The measured galaxy correlation function

Ruth Durrer
Université de Genève
Département de Physique Théorique et Center for Astroparticle Physics

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Outline

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2 What are very large scale galaxy catalogs really measuring?
   - Matter fluctuations per redshift bin, volume perturbations

3 The angular power spectrum and the correlation function of galaxy density fluctuations
   - The transversal power spectrum
   - The radial power spectrum

4 Alcock-Paczyński test

5 Conclusions
Introduction

The CMB

WMAP 7 year CMB sky

The WMAP Team

![CMB Sky Image]

![Graph of Multipole moment vs. $I$]

$1/(l+1)C_l / 2\pi [\mu K^2]$
Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)

from Anderson et al. ’12

SDSS-III (BOSS) power spectrum.

Galaxy surveys $\sim$ matter density fluctuations, biasing and redshift space distortions.
Introduction

The observed Universe is well approximated by a $\Lambda$CDM model, $\Omega_\Lambda \approx 0.72$, $\Omega_m = \Omega_{cdm} + \Omega_b \approx 0.28$, $\Omega_b \approx 0.04$.

![Graph showing BAO measurements from different surveys and their agreement with the $\Lambda$CDM model](image_url)

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For small galaxy catalogs, these effects are not very important, but when we go out to $z \sim 1$ or more, they become relevant. Already for SDSS which goes out to $z \sim 0.2$ (main catalog) or even $z \sim 0.6$ (BOSS).
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- But of course much more for future surveys like DES, bigBOSS and Euclid.
- Whenever we convert a measured redshift or angle into a length scale, we make an assumption about the underlying cosmology.
What are very large scale galaxy catalogs really measuring?

see also Challinor & Lewis, [arXiv:1105:5092].
Relativistic corrections to galaxy surveys are also discussed in:
J. Yoo el al. 2009; J. Yoo 2010
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We can count the galaxies inside a redshift bin and small solid angle, $N(z, n)$ and measure the fluctuation of this count:

$$\Delta(z, n) = \frac{N(z, n) - \bar{N}(z)}{\bar{N}(z)}.$$
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\[
\Delta(z, n) = \frac{N(z, n) - \bar{N}(z)}{\bar{N}(z)}.
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This quantity is directly measurable \(\Rightarrow\) gauge invariant.
Density fluctuation per redshift bin $dz$ and per solid angle $d\Omega$ as $\delta_z(n, z)$.

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\delta_z(n, z) = \frac{\rho(n, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\bar{N}(n, z)}{V(n, z)} - \frac{\bar{N}(z)}{V(z)}
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Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.
We consider a photon emitted from a galaxy ($S$), moving in direction $\mathbf{n}$ into our telescope. The observer ($O$) receives the photon redshifted by a factor

$$1 + z = \frac{(n \cdot u)_S}{(n \cdot u)_O}.$$
Matter fluctuations per redshift bin

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To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1 + z)} = - \left[ (n \cdot \mathbf{V} + \psi)(n, z) + \int_0^{\chi(z)} (\phi + \tilde{\psi}) d\chi \right]$$
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With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3 \int_0^{\chi_S} (\dot{\Psi} + \dot{\Phi})(\mathbf{n}, z(\chi)) d\chi.$$

This quantity is gauge invariant and therefore, in principle, measurable. But when we count numbers of galaxies per solid bangle and per redshift bin, we also have to consider volume perturbations.
The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

\[
\Delta(n, z) = D_g + \Phi + \Psi + \frac{1}{H} \left[ \dot{\Phi} + \partial_X (V \cdot n) \right] \\
+ \left( \frac{\dot{H}}{H^2} + \frac{2}{\chi(z) H} \right) \left( \Psi + V \cdot n + \int_0^{\chi(z)} d\chi (\dot{\Phi} + \dot{\Psi}) \right) \\
+ \frac{1}{\chi(z)} \int_0^{\chi(z)} d\chi \left[ 2 - \frac{\chi(z) - \chi}{\chi} \Delta_\Omega \right] (\Phi + \Psi). 
\]

(C. Bonvin & RD ’11)
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+ \frac{1}{\chi_s} \int_0^{\chi(z)} d\chi \left[ 2(\Phi + \Psi) - \frac{\chi(z)-\chi}{\chi} \Delta_\Omega(\Phi + \Psi) \right].
\]

( C. Bonvin & RD '11)
For fixed $z$, we can expand $\Delta(n, z)$ in spherical harmonics,

$$\Delta(n, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(n), \quad C_{\ell}(z, z') = \langle a_{\ell m}(z)a^*_{\ell m}(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(n, z)\Delta(n', z') \rangle = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}(z, z') P_{\ell}(\cos \theta).$$
What are very large scale galaxy catalogs really measuring?

If we convert the measured $\xi(\theta, z, z')$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales. $r(z, z', \theta) = \sqrt{\chi^2(z) + \chi^2(z') - 2\chi(z)\chi(z') \cos \theta}$.

(Figure by F. Montanari)
What are very large scale galaxy catalogs really measuring?

\[ \Delta(k)/k = k^2 P(k) \]

\( z = 0 \)

True \( \Omega_m = 0.24 \)

Wrong \( \Omega_m = 0.3 \)

Wrong \( \Omega_m = 0.5 \)

(Figure by F. Montanari)
The transversal power spectrum

The transverse power spectrum, $z' = z$ (from Bonvin & RD '11)

From top to bottom $z = 0.1$, $z = 0.5$, $z = 1$ and $z = 3$. 
The transversal power spectrum

Contributions to the transverse power spectrum at redshift $z = 0.1$, $\Delta z = 0.01$ (from Bonvin & RD ’11)

$C_\ell^{DD}$ (red), $C_\ell^{zz}$ (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{\text{Doppler}}$ (cyan), $C_\ell^{\text{lensing}}$ (magenta), $C_\ell^{\text{grav}}$ (black).
The transversal power spectrum

Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$ (from Bonvin & RD '11)

$C^{DD}_\ell$ (red), $C^{zz}_\ell$ (green), $2C^{Dz}_\ell$ (blue), $C^{\text{lensing}}_\ell$ (magenta).
The transversal power spectrum

Contributions to the transversal power spectrum as function of the redshift, $\ell = 20$, $\Delta z = 0$ (from Bonvin & RD ’11)

\[ C_{\ell}^{DD} \text{ (red)}, C_{\ell}^{zz} \text{ (green)}, 2C_{\ell}^{Dz} \text{ (blue)}, C_{\ell}^{\text{lensing}} \text{ (magenta)}, C_{\ell}^{\text{Doppler}} \text{ (cyan)}, C_{\ell}^{\text{grav}} \text{ (black)}. \]
The transversal power spectrum

\[ I(1+1)/2\pi C_1 \]

for \( 0 < z < 2 \)

\( C_{\ell}^{DD} \) (red), \( C_{\ell}^{lensing} \) (magenta).
The transversal correlation function

\[ r(\bar{z},\bar{z},\theta) \text{ [Mpc/h]} \]

\[ \theta^2 \xi(\theta, z, z) \]

Blue: \( C_{DD}^{\ell} \) (real space),
green: flat space approximation for redshift space distortions,
red: \( C_{DD}^{\ell}, C_{zz}^{\ell} \) and \( 2C_{Dz}^{\ell} \) (fully positive!).

(from Montanari & RD '12)
The radial power spectrum $C_\ell(z, z')$
for $\ell = 20$
Left, top to bottom: $z = 0.1, 0.5, 1$
top right: $z = 3$

Standard terms (blue), $C^{\text{lensing}}_\ell$ (magenta), $C^{\text{Doppler}}_\ell$ (cyan), $C^{\text{grav}}_\ell$ (black),
The radial correlation function

\[ \xi^g(\Delta z) \Delta z^2 \times 10^6 \]

![Graph showing the radial correlation function with different curves for different redshift values.]

(from Montanari & RD ’12)

\[ z = 2, \]
\[ z = 1, \]
\[ z = 0.7, \]
\[ z = 0.3. \]

Purely negative for \( \Delta z \gtrsim 0.01. \)
Anisotropic clustering in CMASS galaxies

Contours of equal \( \xi \) as \( r_\sigma \) and \( r_\pi \) (from Reid et al. '12)
Example: Alcock-Paczyński test

(Alcock & Paczyński ’79)

Consider a comoving scale $L$ in the sky.
Horizontally it is projected to the angle $	heta_L = \frac{L}{(1+z)D_A(z)}$.
Radially its ends are at a slightly different redshifts, $\Delta z_L = LH(z)$.

$$\frac{\Delta z_L}{\theta_L} = (1 + z)D_A(z)H(z) = F(z) \equiv \int_0^z \frac{H(z)}{H(z')} \, dz'$$

\[ \theta_{BAO} \text{ [radians]} \]

\[ \Delta z_{BAO} \]

$\bar{z}_{tr}$
Example: Alcock-Paczyński test

\[ F(z)^{AP} \equiv \Delta z_L / \theta_L \text{ measured from the theoretical power spectrum (with Euclid-like redshift accuracies)} \]

\[ F(z) \equiv \int_0^z \frac{H(z)}{H(z')} \, dz'. \]

(from Montanari & RD '12)
Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$. These 1d functions are easier to measure (less noisy) but they require an input cosmology converting redshift and angles to length scales,

$$r = \sqrt{\chi(z)^2 + \chi(z')^2 - 2\chi(z)\chi(z')\cos \theta}.$$
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- But future large & precise 3d galaxy catalogs like Euclid will be able to determine directly the measured 3d correlation function $\xi(\theta, z, z')$ and $C_\ell(z, z')$ from the data.
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- An example is the Alcock-Paczyński test.