Lattice gauge theory problems for tensor networks

Karl Jansen

- Introduction
- Studying strong interaction by a 4-dimensional lattice
  - Breakthrough in Simulation Algorithm
  - Selected Results
- A toy model challenge for tensor networks
- Summary
**strong force**

Particles Interaction

\[
\begin{align*}
\text{u} & \quad \text{Quarks} & \quad \text{Gluon} \\
\text{d} & \quad &
\end{align*}
\]

**elektroweak force**

Particles Interaction

\[
\begin{align*}
\text{Ve} & \quad \text{Photon} \\
\text{e} & \quad \text{W, Z-Boson}
\end{align*}
\]
The complete Particle Spectrum

- there are three generations
- is there a Higgsboson?
- Gravitation, interaction particle $\rightarrow$ Graviton
Why we believe in Quantum Field Theories: Electromagnetic Interaction

Quantum Electrodynamics (QED)

coupling of the electromagnetic interaction is small
⇒ perturbation theory 4-loop calculation

anomalous magnetic moment of the electron

\[ a_e(\text{theory}) = 1159652201.1(2.1)(27.1) \cdot 10^{-12} \]
\[ a_e(\text{experiment}) = 1159652188.4(4.3) \cdot 10^{-12} \]

anomalous magnetic moment of the muon

\[ a_\mu(\text{theory}) = 11659169.6(9.4) \cdot 10^{-10} \]
\[ a_\mu(\text{experiment}) = 11659203.0(8.0) \cdot 10^{-10} \]
The Large Hadron Collider (LHC)

The search of the missing stone of the standard model of particle interaction:

**The Higgs Boson** generates mass of all quarks and leptons
Experimental bounds on the Higgs boson mass

- only small window remains
- excess at $125 - 126\text{GeV}$
- clarification with this year's run (most probably)
Quarks are the fundamental constituents of nuclear matter

\[ f(x, Q^2) \mid x \approx 0.25, Q^2 > 10 \text{GeV} \] independent of \( Q^2 \)

\( x \) momentum of quarks, \( Q^2 \) momentum transfer

Interpretation (Feynman): scattering on single quarks in a hadron \( \rightarrow \) (Bjorken) scaling

Friedman and Kendall, 1972)
Quantum Fluctuations and the Quark Picture

analysis in perturbation theory

$$\int_0^1 dx f(x, Q^2) = 3 \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} - a(n_f) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - b(n_f) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 \right]$$

$- a(n_f), b(n_f)$ calculable coefficients

deviations from scaling $\rightarrow$ determination of strong coupling
Why Perturbation Theory fails for the Strong Interaction

- situation becomes incredibly complicated

- value of the coupling (expansion parameter)
  \[ \alpha_{\text{strong}}(1\text{fm}) \approx 1 \]

⇒ need different ("exact") method
⇒ has to be non-perturbative

- Wilson’s Proposal: Lattice Quantum Chromodynamics
**Lattice Gauge Theory had to be invented**

→ **Quantum ChromoDynamics**

Asymptotic freedom → confinement

Distances \(\ll 1\text{fm}\) → Distances \(\gg 1\text{fm}\)

World of quarks and gluons → World of hadrons and glue balls

Perturbative description → Non-perturbative methods

**Unfortunately, it is not known yet whether the quarks in quantum chromodynamics actually form the required bound states. To establish whether these bound states exist one must solve a strong coupling problem and present methods for solving field theories don’t work for strong coupling.**

Wilson, Cargese Lecture notes 1976
Schwinger model: 2-dimensional Quantum Electrodynamics

Schwinger 1962
(typical system explored in DESY summer student programme;
www.desy.de/summerstudents/)

Quantization via Feynman path integral

\[ Z = \int DA_\mu D\Psi D\bar{\Psi} e^{-S_{\text{gauge}} - S_{\text{ferm}}} \]

Fermion action

\[ S_{\text{ferm}} = \int d^2x \bar{\Psi}(x) [D_\mu + m] \Psi(x) \]

gauge covariant derivative

\[ D_\mu \Psi(x) \equiv (\partial_\mu - ig_0 A_\mu(x)) \Psi(x) \]

with \( A_\mu \) gauge potential, \( g_0 \) bare coupling

\[ S_{\text{gauge}} = \int d^2xF_{\mu\nu}F_{\mu\nu}, \quad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \]

equations of motion: obtain classical Maxwell equations
Properties of the Schwinger Model

- existence of bound states (mass gap)
- asymptotic free \((g_0 \to 0\) for distance between charges going to zero)
- exactly solvable for zero fermion mass (Coleman)
- super-renormalizable

\(\Rightarrow\) valuable test laboratory for QCD
  (simulations can be done on your desk top)
Lattice Schwinger model

introduce a 2-dimensional lattice with lattice spacing $a$

fields $\Psi(x)$, $\bar{\Psi}(x)$ on the lattice sites

$x = (t, x)$ integers

discretized fermion action

$$S \rightarrow a^2 \sum_x \bar{\Psi} \left[ \gamma_\mu \partial_\mu + m \right] \Psi(x)$$

$$\partial_\mu = \frac{1}{2} \left[ \nabla^*_\mu + \nabla_\mu \right]$$

$$\nabla_\mu \Psi(x) = \frac{1}{a} \left[ \Psi(x + a\hat{\mu}) - \Psi(x) \right], \quad \nabla^*_\mu \Psi(x) = \frac{1}{a} \left[ \Psi(x) - \Psi(x - a\hat{\mu}) \right]$$

fermion propagator

$$\tilde{G} = \frac{i}{a} \left[ p_\mu \gamma_\mu + m \frac{1}{a} (i \gamma_\mu \sin p_\mu a) + m \right]^{-1}$$

→ has poles for $p \approx 0$ AND $p \approx \pi/a$

⇒ doubling of spectrum
The Wilson-Dirac Operator

Wilson’s suggestion: add a second derivative term

\[ S \rightarrow a^2 \sum_x \bar{\Psi} \left[ \gamma_{\mu} \partial_{\mu} - r \underbrace{\partial^2_{\mu}}_{\nabla_{\mu}^* \nabla_{\mu}} + m \right] \Psi(x) \]

\[ \rightarrow \tilde{G} = \left[ \frac{1}{a} (i \gamma_{\mu} \sin p_{\mu} a) + \frac{r}{a} \sum_{\mu} (1 - \cos p_{\mu} a) + m \right]^{-1} \]

pole for \( p \approx 0 \)

at \( p \approx \pi/a \): \( \rightarrow \tilde{G}^{-1} \approx \frac{r}{a} \)

\Rightarrow\) unwanted fermion doubler decouples in continuum limit \((a \rightarrow 0)\)

Problem: Wilson term acts as a mass term

\Rightarrow\) breaking of chiral symmetry: \( D\gamma_5 + \gamma_5 D = 0 \) for fermion mass \( m = 0 \)
clash between *chiral symmetry* and *fermion proliferation*

→ Nielsen-Ninomiya theorem:

For any lattice Dirac operator $D$ the conditions

- $D$ is local (bounded by $C e^{-\gamma/a|x|}$)
- $\tilde{D}(p) = i \gamma_\mu p_\mu + O(ap^2)$ for $p \ll \pi/a$
- $\tilde{D}(p)$ is invertible for all $p \neq 0$
- $\gamma_5 D + D \gamma_5 = 0$

cannot be simultaneously fulfilled

*The theorem simply states the fact that the Chern number is a cobordism invariant* (Friedan)
Implementing gauge invariance

Wilson’s fundamental observation: introduce Paralleltransporter connecting the points $x$ and $y = x + a\hat{\mu}$:

$$U(x, \mu) = e^{iaA_\mu(x)} \in U(1)$$

$\Rightarrow$ lattice derivatives

$$\nabla_\mu \Psi(x) = \frac{1}{a} \left[ U(x, \mu) \Psi(x + \mu) - \Psi(x) \right]$$

$$\nabla^*_\mu \Psi(x) = \frac{1}{a} \left[ \Psi(x) - U^{-1}(x - \mu, \mu) \Psi(x - \mu) \right]$$

action gauge invariant under

$$\Psi(x) \rightarrow g(x) \Psi(x), \quad \bar{\Psi}(x) \rightarrow \bar{\Psi}(x)g^*(x),$$

$$U(x, \mu) \rightarrow g(x)U(x, \mu)g^*(x + \mu)$$
Self-interaction of gauge fields

basic object: plaquette:

\[ U_p = U(x, \mu) U(x + \mu, \nu) U^{-1}(x + \nu, \mu) U^{-1}(x, \nu) \rightarrow F_{\mu\nu} F^{\mu\nu}(x) \quad \text{for} \quad a \rightarrow 0 \]

action

\[ S_{\text{gauge}} = a^2 \sum_x \{ \beta \left[ 1 - \text{Re}(U(x,p)) \right] \} \]
Quantization of Theory

\[ Z = \int_{\text{fields}} e^{-S} \]

Partition functions (pathintegral) with Boltzmann weight (action) \( S \)

\[ S = a^2 \sum_x \left\{ \beta \left[ 1 - \text{Re}(U(x,p)) \right] + \bar{\psi} \left[ m_0 + \frac{1}{2} \left\{ \gamma_\mu (\partial_\mu + \partial^*_\mu) - a \partial^*_\mu \partial_\mu \right\} \right] \psi \right\} \]
Physical Observables

expectation value of physical observables $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int_{\text{fields}} \mathcal{O} e^{-S}$$

↓ lattice discretization

01011100011100011110011

↓
Lattice QCD

change:

- 2-d $\rightarrow$ 4-d
- gauge field $U(x, \mu) \in U(1) \rightarrow U(x, \mu) \in SU(3)$
- Pauli matrices $\sigma_\mu \rightarrow$ Dirac-matrices $\gamma_\mu$
- spinors become 12-component complex vectors
- theory needs renormalization
Costs of dynamical fermions simulations, the “Berlin Wall"

see panel discussion in Lattice2001, Berlin, 2001

formula \( C \propto \left( \frac{m_\pi}{m_\rho} \right)^{-z_\pi} (L)^{z_L} (a)^{-z_a} \)

\( z_\pi = 6, \ z_L = 5, \ z_a = 7 \)

“both a \( 10^8 \) increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place.”

(Wilson, 1989)

⇒ need of Exaflops Computers

physical contact to \( \chiPT \) (?)
Why are fermions so expensive?

– need to evaluate

\[ Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\bar{\psi}\{D_{\text{Dirac}}\}\psi} \propto \det[D_{\text{Dirac}}] \]

– bosonic representation of determinant

\[ \det[D_{\text{lattice}}] \propto \int \mathcal{D}\Phi^\dagger \mathcal{D}\Phi e^{-\Phi^\dagger\{D^{-1}_{\text{lattice}}\}\Phi} \]

– need vector \( X = D^{-1}_{\text{lattice}}\Phi \)

– solve linear equation \( D_{\text{lattice}}X = \Phi \)

\( D_{\text{lattice}} \) matrix of dimension 100million \( \otimes \) 100million (however, sparse)

– number of such “inversions”: \( O(100) \) for one field configuration

– want: \( O(1000 - 10000) \) such field configurations
A generic improvement for Wilson type fermions

New variant of HMC algorithm (Urbach, Shindler, Wenger, K.J.)
(see also SAP (Lüscher) and RHMC (Clark and Kennedy) algorithms)

- even/odd preconditioning
- (twisted) mass-shift (Hasenbusch trick)
- multiple time steps

1000 configurations with L=2fm

- comparable to staggered
- reach small pseudo scalar masses ≈ 300MeV
Computer and algorithm development over the years

time estimates for simulating $32^3 \cdot 64$ lattice, 5000 configurations

$\rightarrow O(\text{few months})$ nowadays with a typical collaboration supercomputer contingent
State of the art

**BG/P**

Blue Gene/P system structure

- **Node Card**
  - 32 chips 4x4x2
  - 32 compute, 0-1 IO cards
  - 435 GF/s, 64 GB

- **Rack**
  - 32 Node Cards
  - Cabled 8x8x16
  - 13.9 TF/s, 2 TB

- **System**
  - 72 Racks, 72x32x32
  - 1 PF/s, 144 TB

- **Chip**
  - 4 processors
  - 13.6 GF/s

- **Compute Card**
  - 1 chip
  - 13.6 GF/s
  - 2.0 GB DDR2
  - (4.0GB optional)
Strong Scaling

- Test on 72 racks BG/P installation at supercomputer center Jülich
- using tmHMC code
- **Cyprus (Nicosia)**
  C. Alexandrou, M. Constantinou, T. Korzec, G. Koutsou

- **France (Orsay, Grenoble)**
  R. Baron, B. Bloissier, Ph. Boucaud, M. Brinet, J. Carbonell, V. Drach,
  P. Guichon, P.A. Harraud, Z. Liu, O. Pène

- **Italy (Rome I,II,III, Trento)**
  P. Dimopoulos, R. Frezzotti, V. Lubicz, G. Martinelli, G.C. Rossi, L. Scorzato,
  S. Simula, C. Tarantino

- **Netherlands (Groningen)**
  A. Deuzeman, E. Pallante, S. Reker

- **Poland (Poznan)**
  K. Cichy, A. Kujawa

- **Spain (Valencia)**
  V. Gimenez, D. Palao

- **Switzerland (Bern)**
  U. Wenger

- **United Kingdom (Glasgow, Liverpool)**
  G. McNeile, C. Michael, A. Shindler

- **Germany (Berlin/Zeuthen, Hamburg, Münster)**
  F. Farchioni, X. Feng, J. González López, G. Herdoiza, K. Jansen, I. Montvay,
  G. Münster, M. Petschlies, D. Renner, T. Sudmann, C. Urbach, M. Wagner
Lattice spacing scaling

Pion decay constant

Nucleon mass

→ observe (flat) $O(a^2)$ scaling
Setting the scale

\[ m_{PS}^{\text{latt}} = a m_{PS}^{\text{phys}}, \quad f_{PS}^{\text{latt}} = a f_{PS}^{\text{phys}} \]

\[ \frac{f_{PS}^{\text{phys}}}{m_{PS}^{\text{phys}}} = \frac{f_{PS}^{\text{latt}}}{m_{PS}^{\text{latt}}} + O(a^2) \]

→ setting \( \frac{f_{PS}^{\text{latt}}}{m_{PS}^{\text{latt}}} = 130.7/139.6 \)

→ obtain \( m_{PS}^{\text{latt}} = a139.6[\text{Mev}] \)

→ value for lattice spacing \( a \)
The lattice QCD benchmark calculation: the spectrum

ETMC ($N_f = 2$), BMW ($N_f = 2 + 1$)

$N_f = 2$

$N_f = 2 + 1$
Muon magnetic moment: a tension between theory and experiment

→ signs of new physics?

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Include ISR</th>
<th>Value</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>DHMZ10 (e^+e^-)</td>
<td>incl. ISR</td>
<td>180.2 ± 4.9</td>
<td>[3.6 σ]</td>
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<tr>
<td>DHMZ10 (e^+e^-+τ)</td>
<td>incl. ISR</td>
<td>189.4 ± 5.4</td>
<td>[2.4 σ]</td>
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<tr>
<td>JS11 (e^+e^-+τ)</td>
<td>incl. ISR</td>
<td>179.7 ± 6.0</td>
<td>[3.4 σ]</td>
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<tr>
<td>HLMNT11 (e^+e^-)</td>
<td>incl. ISR</td>
<td>182.8 ± 4.9</td>
<td>[3.3 σ]</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Exclude ISR</th>
<th>Value</th>
<th>Sigma</th>
</tr>
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<tbody>
<tr>
<td>DHea09 (e^+e^-)</td>
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<td>178.8 ± 5.8</td>
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<td>A (e^+e^-+τ)</td>
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<td>173.4 ± 5.3</td>
<td>[4.3 σ]</td>
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<td>B (e^+e^-+τ)</td>
<td>excl. ISR</td>
<td>175.4 ± 5.3</td>
<td>[4.1 σ]</td>
</tr>
</tbody>
</table>

Experiment

BNL-E821 (world average)
208.9 ± 6.3

largest error: non-perturbative QCD contribution

latest analysis
(Benayoun, David, DelBuono, Jegerlehner)

larger than 4σ discrepancy
Some numbers

- experimental value: $a_{\mu, N_f=2}^{\text{hvp,exp}} = 5.66(05)10^{-8}$
- from our old analysis: $a_{\mu, N_f=2}^{\text{hvp,old}} = 2.95(45)10^{-8}$

→ misses the experimental value
→ order of magnitude larger error

- from our new analysis: $a_{\mu, N_f=2}^{\text{hvp,new}} = 5.66(11)10^{-8}$

→ error (including systematics) almost matching experiment

- different volumes
- different values of lattice spacing
- included dis-connected contributions
Dark matter detection

• size of quark content of nucleon important to detect dark matter candidates

• scattering dark matter particle with Higgs boson
  → cross-section changes an order of magnitude with small changes of quark content

\[ y_N \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = 0.082(16)(2) \]

• value much smaller than earlier thought
  → severe consequences for experiments
The $\rho$-meson resonance: dynamical quarks at work

- usage of three Lorentz frames

$$m_{\pi^+} = 330 \text{ MeV}, \ a = 0.079 \text{ fm}, \ L/a = 32$$

$$m_\rho = 1033(31) \text{ MeV}, \ \Gamma_\rho = 123(43) \text{ MeV}$$

fitting $z = (M_\rho + i \frac{1}{2} \Gamma_\rho)^2$
There are dangerous lattice animals
Non-zero temperature and density

suggested phase diagram
Lattice Action with chemical potential

\[ Z = \int_{\text{fields}} e^{-S} \]

Partition functions (pathintegral) with Boltzmann weight (action) \( S \)

\[ S = a^2 \sum_x \left\{ \beta \left[ 1 - \text{Re}(U_{(x,p)}) \right] + \bar{\psi} \left[ m_0 + \frac{1}{2} \{ \gamma_\mu (\partial_\mu + \partial^*_\mu) - a \partial^*_\mu \partial_\mu \} \right] \psi \right\} \]

- adding a chemical potential
  \[ S \rightarrow S + i\tau_3 \bar{\Psi} \Psi \]

- action becomes complex, MC methods not possible

- partial solution: reweighting
**The Schwinger model Hamiltonian**

Recent paper:

Free continuum Hamiltonian

\[
H = \int dx \Psi^\dagger (i\alpha \cdot \partial / \partial x + m_f \beta) \Psi
\]

\[
\alpha = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

fermion field: \( \Psi = \begin{pmatrix} b \\ d^\dagger \end{pmatrix} \)

with \( b^\dagger \) (\( d^\dagger \)) creates fermion (antifermion)
The Schwinger model Hamiltonian on the lattice

Wilson discretization, derivative \((x \rightarrow j \cdot a)\)

\[
\alpha \cdot \partial / \partial x \Psi \rightarrow \frac{1}{2a} \left[ \alpha \cdot (\Psi_{j+1} - \Psi_{j-1} + i\beta \cdot (\Psi_{j+1} + \Psi_{j-1} - 2\Psi_j) \right]
\]

2-component fermion field \(\Psi_j = \begin{pmatrix} \Psi_j^{(1)} \\ \Psi_j^{(2)} \end{pmatrix}\)

“pseudofermion” description: \(\Phi_j = \Psi_j^{(1)}, \bar{\Phi}_j = i\Psi_j^{\dagger(1)}\)

\[
H_{\text{Wilson}} = \frac{1}{2a} \left\{ \sum_j \left( \bar{\Phi}_{j+1}^{\dagger} \Phi_{j+1}^{\dagger} - \Phi_{j+1}^{\dagger} \bar{\Phi}_j^{\dagger} - \bar{\Phi}_j^{\dagger} \Phi_j^{\dagger} + \Phi_j^{\dagger} \Phi_{j+1}^{\dagger} \right) + \text{h.c.} \right. \\
+ \left. 2(m_f a + 1)(\Phi_j^{\dagger} \Phi_j + \bar{\Phi}_j^{\dagger} \bar{\Phi}_j) \right\}
\]

vacuum: \(|0 0 0 0 0 0\rangle \Phi_1^{\dagger} \bar{\Phi}_2^{\dagger} |0 0 0 0 0 0\rangle = |1 0 0 1 0 0\rangle\)
The Schwinger model Hamiltonian on the lattice

Staggered (Kogut-Susskind) discretization

→ associate

upper components $\Psi^{(1)}_j$ with even lattice sites

lower components $\Psi^{(2)}_j$ with odd lattice sites

⇒ introduce single component field $\Phi_j$ with $\{\Phi^\dagger_j, \Phi_{j'}\} = \delta_{j,j'}$

can use naive derivative: $\partial/\partial x \Phi \rightarrow \frac{1}{2a} [\Phi_{j+1} - \Phi_{j-1}]$

special trick for staggered fermions: Jordan-Wigner transformation

$$\Phi(j) = \prod_{j<n} [i\sigma_3] \sigma^-(n)$$

$$\Phi^\dagger(j) = \prod_{j<n} [-i\sigma_3] \sigma^+(n)$$

→ allows for a direct matrix representation of Hamiltonian
Adding gauge fields

\[ H = \int dx \Psi^\dagger (i \alpha \cdot \partial / \partial x - g A_x + m_f \beta) \Psi + \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

As in the pathintegral: introduce parallel transporters

\[ U_x(j) = \exp \left\{ ig \int_{j}^{(j+1)\alpha} A_x(x) dx \right\} \]

for Schwinger model: just a phase factor

\[ U_x(j) = e^{i \theta(n)} \]
Implementing gauge fields

temporal gauge: $A_t = 0 \Rightarrow F_{t,x}(j) = \partial_t A_x(j)$

energy:

$$E(j) = F_{t,x}(j), \text{ vacuum: } E(j)|0\rangle = 0$$

lattice gauge fields $U_x(j)$ introduce a ladder space:

$$U_x(j)^l|0\rangle = |j\rangle_j, \ E(j)|l\rangle_j = l|l\rangle_l$$

Field strength tensor

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} dx \rightarrow \frac{1}{2}g^2 a \sum_j E^2(j)$$
Observables

• bound “positronium” states:
  – vector state:
    \[ |v\rangle = \frac{1}{\sqrt{N}} \sum_j^N \left[ \Phi^\dagger(j)e^{i\theta(n)}\Phi(j + 1) + \text{h.c.} \right] |0\rangle \]
  – scalar state:
    \[ |s\rangle = \frac{1}{\sqrt{N}} \sum_j^N \left[ \Phi^\dagger(j)e^{i\theta(n)}\Phi(j + 1) - \text{h.c.} \right] |0\rangle \]

• average electric field:
  \[ \Gamma = \frac{1}{N} \sum_j E(j) \]

• condensates:
  \[ \Gamma^5 = \langle i\bar{\Psi}\sigma_3\Psi/g \rangle \propto \langle \sum_j \left[ \Phi^\dagger(j)\Phi(j + 1) - \text{h.c.} \right] \rangle_0 \]
DMRG improvement
(Byrnes, Sriganesh, Bursill, Hamer; 2008)

plain Hamiltonian calculations: (Crewther and Hamer, 1980; Irving and Thomas, 1982)

<table>
<thead>
<tr>
<th>m/g</th>
<th>DMRG 2008</th>
<th>plain H 1980</th>
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<tbody>
<tr>
<td>0</td>
<td>0.56419(4)</td>
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<td>16</td>
<td>0.238(5)</td>
<td>0.245(5)</td>
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<tr>
<td>32</td>
<td>0.194(5)</td>
<td>0.197(5)</td>
</tr>
</tbody>
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Summary

• Progress in solving QCD with lattice techniques
  – dramatic algorithm improvements
  – new supercomputer architectures
• offers possibility to
  – reach continuum limit and chiral limit
    → have computed the baryon spectrum
    → anomalous magnetic moment of muon
• challenges
  – cannot reliably simulate chemical potential
  – real time evolution not controlled
• from participants of this workshop
  (started game with M.C. Bañuls, K. Cichy)