Cosmological statistically anisotropic (Gaussian) models are equivalent to non-Gaussian statistically isotropic models

Discuss.

The closet non-Gaussianity of anisotropic Gaussian fluctuations

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In this paper we explore the connection between anisotropic Gaussian fluctuations and isotropic non-Gaussian fluctuations. We first set up a large angle framework for characterizing non-Gaussian fluctuations: large angle non-Gaussian spectra. We then consider anisotropic Gaussian fluctuations in two different situations. Firstly we look at anisotropic space-times and propose a prescription for superimposed Gaussian fluctuations; we argue against accidental symmetry in the fluctuations and that therefore the fluctuations should be anisotropic. We show how these fluctuations display previously known non-Gaussian effects both in the angular power spectrum and in non-Gaussian spectra. Secondly we consider the anisotropic Grischuk-Zel'dovich effect. We construct a flat space time with anisotropic, non-trivial topology and show how Gaussian fluctuations in such a space-time look non-Gaussian. In particular we show how non-Gaussian spectra may probe superhorizon anisotropy.

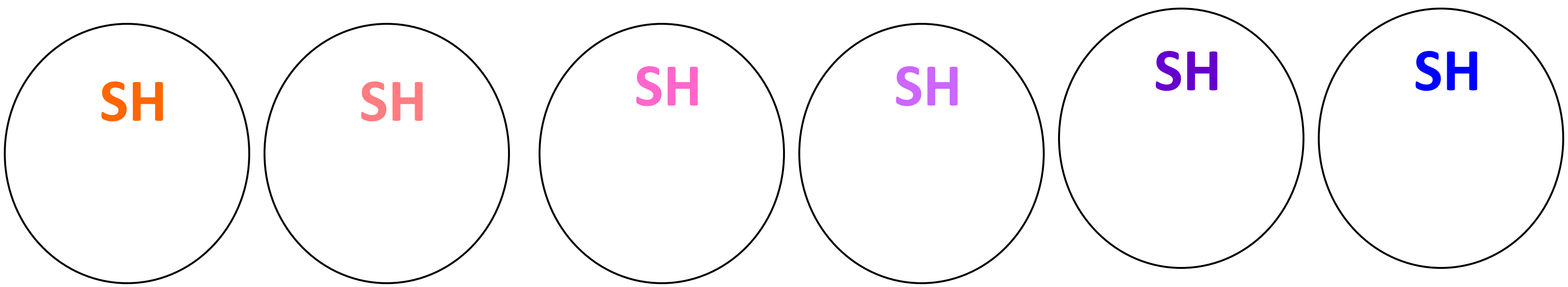
arXiv:astro-ph/9704052v1 5 Apr 1997

$T \sim P(T|\Omega)$ is statistically anisotropic in direction Ω

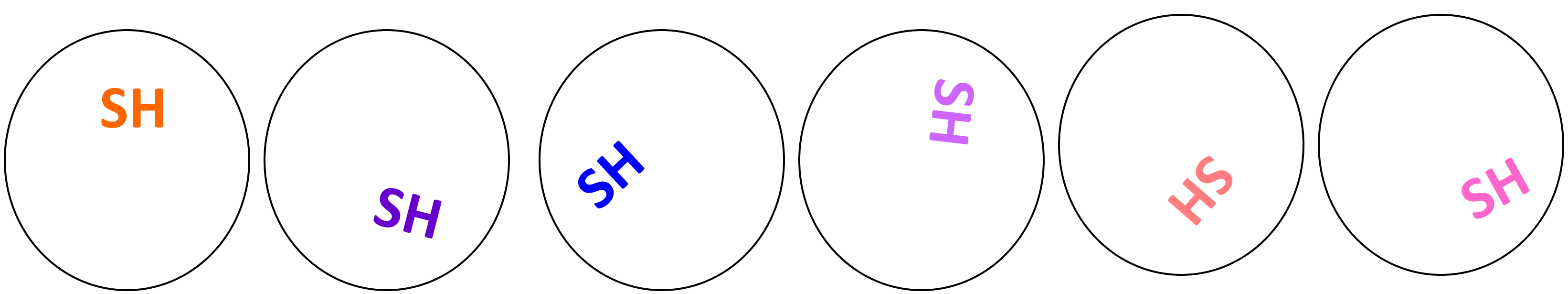
$\Rightarrow\ T \sim \int P(T,\Omega)d\Omega$ is statistically isotropic and non-Gaussian

Gaussian anisotropic models

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$



Or is it a statistically isotropic non-Gaussian model??



Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power

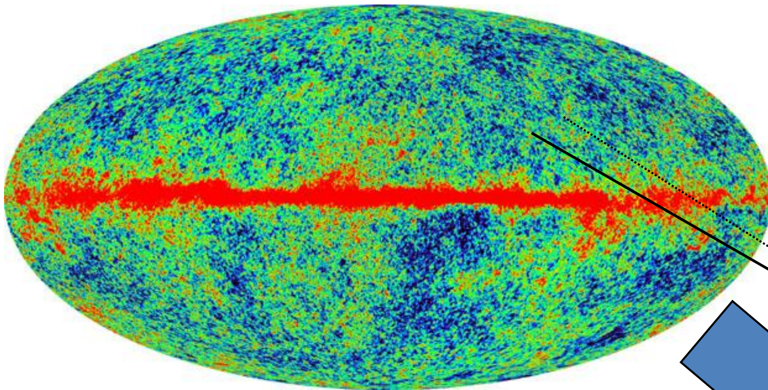
+ various systematics, anisotropic noise, beam-effects, …

Expected signal..

Example: CMB lensing

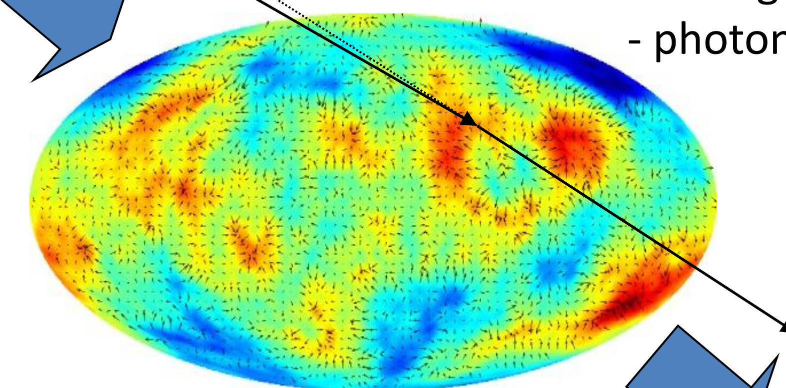
Last scattering surface

$$\boldsymbol{\alpha} = -2\int_0^{\chi_*} \mathrm{d}\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*)} \boldsymbol{\nabla}_\perp \Psi(\chi\hat{\mathbf{n}};\eta_0 - \chi)$$



Gaussian LSS

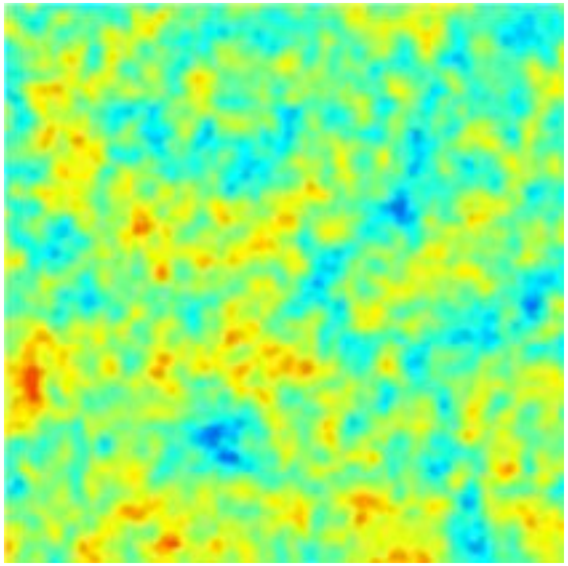
Inhomogeneous universe
- photons deflected

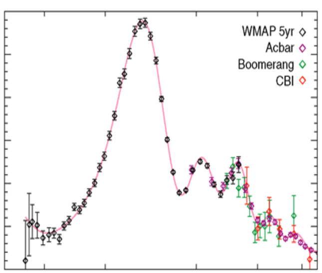


Observer

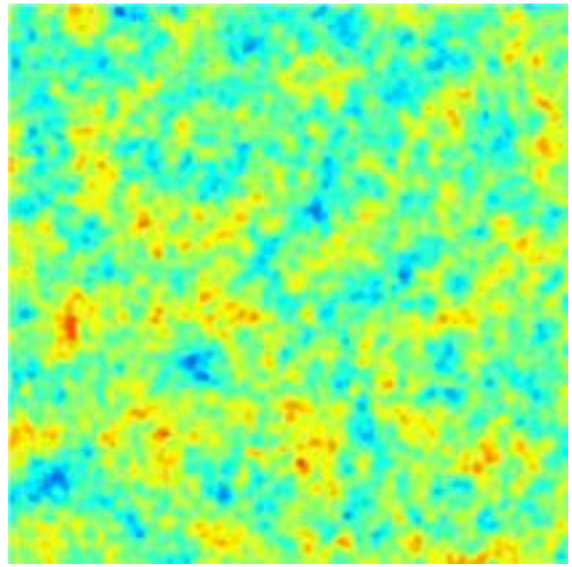
$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

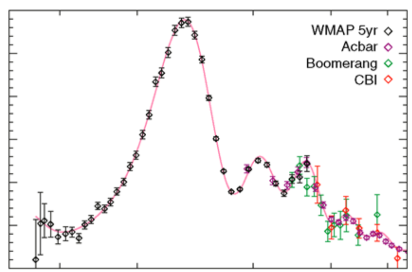
Magnified



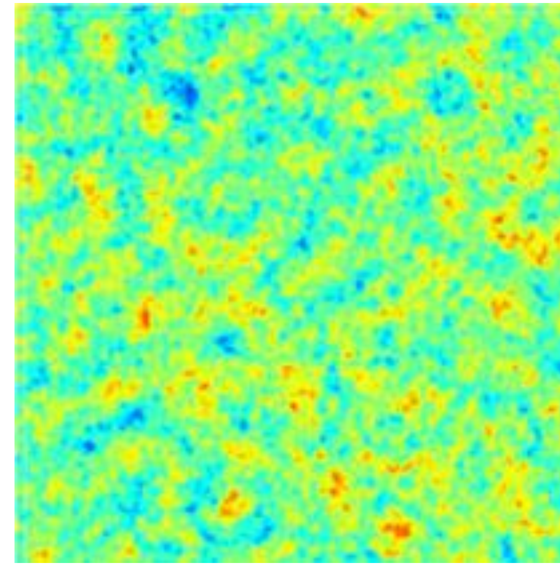


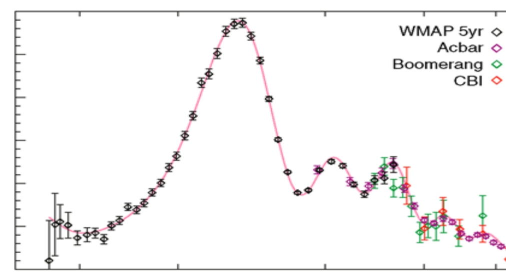
Unlensed





Demagnified





For a given lensing field :

$$T \sim P(T|\psi)$$

- Anisotropic Gaussian temperature distribution

- Different parts of the sky magnified and demagnified
- Re-construct the actual lensing field – infer ψ

Or marginalized over lensing fields:

$$T \sim \int P(T,\psi)d\psi$$

-Non-Gaussian statistically isotropic temperature distribution

- Significant connected 4-point function
- Excess variance to anisotropic-looking realizations

Power asymmetry $\equiv \tau_{NL}$-type trispectrum

e.g. from primordial modulation $\zeta(\mathbf{x}) = \zeta_0(\mathbf{x})[1+\phi(\mathbf{x})]$

Squeezed shape, constant modulation $T(\hat{\mathbf{n}}) \approx T_g(\hat{\mathbf{n}})[1+\phi(\hat{\mathbf{n}}, r_*)]$

Fix $\phi \Rightarrow$ Gaussian power anisotropy
Average $\phi \Rightarrow$ isotropic trispectrum

Easy accurate estimator for τ_{NL} is $\tau_{\rm NL}(L) \equiv \frac{C_L^{\phi}}{C_L^{\zeta_\star}}$

Pearson, Lewis & Regan *arXiv:1201.1010*

e.g. hemispherical power asymmetry found in WMAP ($l_{\max} \sim 64$) $\equiv$

squeezed trispectrum with scale-dependent $\tau_{NL}(L=1)$

Can't you just measure $\langle a_{lm} a_{l'm'} \rangle$?

Anisotropic theory gives $\langle a_{lm} a_{l'm'} \rangle = \mathrm{C}_{\mathrm{lml'm'}}$

But we measure one realisation, so always $a_{lm} a_{l'm'} \neq 0$

Cosmic variance assuming Gaussianity $\sim C_l C_{l'}$.

If you detect signal not compatible with cosmic variance could interpret either as

1. Statistical anisotropy

2. Non-Gaussian covariance of the a_{lm} which makes measured $a_{lm} a_{l'm'}$ acceptable
   $\langle a_{lm} a_{l'm'}\, a_{lm} a_{l'm'} \rangle \sim T(l, l', l, l') + C_l C_{l'} \delta$