Phase diagram and quench dynamics of the Cluster-XY spin chain

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arXiv:1112.4414 (with Alioscia Hamma (PI))

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Perimeter Institute for Theoretical Physics

Motivation

Dinamics of closed quantum systems

- Recent experiments with cold atoms, quantum dots, nanowires
- Foundations of statistical mechanics and thermodynamics: Equilibration, thermalization, closed quantum systems out of equilibrium
- Universal features? (e.g. Kibble-Zurek scaling)

Effective boundary Hamiltonians

Effective behavior of the edge in a non-trivial 2D fermionic symmetry-protected topological state with $\mathbb{Z}2$ symmetry

(Z-C. Gu, X-G. Wen, arXiv:1201.2648v1)

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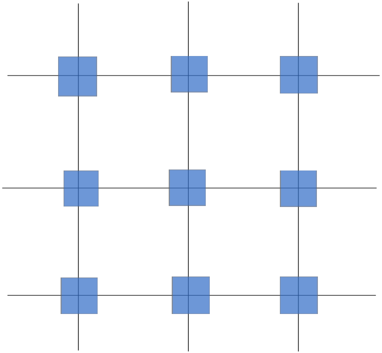
Outline

- Motivation
- Cluster state
- Model and exact solution
- Phase diagram
- Quench dynamics
- Summary

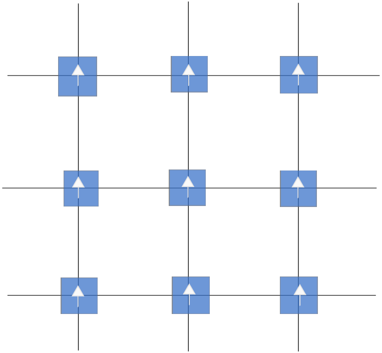
Cluster state

Preparing a cluster state



A. Doherty and S. Bartlett. Phys. Rev. Lett. **103**, 020506 (2009); S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009)

Preparing a cluster state



$$|\psi\rangle = |\uparrow\rangle^{\otimes N}, \qquad \sigma^z |\uparrow\rangle = |\uparrow\rangle$$

A. Doherty and S. Bartlett. Phys. Rev. Lett. **103**, 020506 (2009); S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009)

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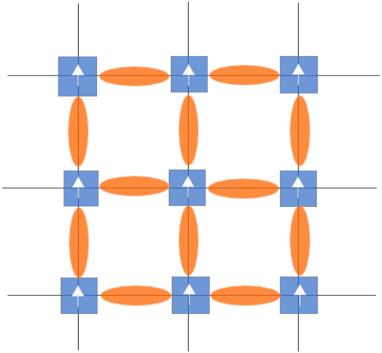
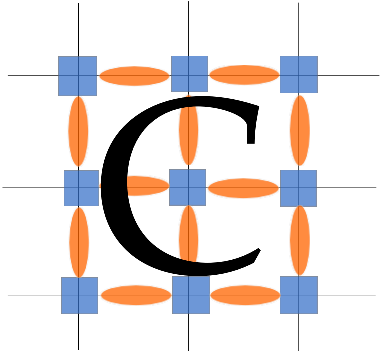


$$U = \exp\left(i\pi \left|+\right\rangle\left\langle+\right| \otimes \left|+\right\rangle\left\langle+\right|\right), \qquad \sigma^x \left|+\right\rangle = \left|+\right\rangle$$

A. Doherty and S. Bartlett. Phys. Rev. Lett. **103**, 020506 (2009); S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009)

Preparing a cluster state



$$|\psi\rangle = |C\rangle$$

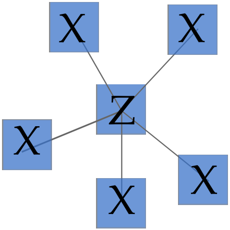
A. Doherty and S. Bartlett. Phys. Rev. Lett. **103**, 020506 (2009); S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009)

Stabilizers

We can also obtain the cluster state as the ground state of a particular stabilizer Hamiltonian:

$$K_\mu = \sigma_\mu^z \prod_{\nu\sim\mu} \sigma_\nu^x$$

$$H_C = -\sum_\mu K_\mu$$



A. Doherty and S. Bartlett. Phys. Rev. Lett. **103**, 020506 (2009); S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009)

Cluster-XY model

Cluster-XY model

Cluster-XY Hamiltonian

$$H(\lambda_x, \lambda_y, h) := -\sum_{i=1}^{N} \sigma_{i-1}^x \sigma_i^z \sigma_{i+1}^x - h \sum_{i=1}^{N} \sigma_i^z + \lambda_y \sum_{i=1}^{N} \sigma_i^y \sigma_{i+1}^y + \lambda_x \sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x$$

For periodic boundary conditions

$$Q = \prod_{i=1}^{N} \sigma_i^z, \qquad [H, Q] = 0, \qquad Q = (-1)^q$$

S. Skrøvseth and S. Bartlett. Phys. Rev. A **80**, 022316 (2009); W. Son, *et. al.* Europhys. Lett. **95**, 50001 (2011).; P. Smacchia, *et. al.* Phys. Rev. A **84**, 022304 (2011)..

Cluster-XY model

Jordan-Wigner transformation

$$c_l^\dagger = \left(\prod_{m=1}^{l-1} \sigma_m^z \right) \sigma_l^+$$

$$\{c_n, c_m\} = 0 \qquad\qquad \{c_n, c_m^\dagger\} = \delta_{nm}$$

Fourier transform

$$c_k = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{ikn} c_n, \qquad k = \frac{\pi}{N}(2m+1-q), \quad m = 0, \cdots, N-1$$

S. Sachdev. *Quantum phase transitions.* Cambridge University Press, 1999

Cluster-XY model

$$H = 2\sum_{k>0}\left[\epsilon_k\left(c_k^\dagger c_k + c_{-k}^\dagger c_{-k}\right) + i\delta_k\left(c_k^\dagger c_{-k}^\dagger + c_k c_{-k}\right)\right]$$

$$\epsilon_k = \cos(2k) - (\lambda_x + \lambda_y)\cos(k) - h, \qquad \delta_k = \sin(2k) - (\lambda_x - \lambda_y)\sin(k)$$

Bogoliubov transformation

$$\gamma_k = \cos(\theta_k/2)c_k - i\sin(\theta_k/2)c_{-k}^\dagger$$

$$\theta_k = -\arctan\left(\frac{\delta_k}{\epsilon_k}\right)$$

Cluster-XY model

Diagonal Hamiltonian

$$H = 2\sum_{k>0} \Delta_k \left(\gamma_k^\dagger \gamma_k + \gamma_{-k}^\dagger \gamma_{-k} - 1 \right)$$

$$\Delta_k = \sqrt{\epsilon_k^2 + \delta_k^2}$$

Ground state

$$|\Omega\rangle = \prod_{k>0} \left(\cos(\theta_k/2) + i \sin(\theta_k/2) c_k^\dagger c_{-k}^\dagger \right) |0\rangle_c$$

Gapless regions

$$\Delta_k = 0, \qquad \text{for some } k$$

Ising planes

$$h = \pm(\lambda_x + \lambda_y) + 1$$

Cluster transitions

$$h = \lambda_y^2 - \lambda_x\lambda_y - 1, \qquad -2 \leq \lambda_x - \lambda_y \leq 2$$

(Video)

S.M., A. Hamma, arXiv:1112.4414

Detecting critical lines

Fidelity

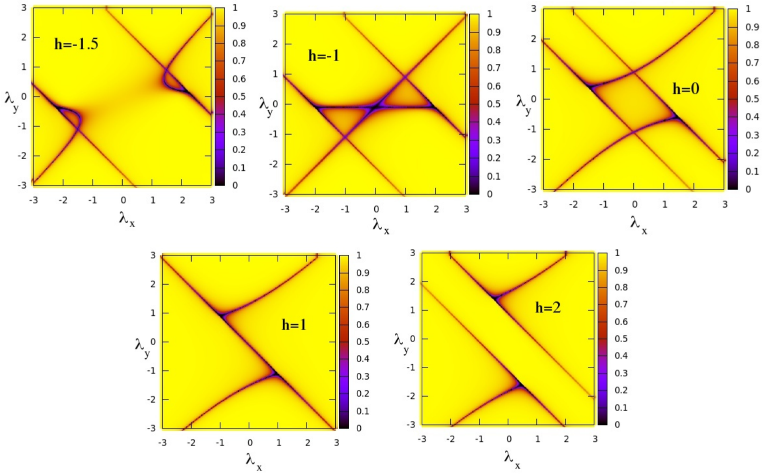
Different phases must be distinguishable from the point of view of quantum mechanics. We can use the fidelity

$$\begin{aligned}\mathcal{F}(\lambda_x,\lambda_y,h;\lambda_x',\lambda_y',h') &= |\,\langle\Omega(\lambda_x,\lambda_y,h)|\Omega(\lambda_x',\lambda_y',h')\rangle\,| \\ &= \prod_{k>0}\left|\cos\left(\frac{\theta_k(\lambda_x,\lambda_y,h)-\theta_k(\lambda_x',\lambda_y',h')}{2}\right)\right|\end{aligned}$$

P. Zanardi and N. Paunković. Phys. Rev. E, **74**, 031123 (2006); L. Campos Venuti and P. Zanardi. Phys. Rev. Lett. **99**, 095701 (2007)

Fidelity

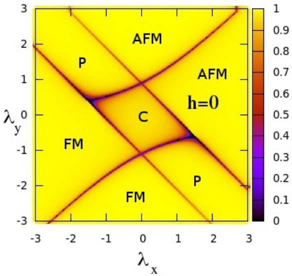
$\mathcal{F}(\lambda_y, \lambda_y + \delta\lambda_y)$, $\delta\lambda_y = 0.05$, $N = 500$



S.M., A. Hamma, arXiv:1112.4414

"Ghost" phases

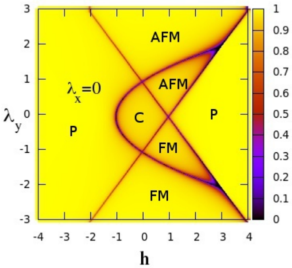
$h = 0$



$$H = -\sum_{i=1}^{N} \sigma_{i-1}^{x}\sigma_{i}^{z}\sigma_{i+1}^{x} + \lambda_y \sum_{i=1}^{N} \sigma_{i}^{y}\sigma_{i+1}^{y} + \lambda_x \sum_{i=1}^{N} \sigma_{i}^{x}\sigma_{i+1}^{x}$$

“Ghost” phases

$\lambda_x = 0$



$$H = -\sum_{i=1}^{N} \sigma^x_{i-1}\sigma^z_i\sigma^x_{i+1} - h\sum_{i=1}^{N}\sigma^z_i + \lambda_y \sum_{i=1}^{N}\sigma^y_i\sigma^y_{i+1}$$

Quench dynamics

Nonequilibrum dynamics of closed quantum systems

Quantum quenches

Local or global change of the parameters of the system.

We would like to study the dynamics and characterize the universal features of a system after a quantum quench.

Here we are interested in instantaneous critical global quenches.

A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore. Rev. Mod. Phys. **83**, 863 (2011).

Loschmidt echo

If we perform a quantum quench, we can compute the fidelity between the initial state and the time-evolved state

$$\mathcal{L}(t) = |\langle\psi_0|U(t)|\psi_0\rangle|^2 ,$$

with

$$U(t) = \exp(-itH_Q).$$

This is known as the Loschmidt echo.

It is related to the study of reversibility in statistical mechanics.

T. Gorin, T. Prosen, T.H. Seligman, and M. Znidaric. Phys. Rep. **435**, 33 (2006); L. Campos Venuti and P. Zanardi. Phys. Rev. A **81**, 022113 (2010); J. Häppölä, G.B. Halász, and A. Hamma. Phys. Rev. A **85**, 032114 (2012).

Proposed lower bound for revival time

Quasiperiodic systems will have revivals after long enough times. These can be detected using the Loschmidt echo.

A proposed lower bound for the revival time in spin chains with (anti)periodic boundary conditions is given by the Lieb-Robinson speed v_{LR}

$$T_{\mathrm{rev}} \approx \frac{N}{2v_{\mathrm{LR}}}.$$

J. Häppölä, G.B. Halász, and A. Hamma. Phys. Rev. A **85**, 032114 (2012)

Loschmidt echo for the cluster-Ising model

We start with

$$|\psi(t=0)\rangle = |\Omega(\lambda_x, \lambda_y, h)\rangle \,.$$

We can now compare the initial state with the time evolution of the quenched Hamiltonian $H = H(\lambda_x', \lambda_y', h')$

$$\begin{aligned}\mathcal{L}(t) &= \prod_{k>0} \left|\cos^2(\chi_k/2) + e^{-i4t\Delta_k} \sin^2(\chi_k/2)\right|^2 \\ &= \prod_{k>0} \left(1 - \sin^2(\chi_k) \sin^2(2t\Delta_k)\right)\end{aligned}$$

where

$$\chi_k = \theta_k(\lambda_x, \lambda_y, h) - \theta_k(\lambda_x', \lambda_y', h'),$$

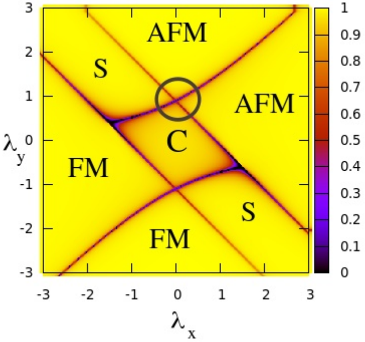
$$\Delta_k = \Delta_k(\lambda_x', \lambda_y', h').$$

First critical point

$h = 0, \;\lambda_x = 0, \;\lambda_y = 1$

The critical Hamiltonian is

$$H = -\sum_{i=1}^{N} \sigma_{i-1}^{x}\sigma_{i}^{z}\sigma_{i+1}^{x} + \sum_{i=1}^{N} \sigma_{i}^{y}\sigma_{i+1}^{y}.$$



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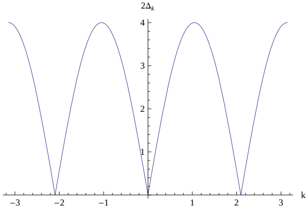
The critical Hamiltonian is

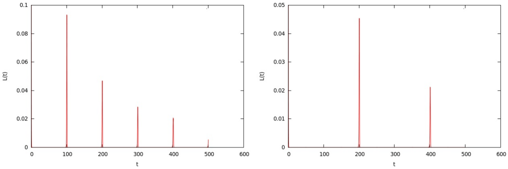
$$H = -\sum_{i=1}^{N} \sigma^x_{i-1}\sigma^z_i\sigma^x_{i+1} + \sum_{i=1}^{N} \sigma^y_i \sigma^y_{i+1}.$$

$$\boxed{v_{LR} \simeq 3.2e/\sqrt{2} = 6.15}$$

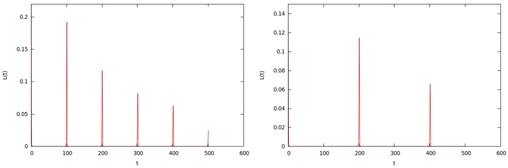


P. Smacchia, L. Amico, P. Facchi, R. Fazio, G. Florio, S. Pascazio, and V. Vedral, Phys. Rev. A **84**, 022304 (2011).

Starting from the cluster state $\lambda_y = 0.8$, $\lambda_x = 0$, $h = 0$, $N = 400$



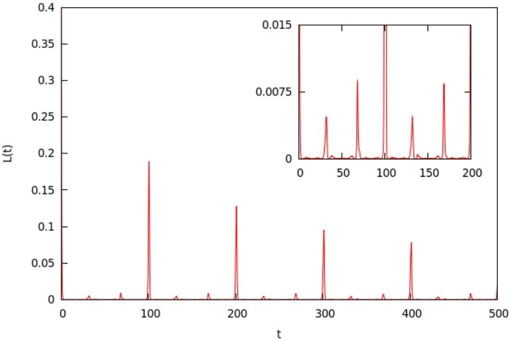
Starting from the AFM state $\lambda_y = 1.2$, $\lambda_x = 0$, $h = 0$,



S.M., A. Hamma, arXiv:1112.4414

Other interactions

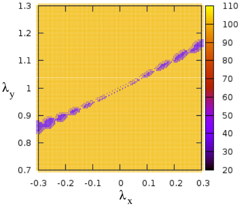
Starting from $\lambda_x = 0.2$, $\lambda_y = 1$, $h = 0$ ($N = 400$, $q = 1$)



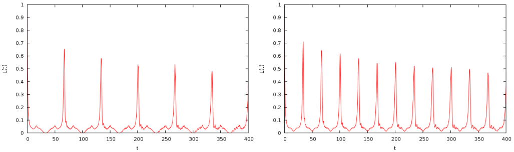
S.M., A. Hamma, arXiv:1112.4414

Along the critical line

Revival times



Loschmidt echo

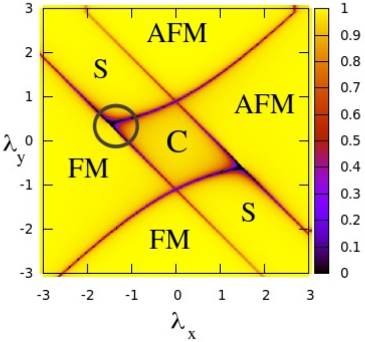


Second critical point

$h = 0,\ \lambda_x = -\frac{3}{2},\ \lambda_y = \frac{1}{2}$

The critical Hamiltonian is

$$H = -\sum_{i=1}^{N} \sigma_{i-1}^{x}\sigma_{i}^{z}\sigma_{i+1}^{x} - \frac{3}{2}\sum_{i=1}^{N} \sigma_{i}^{x}\sigma_{i+1}^{x} + \frac{1}{2}\sum_{i=1}^{N} \sigma_{i}^{y}\sigma_{i+1}^{y}.$$



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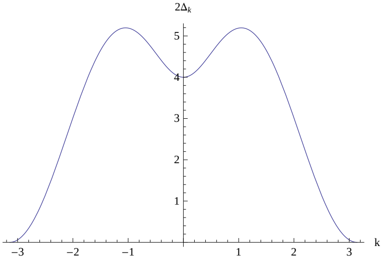
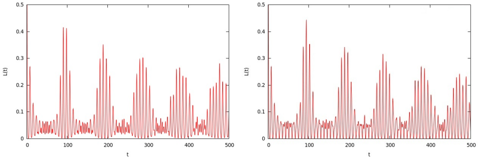
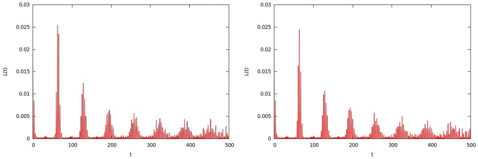
$$H = -\sum_{i=1}^{N} \sigma_{i-1}^x \sigma_i^z \sigma_{i+1}^x - \frac{3}{2} \sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x + \frac{1}{2} \sum_{i=1}^{N} \sigma_i^y \sigma_{i+1}^y .$$



Starting from the cluster state $\lambda_y = \frac{1}{2}$, $\lambda_x = -1.3$ $(N = 400)$

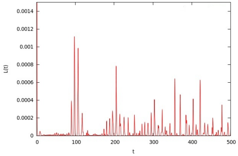


Starting from the cluster state $\lambda_y = \frac{1}{2}$, $\lambda_x = -1$ $(N = 400)$

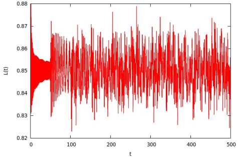


S.M., A. Hamma, arXiv:1112.4414

Starting from $\lambda_y = 0.7$, $\lambda_x = -\frac{3}{2}$ ($N = 400$) - z polarized



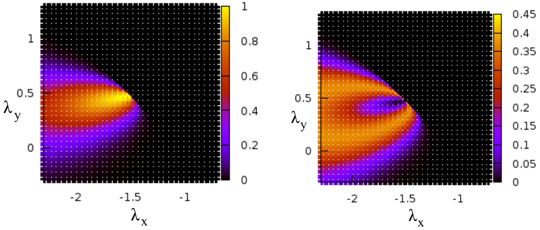
Starting from $\lambda_y = \frac{1}{2}$, $\lambda_x = -1.7$ ($N = 400$) - Ferromagnetic



S.M., A. Hamma, arXiv:1112.4414

Overlaps
Ground state and one-particle states

$$F_1(\lambda_i') = \sum_{0\leq k\leq\pi} \left|\langle\Omega(\lambda_i')|\gamma_k^\dagger\gamma_{-k}^\dagger|\Omega(\lambda_i^{(c)})\rangle\right|^2$$



S.M., A. Hamma, arXiv:1112.4414

Summary

Summary

- The cluster-XY model provides a simple benchmark with a rich phase diagram.
- This model may be useful to test new proposals on the dynamics of composite quantum systems out of equilibrium. In particular, we showed that different critical points have different effects on the Loschmidt echo.
- It would be interesting to extend these ideas even further and characterize the effect of the universality class of a critical point on the behavior of the quench dynamics.

Thank you.