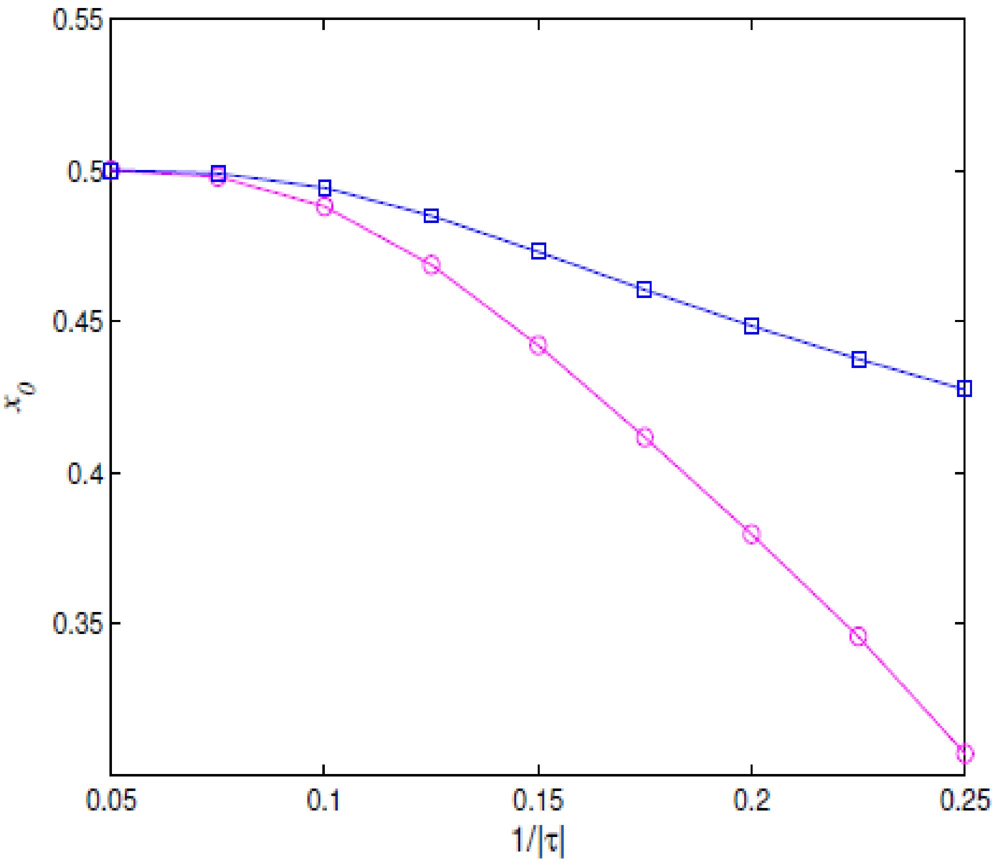
Holographic Mutual Information 2.0



$$x = \frac{|u_1 - v_1||u_2 - v_2|}{|u_1 - u_2||v_1 - v_2|} = \frac{l^2}{(l+d)^2}$$

$$1/|\tau| \propto \frac{z_H}{N} = \frac{l}{N}$$

$$I_{A:B} = \frac{c}{3} \log\left(\frac{x}{(1-x)} F_\nu(x,\tau) \right)$$



Summary

- *We reviewed some aspects of the recently proposal in which MERA tensor networks happens to be some realization of the AdS/CFT duality.*
- *Inspired by this AdS / MERA duality we hypothesized a dual effective geometry for suitably computing the holographic MI between disjoint intervals consisting in an AdS Black Hole.*

Open issues

- *Which coarse-graining procedures has an associated geometry?*
- *Holographic computation of Renyi entropies, entanglement entropy for excited states in a CFT... is there any MERA representations for these settings?*

THANK YOU!