

New Hadronic Form Factors in **Tauola**

arXiv:1203.3955 [hep-ph]

Pablo Roig (IFAE, Barcelona)

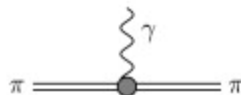
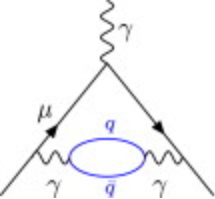
Based on Collaborations with Tomasz Przedzinski, Olga Shekhovtsova, Z. Was **Tauola**
Daniel Gómez-Dumm, Antonio Pich, Jorge Portolés **R_XT**



XL International Meeting on Fundamental Physics & Flavour Mini-Workshop
Centro de Ciencias Pedro Pascual. Benasque, 24 Mayo-3 Junio 2012

CONTENTS

- Motivation *see Matthias' talk*
- Theoretical setting
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 - The project
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 - Conclusions

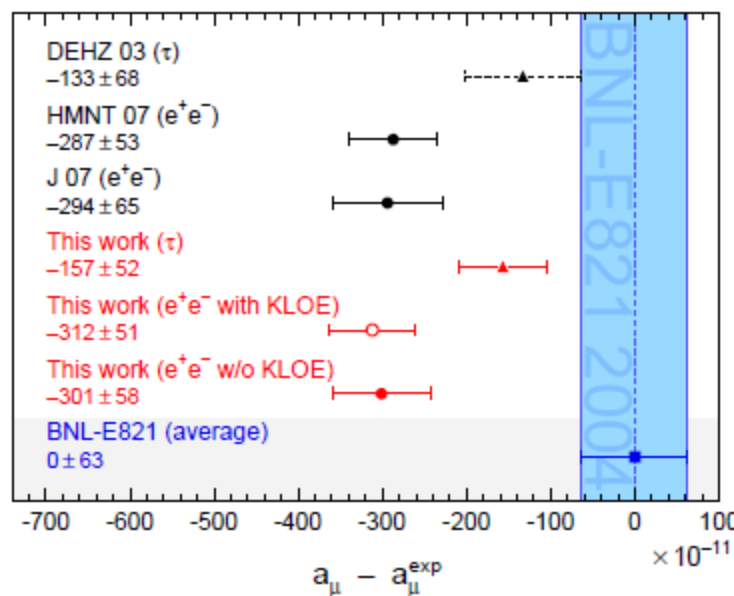


$$a_{\mu}^{\text{exp}} = 11\,659\,208.9 \pm 5.4_{\text{stat}} \pm 3.3_{\text{syst}}$$

MOTIVATION

Units: 10^{-10}

Precise low-energy measurements



Units: 10^{-11}

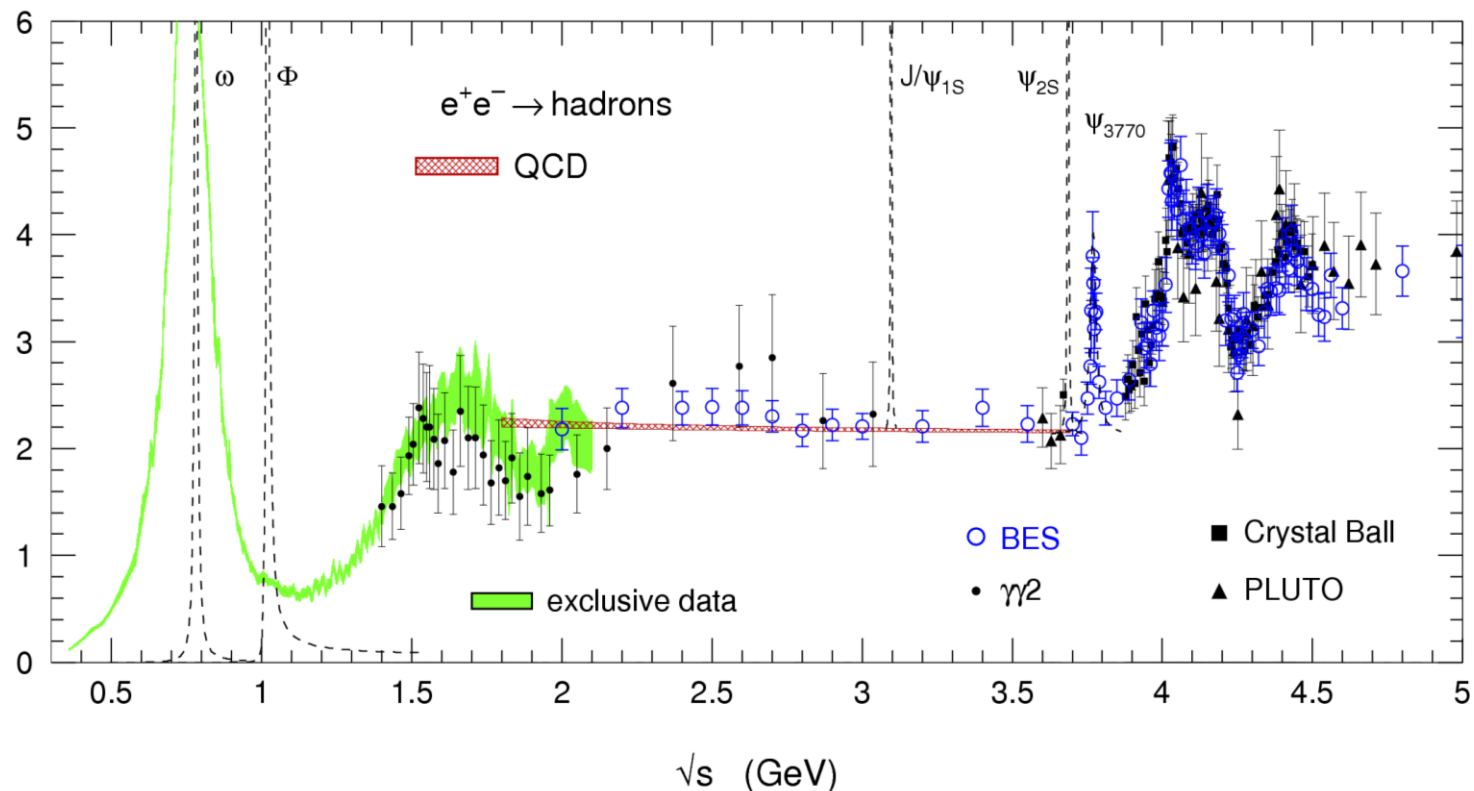
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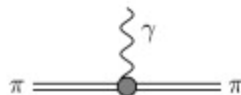
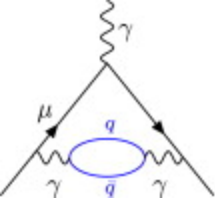
$$R = \frac{\sigma^{(0)}(e^+e^- \rightarrow \text{hadrons})}{\sigma^{(0)}(e^+e^- \rightarrow \mu^+\mu^-)}$$

- Low energies (flavour factories):
Hadronic tau decays

$$a_\mu^{\text{had, LO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma^0(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$a_\mu^{\text{had, } \pi\pi} = \left(\frac{\alpha m_\mu}{6\pi} \right)^2 \int_{4m_\pi^2}^{\Lambda} \frac{ds}{s} \sigma_\pi^3 |F_V^{\pi\pi}(s)|^2 K(s)$$





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73% from $\pi\pi(\gamma)$

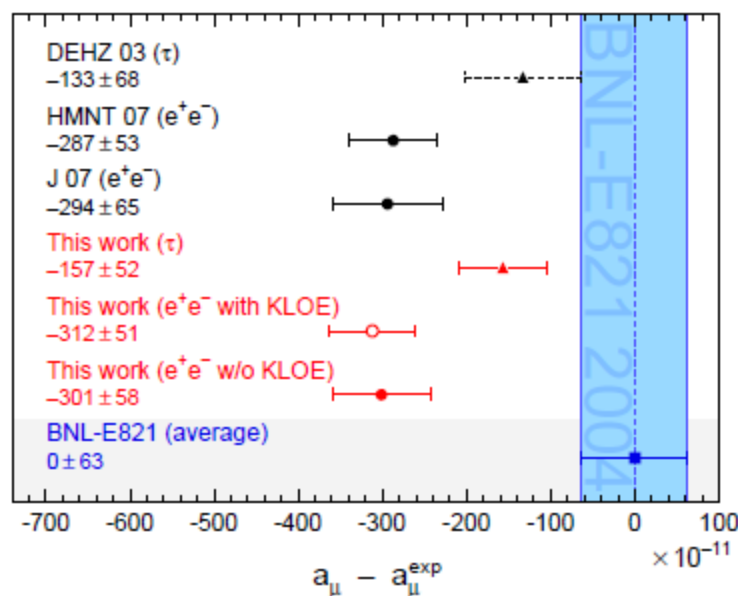
Davier et. al. '09 82% from $\pi\pi(\gamma)$

Including contributions from other exclusive channels at energy below 1.8 GeV as well as the inclusive perturbative QCD calculation at higher energy, one obtains $a_{\mu}^{\text{had,LO}}[\tau] = 705.3 \pm 3.9_{\text{exp}} \pm 0.7_{\text{rad}} \pm 0.7_{\text{QCD}} \pm 2.1_{\text{IB}}$ and $a_{\mu}^{\text{had,LO}}[e^+e^-] = 689.8 \pm 4.3_{\text{exp+rad}} \pm 0.7_{\text{QCD}}$ ($690.9 \pm 5.2_{\text{exp+rad}} \pm 0.7_{\text{QCD}}$). Including further a_{μ}^{QED} , a_{μ}^{weak} , $a_{\mu}^{\text{had,HQ}} = -9.79 \pm 0.08_{\text{exp}} \pm 0.03_{\text{rad}}$ [19] and $a_{\mu}^{\text{LBL}} = 10.5 \pm 2.6$ [20], one gets the total SM predictions $a_{\mu}^{\text{SM}}[\tau] = 11\,659\,193.2 \pm 4.5_{\text{LO}} \pm 2.6_{\text{HQ+LBL}} \pm 0.2_{\text{QED+weak}}$ and $a_{\mu}^{\text{SM}}[e^+e^-] = 11\,659\,177.7 \pm 4.4_{\text{LO}} \pm 2.6_{\text{HQ+LBL}} \pm 0.2_{\text{QED+weak}}$ ($11\,659\,178.8 \pm 5.2_{\text{LO}} \pm 2.6_{\text{HQ+LBL}} \pm 0.2_{\text{QED+weak}}$), which are compared with the direct measurement [1, 2] and other SM predictions [5,

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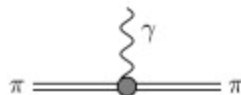
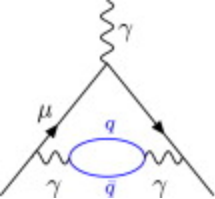
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New Hadronic Form Factors in Tauola



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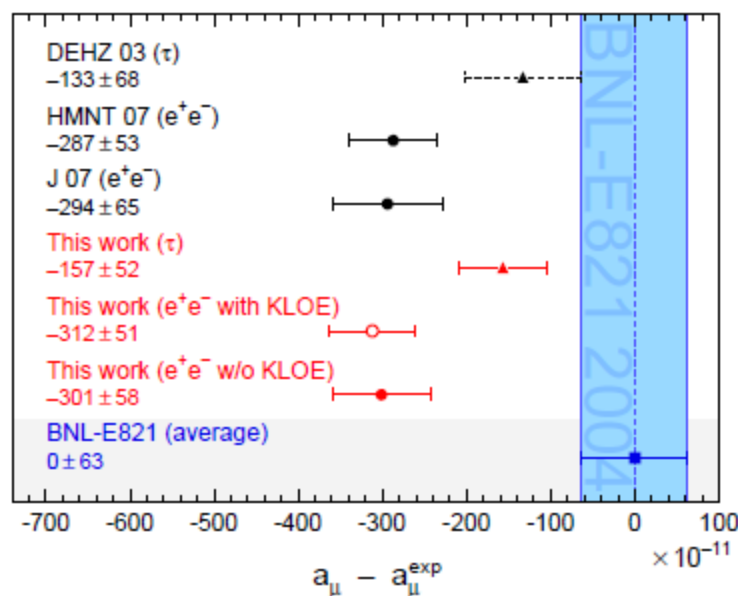
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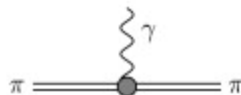
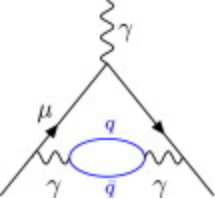
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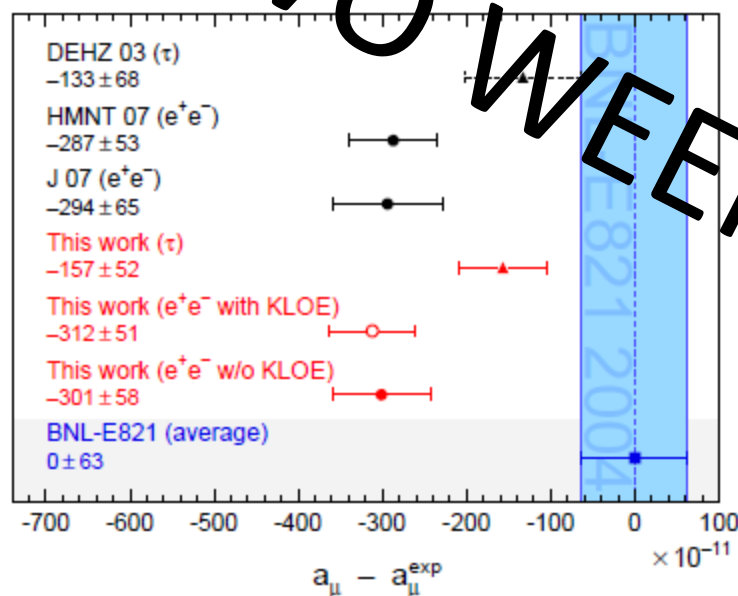
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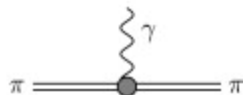
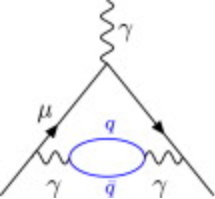
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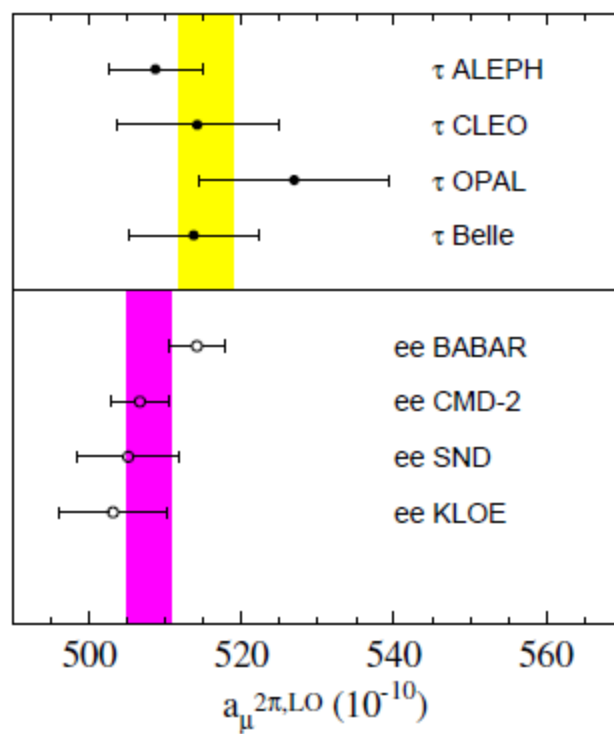


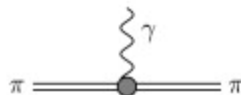
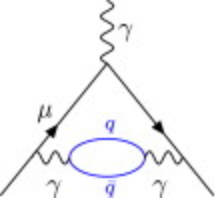
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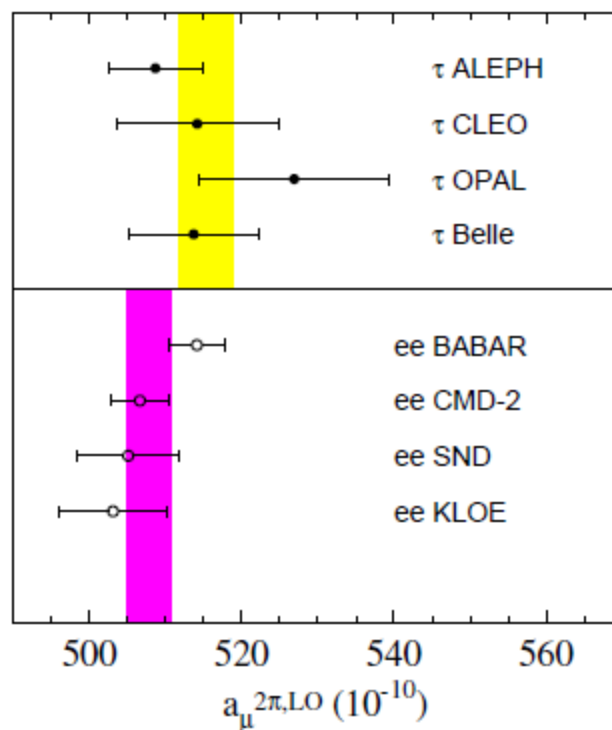


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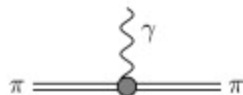
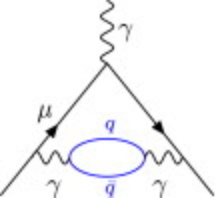
MOTIVATION

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This contribution amounts to $(514.1 \pm 2.2_{\text{stat}} \pm 3.1_{\text{syst}}) \times 10^{-10}$, the most precise result yet from a single experiment. This result brings the contribution estimated from all $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ data combined in better agreement with the τ estimate. When adding all other Standard Model contributions to the present 2π result, in particular using all available *BABAR* data on multihadronic processes, the predicted muon magnetic anomaly is found to be $(11\,659\,186.5 \pm 5.4) \times 10^{-10}$, which is smaller than the direct measurement at BNL by 2.7σ . Adding all previous 2π data increases the deviation to 3.6σ .



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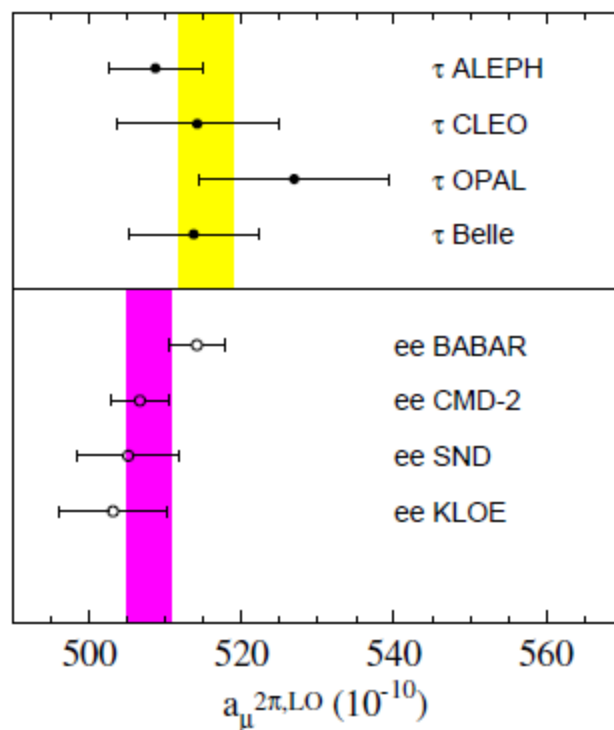


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MOTIVATION

- Low energies (**Flavour factories**):
Hadronic tau decays

See Matthias' talk

$$a_\mu, \Delta\alpha(M_Z^2) \quad \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = - \left(\frac{\alpha M_Z^2}{3\pi} \right) \text{Re} \int_{m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)}$$

CC & NC Universality

~~CP~~ BaBar '11

Resonance Dynamics (NP QCD)

τ hadronic width $\Rightarrow \alpha_s(m_\tau^2) \rightarrow \alpha_s(M_Z^2)$ Rodrigo, Pich, Santamaría '98

Baikov, Chetyrkin, Kuhn '08; Davier, Descotes-Genon, Malaescu, Zhang '08;
Boito et. al. '11, '12

$m_s(m_\tau^2)$ & V_{us} Gámiz, Jamin, Pich, Prades, Schwab '02,'04

MOTIVATION

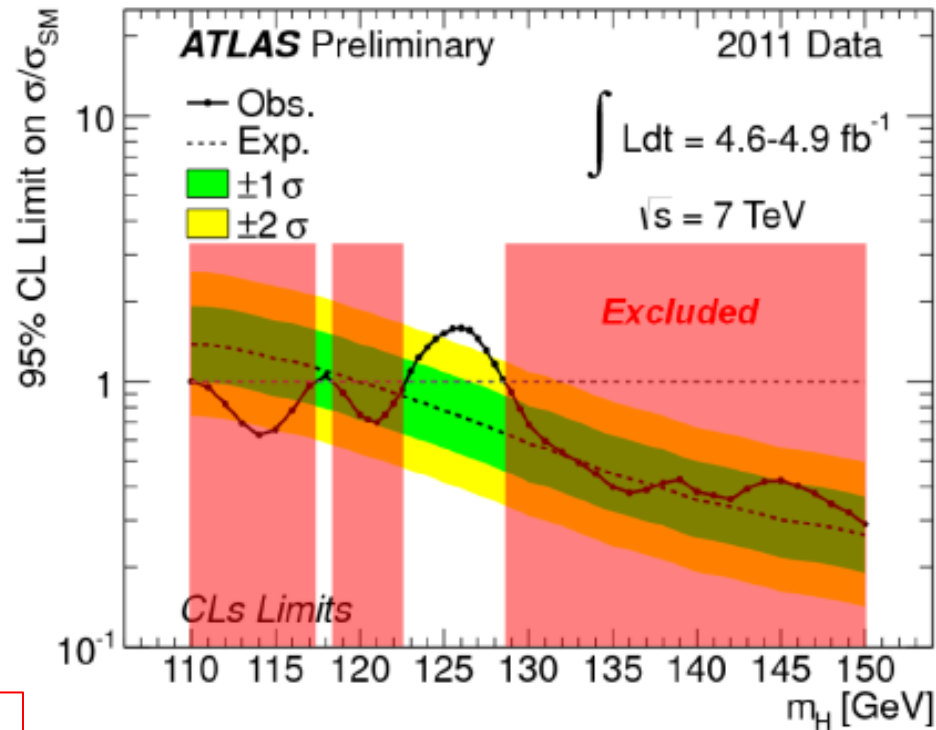
- High energies (**LHC**):
Hadronic tau decays

Search of the scalar sector of the SM, origin of EWSB

MOTIVATION

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Search of the scalar sector of the SM, origin of EWSB



CMS(preliminary)

Allowed Scalar Boson mass has been squeezed into a tiny region:

117.5-118.5 GeV or 122.5-129 GeV

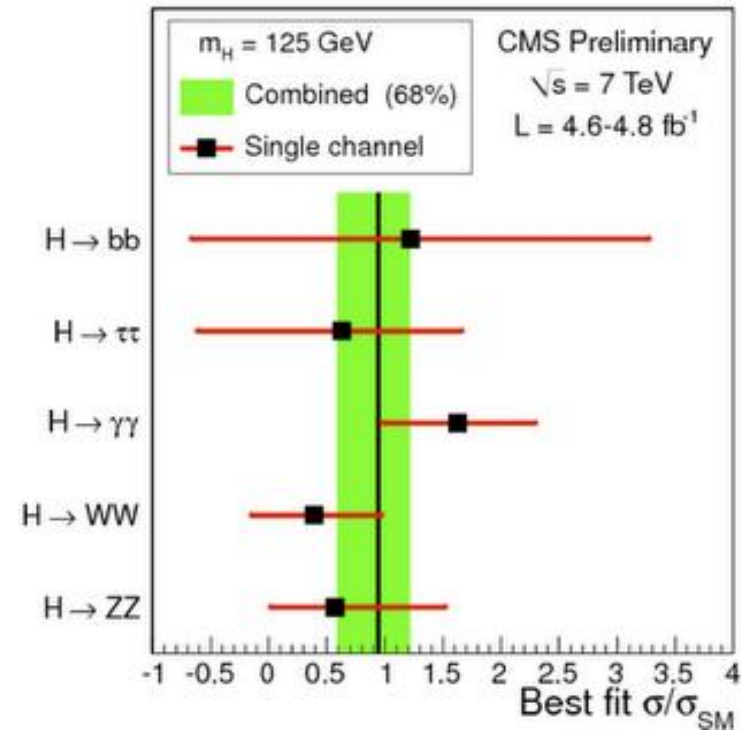
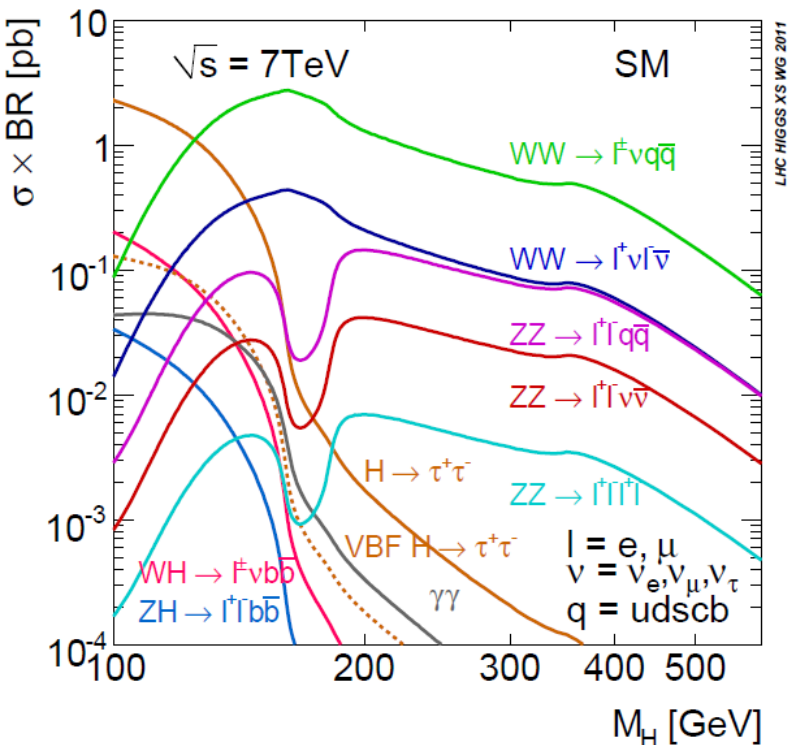
MOTIVATION

See last TAUOLA Workshop (Krakow, May 2012) devoted talks
<http://indico.cern.ch/conferenceDisplay.py?confId=188018>

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Search of the scalar sector of the SM, origin of EWSB

→ In the interesting low-E region the di- τ decay channel can give valuable information



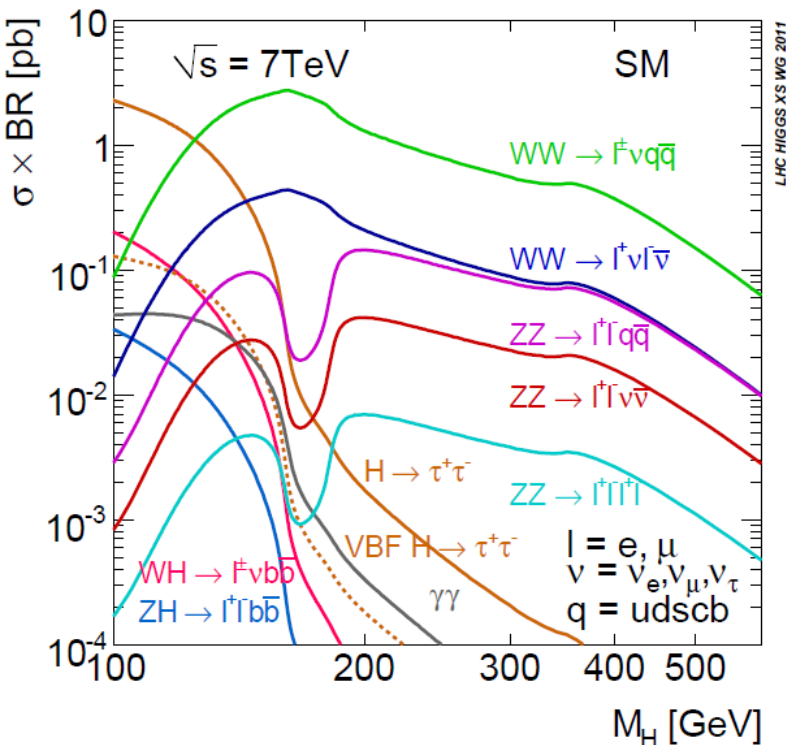
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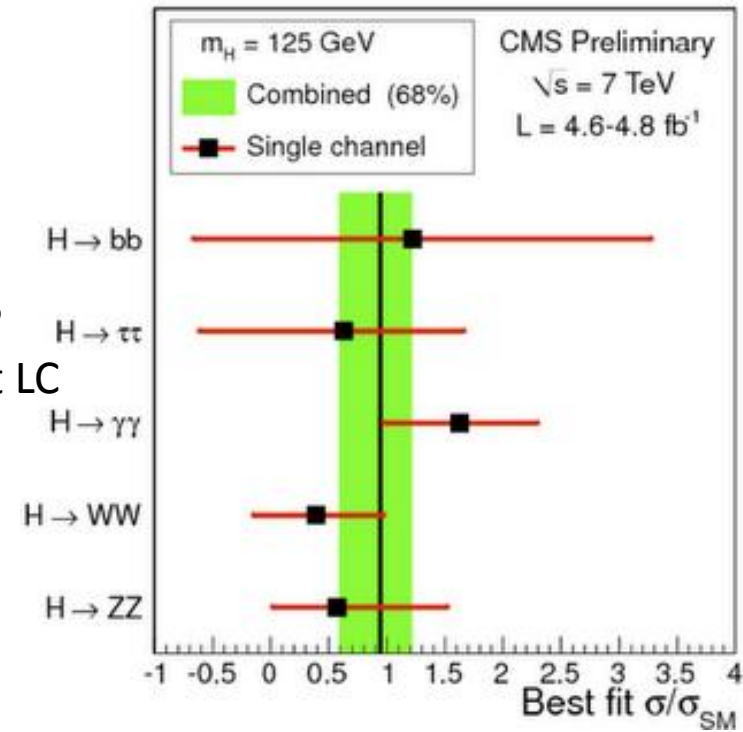
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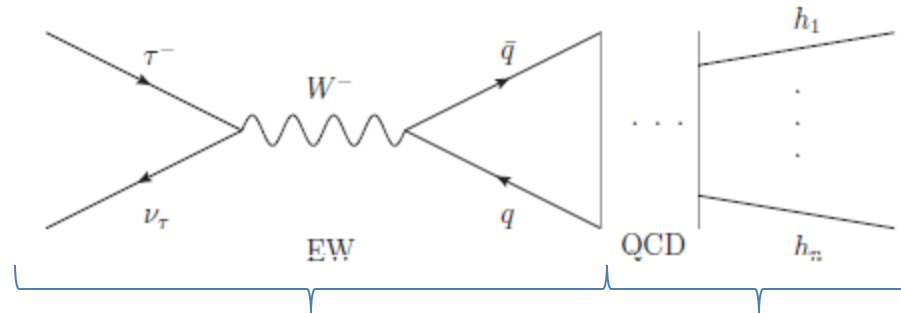
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Polarization, CP
Although better at LC



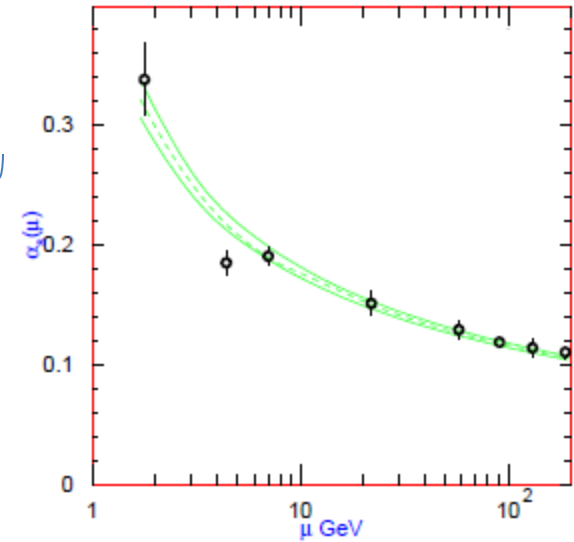
THEORETICAL SETTING



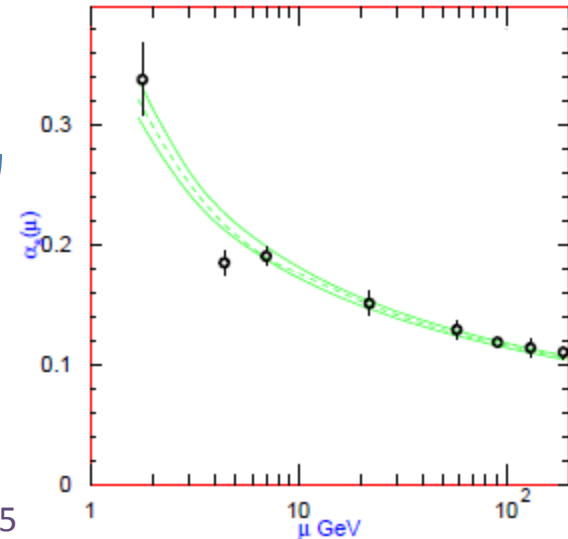
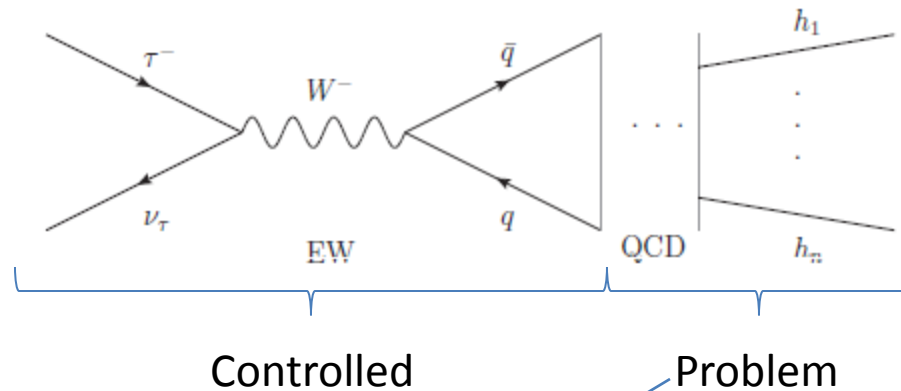
Controlled

Problem

Hadronization in $2m_\pi \leq E \leq M_\tau$



THEORETICAL SETTING



→ At very low energies, χ PT is the **EFT** of **QCD** Gasser, Leutwyler '85

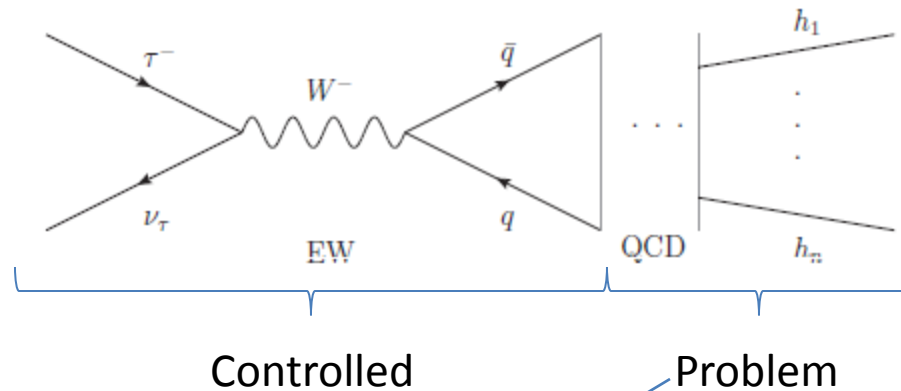
Only light quarks (u, d, s). In the massless limit $G \equiv SU(n_f)_L \otimes SU(n_f)_R$

Spontaneous Symmetry Breaking $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$

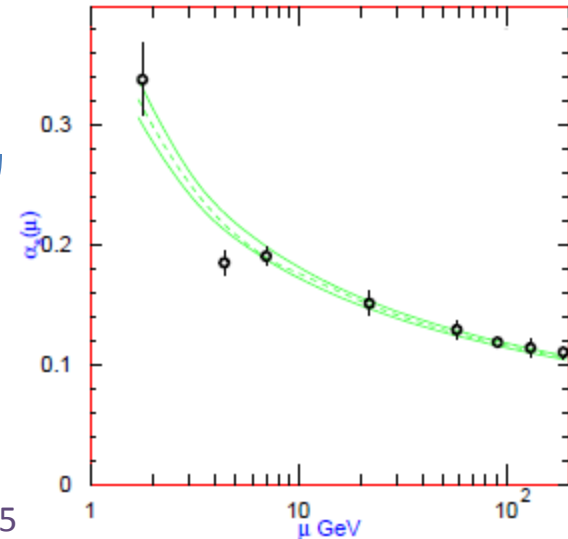
Approximately massless pGbs $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$

Small masses ($m < M_\rho, M_{a_1}$) by explicit Symmetry Breaking

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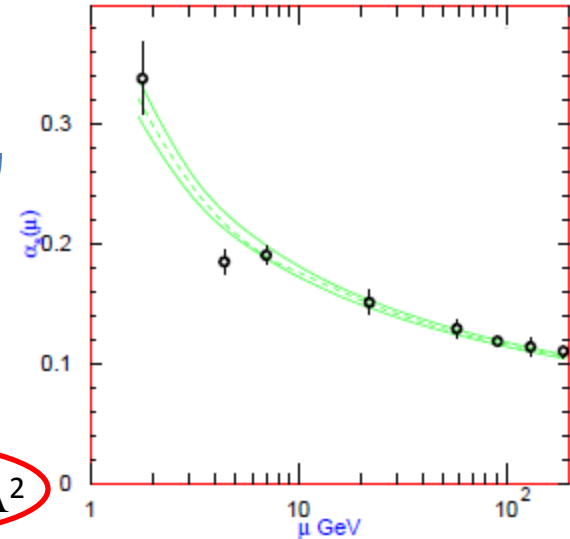
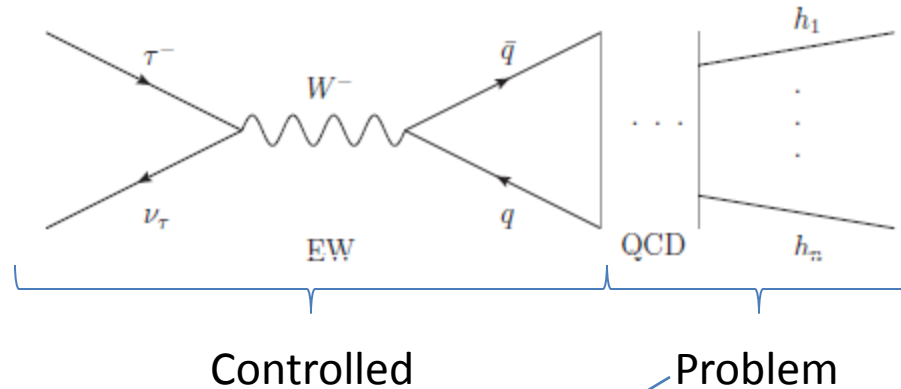
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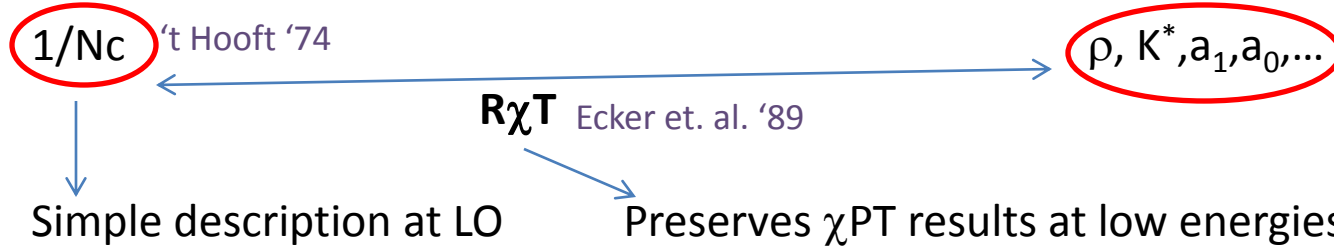
Approximately massless pGBs $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$ EFT: $(p^2, m^2)/\Lambda^2$

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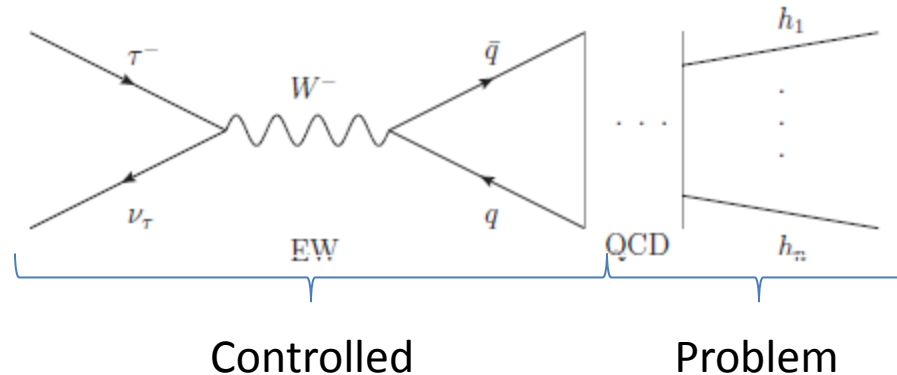
THEORETICAL SETTING



- At very low energies, χPT is the **EFT** of **QCD** EFT: $(p^2, m^2)/\Lambda^2$
- At larger energies the expansion breaks down and new dofs are needed



THEORY AT WORK



$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}(\nu_\tau) \gamma^\mu (1 - \gamma_5) u(\tau) T_\mu$$

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V} - \mathbf{A})_\mu e^{iS_{QCD}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

$$\langle \pi^-(p_{\pi^-}) \pi^0(p_{\pi^0}) | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} (p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

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Different approaches to deal with the diverse energy regimes

- For $E < M_\rho \rightarrow \chi$ PT up to $O(p^6)$ Gasser, Leutwyler'85, Bijnsens, Colangelo, Talavera '98, Bijnsens, Talavera'02

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Guerrero, Pich '97
- For $M_\rho \leq E \leq 1 \text{ GeV} \rightarrow$ Match χ PT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation Pich, Portolés '01

Unitarization approach Trocóniz, Ynduráin '01, Oller, Oser, Palomar '01

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Tower of resonances based on dual QCD Sanz-Cillero, Pich '03
Domínguez '01, Bruch, Khodjamirian, Kuhn '05

THEORY AT WORK

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Sanz-Cillero, Pich '03

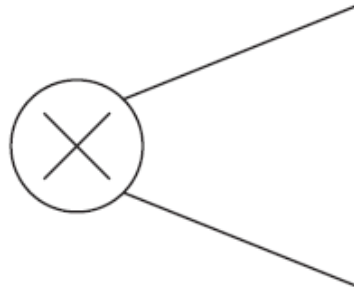
Tower of resonances based on dual QCD

Domínguez '01, Bruch, Khodjamirian, Kuhn '05

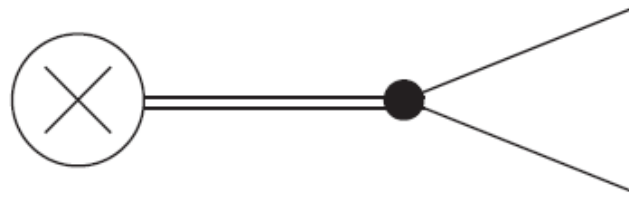
THEORY AT WORK

$$\langle \pi^-(p_{\pi^-}) \pi^0(p_{\pi^0}) | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} (p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Guerrero, Pich '97



$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$



$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$u_\mu = i \{ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \}$$

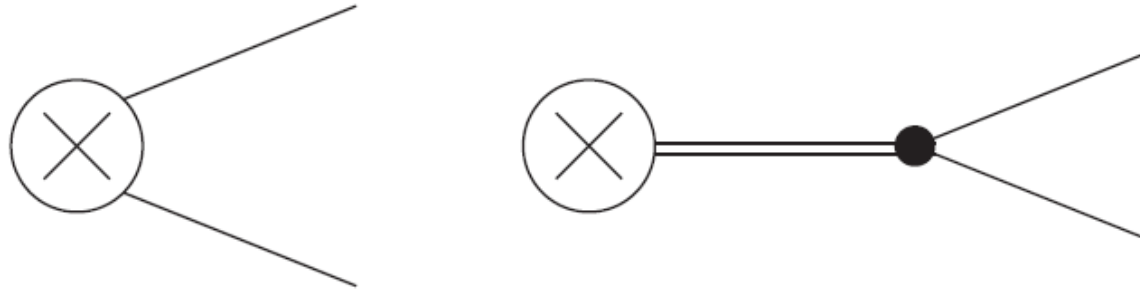
$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$$

$$u(\varphi) = \exp \left\{ i \frac{\Phi}{\sqrt{2} F} \right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}} \sum_{a=1}^8 \lambda_a \varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix}$$

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Guerrero, Pich '97

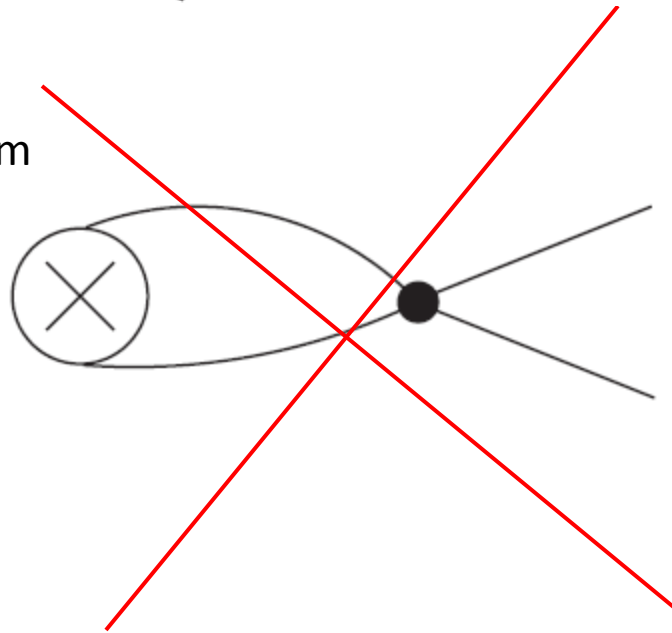


Antisymmetric tensor formalism
for spin-one resonances

Ecker et al.

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

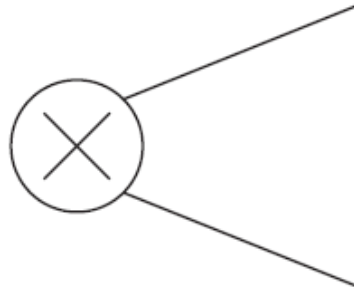


To avoid double counting

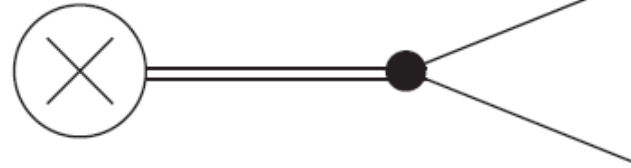
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➡ $F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2} \frac{s}{M_\rho^2 - s}$

Short-distance constraints

$$F(s) \rightarrow 0, \text{ for } s \rightarrow \infty \Rightarrow F_V G_V / f_\pi^2 = 1$$

⇓

$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

THEORETICAL SETTING:

χ PT, Large N_c , $R\chi T$

Portolés '10

Gasser, Leutwyler, '84, '85

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- Ecker, Gasser, Pich, De Rafael '89 Ecker, Gasser, Leutwyler, Pich, De Rafael '89
- Finally, **QCD high-energy** behaviour imposed to the **Green functions** or **form factors**.

Ruiz-Femenía, Pich, Portolés '03

Cirigliano, Ecker, Eidemüller, Pich, Portolés '04

Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés '05, '06

THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

- Tauola is the standard library for MC generation of tau lepton decays. Jadach, Kuhn, Was '90
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The parametrizations used by experimental collaborations (based on 1997-1998 data):

1. Alain Weinstein : http://www.cithec.caltech.edu/~ajw/korb_doc.html#files (*cleo version*)
2. B. Bloch, private communication (*aleph version*) **MOST USED NOWADAYS**

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See Guo, Roig, '10 for the radiative decays and the definition of the one-meson decay width

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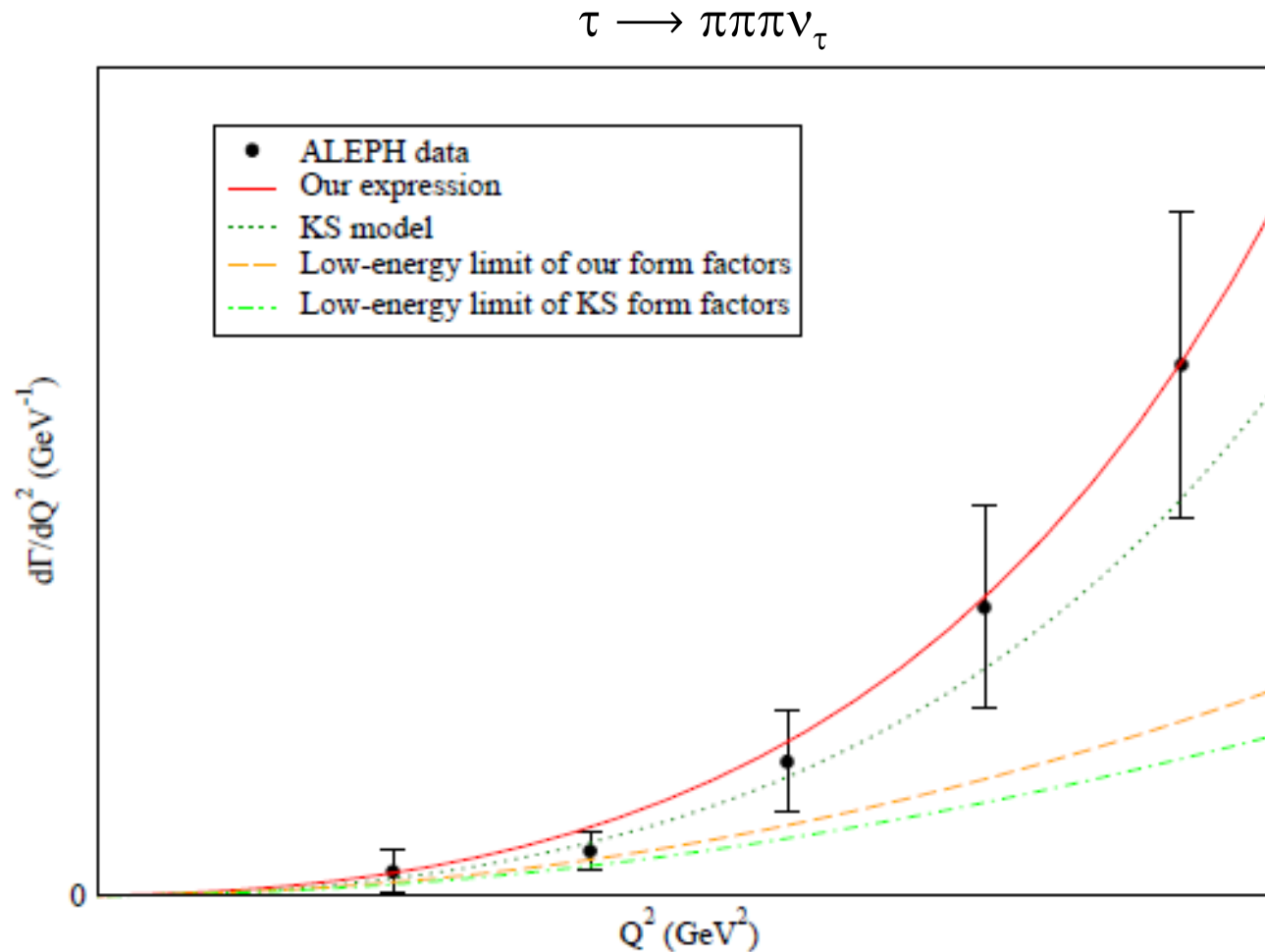
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Is this important?

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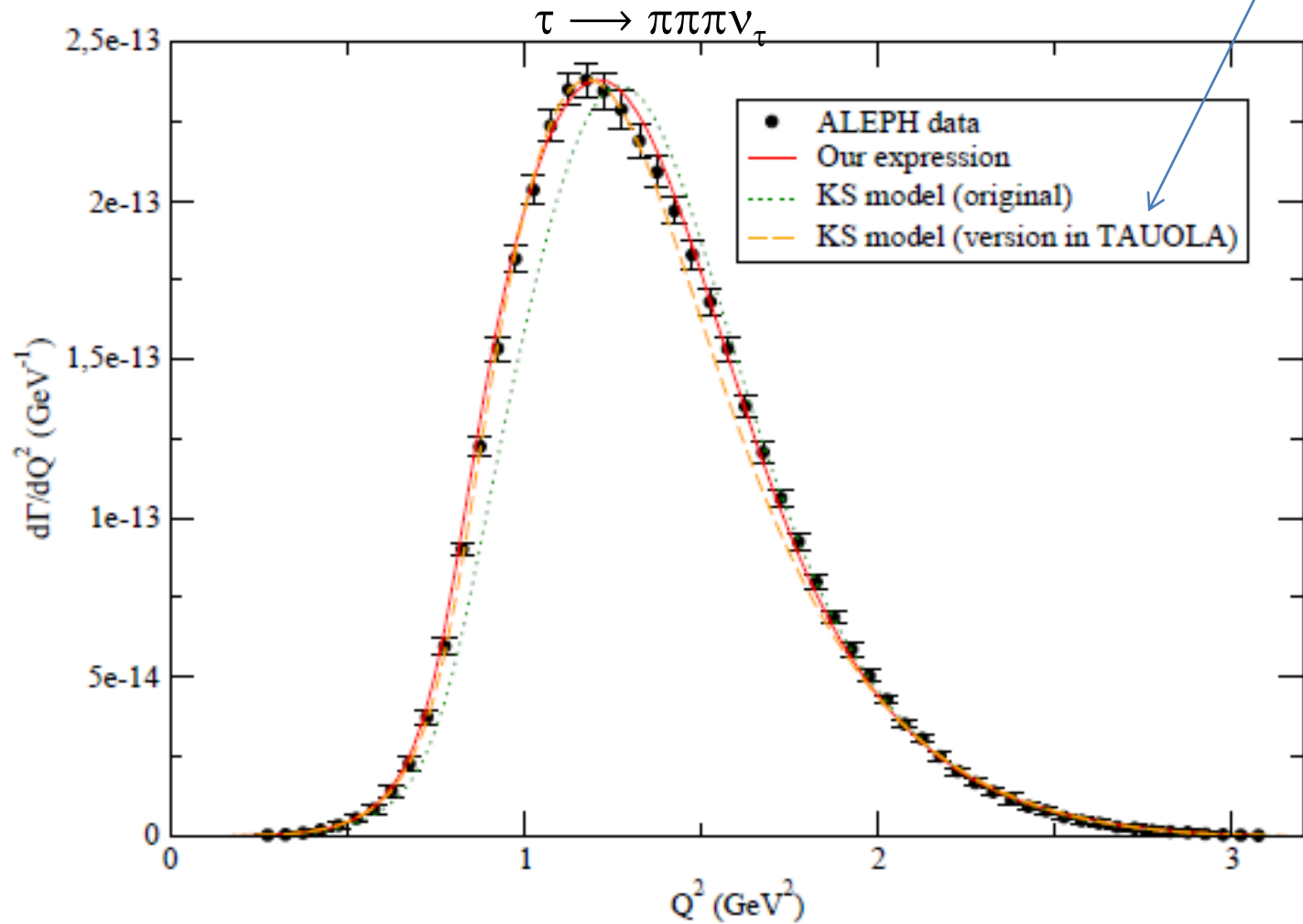
Shekhovtsova, Przedzinski, Roig, Was



THE PROJECT

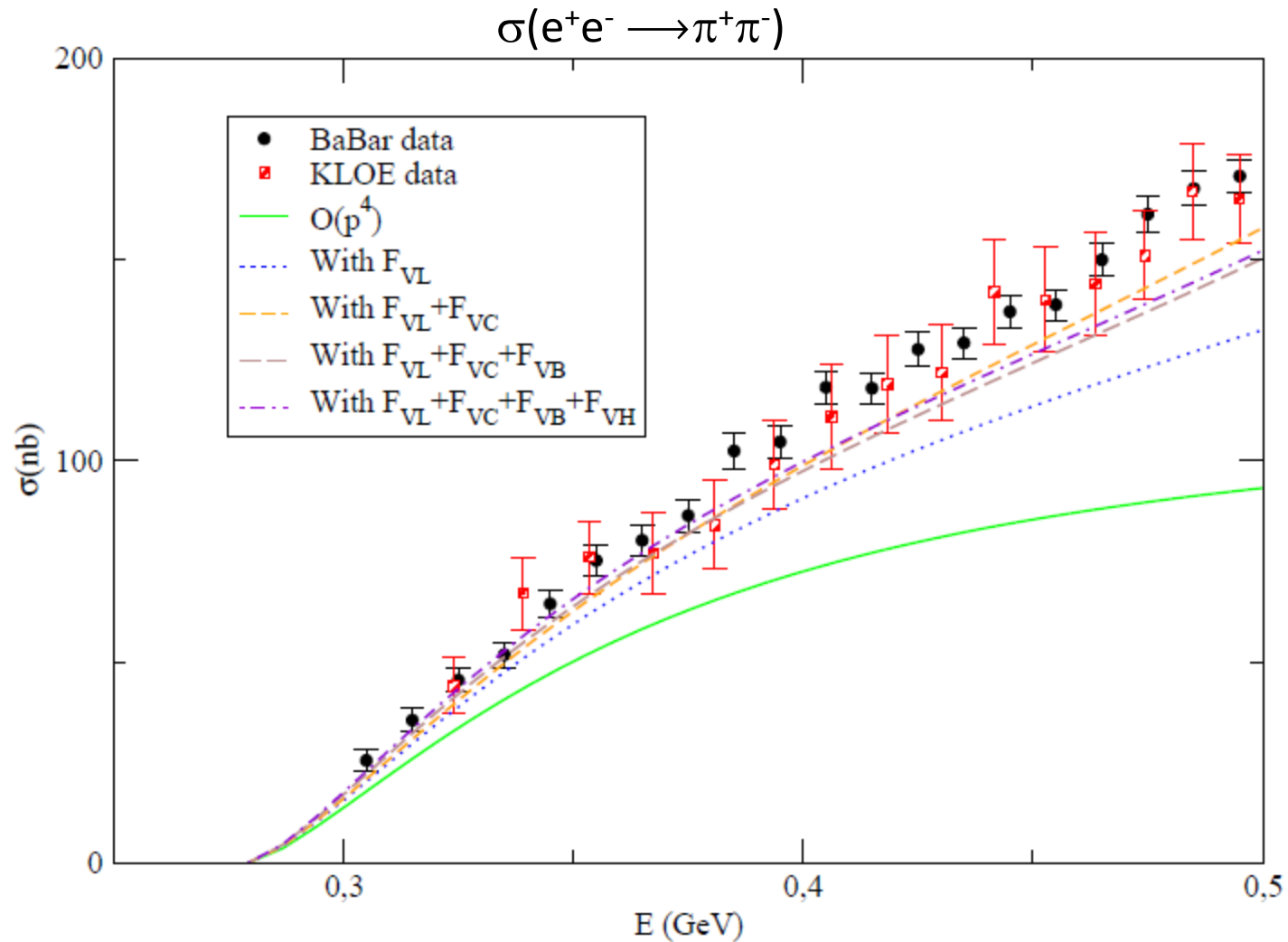
Shekhovtsova, Przedzinski, Roig, Was

Private old version



THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

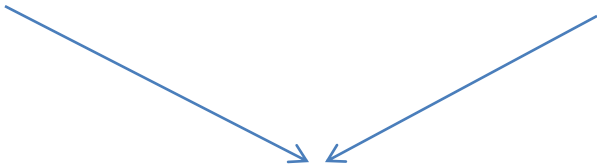


THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

$$\tau \longrightarrow \pi\pi\pi\nu_\tau$$

$$\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$$



Chiral dynamics is important

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NEW HADRONIC FORM FACTORS

$$T_\mu = \langle \text{Hadrons} | (\mathbf{V}-\mathbf{A})_\mu e^{iS_{\text{QCD}}} | 0 \rangle = \sum_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Two mesons $h_1(p_1), h_2(p_2)$: $J^\mu = N[(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)]$ $s = (p_1 + p_2)^2$
 $(T_\mu \sim J_\mu)$

Three mesons $h_1(p_1), h_2(p_2), h_3(p_3)$: $J^\mu = N \left\{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1 + c_2(p_3 - p_1)^\nu F_2 + c_3(p_1 - p_2)^\nu F_3] \right.$
 $\left. + c_4 q^\mu F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^\mu{}_{\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5 \right\}.$

$$T_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2 \qquad q^\mu = (p_1 + p_2 + p_3)^\mu$$

$$\underline{q^2 = (p_1 + p_2 + p_3)^2}$$

$$\underline{s_1 = (p_2 + p_3)^2}$$

$$\underline{s_2 = (p_1 + p_3)^2}$$

$$s_3 = (p_1 + p_2)^2$$

More mesons $\sim 4\pi$ (Fischer, Wess and Wagner '80; Bondar et. al. '02)

Form factors

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$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s} \quad \text{Guerrero, Pich '97}$$

➔ $F(s)_{\text{O(p}^4\text{)}}^{\text{ChPT}} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2} \left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2} A(m_K^2/s, m_K^2/\mu^2) \right]$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln(m_P^2/\mu^2) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \quad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

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Unitarity+Analyticity Omnés, '58



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O(p²) result for $\delta_1^1(s)$

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Gómez-Dumm, Pich, Portolés '00

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$$\boxed{F(s)} = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re} A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

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Starting point

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Match χ PT results to VMD using an Omnés solution for dispersion relation

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Guerrero, Pich '97

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies

- SU(2)
- Analyticity and unitarity constraints (NNLO)



Idea: Follow the approach of Jamin, Pich, Portolés '06 including excited resonances while retaining (some of) these nice properties

NEW HADRONIC FORM FACTORS

Starting point

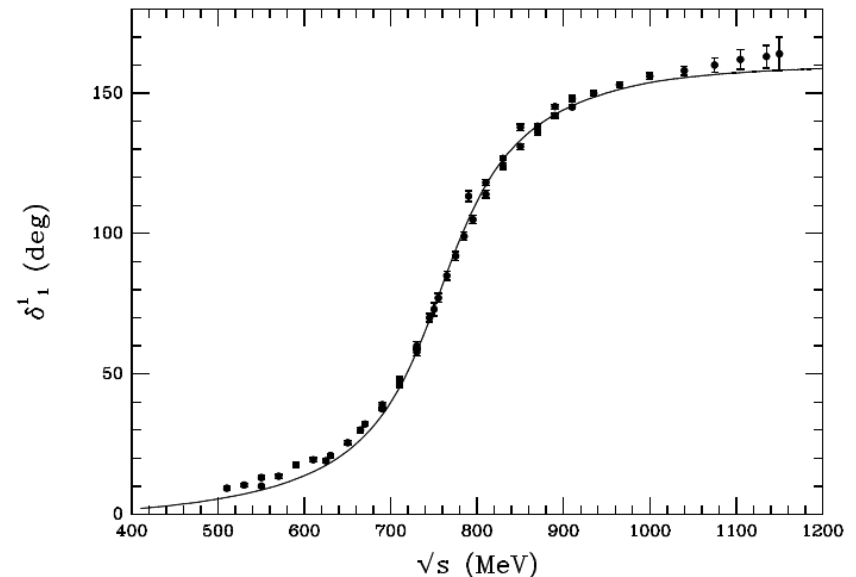
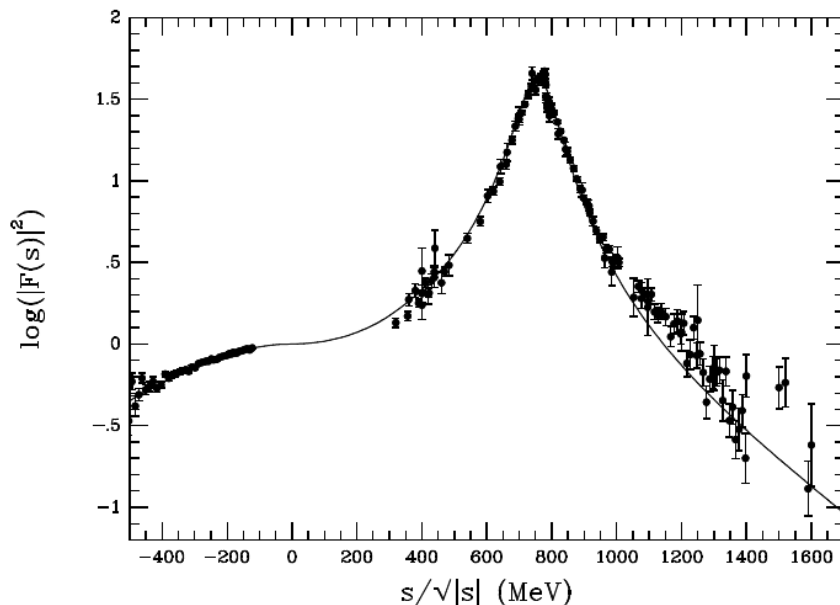
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Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right] \\ - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp \left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right] \\ - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp \left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].$$

- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
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- Analyticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of $\rho' + \rho''$

NEW HADRONIC FORM FACTORS

Starting point

Guerrero, Pich '97

Match χ PT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\text{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} \text{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Our formula $F_V^-(s)$ Roig '11

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right] \\ - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right] \\ - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right] .$$

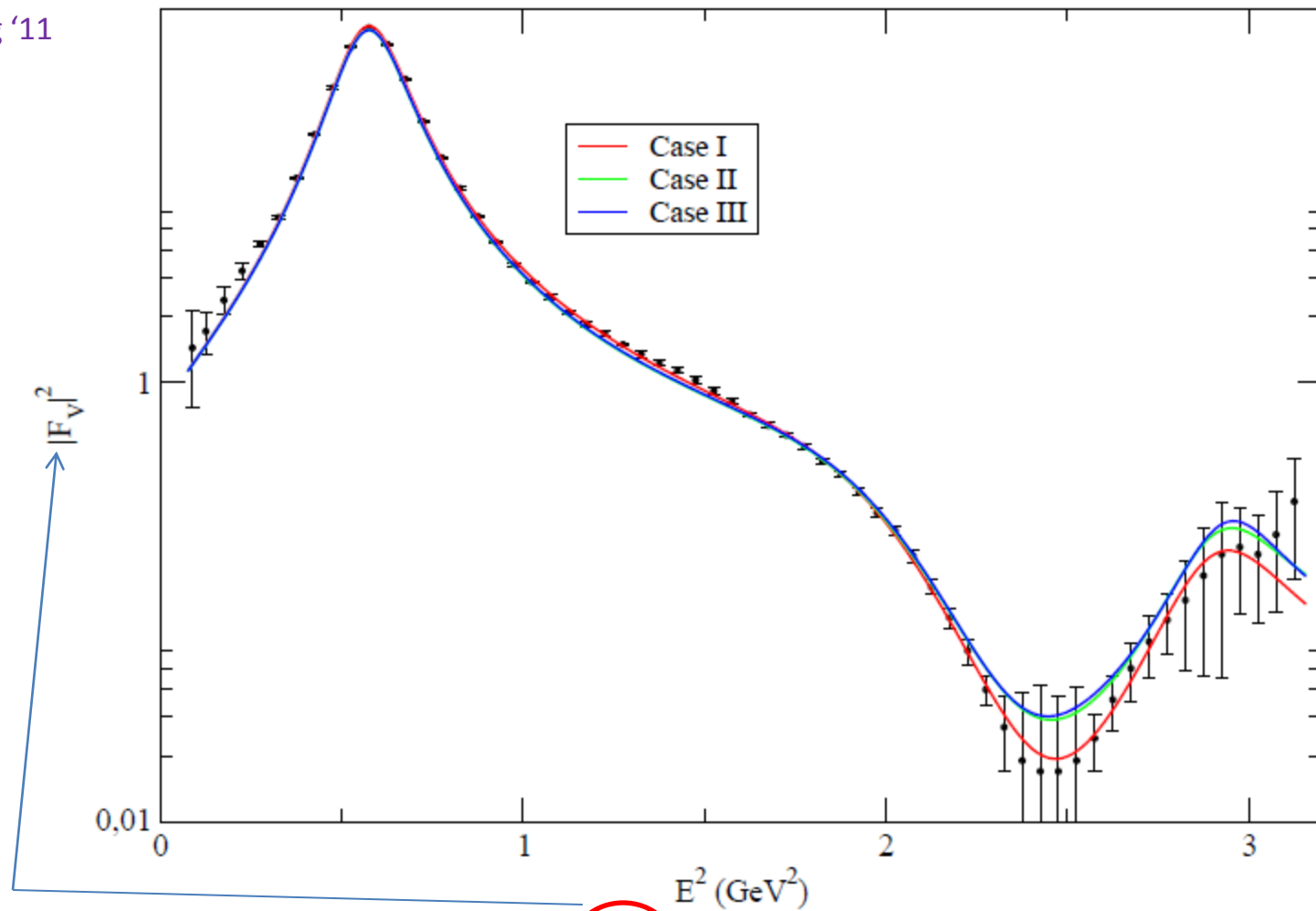
- χ PT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)

- Analyticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of $\rho' + \rho''$

This is what is included in TAUOLA right now

NEW HADRONIC FORM FACTORS

Roig '11



$$\langle \pi^-(p_{\pi^-}) \pi^0(p_{\pi^0}) | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} (p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

NEW HADRONIC FORM FACTORS

On the inclusion of excited resonances

Our formula $F_V^-(s)$ =

Roig '11

$$= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right]$$

$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right]$$

$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right] .$$

$$\gamma \equiv -F'_V G'_V / F^2 \quad \delta \equiv -F''_V G''_V / F^2 \quad F_V G_V + F'_V G'_V + F''_V G''_V + \dots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

NEW HADRONIC FORM FACTORS

On the inclusion of excited resonances

Our formula $F_V^-(s)$ =

$$\begin{aligned}
 & \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left[\frac{-s}{96\pi^2 F_\pi^2} (\Re A_\pi(s) + \Re A_K(s)/2) \right] \\
 & - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left[\frac{-s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re A_\pi(s) \right] \\
 & - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left[\frac{-s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re A_\pi(s) \right].
 \end{aligned}$$

Roig '11

$$\gamma \equiv -F'_V G'_V / F^2 \quad \delta \equiv -F''_V G''_V / F^2 \quad F_V G_V + F'_V G'_V + F''_V G''_V + \dots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

→ Easy to implement for two meson modes. For three meson modes a number of new couplings (involving new operator structures) appear. At which stage shall we include them?

NEW HADRONIC FORM FACTORS

Similar philosophy for other two meson tau decay modes

$$F_{PQ}^V(s) = F^{VMD}(s) \exp \left[\sum_{P,Q} N_{loop}^{PQ} \frac{-s}{96\pi^2 F^2} \text{Re} A_{PQ}(s) \right]$$

$$F_{KK}^V(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 F^2} \left[\text{Re} A_\pi(s) + \frac{1}{2} \text{Re} A_K(s) \right] \right\}$$

Guerrero, Pich '97, Arganda, Herrero, Portolés '08

$$F_+^{K\pi}(s) = \left[\frac{M_{K^*}^2 + \gamma s}{M_{K^*}^2 - s - iM_{K^*} \Gamma_{K^*}(s)} - \frac{\gamma s}{M_{K^{*'}}^2 - s - iM_{K^{*'}} \Gamma_{K^{*'}}(s)} \right] e^{\frac{3}{2} \text{Re} [\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)]}$$

Jamin, Pich, Portolés '06

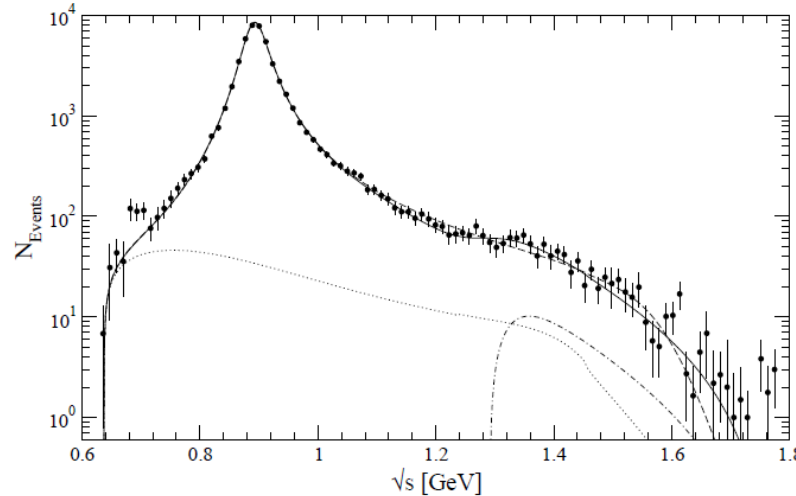
$$\Gamma_{K^*}(q^2) = \frac{M_{K^*} q^2}{128\pi F^2} \left[\lambda^{3/2} \left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2} \right) \theta(q^2 - \text{thr}_{K\pi}) + \lambda^{3/2} \left(1, \frac{m_K^2}{q^2}, \frac{m_\eta^2}{q^2} \right) \theta(q^2 - \text{thr}_{K\eta}) \right]$$

$$\langle \pi^-(p) | \bar{s} \gamma^\mu u | K^0(k) \rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu \right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} (k-p)^\mu F_0(q^2)$$

Jamin, Oller, Pich '01, '06

NEW HADRONIC FORM FACTORS

Similar philosophy for other two meson tau decay modes



$$F_+^{K\pi}(s) = \left[\frac{M_{K^*}^2 + \gamma s}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma s}{M_{K^{*'}}^2 - s - iM_{K^{*'}}\Gamma_{K^{*'}}(s)} \right] e^{\frac{3}{2}\text{Re}[\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)]}$$

Jamin, Pich, Portolés '06

$$\Gamma_{K^*}(q^2) = \frac{M_{K^*} q^2}{128\pi F^2} \left[\lambda^{3/2} \left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2} \right) \theta(q^2 - thr_{K\pi}) + \lambda^{3/2} \left(1, \frac{m_K^2}{q^2}, \frac{m_\eta^2}{q^2} \right) \theta(q^2 - thr_{K\eta}) \right]$$

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Jamin, Oller, Pich '01, '06

NEW HADRONIC FORM FACTORS

$$\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_{+,0}(0)}$$

With new improvements also Boito, Escribano, Jamin '08
Exact analyticity and unitarity

$$\bar{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*} \bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*'}}, \gamma_{K^{*'}})}$$

$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \text{Re} \bar{A}_{K\pi}(s) - i m_n \gamma_n(s),$$

$$\gamma_{K^*}(s) = \gamma_{K^*} \frac{s}{m_{K^*}^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \quad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3} \frac{\gamma_{K^*}}{m_{K^*}}, \quad F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa \bar{A}_{K\pi}(0)}$$

$$\sigma_{K\pi}(s) = \sqrt{\left(1 - \frac{(m_K + m_\pi)^2}{s}\right) \left(1 - \frac{(m_K - m_\pi)^2}{s}\right)} \theta(s - (m_K + m_\pi)^2)$$

$$\delta^{PQ}(s) = \text{Im} \left[F_V^{PQ}(s) \right] / \text{Re} \left[F_V^{PQ}(s) \right]$$

$$F_V^{PQ}(s) = \exp \left\{ \alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi} \int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3 (s' - s - i\epsilon)} \right\}$$

Other approaches: Jamin, Pich, Portolés '06, '08

Boito, Escribano, Jamin '06

Moussallam '07

Bernard, Boito, Passemar '11

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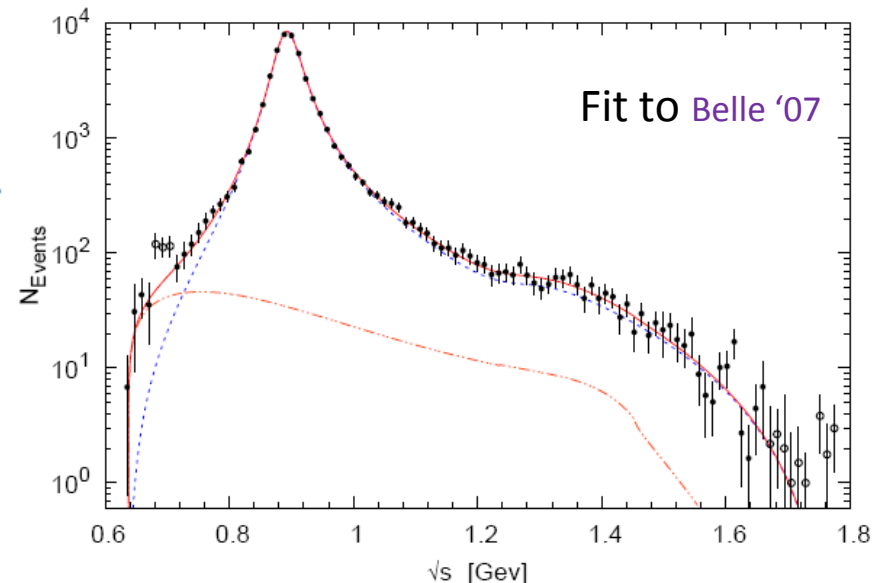
$$F_V^{PQ}(s) = \exp \left\{ \alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi} \int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3 (s' - s - i\epsilon)} \right\}$$

Other approaches: Jamin, Pich, Portolés '06, '08

Boito, Escribano, Jamin '06

Moussallam '07

Bernard, Boito, Passemar '11



NEW HADRONIC FORM FACTORS

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Exact analyticity and unitarity

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$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \text{Re} \bar{A}_{K\pi}(s) - i m_n \gamma_n(s),$$

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We will include also the
 dispersive approach for other
 two-meson decay channels

Other approaches: Jamin, Pich, Portolés '06, '08

Boito, Escribano, Jamin '06

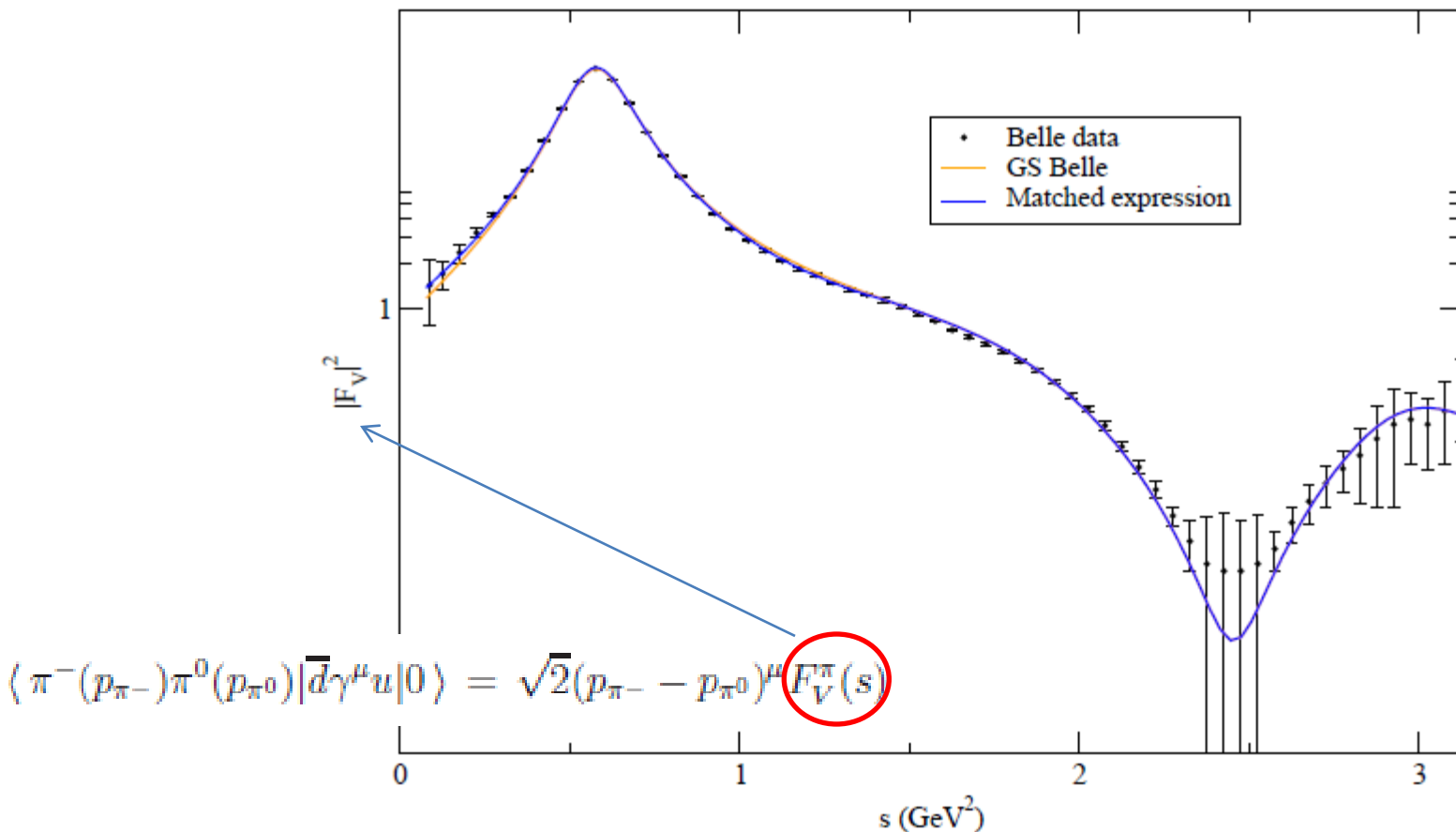
Moussallam '07

Bernard, Boito, Passemar '11

NEW HADRONIC FORM FACTORS

With new improvements also
Exact analyticity and unitarity

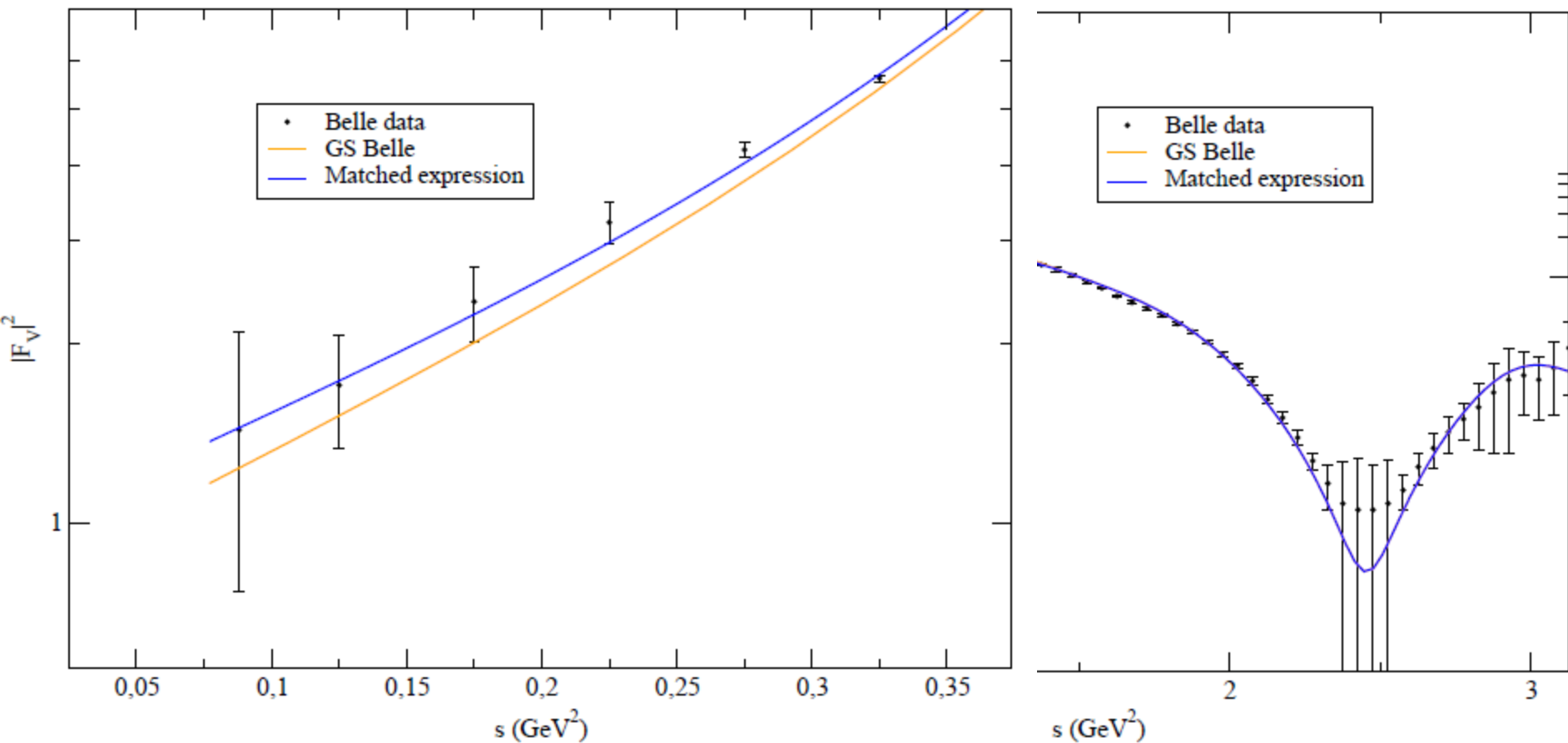
$$\delta^{PQ}(s) = \text{Im} [F_V^{PQ}(s)] / \text{Re} [F_V^{PQ}(s)] \longrightarrow F_V^{PQ}(s) = \exp \left\{ \alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi} \int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s' - s - i\epsilon)} \right\}$$



NEW HADRONIC FORM FACTORS

With new improvements also
Exact analyticity and unitarity

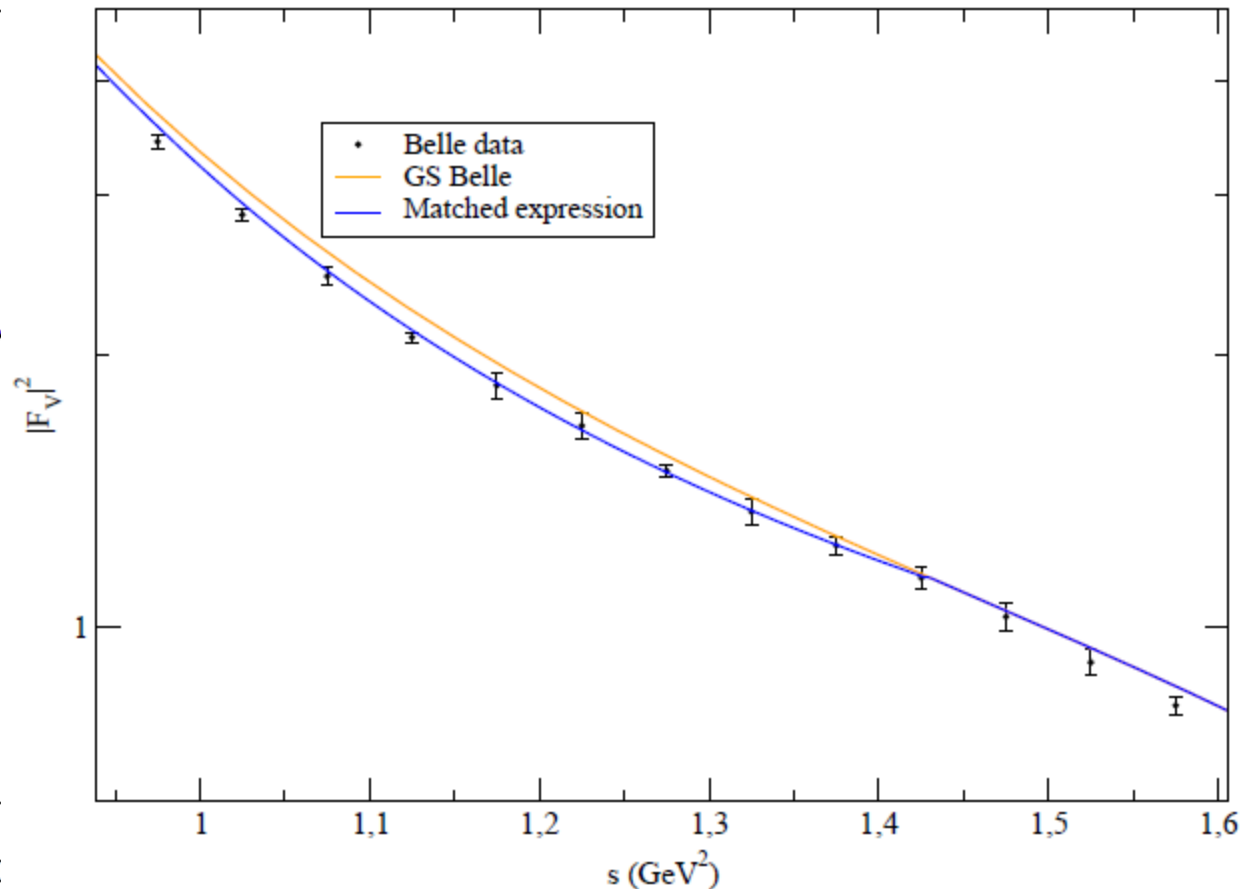
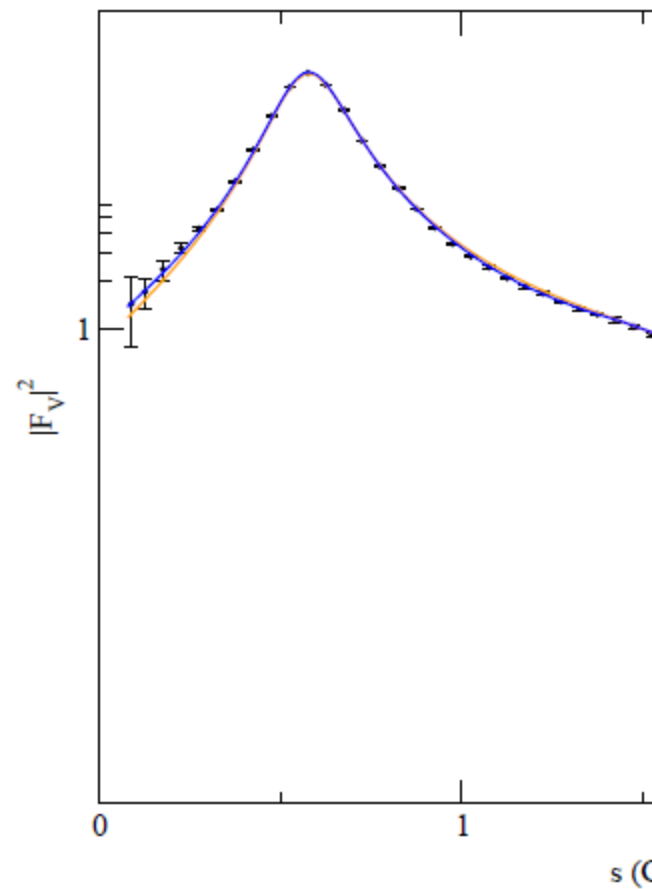
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NEW HADRONIC FORM FACTORS

With new improvements also
Exact analyticity and unitarity

$$\delta^{PQ}(s) = \text{Im} \left[F_V^{PQ}(s) \right] / \text{Re} \left[F_V^{PQ}(s) \right] \longrightarrow F_V^{PQ}(s) = \exp \left\{ \alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi} \int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3 (s' - s - i\epsilon)} \right\}$$

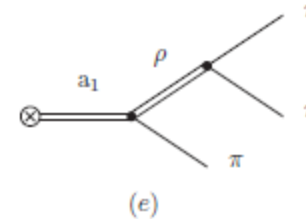
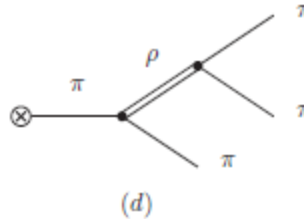
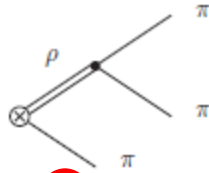
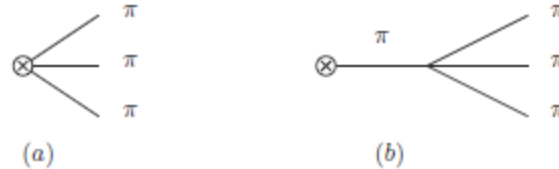


NEW HADRONIC FORM FACTORS

Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés '09

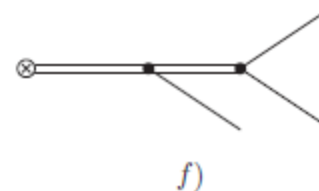
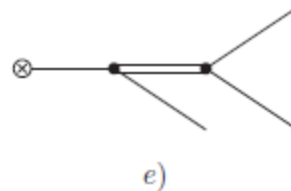
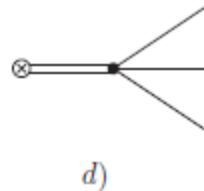
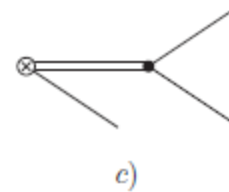
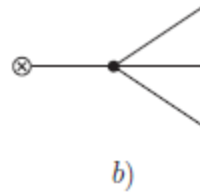
$$\tau \longrightarrow \pi\pi\pi\nu_\tau$$



$$\mathcal{L}_4^V = \sum_{i=1}^5 \frac{g_i}{M_V} \mathcal{O}_{VPPP}^i + \sum_{i=1}^7 \frac{c_i}{M_V} \mathcal{O}_{VJP}^i$$

$$\mathcal{L}_2^{RR} = \sum_{i=1}^5 \lambda_i \mathcal{O}_{VAP}^i + \sum_{i=1}^4 d_i \mathcal{O}_{VVP}^i$$

$$\tau \longrightarrow KK\pi\nu_\tau$$



Gómez-Dumm, Roig, Pich, Portolés '09

Phys.Rev.D81:034031,2010

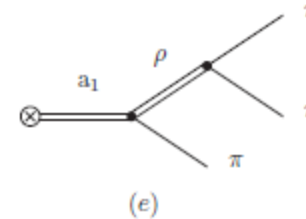
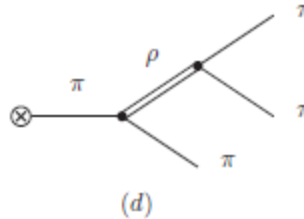
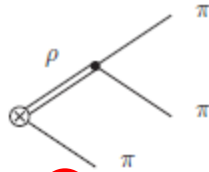
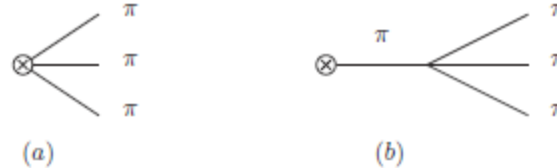
NEW HADRONIC FORM FACTORS

Only one independent (relevant) form factor $\tau \longrightarrow \pi\pi\pi V_\tau$

Only axial-vector current

Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés '09



$$\mathcal{L}_4^V = \sum_{i=1}^5 \frac{g_i}{M_V} \mathcal{O}_{VPPP}^i + \sum_{i=1}^7 \frac{c_i}{M_V} \mathcal{O}_{VJP}^i$$

$$\mathcal{L}_2^{RR} = \sum_{i=1}^5 \lambda_i \mathcal{O}_{VAP}^i + \sum_{i=1}^4 d_i \mathcal{O}_{VVP}^i$$

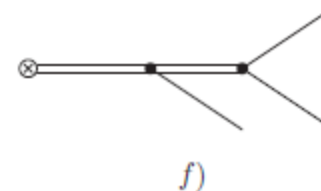
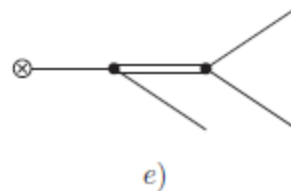
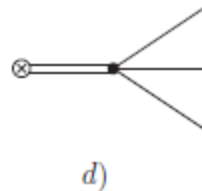
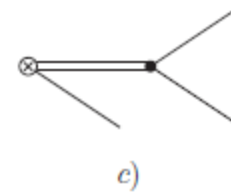
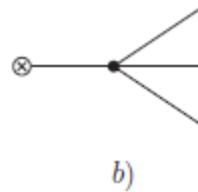
Three independent (relevant) FFs

$\tau \longrightarrow KK\pi V_\tau$

Both axial-vector and vector current

Gómez-Dumm, Roig, Pich, Portolés '09

Phys.Rev.D81:034031,2010



NEW HADRONIC FORM FACTORS

Phys.Rev.D69:073002,2004

Gómez-Dumm, Pich, Portolés '03

Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés '09

$$F_{\pm i} = \pm (F_i^X + F_i^R + F_i^{RR}) , \quad i = 1, 2$$

$$F_2(Q^2, s, t) = F_1(Q^2, t, s)$$

$$F_1^X(Q^2, s, t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^R(Q^2, s, t) = \frac{\sqrt{2} F_V G_V}{3 F^3} \left[\frac{3s}{s - M_V^2} - \left(\frac{2G_V}{F_V} - 1 \right) \left(\frac{2Q^2 - 2s - u}{s - M_V^2} + \frac{u - s}{t - M_V^2} \right) \right]$$

$$F_1^{RR}(Q^2, s, t) = \frac{4 F_A G_V}{3 F^3} \frac{Q^2}{Q^2 - M_A^2} \left[-(\lambda' + \lambda'') \frac{3s}{s - M_V^2} + H(Q^2, s) \frac{2Q^2 + s - u}{s - M_V^2} + H(Q^2, t) \frac{u - s}{t - M_V^2} \right]$$

$$H(Q^2, x) = -\lambda_0 \frac{m_\pi^2}{Q^2} + \lambda' \frac{x}{Q^2} + \lambda''$$

Relations from short-distance QCD:

$$F_V G_V = F^2$$

$$F_V^2 - F_A^2 = F^2$$

$$F_V^2 M_V^2 = F_A^2 M_A^2$$

$$\lambda' = \frac{F^2}{2\sqrt{2} F_A G_V} = \frac{M_A}{2\sqrt{2} M_V},$$

$$\lambda'' = \frac{2 G_V - F_V}{2\sqrt{2} F_A} = \frac{M_A^2 - 2M_V^2}{2\sqrt{2} M_V M_A}$$

$$4\lambda_0 = \lambda' + \lambda'' = \frac{M_A^2 - M_V^2}{\sqrt{2} M_V M_A},$$

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09

$\Gamma_{a_1}(Q^2)$:



$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^{\pi}(Q^2) \theta(Q^2 - 9m_{\pi}^2) + \Gamma_{a_1}^K(Q^2) \theta(Q^2 - (2m_K + m_{\pi})^2) ,$$

$$\Gamma_{a_1}^{\pi,K}(Q^2) = \frac{-S}{192(2\pi)^3 F_A^2 M_{a_1}} \left(\frac{M_{a_1}^2}{Q^2} - 1 \right)^2 \int ds dt T_{1+}^{\pi,K\mu} T_{1+\mu}^{\pi,K*} .$$

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09

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$$\frac{1}{M_{\rho}^2 - q^2 - iM_{\rho}\Gamma_{\rho}(q^2)} \longrightarrow \frac{1}{1 + \beta_{\rho'}} \left[\frac{1}{M_{\rho}^2 - q^2 - iM_{\rho}\Gamma_{\rho}(q^2)} + \frac{\beta_{\rho'}}{M_{\rho'}^2 - q^2 - iM_{\rho'}\Gamma_{\rho'}(q^2)} \right]$$

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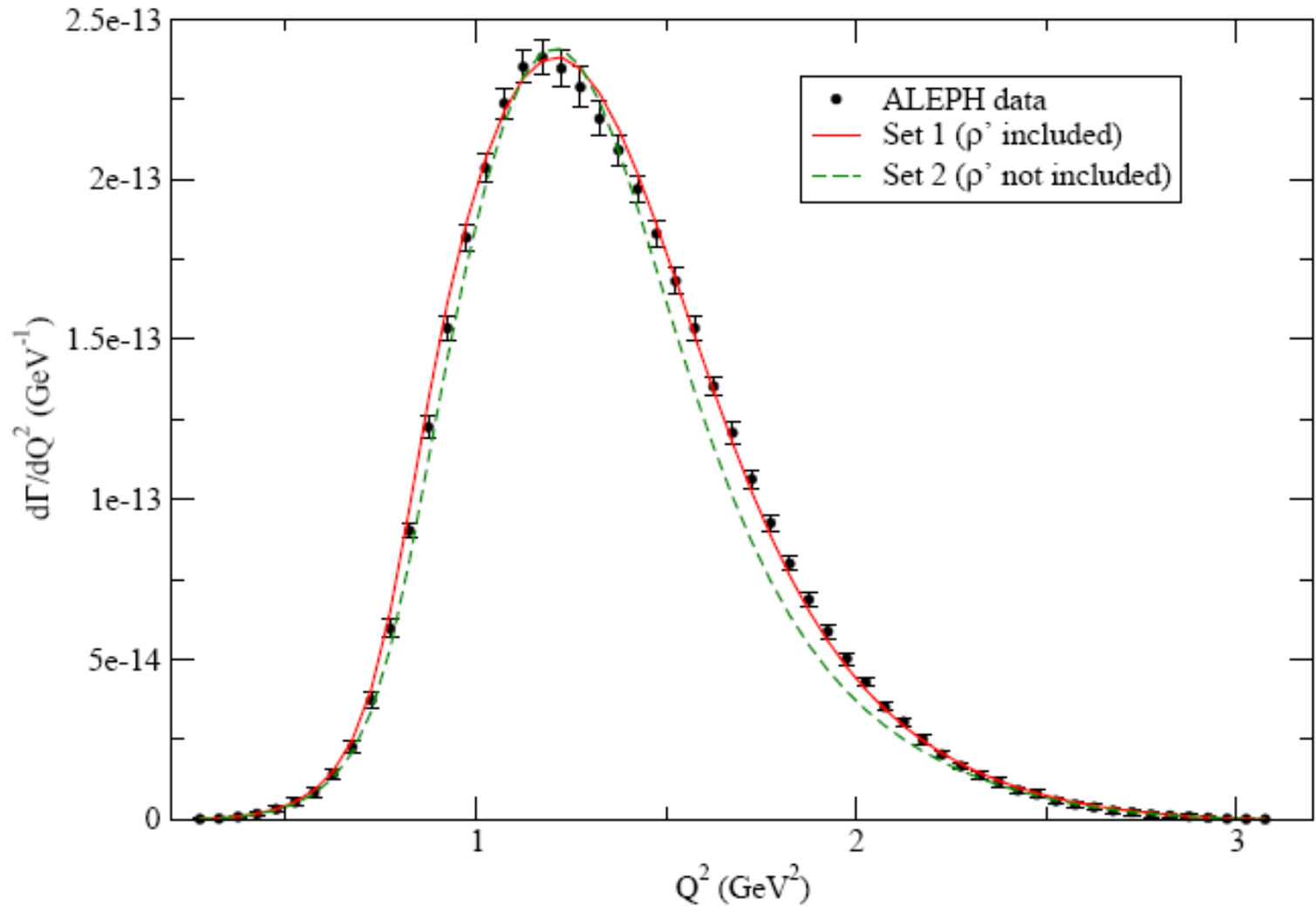
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→ Inclusion from a Lagrangian would imply 3 coups. instead of $\beta_{\rho'}$

F_V', G_V', F_A'

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09



NEW HADRONIC FORM FACTORS

Additional high-energy constraints are found on the $\mathbf{KK}\pi$ channels:

$$\begin{aligned}c_1 - c_2 + c_3 &= 0, \\c_1 - c_2 - c_3 + 2c_6 &= -\frac{N_C}{96\pi^2} \frac{F_V M_V}{\sqrt{2} F^2}, \\d_3 &= -\frac{N_C}{192\pi^2} \frac{M_V^2}{F^2}, \\g_1 + 2g_2 - g_3 &= 0, \\g_2 &= \frac{N_C}{192\sqrt{2}\pi^2} \frac{M_V}{F_V}.\end{aligned}$$

- We do not have any hint on the value of two of the odd-intrinsic parity couplings.
- Up to now, excited resonances have not been implemented.
- Through the framework provided by **TAUOLA**, with the new currents from R χ T installed, it will be much easier to learn about the unknown couplings and to estimate properly the size of the excited resonances contribution.

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- Through the framework provided by **TAUOLA**, with the new currents from R χ T installed, it will be much easier to learn about the unknown couplings and to estimate properly the size of the excited resonances contribution.
- **Essential to study the most relevant channels in unified framework for signal/background splitting and data analysis: New TAUOLA currents.**

COMPARISONS

MC-Tester Davidson, Golonka, Przedzinski, Was '08

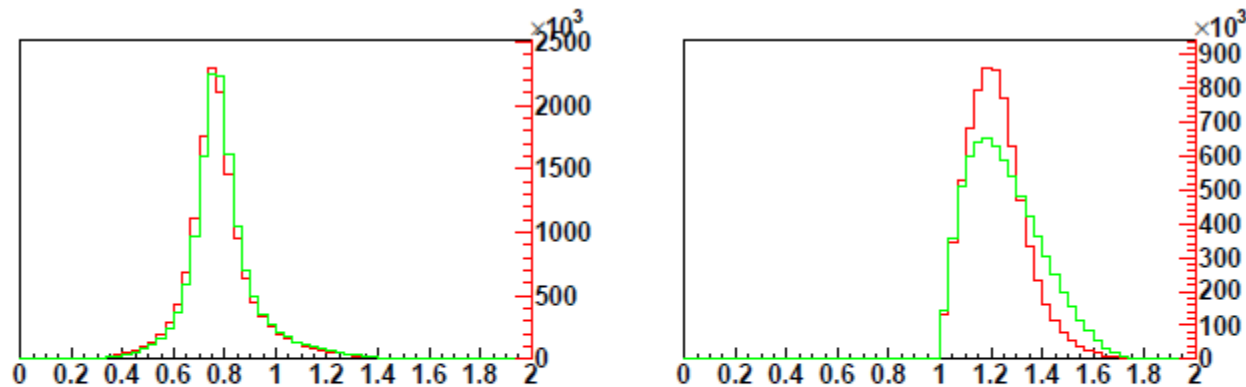


Figure 6: $\tau \rightarrow \pi^- \pi^0 \nu_\tau$ and $\tau \rightarrow K^- K_S^0 \nu_\tau$ decays: Comparison of distributions for TAUOLA cleo current [21] and for our new current. On the left-hand side, plot of $\pi^- \pi^0$ invariant mass is shown and on the right-hand side $K^- K_S^0$ are for new current, red (darker grey) are for TAUOLA cleo.



Shekhovtsova, Przedzinski, Roig, Was '12

COMPARISONS

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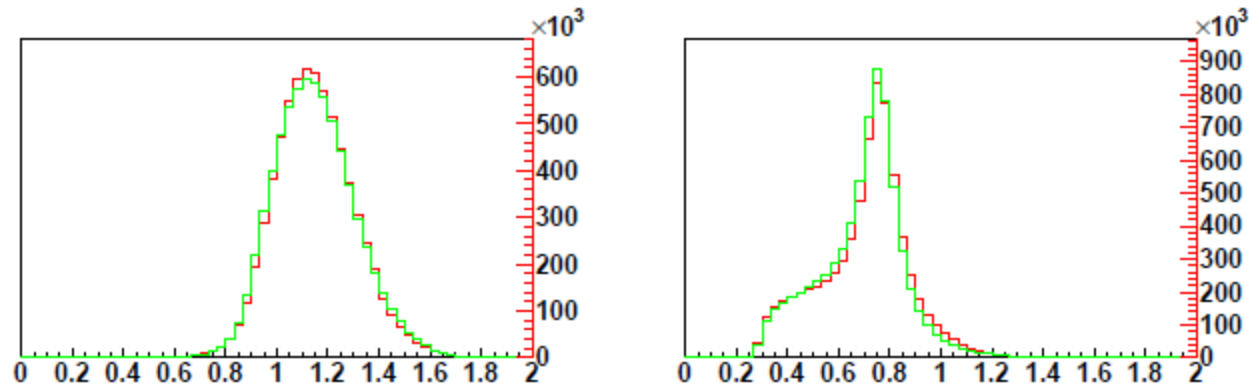


Figure 2: The $\tau \rightarrow \pi^+\pi^-\pi^-\nu_\tau$ decay: comparison of distributions for TAUOLA cleo current [21] and for our new current. On the left-hand side, the plot of $\pi^+\pi^-\pi^-$ invariant mass is shown and on the right-hand side $\pi^+\pi^-$ invariant mass is given. Green histograms (light grey) are for the new current, red (darker grey) are for TAUOLA cleo. Distributions for the $\tau \rightarrow \pi^-\pi^0\pi^0\nu_\tau$ decay coincide with the ones for $\tau \rightarrow \pi^+\pi^-\pi^-\nu_\tau$.



Shekhovtsova, Przedzinski, Roig, Was '12

<http://annapurna.ifj.edu.pl/~wasm/RChL/RChL.htm>

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MC-Tester Davidson, Golonka, Przedzinski, Was '08

Shekhovtsova, Przedzinski, Roig, Was '12

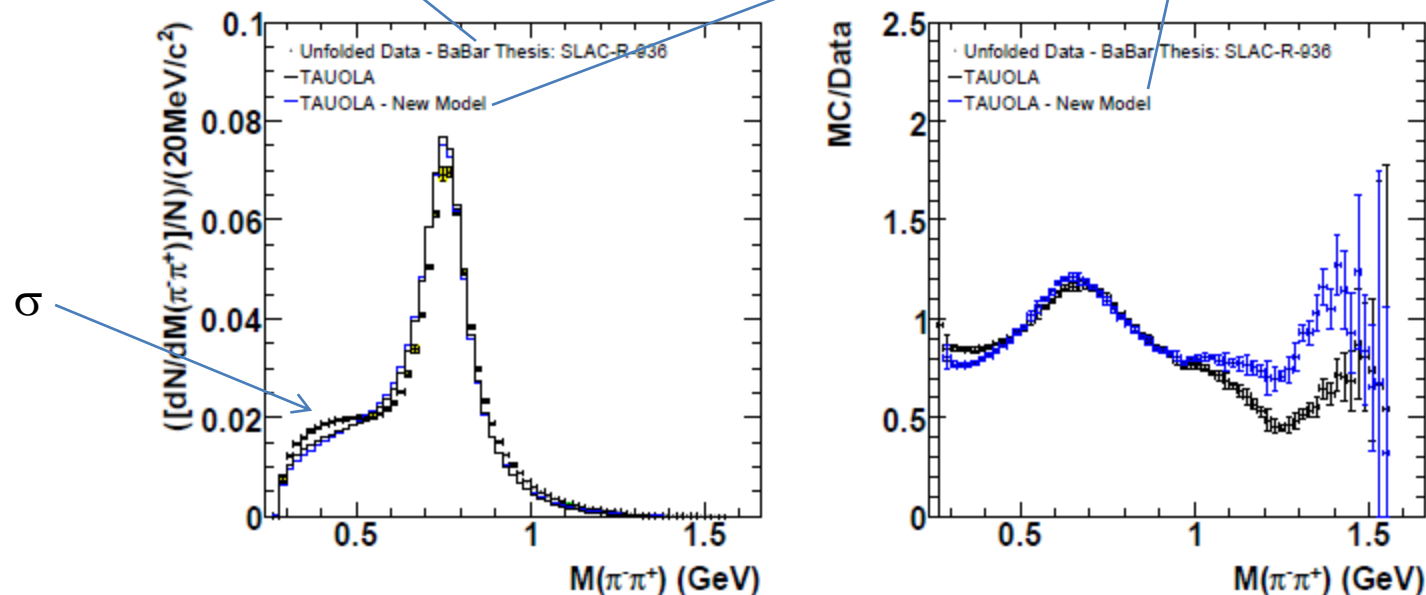


Figure 8: Invariant mass distribution of the $\pi^+\pi^-$ pair in $\tau \rightarrow \pi^+\pi^-\pi^-\nu_\tau$ decay. Lighter grey histogram is from our model, darker grey is from default parametrization of TAUOLA c1eo. The unfolded BaBar data are taken from Ref. [57]. The plot on the left-hand side corresponds to the differential decay distribution, and the one on the right-hand side to plot ratios between Monte Carlo results and data. Courtesy of Ian Nugent.

<http://annapurna.ifj.edu.pl/~wasm/RChL/RChL.htm>

COMPARISONS

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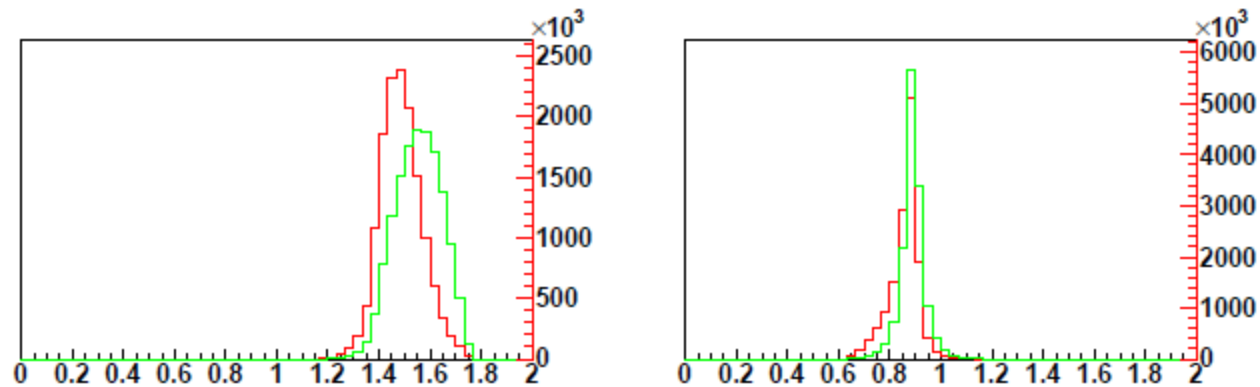


Figure 3: The $\tau \rightarrow K^+ K^- \pi^- \nu_\tau$ decay: comparison of distributions for TAUOLA cleo current [21] and for our new current. On the left-hand side, plot of $K^+ K^- \pi^-$ invariant mass is shown and on the right-hand side $K^+ \pi^-$ invariant mass is given. Green histograms (light grey) are for the new current, red (darker grey) are for TAUOLA cleo.

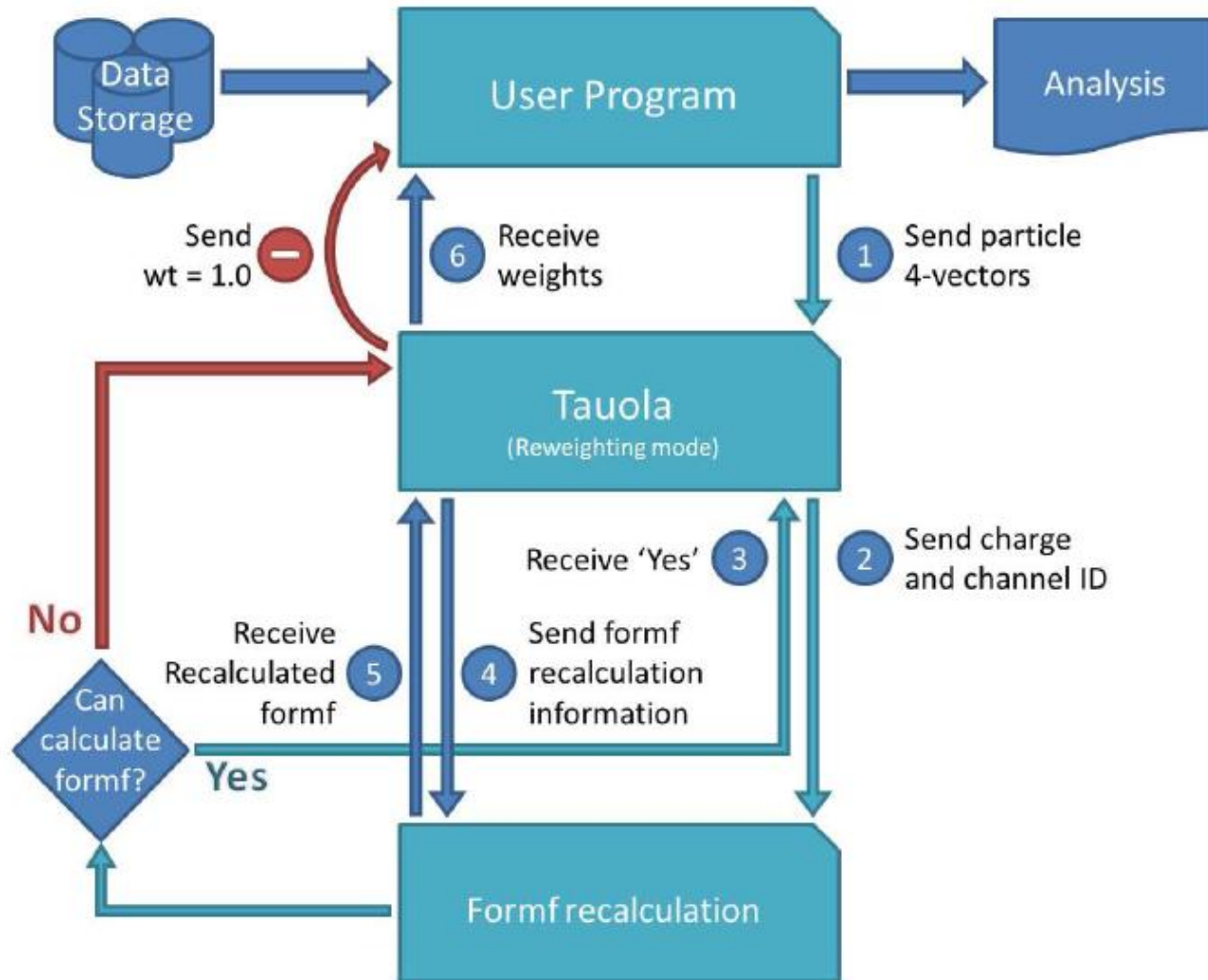


Shekhovtsova, Przedzinski, Roig, Was '12

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COMPARISONS

MC-Tester Davidson, Golonka, Przedzinski, Was '08



CONCLUSIONS

Shekhovtsova, Przedzinski, Roig, Was '12

Hadronic currents for the modes: $\pi^-\pi^0$, $(K\pi)^-$, K^-K^0 , $(\pi\pi\pi)^-$, $(KK)\pi^-$, $K^-K^0\pi^0$ - from $R\chi T$ have been implemented in **TAUOLA** (88% of hadronic decays of the tau lepton). They are ready for precise confrontation with data amassed at Belle and BaBar (and future Belle II & Frascati superB data). Collaboration with experimentalists is essential for the success of the project.

In order to obtain the maximum possible information from experiments, the theory input to the MC has to be as accurate as possible with known properties respected (χPT results at low energies, smooth behaviour of FF at short distances, unitarity, analyticity,...). That is why our effort is and will be worth.

There are improvements to be done in all modes...

Ongoing/Future improvements

- ➡ • Resummation in the two meson modes. ε below 3%.
- ➡ • FSI in three meson modes. Relevant in $d\Gamma/ds$ ($\sim 10\%$).
- ➡ • SU(3) breaking terms in the Lagrangian. ($KK\pi$, $\sim 30\%$)
- ➡ • Spin zero resonance contributions. ($\pi\pi\pi$)
- ➡ • Excited resonance contribution in $KK\pi$ channels.
- ➡ • Complete $O(p^6)$ χ PT, i.e. NNLO, in $\pi\pi$ channels.
- ➡ • SU(2) breaking in $\pi\pi$ channels.
- Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, SFF in $K\pi$...
- Remaining two meson modes: $\pi\eta^{(\prime)}$, $K\eta^{(\prime)}$

Moussallam
Eur.Phys.J.C53:401-412,2008

Bijnens, Colangelo, Talavera
JHEP 9805:014,1998

López Castro et. al. Phys.Rev.D74:071301,2006

Cirigliano, Ecker, Neufeld

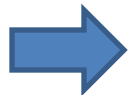
Phys.Lett.B513:361-370,2001 JHEP 0208:002,2002

Jamin, Oller, Pich '01,'06

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- Complete $O(p^6)$ χ PT, i.e. NNLO, in $\pi\pi$ channels.

López Castro et. al. *Phys.Rev.D***74:071301,2006**



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Cirigliano, Ecker, Neufeld
*Phys.Lett.B***513:361-370,2001** *JHEP* **0208:002,2002**

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Jamin, Oller, Pich '01,'06

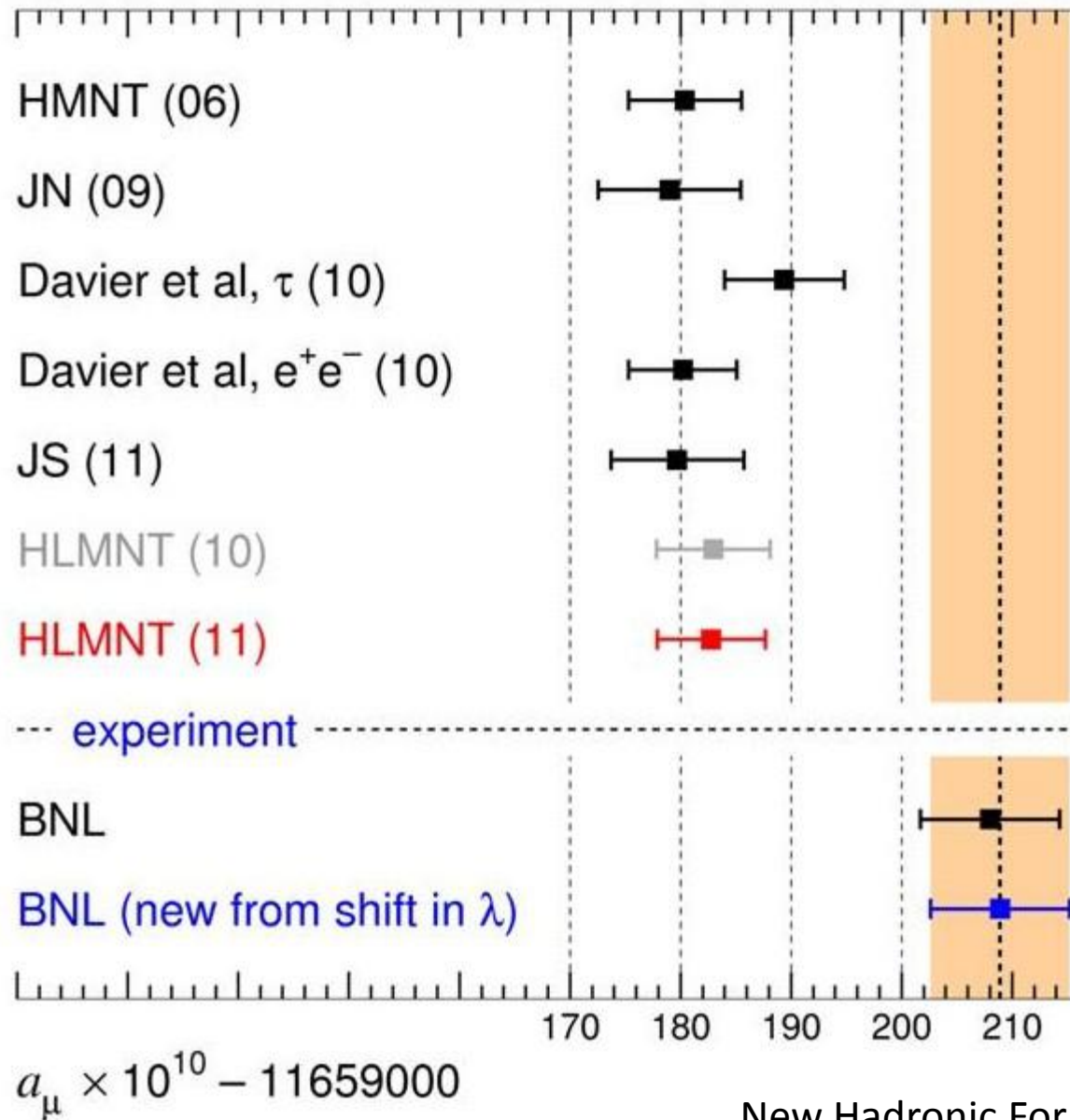
Moussallam
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**THANK
YOU!**

SKIPPED SLIDES

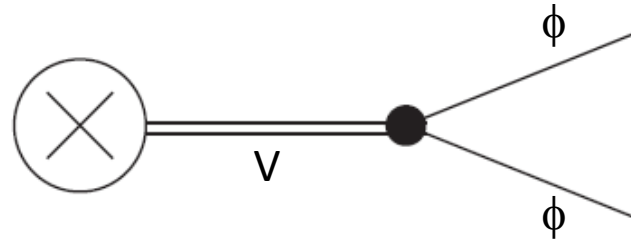
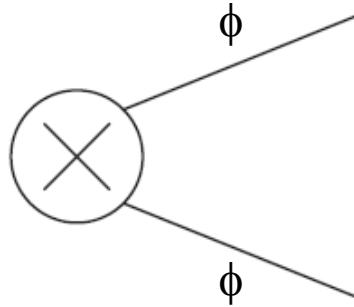
AMMM



Form factors in two meson τ decays

$\phi = \pi, K$

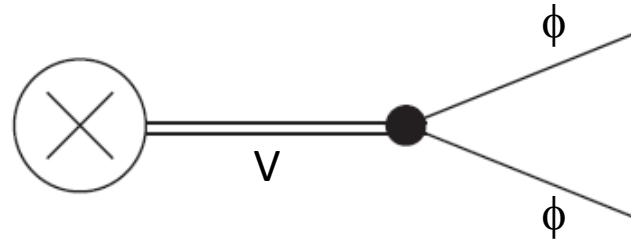
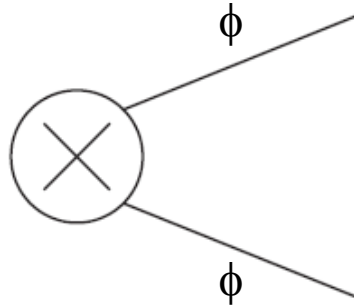
$V = \rho, K^*$



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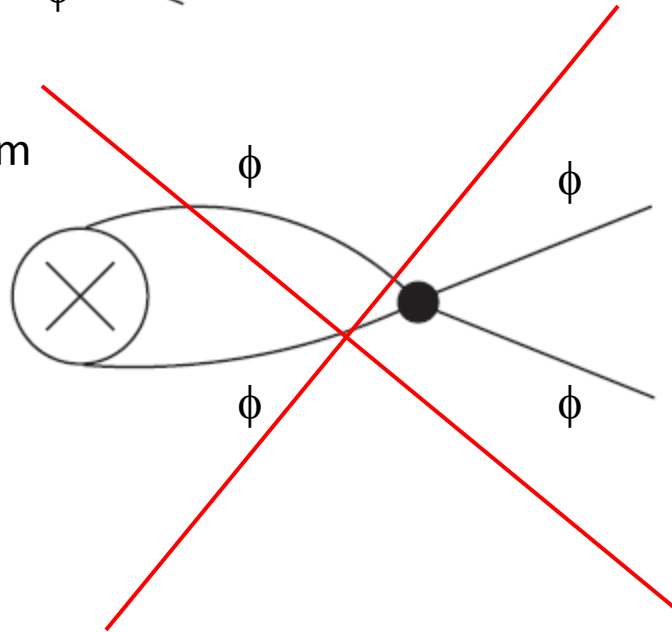


Antisymmetric tensor formalism
for spin-one resonances

Ecker et al.

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989



To avoid double counting

Discussion on the errors

- $\varepsilon(1/N_c) \sim 1/3$?

't Hooft '74, Witten '79

Nucl.Phys.B72:461,1974 Nucl.Phys.B160:57,1979

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→ We cannot specify the expansion parameter ($\sim 1/N_c$)

Ecker *et al.* '88, '89

QED: $\alpha \equiv e^2/(4\pi)^2$; χ PT: $(p,m)^2/(4\pi F, M_V)^2$; $R\chi$ T: ($\sim 1/N_c$)

Gasser, Leutwyler '83, '84

Annals Phys.158:142,1984 Nucl.Phys.B250:465,1985

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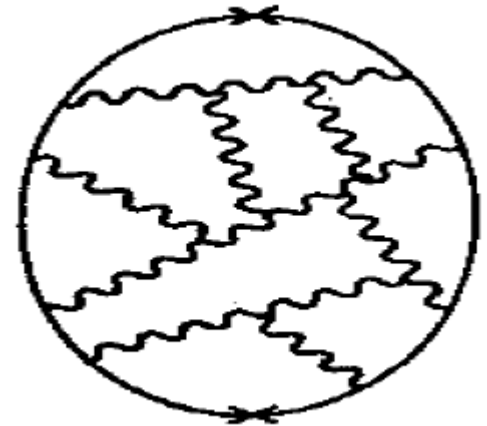
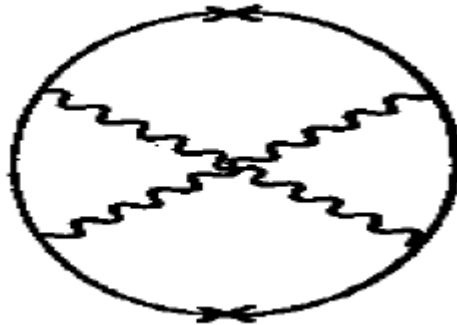
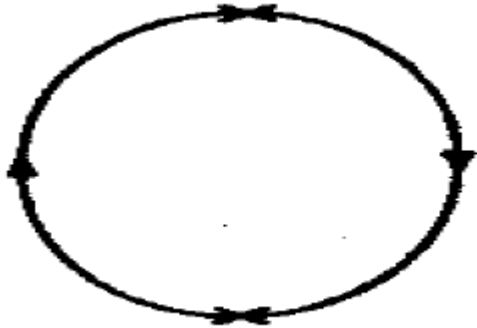
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LO in $1/N_c$ QCD



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Pich, Rosell, Sanz Cillero '04,'06,'08,'10 Portolés, Rosell, Ruiz-Femenía '06

JHEP 0408:042,2004 JHEP 0701:039,2007 JHEP 0807:014,2008 JHEP 1102:109,2011 Phys.Rev.D75:114011,2007

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→ Good description of the data

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07 Gómez-Dumm, Roig, Pich, Portolés '09

Phys.Lett.B664:78-83,2008 Eur.Phys.J.C59:821-829,2009 Phys.Lett.B685:158-164,2010

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$$F_{PQ}^V(s) = F^{VMD}(s) \exp \left[\sum_{P,Q} N_{loop}^{PQ} \frac{-s}{96\pi^2 F^2} \text{Re} A_{PQ}(s) \right]$$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V \Gamma_V(s)}$$

In this way (exponentiation of $\text{Re} A_{PQ}(s)$) unitarity is violated at $O(p^6)$, i.e. NNLO

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Alternatively:

Exact Unitarity

$$\left\{ \begin{array}{l} F_V(s) = \frac{M_V^2}{M_V^2 \left[1 + \sum_{P,Q} N_{loop}^{PQ} \frac{s}{96\pi^2 F^2} A_{PQ}(s) \right] - s} \\ \delta^{PQ}(s) = \text{Im} \left[F_V^{PQ}(s) \right] / \text{Re} \left[F_V^{PQ}(s) \right] \\ F_V^{PQ}(s) = \exp \left\{ \alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi} \int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3 (s' - s - i\epsilon)} \right\} \end{array} \right\}$$

Tiny differences in observables between both approaches

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07

Discussion on the errors

- $\varepsilon(1/N_c) < 1/3$
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$$F_I^{\text{scal}}(x) = F_I^{\text{resonant}}(x) + R_I^{\text{scal}}(x)$$

$$\begin{aligned} F_+ &= F_+^{\chi} + F_+^R + F_+^{RR} + \sqrt{2} [R_0^{\text{scal}}(s) + R_0^{\text{scal}}(t)] + R_2^{\text{scal}}(s) + R_2^{\text{scal}}(t), \\ F_- &= -(F_+^{\chi} + F_+^R + F_+^{RR}) - [R_0^{\text{scal}}(s) + R_0^{\text{scal}}(t)] + \sqrt{2} [R_2^{\text{scal}}(s) + R_2^{\text{scal}}(t)] \end{aligned}$$

Isidori, Maiani, Nicolacci, Pacetti

JHEP 0605 (2006) 049

$$R_0(x) = \left\{ \frac{\alpha_0}{Q^2} + \frac{\alpha_1}{Q^4} (x - M_{f_0}^2) + \mathcal{O}[(x - M_{f_0}^2)^2] \right\} e^{i\delta_0(x)}$$

Schenk **Nucl.Phys. B363 (1991) 97-116**

Colangelo, Gasser, Leutwyler

Nucl.Phys. B603 (2001) 125-179

$$\tan \delta_I(x) = \sigma_\pi(x) (A_0^I + B_0^I q^2 + C_0^I q^4 + D_0^I q^6) \frac{4m_\pi^2 - x_0^I}{x - x_0^I}$$

We have to check if this approach is enough to confront the data successfully.

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 - SU(3) breaking terms in the Lagrangian.
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- Excited resonance contribution in $KK\pi$ channels.
- Complete $O(p^6)$ χ PT, i.e. NNLO, in $\pi\pi$ channels.
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Jamin, Oller, Pich '01,'06

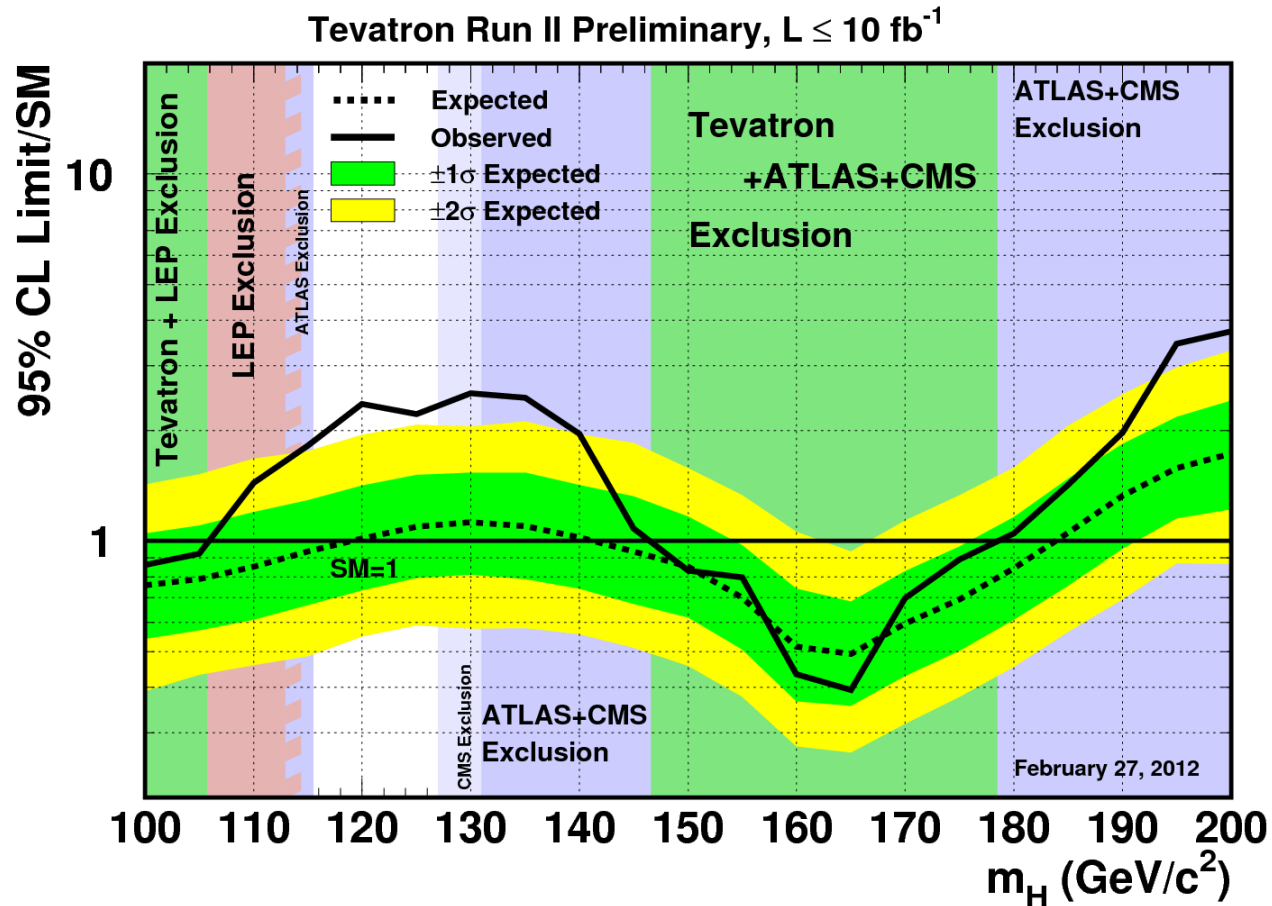
Nucl.Phys.B622:279-308,2002 Phys.Rev.D74:074009,2006



MOTIVATION

- High energies (LHC):
Hadronic tau decays

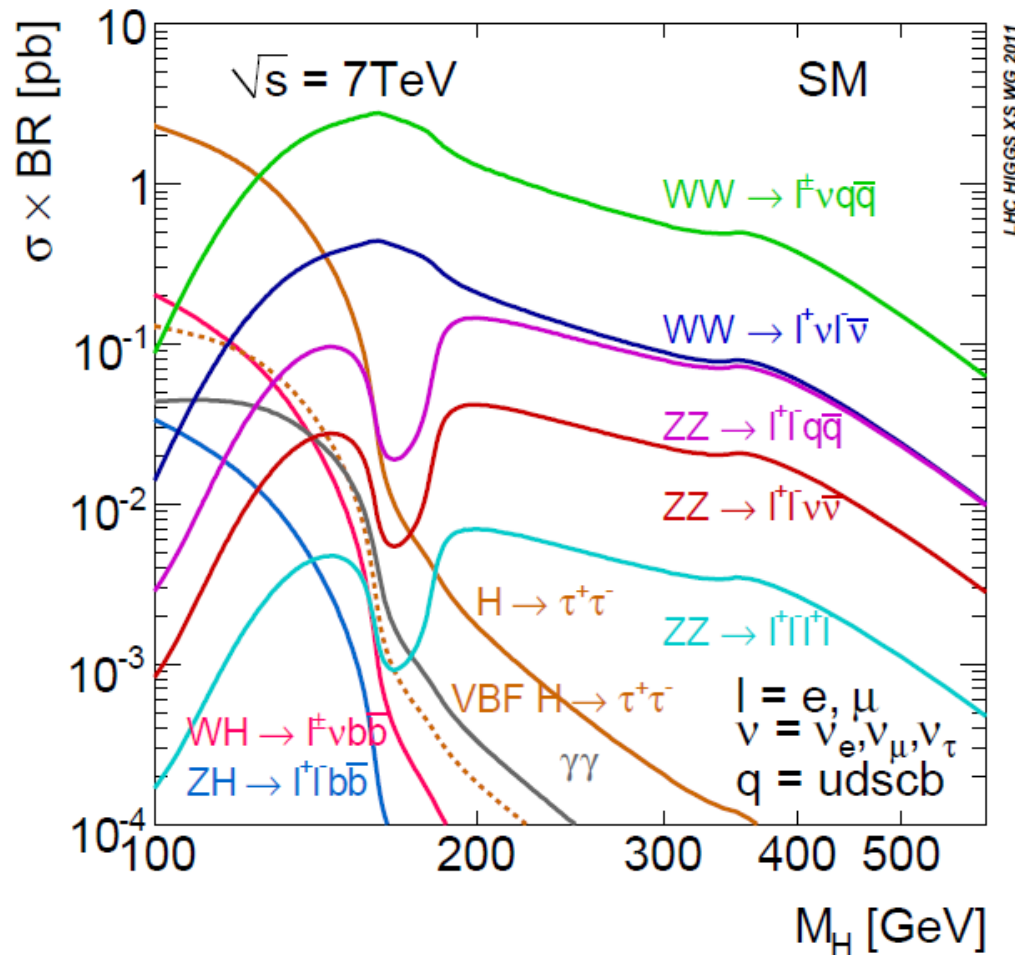
Search of the scalar sector of the SM, origin of EWSB



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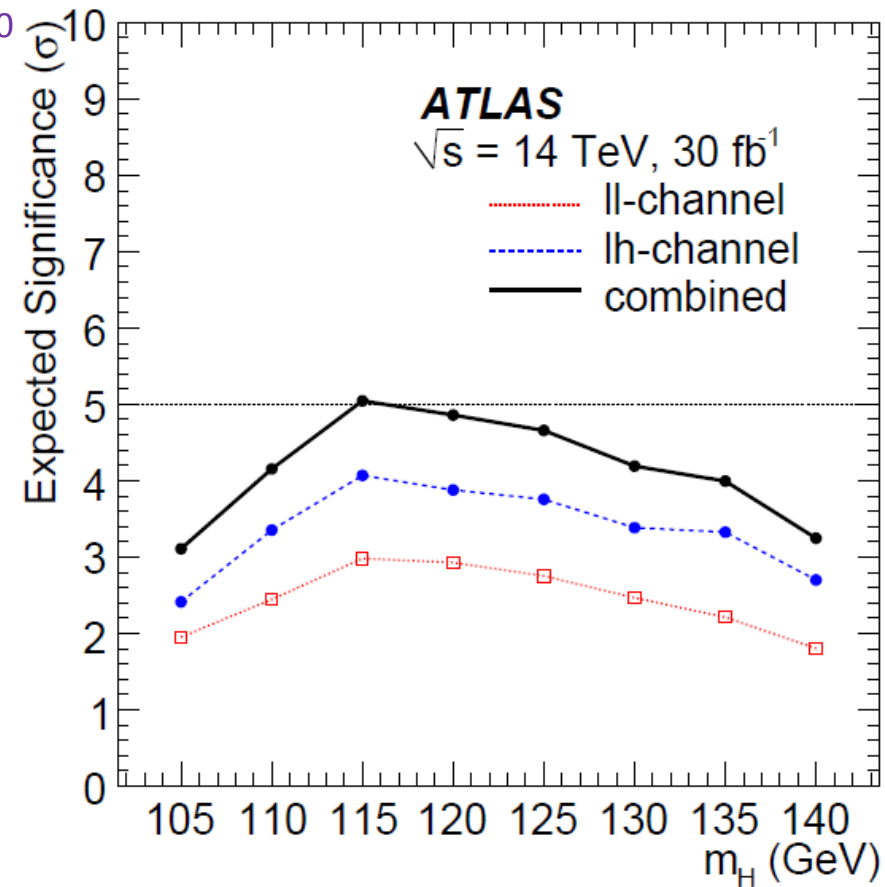
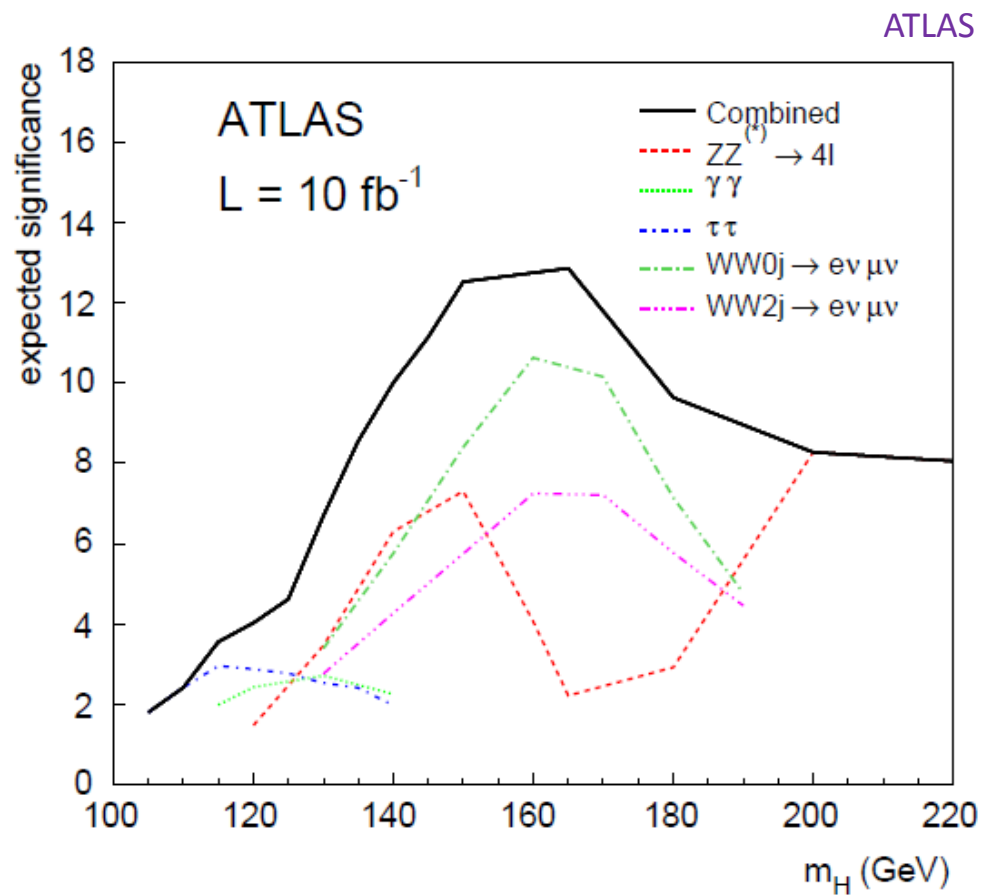
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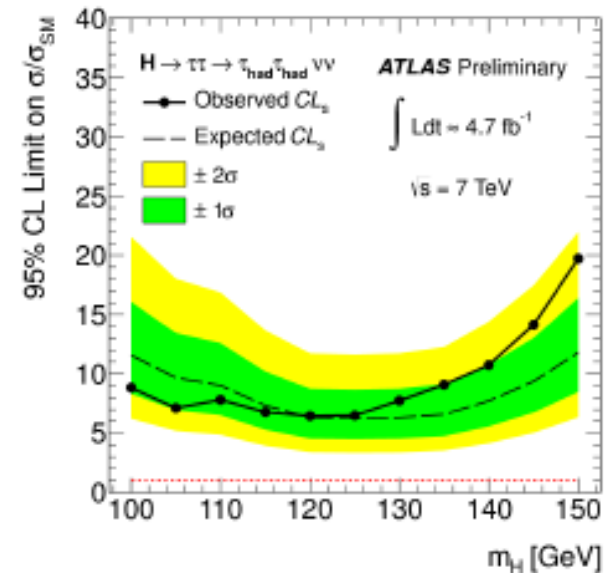
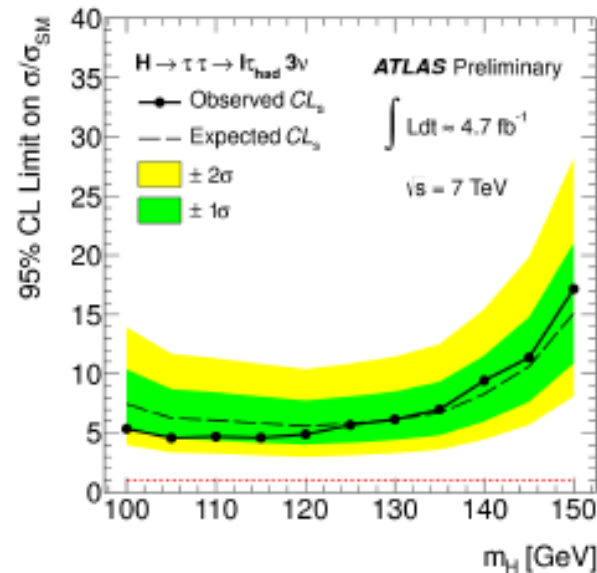
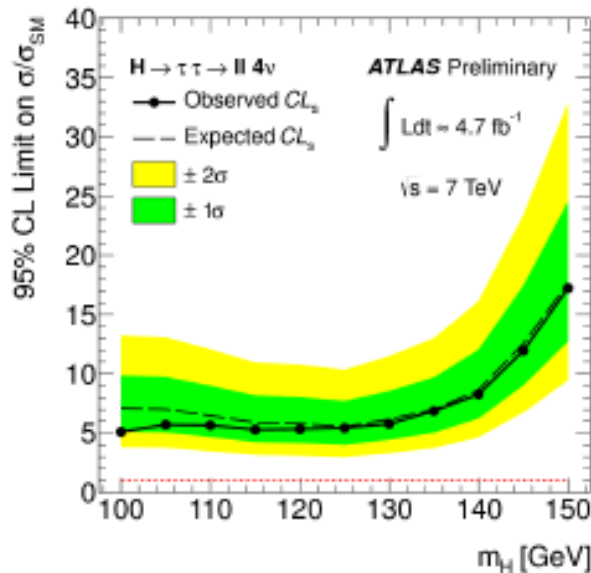
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