Gravitational Waves from Fermions

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Motivation

- Fermions are produced in the early Universe.
  - Fermionic preheating? Phase transition? Other?
- GW production by bosonic sources have been studied before.
- We have studied for the very first time GW produced by fermionic source.
  - Our formalism can be applied to **fermionic preheating**, **fermionic decay of curvaton** and other fermion production scenarios in the early universe.

Gravitational Waves

- GW are the transverse-traceless (TT) part of the metric perturbations:

$$ds^2 = a^2(t)\left[-dt^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right], \; \partial_i h_{ij} = h_{ii} = 0$$

- GW spectrum at sub-horizon scales ($k \gg \dot{a}/a$):

$$\frac{d\rho_{gw}}{d\log k}(k,t) = \frac{M_p^2 k^3 \left|\dot{h}_k(t)\right|^2}{8\pi^2 a^2(t)} =$$

$$\frac{k^3}{4\pi^2 M_p^2 a^4(t)} \int dt' \int dt'' a^3(t')\, a^3(t'') \cos\left[k\left(t' - t''\right)\right] \Pi^2\left(k, t', t''\right)$$

  - $\left\langle \Pi_{ij}^{TT}(\mathbf{k}, t)\, \Pi_{ij}^{TT*}(\mathbf{k}', t')\right\rangle = (2\pi)^3\, \Pi^2(k, t, t')\, \delta(\mathbf{k} - \mathbf{k}')$
  - Π_{ij}^{TT} is the TT part of the anisotropic stress tensor.

Fermionic Source

- The fermionic energy-momentum tensor:

$$T_{ij}(\mathbf{x},t) = \frac{i}{2a^4(t)}\left[\bar{\psi}\gamma_{(i}\overrightarrow{D_{j)}}\psi - \bar{\psi}\overleftarrow{D_{(i}}\gamma_{j)}\psi\right]$$

- The spin-$\frac{1}{2}$ fermion field:

$$\psi(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}}\left[a_{\mathbf{k},r}\mathbf{u}_{\mathbf{k},r} + b^{\dagger}_{-\mathbf{k},r}\mathbf{v}_{\mathbf{k},r}\right],$$

$$\left\{a_{\mathbf{k},r}, a^{\dagger}_{\mathbf{k}',r'}\right\} = \left\{b_{\mathbf{k},r}, b^{\dagger}_{\mathbf{k}',r'}\right\} = (2\pi)^3\,\delta^3(\mathbf{k}-\mathbf{k}')\,\delta_{r,r'}$$

$$\mathbf{u}_{\mathbf{k},r} \equiv \begin{pmatrix} u_{\mathbf{k},+}S_r & u_{\mathbf{k},-}S_r \end{pmatrix}^T,\ \mathbf{v}_{\mathbf{k},r} \equiv \begin{pmatrix} v_{\mathbf{k},+}S_{-r} & v_{\mathbf{k},-}S_{-r} \end{pmatrix}^T$$

The GW Spectrum from Fermionic Source

$$\boxed{\frac{d\rho_{gw}}{d\log k}(k,t) = \frac{k^3}{8\pi^4 M_p^2 a^4(t)} \int dp d\theta p^4 \sin^3\theta F(k,p,\theta)}$$

$$F(k,p,\theta) = \left| I_+^{(c)} - I_-^{(c)} \right|^2 + \left| I_+^{(s)} - I_-^{(s)} \right|^2$$

$$I_\pm^{(c)}(k,p,\theta) = \int_{t_i}^{t} \frac{dt'}{a(t')} \cos(kt')\, u_{\mathbf{k}-\mathbf{p},\pm}(t')\, u_{\mathbf{p},\pm}(t')$$

Regularization of the Source

- Straightforward calculation $\Longrightarrow$ UV divergence
- Time-dependent normal-ordering:

$$\left\langle \Pi_{ij}^{TT}(t) \right\rangle_{reg} \equiv \left\langle 0 \left| \Pi_{ij}^{TT}(t) \right| 0 \right\rangle - \left\langle 0_t \left| \Pi_{ij}^{TT}(t) \right| 0_t \right\rangle$$

- This amounts to replacement $u_{\mathbf{p},\pm} u_{\mathbf{p}',\pm} \rightarrow \left(u_{\mathbf{p},\pm} u_{\mathbf{p}',\pm} \right)_{reg} = |\beta_{\mathbf{p}}| \left| \beta_{\mathbf{p}'} \right| u_{\mathbf{p},\pm} u_{\mathbf{p}',\pm} - \left(\beta_{\mathbf{p}} \beta_{\mathbf{p}'} u_{\mathbf{p},\mp} u_{\mathbf{p}',\mp} \right)^*$
  - $|\beta_{\mathbf{p}}|^2 = 1 - |\alpha_{\mathbf{p}}|^2 =$ occupation number of fermions
  - α and β are the canonical Bogoliubov coefficients.

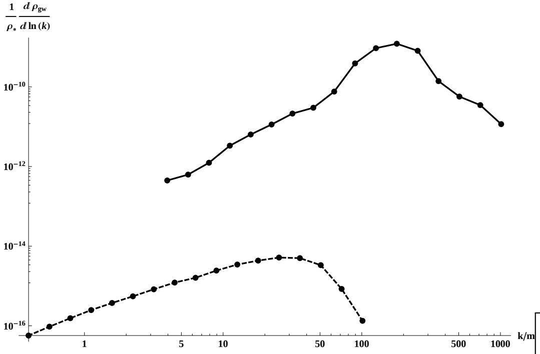
Fermionic Preheating

- Preheating is non-perturbative particle production stage instantly after inflation.
- Yukawa interaction $h\varphi\psi\bar{\psi}$.
- $M_{eff}(t) = h\varphi(t)$
  - $\varphi(t) = \Phi(t) F(t)$
  - The amplitude $\Phi(t)$ decreases.
  - $F(t + 2\pi/\omega_0) = F(t)$

Fermionic Preheating

- $M_{eff}(t) \sim 0 \implies$ fermionic modes are excited effectively and rapidly.
- Fermions fill up a "Fermi-sphere" with radius $k_F \sim a(t)\, q^{1/4}\omega_0$.
    - $q \equiv \frac{h^2\Phi^2}{\omega_0^2}$ (the resonant parameter)
    - Pauli blocking $\implies$ occupation number < 1
- Fermion excitations $\implies \Pi_{ij}^{TT} \neq 0 \implies$ GWs

Spectra of GW from Fermionic Preheating



$k_{peak}/\omega_0 \sim q^{1/4}$

- GW spectra after production $\left(E_I \sim 10^{16}\mathrm{GeV},\, \omega_0 \sim 10^{14}\mathrm{GeV}\right)$
  - $q = 10^2$ (dashed line, $k_{peak}/\omega_0 = 8q^{1/4} \sim 30$)
  - $q = 10^6$ (solid line, $k_{peak}/\omega_0 = 6q^{1/4} \sim 200$)

GW from Fermionic Preheating - Today

- The present frequency (f) and amplitude ($h^2\Omega_{gw}$) of GW:

	$q = 10^2$	$q = 10^6$
$E_I \sim 10^{15}\mathrm{GeV}$ $\omega_0 \sim 10^{12}\mathrm{GeV}$	$f \sim 10^8\mathrm{Hz}$, $h^2\Omega_{gw} \sim 10^{-28}$	$f \sim 10^9\mathrm{Hz}$, $h^2\Omega_{gw} \sim 10^{-22}$
$E_I \sim 10^{16}\mathrm{GeV}$ $\omega_0 \sim 10^{14}\mathrm{GeV}$	$f \sim 10^9\mathrm{Hz}$, $h^2\Omega_{gw} \sim 10^{-20}$	$f \sim 10^{10}\mathrm{Hz}$, $h^2\Omega_{gw} \sim 10^{-14}$

E_I: the initial energy scale
ω_0: the angular frequency of the inflaton

GW from Fermions

- The amplitude of GW background from fermions is significant.
- Typical frequencies are too high for the range of probed GW observatories $\lesssim 10^4 \mathrm{Hz}$.
  - VIRGO, LIGO, eLISA / NGO, BBO, DECIGO
- Curvaton decay to fermions produce qualitatively similar GW spectrum as fermionic preheating.
- Other scenarios of fermion production may produce GW in the observable range.
  - Small-coupling / low energy scale models
  - Hybrid inflation

Summary

- Fermions created in the early universe produce significant GW background.
- In the studied cases, the frequencies are too high for the planned GW detectors.
    - Some small-coupling / low energy scale models may produce GW in the observable range.