On the nature of the 4th family ν

(in collaboration with A. Aparici, N. Rius and A. Santamaría)

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3. **The fourth generation neutrino sector**
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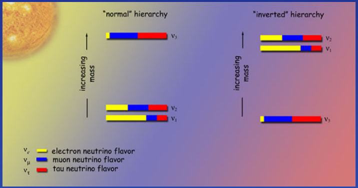
1. Introduction

Introduction: light neutrino masses ask for new physics

- From cosmology $\sum m_\nu \lesssim 1$ eV, but scale is unknown.
- From oscillation experiments, the possible hierarchies are:

1. NH (if $m_1 = 0$): $m_3 \approx 0.05$ eV & $m_2 \approx 0.01$ eV.
2. IH (if $m_3 = 0$): $m_2 \approx 0.05$ eV & $m_1 \approx 0.04$ eV.
3. Quasi-degenerate: $m_1 \simeq m_2 \simeq m_3 \lesssim 0.3\,\text{eV} \sim 10^{-6} m_e$.



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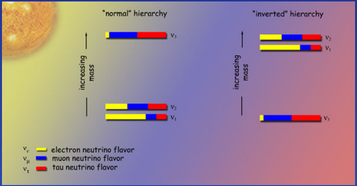

New physics is needed (in SM neutrinos are massless):

- Many models have been proposed (see later).
- Here we study the ν sector in the presence of a 4th family.

2. Mechanisms for light neutrino masses

Light Dirac neutrino masses

- Add ν_R (at least 2) to the SM content.
- Impose by hand a global symmetry such as $B - L$.

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No explanation for the smallness of neutrino masses:

- Tiny Yukawa couplings, 6 (11) orders of magnitude smaller than the electron (top) one: it does not seem very natural.

Light Majorana neutrino Masses

- Majorana masses parametrized by the Weinberg operator:

$$\mathcal{L}_5 = -\frac{1}{2}\frac{c_{\alpha\beta}}{\Lambda_W}(\overline{\ell_\alpha}\tilde{\phi})\,(\phi^\dagger\tilde{\ell_\beta}) + \mathrm{H.c.}\,,$$

where $\tilde{\ell} = i\tau_2\ell^c$ and $\Lambda_W \gg v_\phi$ is the scale of NP.

- $c_{\alpha\beta}$ are model-dependent which can carry extra loop factors and/or ratios of masses.
- Upon EW SSB, it leads to:

$$m_\nu = c\,\frac{v_\phi^2}{\Lambda_W}\,,$$

with $\langle\phi\rangle = v_\phi = 174$ GeV.

Seesaw type I (SS I) ((and type III, SS III))

- Add n $Y = 0$ SM fermion singlets (ν_R), at least 2.
- Majorana masses for ν_R are allowed by gauge invariance:

$$\mathcal{L}_{\nu_\mathrm{R}} = i\,\overline{\nu_\mathrm{R}}\gamma^\mu\partial_\mu\nu_\mathrm{R} - \left(\frac{1}{2}\overline{\nu^\mathrm{c}_\mathrm{R}}M\nu_\mathrm{R} + \bar{\ell}\,\tilde{\phi}\,Y\,\nu_\mathrm{R} + \mathrm{H.c.}\right)\,,$$

where $\tilde{\phi} = i\,\tau_2\,\phi^*$. After SSB:

$$\mathcal{L}_{\nu\,\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_\mathrm{L}} & \overline{\nu^\mathrm{c}_\mathrm{R}}\end{pmatrix}\begin{pmatrix}0 & m_\mathrm{D} \\ m_\mathrm{D}^\mathrm{T} & M\end{pmatrix}\begin{pmatrix}\nu^\mathrm{c}_\mathrm{L} \\ \nu_\mathrm{R}\end{pmatrix} + \mathrm{H.c.}\,,$$

where $m_D = Yv_\phi$. If $M \gg m_\mathrm{D}$, one obtains n heavy leptons (mainly SM singlets), with masses $\sim M$, and light ν masses,

$$m_\nu \simeq -m_\mathrm{D}\,M^{-1}\,m_\mathrm{D}^\mathrm{T}\,,$$

which naturally explains the smallness of m_ν. Similarly SS III, with fermion triplets of masses > 100 GeV.

- Add to the SM one scalar triplet with hypercharge $Y = 1$ and $L = -2$. In the doublet representation of $SU(2)_{\mathrm{L}}$ the triplet is a 2×2 matrix:

$$\chi = \begin{pmatrix} \chi^+/\sqrt{2} & \chi^{++} \\ \chi_0 & -\chi^+/\sqrt{2} \end{pmatrix}$$

Gauge invariance allows a Yukawa coupling of the scalar triplet to 2 lepton doublets,

$$\mathcal{L}_\chi = -\left((Y_\chi)_{\alpha\beta}\, \bar{\tilde{\ell}}_\alpha \chi \ell_\beta + \mathrm{H.c.} \right) - V(\phi, \chi)\,,$$

where Y_χ is a symmetric matrix and $\tilde{\ell} = i\tau_2\, \ell^c$. The scalar potential has the following terms:

$$V(\phi, \chi) = m_\chi^2\, \mathrm{Tr}[\chi\chi^\dagger] + \left(\mu\, \tilde{\phi}^\dagger \chi^\dagger \phi + \mathrm{H.c.} \right) + \ldots$$

SS II

- The μ coupling violates L and induces a VEV for the triplet via v_ϕ, even if $m_\chi > 0$. In the limit $m_\chi \gg v_\phi$:

$$m_\nu = 2Y_\chi v_\chi = 2Y_\chi \frac{\mu v_\phi^2}{m_\chi^2}$$

- m_ν are thus proportional to both Y_χ and μ, since the breaking of L results from their simultaneous presence.
- If m_χ^2 is positive and large, v_χ will be small, in agreement with the ρ parameter, $v_\chi \lesssim 6\ \mathrm{GeV}$.
- Moreover, μ can be naturally small, because in its absence L is recovered, increasing the symmetry.

Other mechanisms

- Induced by radiative corrections. On top of loop factors $1/(4\pi)^2$, there can be extra suppressions due to couplings or ratios of masses, so Λ_{NP} can be EW scale.
- Supersymmetry by R-parity breaking. The SM doublet neutrinos mix with the neutralinos. Majorana masses for ν's (generated at tree level and at one loop) are naturally small because they are proportional to the small R-parity-breaking parameters.

3. The fourth generation neutrino sector

Fourth generation neutrino masses

- New active neutrino must be much heavier than light ones. Lower bounds on $m_{\nu 4} \equiv m_4$ are (in GeV):

1. Unstable (LEP II): $m_4 > 80.5$ (M), 90.3 (D), 62.1 (both).
2. Stable (inv. Γ_Z): $m_4 > 39.5$ (M), 45 (D), 33.5 (both).

Possible mechanisms:

- 1) **Dirac:** quite natural for ν_4 as long as L is conserved. However, this is not the case if the light ν are Majorana (as in most of models) where they can mix with the 4th family.
- 2) **Weinberg Operator:** masses $\mathcal{O}(v_\phi^2/\Lambda_W)$ which should be $\gtrsim m_Z/2$, so Λ_W can not be $\gg v_\phi$ and the effective theory does not provide a useful parametrization.

Fourth generation neutrino masses

- 3) **Seesaw type I:** if $m_R \gg m_D$, $m_4 \sim m_D^2/m_R$, which must be heavier than $\sim m_Z/2$. Therefore $m_R < m_D^2/m_Z$, so m_R cannot be $\gg$EW scale. If $m_R \ll m_D$ there are 2 almost degenerate ν (PD limit), which is OK.
- 4) **Seesaw type II:** $v_\chi \lesssim 6\ \mathrm{GeV}$ will yield ν_4 masses too small. Not viable.
- 5) **Seesaw type III:** the charged fermions must be $> 100\ \mathrm{GeV}$, so the PD limit is not possible. These new fermions have to be $\lesssim$ few TeV, so it is viable but much more constrained than type I.
- 6) **Others:** radiative mechanisms and SUSY with broken R parity. m_ν in these models are strongly suppressed with respect to the EW scale by either loop factors, couplings and/or ratios of masses. Not viable.

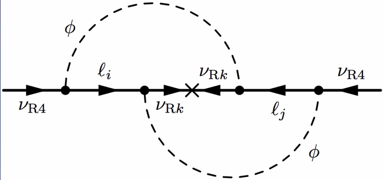
Fourth generation neutrino masses

Conclusion:

- Only Dirac, SS I (which includes Dirac in some limit) and SS III are good mechanisms for ν_4:
- One needs at least one ν_R (either SM singlet or neutral component of the triplet) which has standard Yukawa couplings to the doublets.
- In general, a m_R for the 4th gen. ν_R is allowed by symmetry, and naturality arguments set a lower bound for it (unless some symmetry for the 4th gen. is invoked).

Natural fourth generation neutrino masses in SS I

- Suppose SS I for light neutrino masses, and we have ν_{R4}.
- Is it stable under radiative corrections to set $m_{R4} = 0$, given that m_{R4} does not increase the symmetry?



Above the $m_{\mathrm{R}k}$ scale, $m_{\mathrm{R}4}$ and $m_{\mathrm{R}k}$ mix under renormalization. Even if $m_{\mathrm{R}4} = 0$ at some scale $\Lambda_C > m_{\mathrm{R}k}$, $m_{\mathrm{R}4}$ will be generated by running from Λ_C to $m_{\mathrm{R}k}$. Barring accidental cancellations, one should require:

$$m_{\mathrm{R}4} \gtrsim \frac{1}{(4\pi)^4} \sum_{ijk} Y_{i4} Y^*_{ik} m_{\mathrm{R}k} Y^*_{jk} Y_{j4} \ln(\Lambda_C / m_{\mathrm{R}k})$$

where $i, j = 1, 2, 3, 4$, $k = 1, 2, 3$.

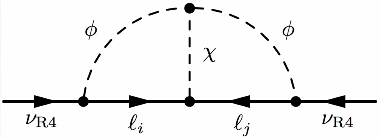
Natural fourth generation neutrino masses in SSI

As $(m_\nu)_{ij} \sim \sum_k Y_{ik} Y_{jk} v_\phi^2 / m_{\mathrm{R}k}$, by taking all $m_{\mathrm{R}k}$ of the same order we can rewrite the bound as (taking $\ln(\Lambda_C / m_{\mathrm{R}k}) \gtrsim 1$):

$$m_{\mathrm{R}4} \gtrsim \sum_{ij} \frac{Y_{i4} (m_\nu)_{ij} Y_{j4}}{(4\pi)^4} \frac{m_{\mathrm{R}k}^2}{v_\phi^2}$$

- The same result is obtained for SS III.
- For $m_\nu = 0.01\ \mathrm{eV}$ and $Y_{k4} = 0.01$ (suppressed due to universality and LFV constraints) we obtain that $m_{\mathrm{R}4}$ is of order keV, GeV, PeV for $m_{\mathrm{R}k} = 10^9, 10^{12}, 10^{15}\ \mathrm{GeV}$ resp.

Natural fourth generation neutrino masses in SSII



In SS II (for light ν's) plus one ν_R for the 4th gen., m_{R4} and the trilinear coupling of the triplet μ, mix under renormalization, so, as before (taking $\ln(\Lambda_C/m_\chi) \gtrsim 1$):

$$m_{\rm R4} \gtrsim \frac{\mu}{(4\pi)^4} \sum_{ij} Y_{i4} (Y_\chi)_{ij} Y_{j4} \,,$$

where Y_χ are the Yukawa couplings of the triplet to the lepton doublets. Expressed in terms of $(m_\nu)_{ij} \sim (Y_\chi)_{ij} \mu v_\phi^2 / m_\chi^2$:

$$m_{\rm R4} \gtrsim \sum_{ij} \frac{Y_{i4} (m_\nu)_{ij} Y_{j4}}{(4\pi)^4} \frac{m_\chi^2}{v_\phi^2} \,,$$

which is very similar to that obtained for SS I.

Natural fourth generation neutrino masses in SS

Conclusion:

So in general, one expects that (using NDA):

$$m_{\mathrm{R}4} \sim \frac{\Lambda_W}{(4\pi)^4} \sum_{ij} Y_{i4} c_{ij} Y_{j4} \sim \frac{Y_{i4}(m_\nu)_{ij} Y_{j4}}{(4\pi)^4} \frac{\Lambda_W^2}{v_\phi^2},$$

which is the result obtained in the see-saw models if one identifies $\Lambda_W \sim m_{\mathrm{R}k}, m_\chi$.

The minimal scenario with four generations

So in the minimal 4G scenario, one needs:

- one relatively “light” ν_{R}, to give Dirac mass terms to ν_4’s.
- the Weinberg operator to parametrize light m_ν.

$$\mathcal{L}_Y = -\bar{\ell} Y_e e_{\mathrm{R}} \phi - \bar{\ell} y \nu_{\mathrm{R}} \tilde{\phi} - \frac{1}{2} \overline{\nu_{\mathrm{R}}^{c}} m_{\mathrm{R}} \nu_{\mathrm{R}} - \frac{1}{2 v_\phi^2} (\bar{\ell} \tilde{\phi}) m_{\mathrm{L}} (\phi^\dagger \tilde{\ell}) + \mathrm{H.c.}\ ,$$

where ℓ and e_{R} contain the 4 gen. comp., Y_e is a general 4×4 complex matrix, y is a 4 comp. column vector, m_{R} is a number and m_{L} is a general complex symmetric 4×4 matrix.

- $m_{\mathrm{L}} \ll v_\phi$, while m_{R} cannot be very large so $m_4 > m_Z/2$, and we do not expect it vanish if $m_{\mathrm{L}} \neq 0$.

4. Phenomenological implications

Heavy sector

The heavy ν sector consists of two Majorana ν's:

$$\nu_4 = i\cos\theta(-\nu_4' + \nu_4'^{c}) + i\sin\theta(\nu_R - \nu_R^c)$$

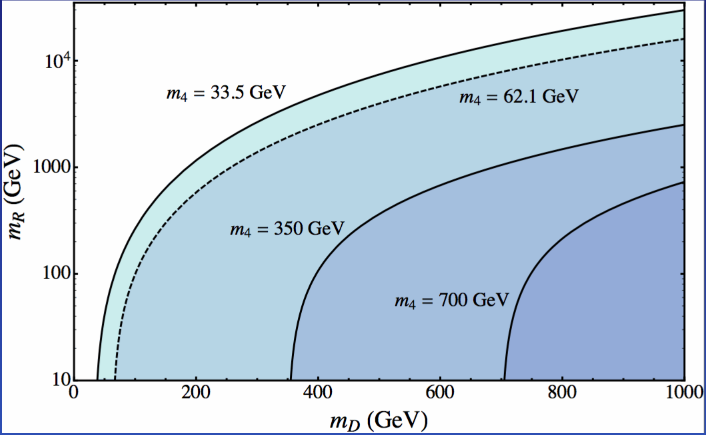
$$\nu_{\bar{4}} = -\sin\theta(\nu_4' + \nu_4'^{c}) + \cos\theta(\nu_R + \nu_R^c)$$

with masses:

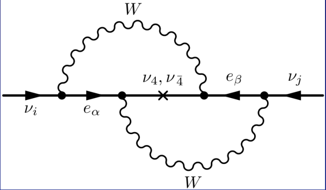
$$m_{4,\bar{4}} = \frac{1}{2}\left(\sqrt{m_R{}^2 + 4m_D^2} \mp m_R\right) ,$$

and mixing angle $\tan^2\theta = m_4/m_{\bar{4}}$.

m_D versus m_R plane



Light neutrino masses induced by the extra family



Two-loop corrections induced by the 4th gen. fermions generate light m_ν even if they were not present at tree level:

$$(m_\nu)^{(2)}_{ij} = -\frac{g^4}{m_W^4} m_\mathrm{R} m_\mathrm{D}^2 \sum_\alpha V_{\alpha i} V_{\alpha 4} m_\alpha^2 \sum_\beta V_{\beta j} V_{\beta 4} m_\beta^2 I_{\alpha\beta} \, ,$$

where the sums run over the charged leptons $\alpha, \beta = e, \mu, \tau, E$ while $i, j = 1, 2, 3$, and $I_{\alpha\beta}$ is a loop integral.

- When $m_\mathrm{R} = 0$, $(m_\nu)^{(2)}_{ij} = 0$, because then L is conserved.
- Also when $m_\mathrm{D} = 0$ we obtain $(m_\nu)^{(2)}_{ij} = 0$, since then ν_R decouples completely and L is again conserved.

Light neutrino masses induced by the extra family

Defining

$$N_\alpha \equiv V_{\alpha 4} = \frac{y_\alpha}{\sqrt{\sum_\beta y_\beta^2}}\,,$$

for $m_E \gg m_{4,\bar{4}} \gg m_W$, the largest contribution is approximately:

$$(m_\nu)^{(2)}_{33} \approx \frac{g^4}{2(4\pi)^4}(N_e^2 + N_\mu^2 + N_\tau^2)\, m_\mathrm{R}\, \frac{m_\mathrm{D}^2 m_E^2}{m_W^4} \ln \frac{m_E}{m_{\bar{4}}}$$

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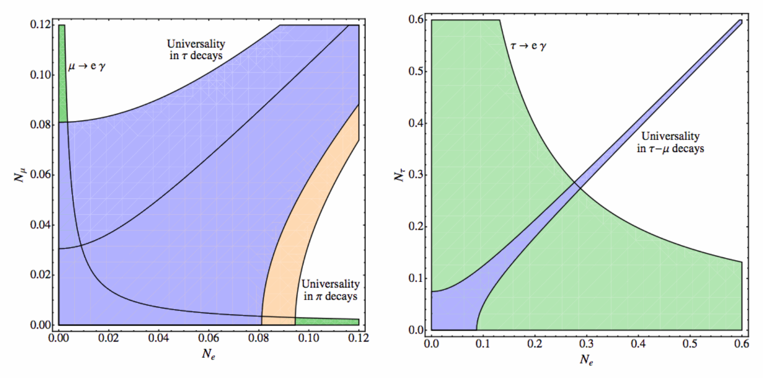
Could these radiative corrections explain by themselves the observed spectrum of masses and mixings?

- No, because the eigenvalues are $\propto m_\mu^4,\ m_\tau^4,\ m_D^2 m_E^2$, which gives a huge hierarchy between neutrino masses.
- However, they lead to a strong constraint for a 4th family.

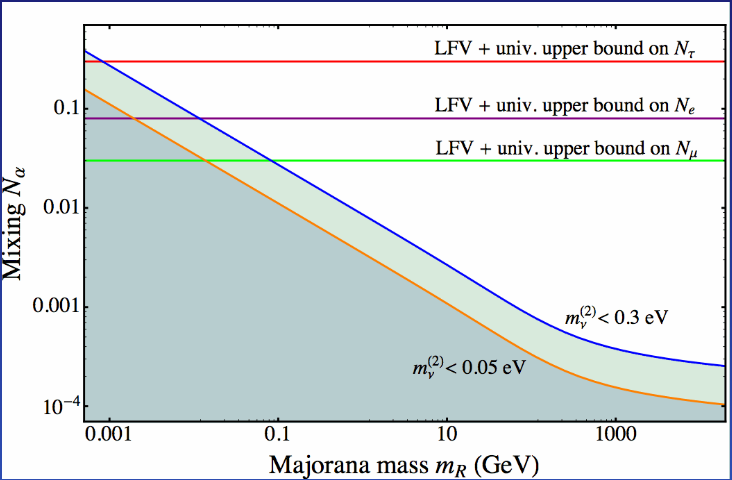
Universality and LFV constraints

The bounds on the mixings of the light families are, at 90% C.L.:

$$N_e < 0.08, N_\mu < 0.03, N_\tau < 0.3$$



Light neutrino mass bound



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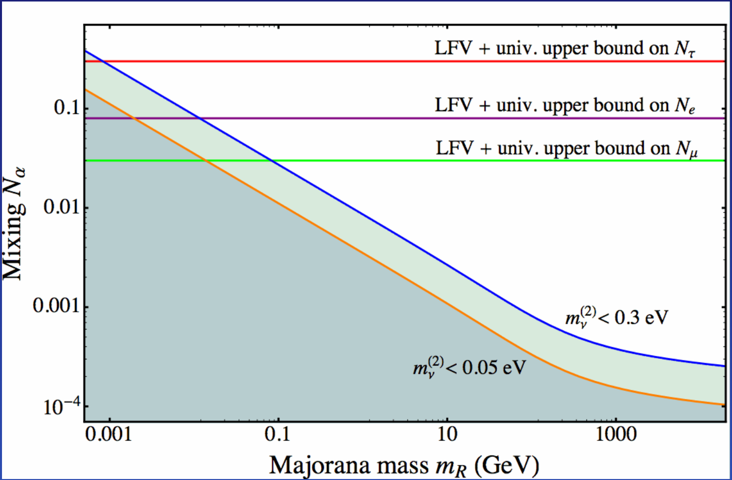

Conclusion:

Light neutrino masses give the strongest bound on the mixings.

$$A \approx A_L + A_4 \ ,$$

where A_L is the light ν contribution (i.e., $m_k \ll p_{\rm eff} \sim 100$ MeV):

$$A_L \propto \sum_k^{\rm light} m_k U_{ek}^2 M^{0\nu 2\beta}(m_k) \simeq m_{ee} M^{0\nu 2\beta}(0) \ ,$$

with $M^{0\nu 2\beta}(0) \propto 1/p_{\rm eff}^2$ the nuclear matrix element.

$$\begin{aligned} A_4 &\propto N_e^2 \left(m_4 \cos^2\theta M^{0\nu 2\beta}(m_4) - m_{\bar{4}} \sin^2\theta M^{0\nu 2\beta}(m_{\bar{4}}) \right) \\ &\propto N_e^2 \left(\frac{\cos^2\theta}{m_4} - \frac{\sin^2\theta}{m_{\bar{4}}} \right) = N_e^2 \frac{m_{\rm R}}{m_{\rm D}^2} \end{aligned}$$

- The largest contributions to A_4 correspond to small m_D.

The contribution of ν_4's to the $0\nu2\beta$ amplitude is dominant if:

$$N_e^2 m_\mathrm{R}/m_\mathrm{D}^2 > m_{ee}/(100\,\mathrm{MeV})^2.$$

One can use the dependence on $N_e^2 m_\mathrm{R}$ of both A_4 and $(m_\nu)_{33}^{(2)}$ to constrain the ν_4 contribution to the $0\nu2\beta$ decay amplitude:

$$A_4 \leq \left(\frac{4\pi m_W}{g m_\mathrm{D}}\right)^4 \frac{2(m_\nu)_{33}^{(2)}}{m_E^2 \ln \frac{m_E}{m_{\bar{4}}}} \lesssim 190 (m_\nu)_{33}^{(2)} \left(\frac{50\,\mathrm{GeV}}{m_\mathrm{D}}\right)^4 \mathrm{GeV}^{-1},$$

where we have used the LEP limit, $m_E \gtrsim 100$ GeV.

Imposing that $(m_\nu)^{(2)}_{33} \leq 0.05$ eV:

$$A_4 < 10^{-8}(\frac{50\,\mathrm{GeV}}{m_\mathrm{D}})^4\,\mathrm{GeV}^{-1}.$$

The non-observation of $0\nu2\beta$ implies that:

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So the constraint from light neutrino masses implies that:

- the contribution of $\nu_4's$ to $0\nu2\beta$ can reach observable values only if $m_\mathrm{D} \lesssim 100$ GeV (although it can be the dominant one for other values of masses and mixings).

4G Higgs sector

- σ_H through *gg* fusion at LHC is enhanced by a factor of 9.
- Some Higgs decay channels behave very differently.
- To distinguish and discover/exclude one must look to $\sigma_H \cdot B_r$, for example, to the $\gamma\gamma$ and *WW* channels, which are very different for SM3 and SM4.
- If ν_4's are light enough ($m_W/2 \lesssim m_{4,\bar{4}} \lesssim m_W$), the decay mode of the Higgs into ν_4's can be dominant.
- However, even if the SM-like 4G Higgs is excluded, many possibilities arise, for instance, with an extra Higgs doublet.

5. Summary and conclusions

Summary and conclusions

1. We have considered the case where light ν are Majorana, so their tiny mass is naturally understood.
2. If a 4th generation exists, at least one ν_R is needed.

$\longrightarrow$ 1) $+$ 2) imply that the ν_R should naturally have a Majorana mass m_R whose size ($\lesssim$TeV) depends on the LNV mechanism (if set $m_R = 0$ at tree level, it is generated at 2-loops).

- We have analyzed the phenomenology of the minimal 4G scenario: universality, charged LFV processes and $0\nu\beta\beta$.
- Strongest constraint: the 4th generation induces two-loop contributions to the light m_ν, which can easily exceed the atmospheric or the cosmological scale.

BACK-UP SLIDES

New families are a natural SM extension

- New families are allowed and testable at LHC.
- Theoretically: $\beta_{QCD} < 0 \Longrightarrow n_{gen} \leq 8$.
- Fits: 3 & 4 gens. give roughly same χ^2.
- A heavy Higgs would fit better (but hint of a 126 GeV one).
- Baryogenesis: more CPV.
- DM: hadrons, heavy neutrinos/singlets if stable.
- Composite Higgs & dynamical EW symm. breaking: no hierarchy problem!
- Might help to solve flavor discrepancies.

Seesaw type III (SS III)

- The SM is extended by fermion $SU(2)_{\mathrm{L}}$ triplets Σ_i with $Y = 0$ (at least 2).

The new terms in the Lagrangian are given by:

$$\mathcal{L}_\Sigma = i\,\mathrm{Tr}\left[\overline{\Sigma}\gamma^\mu D_\mu \Sigma\right] - \left(\frac{1}{2}\mathrm{Tr}\left[\overline{\Sigma} M \Sigma^{\mathrm{c}}\right] + \sqrt{2}\, Y_{\alpha\beta}\, \bar{\ell}_\alpha\, \Sigma_\beta \tilde{\phi} + \mathrm{H.c.}\right) ,$$

where Y is the Yukawa coupling of the fermion triplets to the SM lepton doublets and the Higgs and M their Majorana mass matrix, which can be chosen to be diagonal and real.

SS III

After SSB the neutrino mass matrix can be written as

$$\mathcal{L}_{\nu\,\mathrm{mass}} = -\frac{1}{2}\left(\overline{\nu_{\mathrm{L}}} \quad \overline{\Sigma_0^{\mathrm{c}}}\right) \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}}^{\mathrm{T}} & M \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}}^{\mathrm{c}} \\ \Sigma_0 \end{pmatrix} + \mathrm{H.c.}\,,$$

and leads to a light neutrino Majorana mass matrix

$$m_\nu \simeq -m_{\mathrm{D}}\, M^{-1}\, m_{\mathrm{D}}^{\mathrm{T}}$$

However, since the triplet has also charged components with the same Majorana mass, there are stringent lower bounds:

$$M \gtrsim 100\ \mathrm{GeV}$$

Two-loop integral

$$I_{kn} = \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{p\cdot q}{(p^2-m_k^2)(q^2-m_n^2)((p+q)^2-m_1^2)((p+q)^2-m_2^2)} \times$$

$$\times \left[\frac{1}{p^2q^2} - \frac{3}{4}\frac{1}{(p^2-M_W^2)(q^2-M_W^2)}\right]$$

If we take $m_E \gg m_{4,\bar{4}} > m_W$, we obtain:

$$I_0 \approx -\frac{1}{2^{10}\pi^4 m_{\bar{4}}^2} \ln\frac{m_{\bar{4}}^2}{m_4^2}\,, \qquad k,n = e,\mu,\tau$$

$$I_E \approx -\frac{1}{2^{10}\pi^4 m_E^2} \ln\frac{m_E^2}{m_{\bar{4}}^2}\,, \qquad k \text{ and/or } n = E$$

A renormalizable model

- We build a model in the context of the 4G SM in which m_{R4} is generated radiatively and finite.
- 4G + 4$\nu_{\mathrm{R}i}$ ($i = 1, \cdots, 4$): 3 of them very heavy while one of them should be much lighter in order to avoid a too light ν_4.
- So let the 4th ν_R be massless at tree level and let its mass be generated by radiative corrections.
- We add 3 extra chiral singlets $s_{\mathrm{L}a}$ ($a = 1, \cdots, 3$), and in order to break L, also a complex scalar singlet σ.

We assign L:

$$\ell_j \to e^{i\alpha}\ell_j\,, \quad e_{\mathrm{R}j} \to e^{i\alpha}e_{\mathrm{R}j}\,, \quad \nu_{\mathrm{R}j} \to e^{i\alpha}\nu_{\mathrm{R}j}\,, \quad \sigma \to e^{i\alpha}\sigma$$

The $s_{\mathrm{L}a}$ do not carry L.

A renormalizable model

We have:

$$\mathcal{L}_Y = -\bar{\ell}\, Y_e e_{\rm R}\phi - \bar{\ell}\, Y_\nu \nu_{\rm R}\tilde{\phi} - \sigma\, \overline{\nu_{\rm R}}\, y\, s_{\rm L} - \frac{1}{2}\, \overline{s_{\rm L}^{\rm c}} M s_{\rm L} + {\rm H.c.}\ ,$$

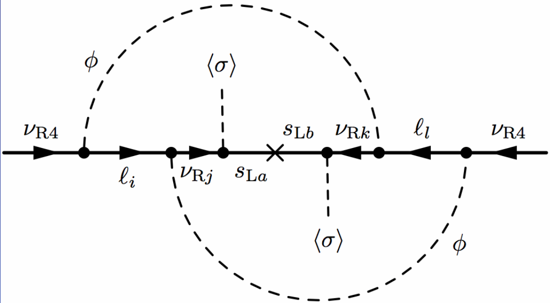
where y_{ia} is a general 4×3 matrix while M is a symmetric 3×3 matrix, diagonal and positive.

- Before SSB only $s_{{\rm L}a}$ are massive.
- After σ gets a VEV, if $y\, v_\sigma \ll M$ the 4 ν_R's will get a 4×4 Majorana mass matrix:

$$M_{\rm R}^{(0)} \simeq v_\sigma^2\, y M^{-1} y^{\rm T}$$

- This is basically the see-saw formula but applied to ν_R's with $v_\phi \longrightarrow v_\sigma$. This matrix has rank 3 and, therefore, only 3 ν_R's will obtain a tree-level mass.

- The other neutrino will remain massless at tree level. However, at two loops, also ν_{R4} will acquire a Majorana mass, via the following 2-loop diagram:



The diagram is obviously finite by power counting, so:

$$M_{\rm R}^{(2)} \sim \frac{v_\sigma^2}{(4\pi)^4}\, Y_\nu Y_\nu^\dagger\, y M^{-1} y^{\rm T}\, (Y_\nu Y_\nu^\dagger)^{\rm T} \ln\left(\frac{M}{y v_\sigma}\right)$$