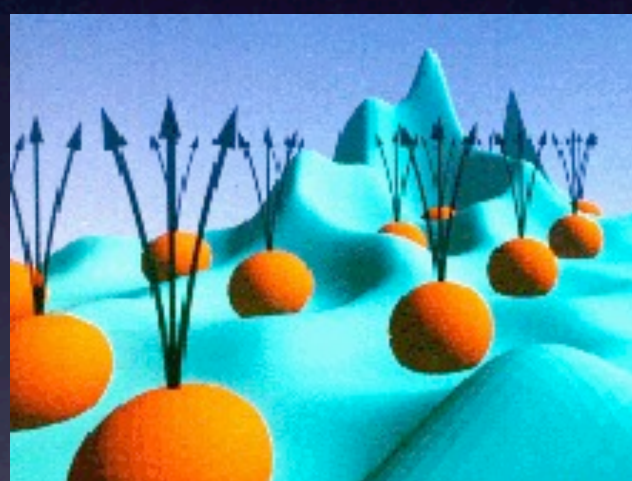
Entanglement, fractional magnetization and long range interactions

Andrea Cadarso-Rebolledo
joint work with D. Pérez-García, M. Sanz, M. Wolf and J.I. Cirac

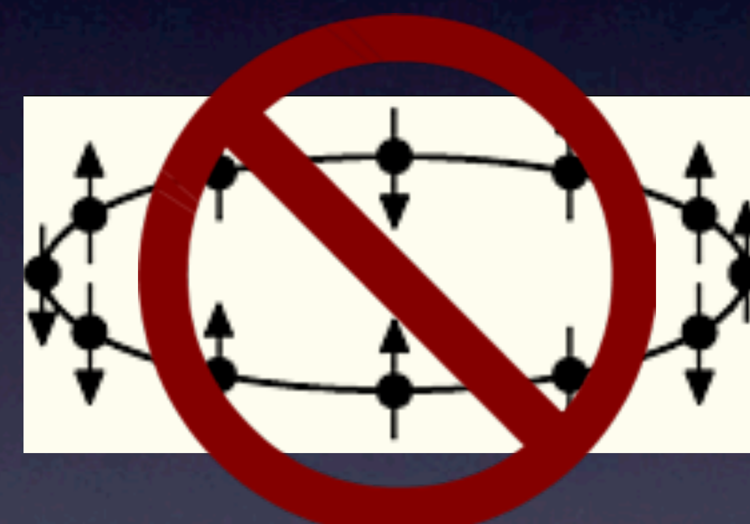
Networking Tensor Networks, Benasque (10-05-2012)

Our problem

Fractionalization of a quantum number



Impossibility of being well-approximated by the GS of a local Hamiltonian / long-range interactions



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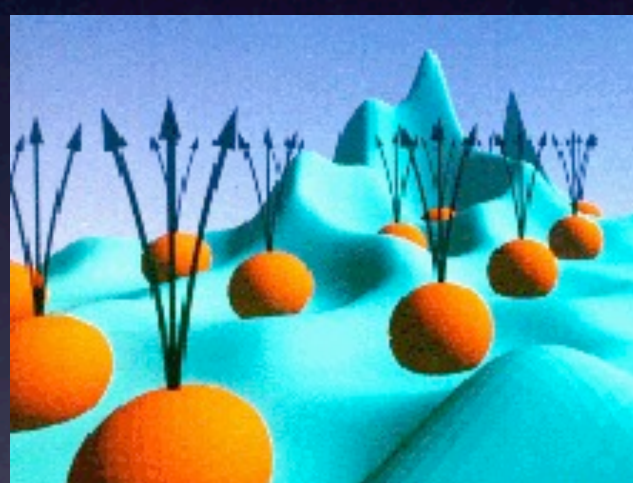

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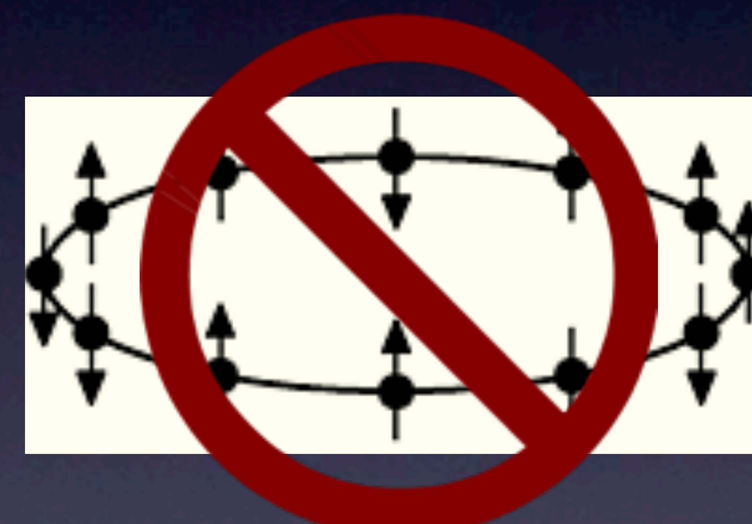
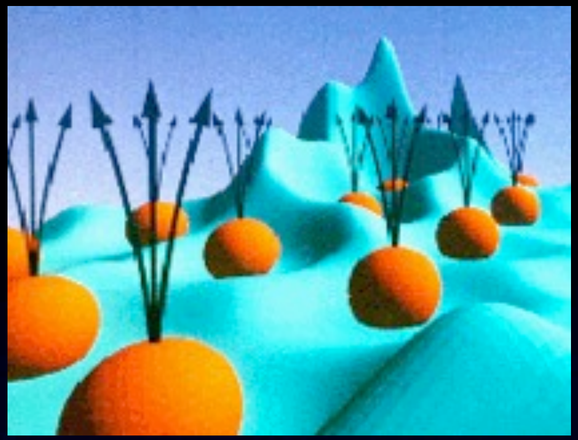


Entanglement?

Large fractionalization of a quantum number



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Large entanglement in a quantum state

MPS assumptions for our problem

We consider traslational invariant spin chains, that is, states of the form

$$|\psi_A\rangle = \sum_{i_1,\ldots,i_N=1}^{d} \mathrm{tr}[A_{i_1}\ldots A_{i_N}]|i_1\ldots i_N\rangle$$

$A_i \in \mathcal{M}_{D\times D},\ d:$ dimension of the Hilbert space corresponding to the physical system.

M. Fannes, B. Nachtergaele and R. F. Werner, Commun. Math. Phys. 144, 443-490 (1992).

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An MPS $|\psi_A\rangle$ is injective if there exists an L such that for regions of size L or larger, different boundary conditions give rise to different states.

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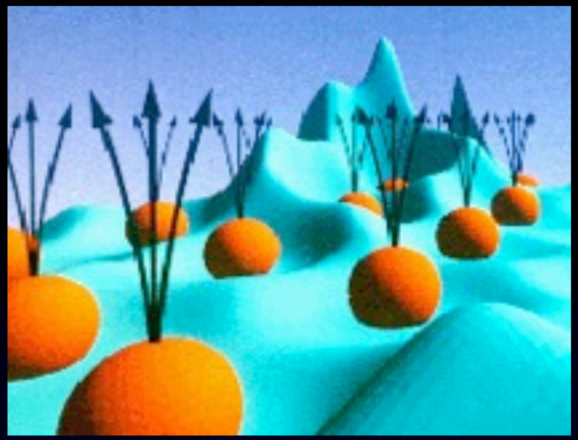

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Fractional Quantum Hall Effect

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D. C. Tsui

H. Störmer

It is a physical phenomenon concerning the collective behaviour in a two-dimensional system of electrons.

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Large fractionalization requires large entanglement

Let $|\psi\rangle$ be a spin J, $U(1)$ invariant MPS with magnetization per particle m verifying $J - m = \frac{q}{p}$ (p and q relatively prime).

Then there exists a multiple of p, which we denote by $\tilde{p}$, such that the entropy of the reduced density matrix of any region of size $L = k\tilde{p}$ ($\forall k$) verifies

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D. Pérez-García, F. Verstraete, M.M. Wolf, J.I. Cirac, Quantum Inf. Comput. 7, 401 (2007)

M. Sanz, M.M. Wolf, D. Perez-García and J. I. Cirac, Phys. Rev. A 79, 042308 (2009).

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Given two injective MPS, $|\psi_A\rangle$ and $|\psi_B\rangle$, then $\||\psi_A\|, \||\psi_B\rangle\| = 1$ up to an exponentially (in N) small correction. Moreover, either both are equal for all N, or $|\langle\psi_A|\psi_B\rangle| = 0$ up to an exponentially (in N) small correction.

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- If $|\psi\rangle = \sum_{r=1}^{n} \lambda_r |\psi_r\rangle$ where $|\psi_r\rangle$ are different injective MPS, then ρ_L is **"close"** to $\oplus_r |\lambda_r|^2 \rho_r$, being ρ_r the reduced density matrix of $|\psi_r\rangle$.

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If p is a factor of N, then the state can be written as a superposition of p p-periodic *different and injective* MPS with bonds D_i (and with the property that $\sum_i D_i = D$).

Otherwise, if p is not a factor of N, then $|\psi_A\rangle = 0$.

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  We must take into account that:
  - A condition on the number of blocks implies a restriction on $p(J - m) = q$.

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Let p be the smallest integer such that, after blocking p sites together, $|\psi\rangle$ has a block-diagonal representation with injective blocks. Then $p(J-m)=q$, with q an integer.

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Let m be any rational number and $p \in \mathbb{N}$ such that there exist two quantum states of (local spin J and) pN and $(N+1)p$ particles respectively, for some N, having both of them magnetization per particle m. Then $p(J-m)=q$ with q integer.

D. Perez-Garcia, M. Sanz, C.E. Gonzalez-Guillen, M.M. Wolf, J.I. Cirac, New J. Phys. 12 (2010) 025010.

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Let us assume that $J - m = \frac{q}{p}$ with $\gcd(p, q) = 1$ in a $U(1)$ symmetric MPS, then the MPS has only $\tilde{p}$-periodic blocks with $\tilde{p}$ a multiple of p. Moreover, states belonging to blocks of different periods are different.

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Sketch of the proof

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$|\psi\rangle$, J, m, $J-m=\frac{p}{q}$

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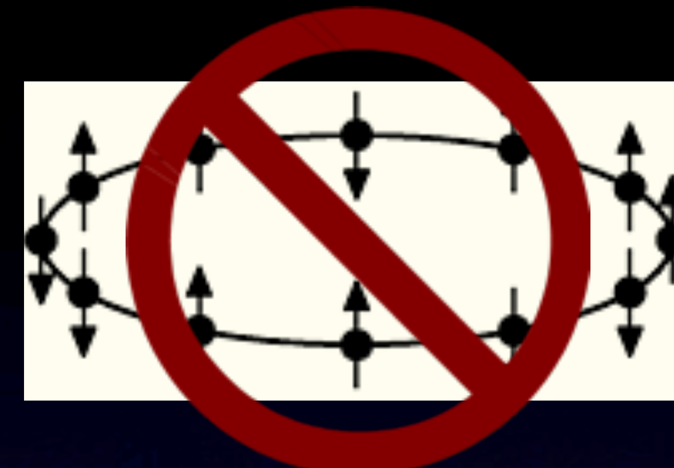

Large entanglement in a quantum state

Large interaction length implies large entanglement

Given an MPS $|\psi_A\rangle$ such that, for $\alpha = \frac{1}{6}$, we can upper-bound the α-Renyi entropy by

$$S_\alpha(\rho_A^L) \leq \frac{4}{5}\log\epsilon + \frac{1}{10}(L\log d - \log L) - \log\frac{d}{4}$$

where ρ_A^L is the reduced density matrix of a region of a certain size L, there exists another MPS $|\psi_{\tilde{A}}\rangle$ with the following properties:

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- $|\psi_A\rangle = \sum_{i_1,\ldots,i_L} \mathrm{tr}[A_{i_1} \ldots A_{i_L}] |i_1 \ldots i_L\rangle$ then the **normalized reduced density matrix** (L particles) is

$$\rho_A^L = \sum_{\substack{i_1,\ldots,i_L \\ cj_1,\ldots,j_L c}} \mathrm{tr}[A_{j_L}^\dagger \cdots A_{j_1}^\dagger \Lambda A_{i_1} \cdots A_{i_L}] |i_1 \cdots i_L\rangle\langle j_1 \cdots j_L|$$

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- Given a density operator ρ. If $0 < \alpha < 1$, then

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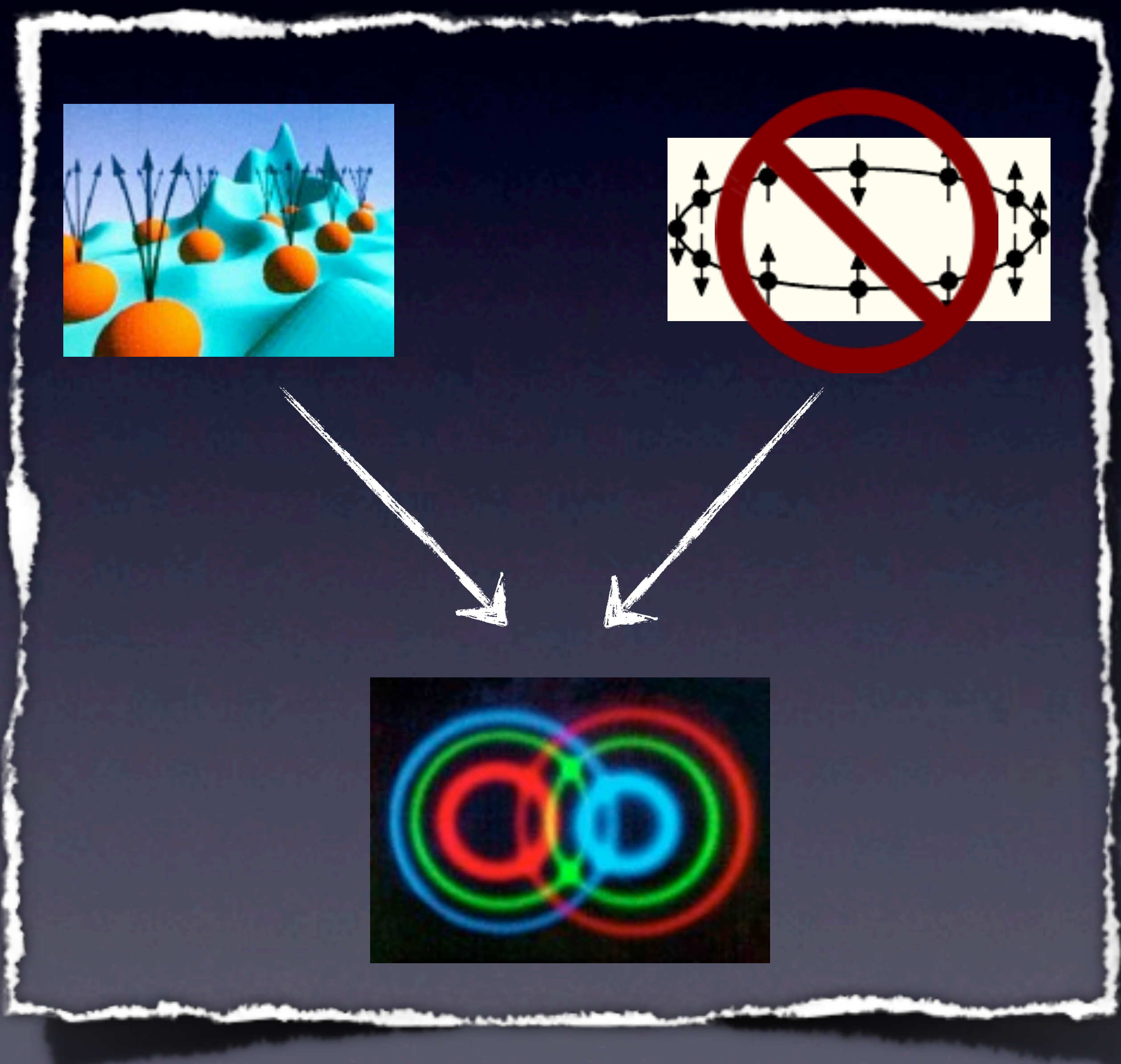
Summary



- A large fractionalization in the magnetization requires large entanglement in a quantum system.
- The absence of a local model implies a large entanglement in a quantum system.
- MPS allow us to work formally with these physical concepts and deduce consequences in full generality.

Thank you!

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