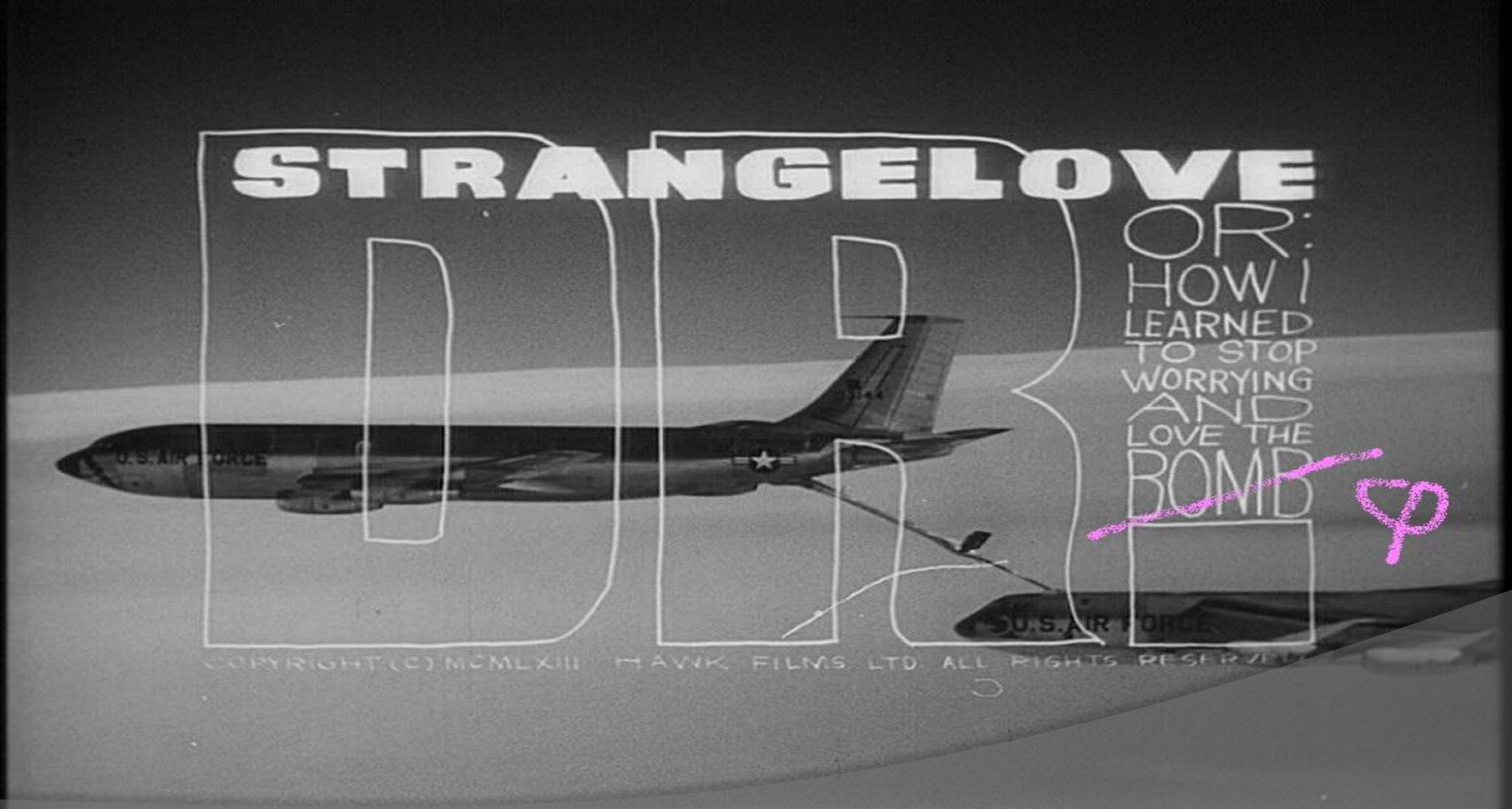
STRANGELOVE
DR.
OR:
HOW I
LEARNED
TO STOP
WORRYING
AND
LOVE THE
~~BOMB~~
φ
U.S. AIR FORCE
U.S. AIR FORCE
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Gen. Jack D. Ripper

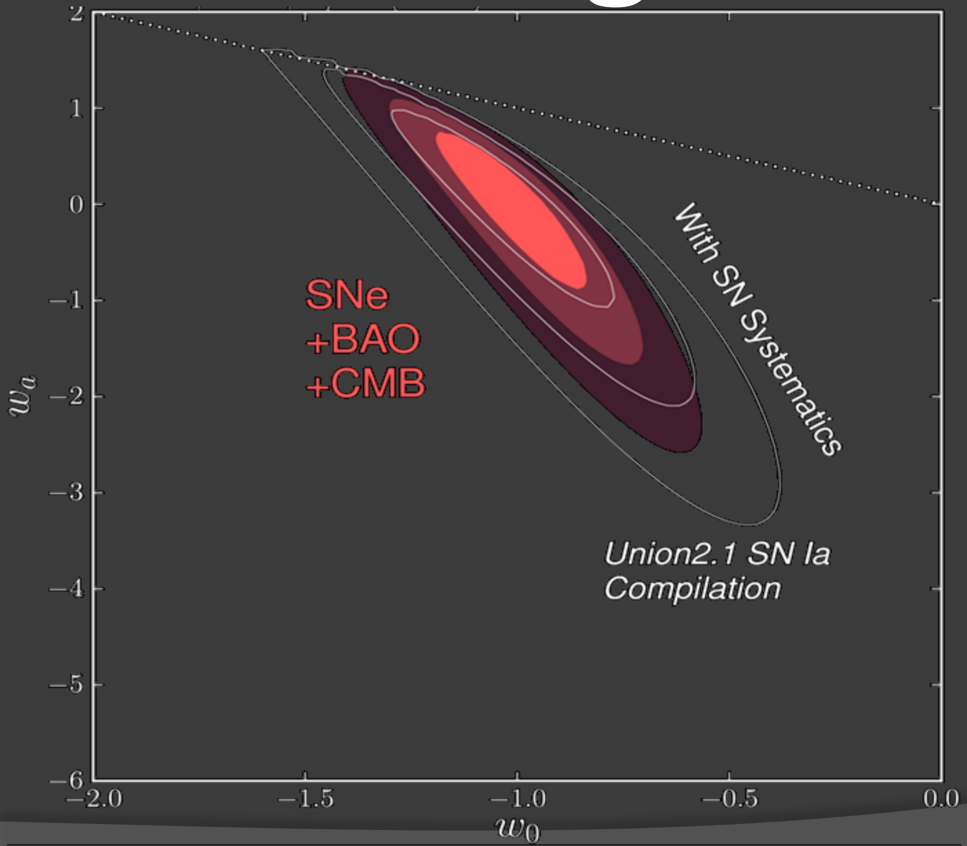
“God willing, we will prevail, in peace and freedom from fear, and in true health, through the purity and ***k-essence*** of our natural... ***fluids***. God bless you all.”

Conclusions

- There is a ***fundamental problem*** with "phantom" backgrounds
  - Standard scalar models unstable when $w < -1$
  - As it is, perturbation calculations are meaningless in such situations
- ***Galileon models*** are a (relatively) new consistent extension
  - Stable violations of the NEC are now possible, even when minimally coupled (phantom DE, bounces)
  - They represent a particular sort of expansion away from a perfect fluid
- [We will have shown how to deal with structure formation in these much more complex setups
  - The energy-momentum tensor and the fluid language are much simpler and more physical than dealing directly with the equation of motion]

The background…

Supernova Cosmology Project
Suzuki et al. (2011)



$$w = w_0 + w_a(1 - a)$$

Ratra & Peebles, Wetterich

Armendaríz-Picón, Mukhanov, Steinhardt

Non-canonical Scalars

$$X \equiv 1/2\left(\partial_\mu \phi\right)^2$$

- ***Quintessence*** is a canonical scalar field

  $$\mathcal{L}_\phi = X - V(\phi)$$

  - $w(t) = \frac{X-V}{X+V} > -1$
  - Speed of sound is $c_s = 1$
- Problem: mass $m \sim H$

- ***k-essence***

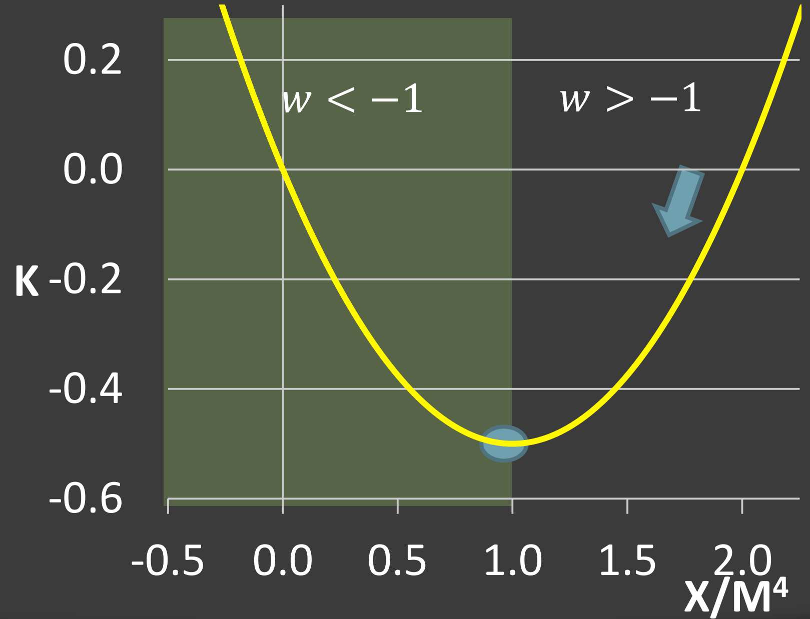
  $$\mathcal{L}_\phi = K(X, \phi)$$

  - This is still one degree of freedom
  - $c_s^2 \neq 1$

Arkani-Hamed, Cheng, Luty, Mukohyama

e.g. Ghost Condensation

$$K = -X + X^2/M^4$$



$$\mathcal{P} = K$$

$$\mathcal{E} = 2X\cancel{K_{,X}} - K$$

- Get de Sitter with broken Lorentz
  - Effective c.c. $\sim M^4$
  - $c_s = 0$: DE perturbations
  - Massless!

Garriga, Mukhanov

k-essence a Perfect Fluid?

$$T_{\mu\nu} = K_{,X}\nabla_\mu\phi\nabla_\nu\phi + K g_{\mu\nu}$$

$$\Downarrow$$

$$T_{\mu\nu} = (\mathcal{E} + \mathcal{P})u_\mu u_\nu + \mathcal{P} g_{\mu\nu}$$

$$\begin{pmatrix} \mathcal{E} & & & \\ & \mathcal{P} & & \\ & & \mathcal{P} & \\ & & & \mathcal{P} \end{pmatrix}$$

$$\partial_\mu \tau = u_\mu \equiv \frac{\nabla_\mu\phi}{\sqrt{\nabla^\alpha\phi\nabla_\alpha\phi}}$$

- ***Only*** have a fluid interpretation if u^μ is a velocity (time-like)
- ϕ is a clock for the lagrangian observer ***if*** u^μ is time-like

Can firm this up: hydro at $T = 0$

- Dependence on ϕ is a dependence on ***internal time***
  - Shift symmetry $\Leftrightarrow$ ϕ independence
- Define a ***Noether charge***

$$n \equiv \sqrt{2X}K_{,X}$$

- Equation of motion is just ***conservation*** of this charge

$$\dot{n} + 3Hn = \mathcal{L}_{,\phi}$$

- Continuity equation $\Leftrightarrow$ ***1st Law***

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + \sqrt{2X}\mathrm{d}\mathcal{N}$$

- Euler equation $\Leftrightarrow$ ***Euler relation***

$$\mathcal{E} = \sqrt{2X}n - \mathcal{P}$$

- ***Chemical potential***

$$\boxed{\mu \equiv \left(\frac{\partial \mathcal{E}}{\partial n}\right)_{\tau,V} = \sqrt{2X}}$$

- Alternative interpretation

$$\mu \Leftrightarrow T$$
$$n \Leftrightarrow s$$

Does this go both ways?

- Lagrangian for ***sound mode*** in radiation

$$\mathcal{L} = X^2 \qquad \mathcal{P} = \frac{1}{4}T^4, w = c_s^2 = \frac{1}{3}$$

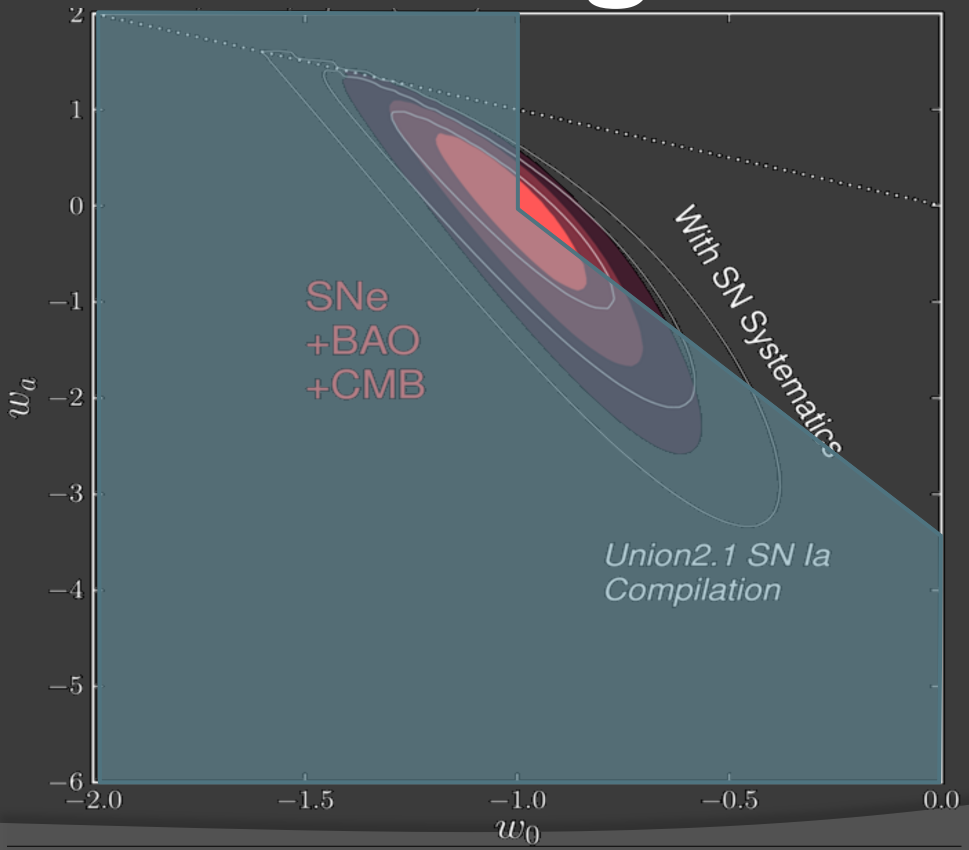
- Sound in ***electrons*** at zero temperature

$$\mathcal{L} = \frac{3}{2}\ln\left(\sqrt{2X} + \sqrt{2X-1}\right) + \\ +\sqrt{4X^2 - 2X}\left(2X - \frac{5}{2}\right)$$

$$\boxed{2X = \varepsilon_\mathrm{F}}$$

The background…

Supernova Cosmology Project
Suzuki et al. (2011)

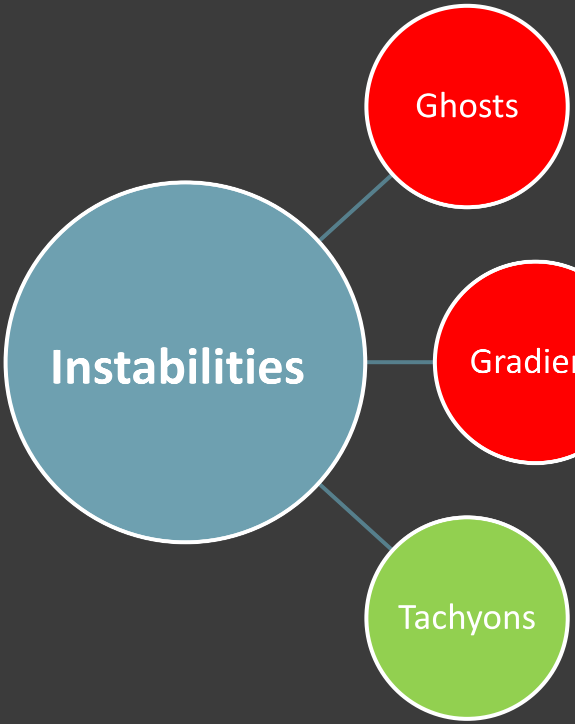


$$w = w_0 + w_a(1 - a)$$

$$w < -1$$

~~NEC~~

k-*Essence*: Phantom ⇒ Unstable



- Negative KE: $A\delta\dot{\phi}^2 < 0$

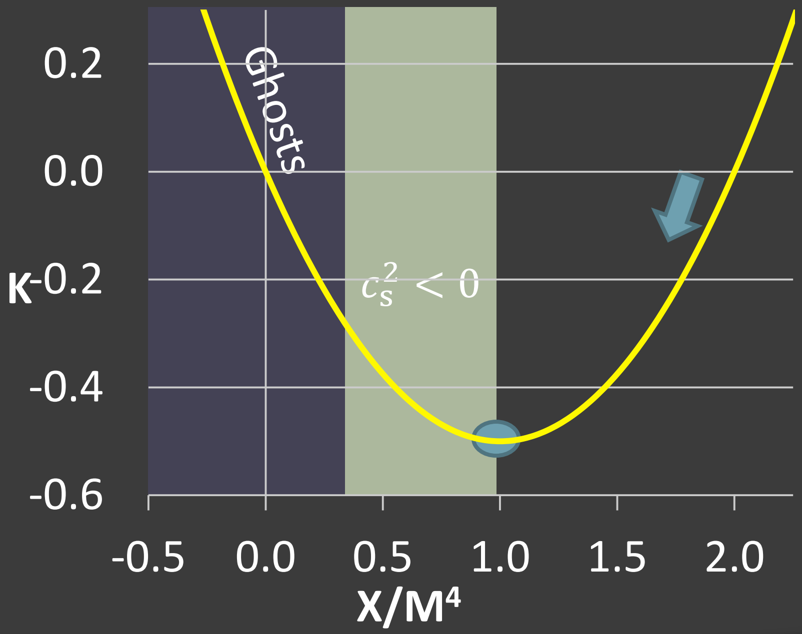
- $c_s^2 < 0$
- $\delta\phi \sim \mathrm{e}^{ic_s kt}$

- $m^2 < 0$
- Timescale m

$$\mathcal{E} + \mathcal{P} = \dot{\phi}^2 A c_s^2$$

Ghost Condensation (cont'd)

$$K = -X + X^2/M^4$$



- Solution ***unstable*** when phantom
- Can we ***actually*** model a theory with $w < -1$?

Gallileons: Imperfect Fluids

Can we do more?

- Ostrogradsky procedure
  - EoM of higher order than 2 $\Rightarrow$ ghost
  - Avoid this!

- There are some lucky exceptions
  - e.g. $f(R)$, $f(\text{Gauss Bonnet})$, non-local

- We ***used to*** think that this is it

Nicolis, Ratazzi, Trincherini (2009)

Galileon: Flat Space

$$B_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi$$

$\mathcal{L}_2$

- $\dfrac{\delta}{\delta\phi} X = \Box\phi = B$

$\mathcal{L}_3$

- $\dfrac{\delta}{\delta\phi} X \Box\phi = B^2 - B_{\mu\nu} B^{\mu\nu}$

$\mathcal{L}_4$

- $\dfrac{\delta}{\delta\phi} X\left(B^2 - B_{\mu\nu} B^{\mu\nu}\right) = 3 B_{\mu\nu} B^{\mu\nu} B - B^3$

$$\phi \to \phi + c + v_\mu x^\mu$$

Deffayet, Pujolàs, IS, Vikman (2010)
Deffayet, Gao, Steer, Zahariade (2011)
Horndeski (1974)

Horndeski Lagrangian

$B_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi$

$\mathcal{L}_2$

- $\frac{\delta}{\delta\phi} K(X) = K_X \Box\phi + K_{XX} B_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi$

$\mathcal{L}_3$

- $\frac{\delta}{\delta\phi} G(X) \Box\phi = G_X R_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi +$
  $+ G_X(\mathcal{O}_1(B^2)) + G_{XX}\left(\mathcal{O}_{2\mu\nu}(B^2)\right) \nabla^\mu \phi \nabla^\nu \phi$

$\mathcal{L}_4$

- $\frac{\delta}{\delta\phi} G_X(\mathcal{O}_1(B^2)) - GR = \text{Gravity} +$
  $+ G_{XX}(\mathcal{O}_1(B^3)) + G_{XXX}\left(\mathcal{O}_{2\mu\nu}(B^3)\right) \nabla^\mu \phi \nabla^\nu \phi$

Deffayet, Pujolàs, IS, Vikman (2010)
Pujolàs, IS, Vikman (2011)

Kinetic Gravity Braiding

$$S_\phi = \int \mathrm{d}^4 x \sqrt{-g}\, K(\phi, X) + \boxed{G(\phi, X)\Box\phi}$$

- Equations of motion ***second order***
- Most general minimally coupled theory

- From now on: shift symmetry
  - Invariant under $\phi \to \phi + \mathrm{const}$
  - ***Not*** invariant under $\phi \to -\phi$

Kinetic Gravity Braiding?

- ***Essential*** mixing of metric and scalar

$$G_{\mu\nu}(\dots,\partial\partial g_{\alpha\beta},\dots) = T_{\mu\nu}(\dots,\partial\partial\phi,\dots)$$
$$\mathrm{EOM}_{\phi}(\dots,\partial\partial\phi,\partial\partial g_{\alpha\beta},\dots) = 0$$

- There is ***no Einstein frame***
  - Cannot diagonalise scalar and metric
- EMT contains second derivatives of ϕ

The Imperfect Fluid

$$\kappa \equiv 2XG_X$$

$$T_{\alpha\beta} = (\mathcal{E} + \mathcal{P})u_\alpha u_\beta + \mathcal{P} g_{\alpha\beta} + \boxed{u_\alpha q_\beta + u_\alpha q_\beta}$$

Energy $\mathcal{E} = E_0 \boxed{+ \kappa\mu\theta}$

Pressure $\mathcal{P} = P_0 \boxed{- \kappa\dot{\mu}}$

Energy Flow $q_\lambda = \boxed{-\kappa\dot{u}_\lambda}$

$$u_\alpha \equiv \frac{\partial_\alpha \phi}{\mu}$$

$$\theta \equiv \nabla_\alpha u^\alpha \sim 3H$$

All corrections: second derivatives of ϕ

(Conserved) Charge Number

- Charge density depends on ***expansion***

$$n = n_0 + \boxed{3H\kappa}$$

- EoM is equation for evolution of n
  - In cosmology: $\dot{n} + 3Hn = 0 \Rightarrow n \propto a^{-3}$
  - Vacua are ***cosmological attractors*** but ***NON-TRIVIAL***
- Hydrodynamics discussion still goes through

Diffusion

$$\bar{\nabla}_\alpha \equiv \perp_\alpha^\beta \nabla_\beta$$

- 1st Fick's Law:
  - we have ***spatial charge flux*** proportional to gradient of chemical potential

$$J_\alpha = n u_\alpha - \frac{\kappa}{\mu} \bar{\nabla}_\alpha \mu$$

- In incompressible limit, scalar EoM is the ***diffusion equation***

$$\mathfrak{D} \equiv -\frac{\kappa}{\mu n_{,\mu}}$$

$$\dot{n} = -\bar{\nabla}_\alpha (\mathfrak{D} \bar{\nabla}^\alpha n) + \mathfrak{D} \dot{u}^\alpha \bar{\nabla}_\alpha n$$

Simplest Interesting Model

$$\mathcal{L}_\phi = -X + gX\Box\phi$$

$$g^{-1/3} \sim (1000\ \mathrm{km})^{-1}$$

$$n = \mu(3gH\mu - 1)$$

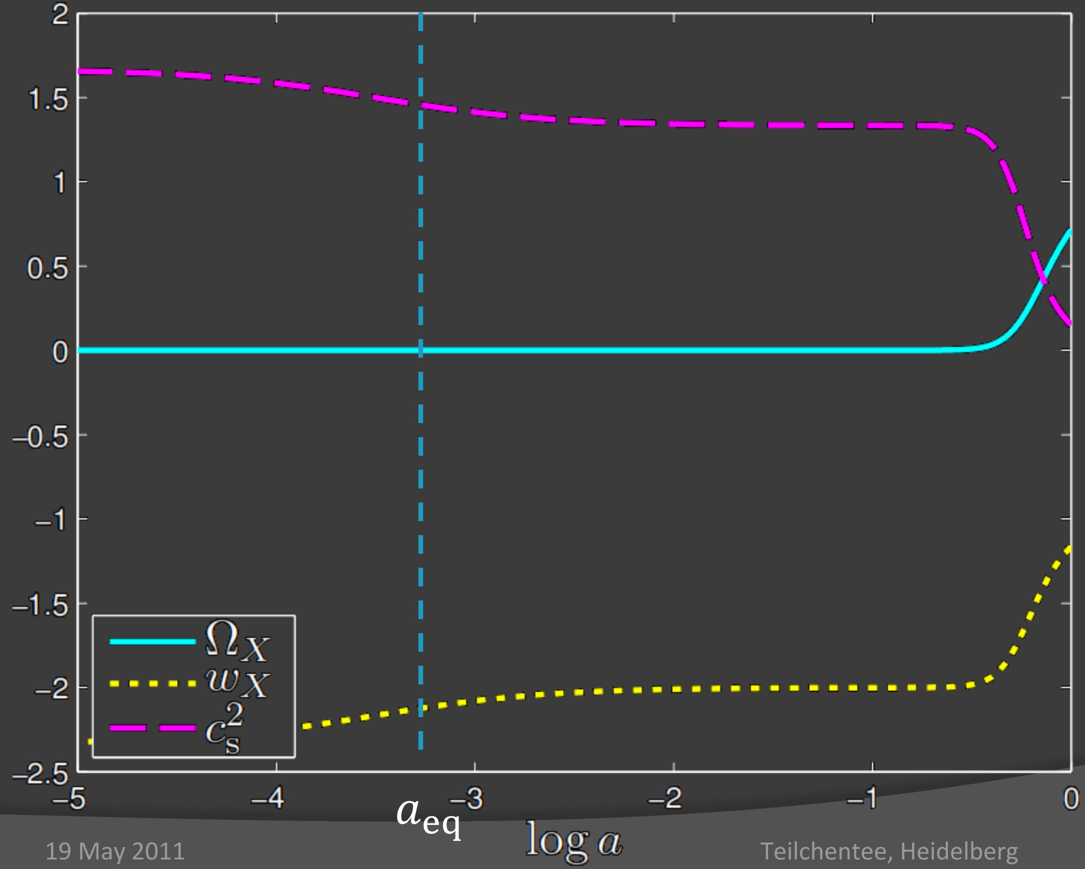
$$\mu = \sqrt{2X}$$

$$n_* = 0$$

$$\mu_* = \frac{1}{3gH}$$

$$H^2 = \frac{1}{6}\left(\rho_{\mathrm{ext}} + \sqrt{\rho_{\mathrm{ext}}^2 + \frac{2}{3g^2}}\right)$$

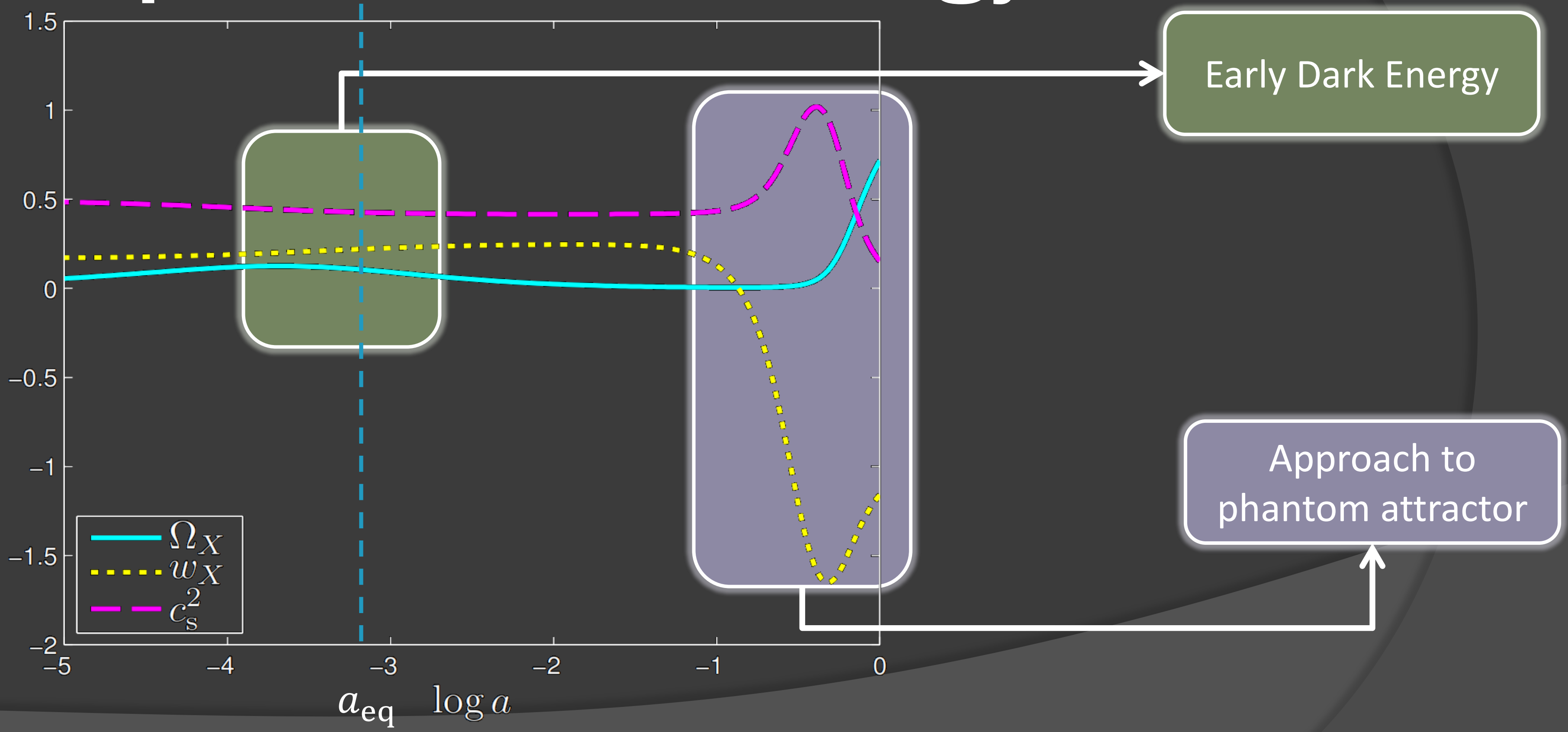
On-Attractor Behaviour



Not a ghost

$A \equiv 1 + \Omega_{\mathrm{DE}}$

Imperfect Dark Energy



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