Invariant Theory for Matrix Product States

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Overview

- History of the graphical notation (Penrose 1960’s)
- Recent results
- Invariants
Fig. 29.5 Diagrammatic notation for density matrices constructed by ‘summing over unknown states’. The normalized ket-vector $|\psi\rangle$ is expressed as $\psi^\alpha$ where ‘$\alpha$’ refers to ‘here’ (Earth) and ‘$\sigma$’ refers to ‘there’ (Titan). The Hermitian conjugate (bra-vector $\langle\psi|$) is $\bar{\psi}_\alpha$ and the normalization is $\bar{\psi}_\alpha\psi^\alpha = 1$. The density matrix used ‘here’ is $D^\beta_\alpha = \psi^\alpha\psi^\beta$ while that used ‘there’ is $\bar{D}_\rho^\sigma = \bar{\psi}_\alpha\psi^{2\sigma}$.


Roger Penrose, Theory of quantized directions, 1967

Zig-zag equation
Tensor networks for entanglement evolution
Sebastian Meznaric and JDB
in review, (2012)
http://arxiv.org/abs/1204.3599

Tensor networks and graphical calculus for open quantum systems
Christopher J. Wood, JDB and David G. Cory
in review (2011)
http://arxiv.org/abs/1111.6950
Version of map-state duality II
Tensor networks and graphical calculus for open quantum systems
Christopher J. Wood, JDB and David G. Cory
in review (2011)
http://arxiv.org/abs/1111.6950

FIG. 42. Graphical proof of Fig. 41 for the construction of a Kraus representation \( \{ K_\alpha \} \) for a CPTP-map \( \mathcal{E} \) from the Choi-matrix representation \( \Lambda \).
Boolean tensor network states

**Theorem 3.3** (Boolean tensor network states). A tensor network representing a Boolean quantum state is determined from the classical network description of the corresponding function.

Theorem 3.3 was given in [2], where the quantum tensor networks are found by letting each classical gate act on a linear space and from changing the composition of functions, to the contraction of tensors.

**Example 3.4** (AND-tensors). As an example of a Boolean logic tensor, consider the AND-tensor defined as

\[ \text{AND}_{jk}^i = |00\rangle\langle 0| + |01\rangle\langle 0| + |10\rangle\langle 0| + |11\rangle\langle 1| \]

We depict the contraction of the output of the AND-tensor with |1\rangle and |0\rangle as

In (a) the contraction results in creation of the product state |11\rangle and in (b) the contraction yields |00\rangle + |01\rangle + |10\rangle.
FIG. 3. Read top to bottom. A presentation of the linear fragment of the Boolean calculus. The plus (⊕) dots are XOR and the black (●) dots represent COPY. The details of (a)-(g) will be given in Sections III and IV. For instance, (d) represents the bialgebra law and (g) the Hopf law (in the case of qubits $x \oplus x = 0$, in higher dimensions the units ($+$) becomes $0| + (1| + \cdots + (d-1)|$).

FIG. 4. Diagrams read top to bottom. A presentation of the Boolean calculus with Figure 3. The details of (a)-(g) will be given in Sections III and IV. For instance, (h) represents distributivity of AND($\wedge$) over XOR ($\oplus$), and (d) shows that $x \wedge x = x$. 

Boolean tensor network states
Contracting tensor networks to solve problems

Remark 3.1 (Graphical depiction) The algorithm is depicted below. (a) gives a network realization of the function and determining if the network in (b) contracts to a value greater than zero solves a SAT instance.

Related also to work in progress, Problem solving by strongly simulating quantum circuits

T. H. Johnson J. D. Biamonte, S. R. Clark, and D. Jaksch and also related work with Jason Morton
When a tensor network represents a physical state

Theorem 3.2 (Penrose, 1967) The norm of a tensor network vanishes iff the physical situation it represents is forbidden by the rules of quantum mechanics.
Deciding if tensors are physical?

Example 3.3 (Examples of Penrose’s theorem) For example, consider a Bell state $\Phi^+ = |00\rangle + |11\rangle$. The amplitude of the first party measuring $|0\rangle$ followed by the second party measuring $|1\rangle$ is zero. This vanishing tensor network contraction is given by $\langle 01, \Phi^+ \rangle$. A second example is found by considering the norm of a state $|\psi\rangle$ formed by a network of connected tensors, by taking an inner product with a conjugated copy of itself $\langle \psi, \psi \rangle$. If this inner product vanishes, the network necessarily represents a non-physical quantum state, by Penrose’s theorem.

Corollary 3.4 All physical Boolean states are satisfiable.


Problem solving by strongly simulating quantum circuits T. H. Johnson J. D. Biamonte, S. R. Clark, and D. Jaksch and also related work with Jason Morton
Invariant Theory for Matrix Product States
Jacob Biamonte, Ville Bergholm and Marco Lanzagorta
(2012)

Graphical SVD

FIG. 3. Diagrammatic Schmidt decomposition for $|\psi\rangle \in A \otimes B$. 
Entanglement topology

(a)  

(b)  

(c)  

MPS

References for MPS in the paper
Polynomial invariants

- Written in terms of a coefficients of a state
Contracting permutations
Alternatives to $\text{Tr} \rho^n$

(a)  
(b)  
(c)
Returning to the diagrammatic SVD
Example
Example part II
References