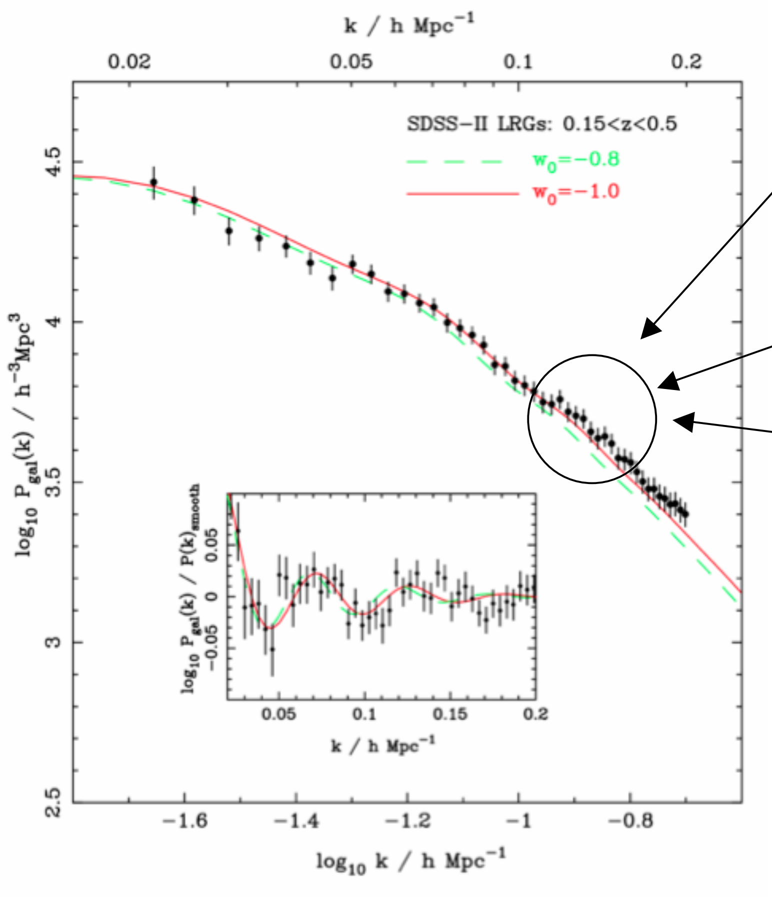
Embracing Nonlinearities

Martin Crocce
Institute for Space Science (IEEC/CSIC) - Barcelona

Benasque - 2012

Motivation can be rather obvious

- Given all these nice datasets how do we extract the most out of them ?
- At smaller scales S/N is much better but we have to “embrace non-linearities”



Nonlinear Gravitational Clustering

Red-shift Space Distortions

Galaxy (or cluster) biasing

They all quick in were they should not.

Nonlinear Gravitational Clustering

scales much smaller than the Horizon (Hubble radius) $\longrightarrow$ Newtonian gravity

scales larger than strong clustering regime $\longrightarrow$ *single stream approximation*

no velocity dispersion or pressure
(prior to virialization and shell crossing)

$$\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\ \mathcal{H}^2(\tau)\ \delta(\mathbf{x},\tau)$$

$$\frac{\partial\delta(\mathbf{x},\tau)}{\partial\tau} + \nabla\cdot\{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0$$

$$\frac{\partial\mathbf{u}(\mathbf{x},\tau)}{\partial\tau} + \mathcal{H}(\tau)\ \mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau)$$

velocity field can be assumed irrotational $\theta(\mathbf{x},\tau)\ \equiv\ \nabla\cdot\mathbf{u}(\mathbf{x},\tau)$

$$\frac{\partial\tilde\delta(\mathbf{k},\tau)}{\partial\tau} + \tilde\theta(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\tilde\theta(\mathbf{k}_1,\tau)\,\tilde\delta(\mathbf{k}_2,\tau),$$

$$\frac{\partial\tilde\theta(\mathbf{k},\tau)}{\partial\tau} + \mathcal{H}(\tau)\tilde\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\tilde\delta(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\,\tilde\theta(\mathbf{k}_1,\tau)\,\tilde\theta(\mathbf{k}_2,\tau)$$

Reformulating the equations (Scoccimarro 1997)

$$\Psi_a(\mathbf{k},\eta) \equiv \left(\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta)/\mathcal{H}\right), \qquad \eta \equiv \ln a(\tau).$$

The solution to the nonlinear equations of motion can be formally written as

$$\Psi_a(\mathbf{k},\eta) = g_{ab}(\eta)\ \phi_b(\mathbf{k}) + \int_0^{\eta} d\eta'\ g_{ab}(\eta-\eta')\ \gamma^{(s)}_{bcd}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\ \Psi_c(\mathbf{k}_1,\eta')\Psi_d(\mathbf{k}_2,\eta')$$

Initial Conditions

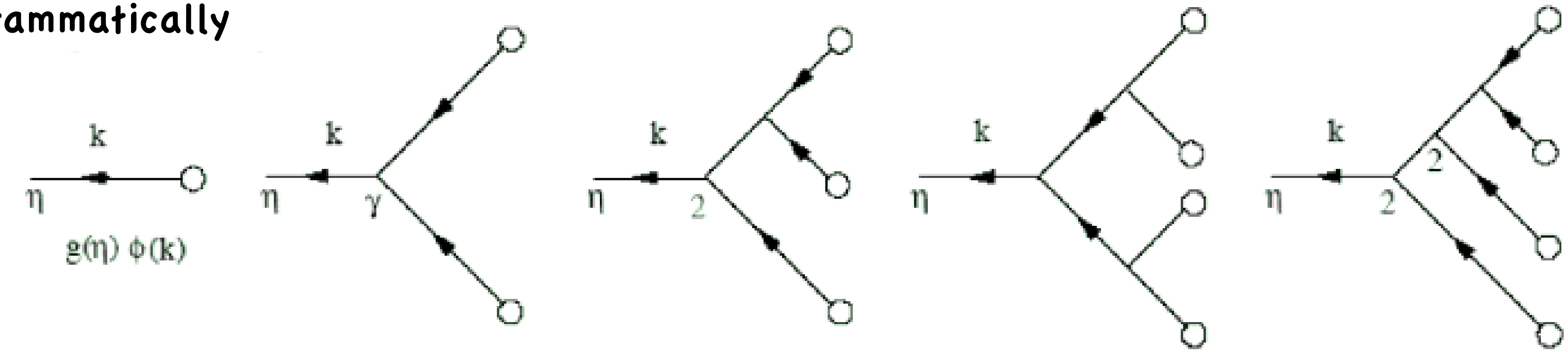
Linear propagator $$g_{ab}(\eta) = \frac{e^{\eta}}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{e^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix},$$

growing mode
$\phi_a(\mathbf{k}) \propto (1,1)$

decaying mode
$\phi_a(\mathbf{k}) \propto (1,-3/2)$

Diagrammatically

Ψ_a



Linear order $\delta_L(k,z) = D_+(z)\delta_0(k) \xrightarrow[\langle\delta(\mathbf{k})\delta(-\mathbf{k})\rangle]{} P_{\rm lin}(k,z) = [D_+(z)]^2 P_0(k)$.

Standard perturbation theory expands the density contrast in terms of the linear solution,

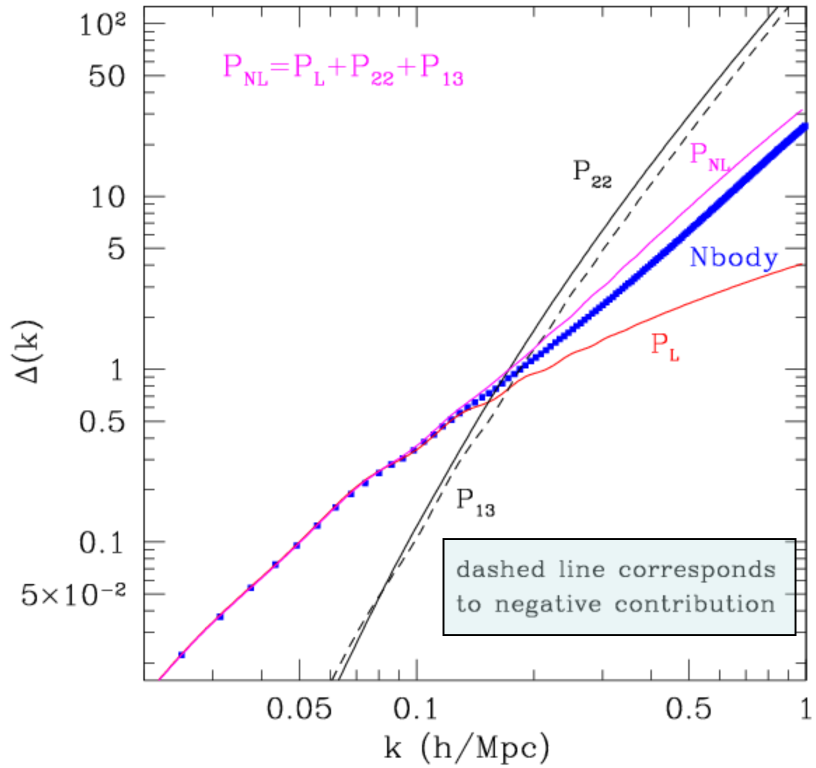
$$P(k,z) = D_+^2(z)P_0(k) + \underbrace{P_{13}(k,z) + P_{22}(k,z)}_{\text{1 - loop terms}} + \ldots$$

$$P_{22}(k,\tau) \equiv 2\int [F_2^{(s)}(\mathbf{k}-\mathbf{q},\mathbf{q})]^2 P_L(|\mathbf{k}-\mathbf{q}|,\tau)P_L(q,\tau)\mathrm{d}^3\mathbf{q},$$

$$P_{13}(k,\tau) \equiv 6\int F_3^{(s)}(\mathbf{k},\mathbf{q},-\mathbf{q})P_L(k,\tau)P_L(q,\tau)\mathrm{d}^3\mathbf{q}.$$

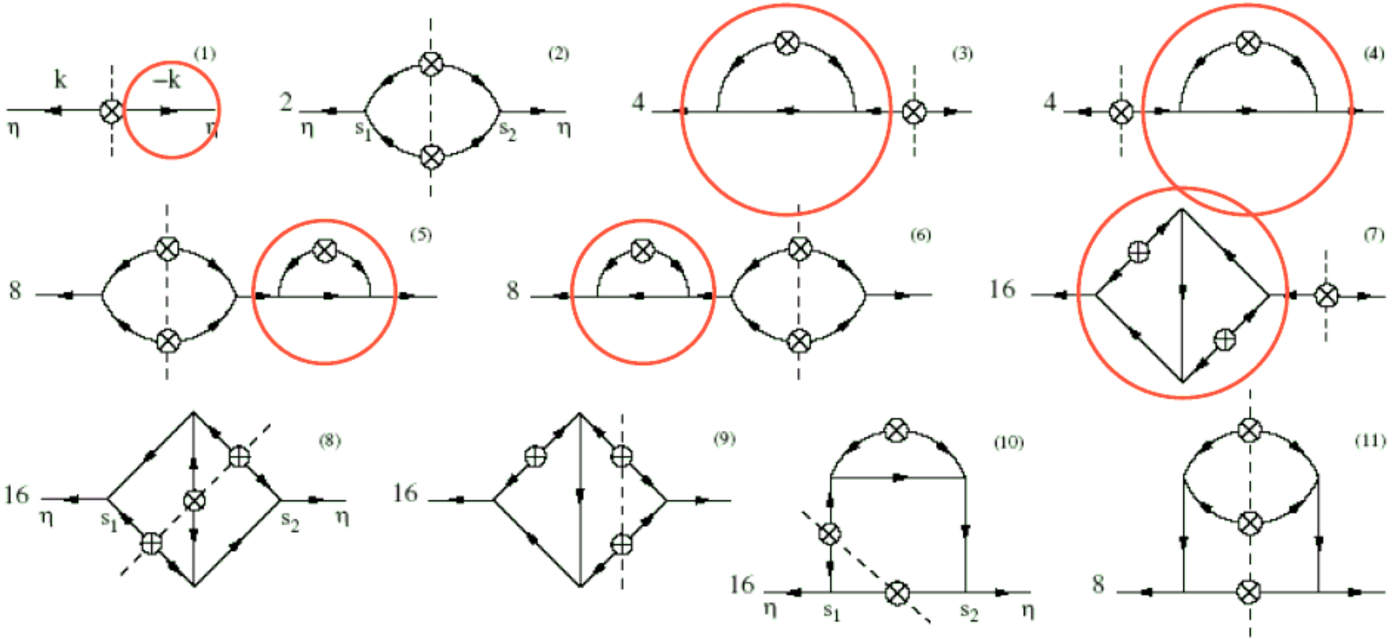
This expansion is valid at large scales where fluctuations are small, but it brakes down when approaching the nonlinear regime where $\Delta_{\rm lin} \gtrsim 1$.

Way out is to sum up all orders (!)



Power spectrum

$$\langle \Psi(k,a)\Psi(-k,a)\rangle$$

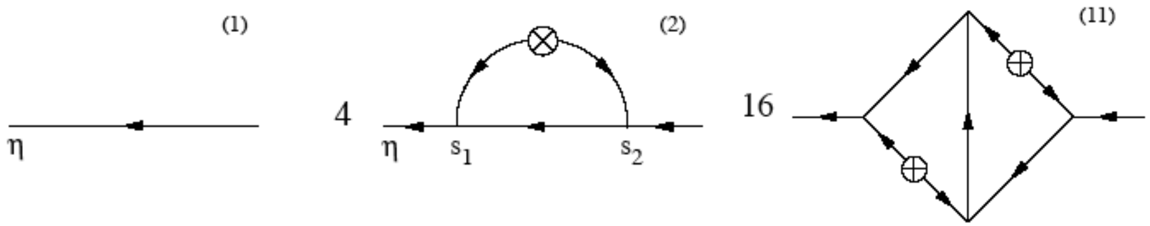


all orders can be systematically incorporated

Nonlinear Propagator

(Crocce & Scocimarro 2006)

$$G(k,\eta) \equiv \left\langle \frac{\delta \Psi_a(\mathbf{k},\eta)}{\delta \phi_b(\mathbf{k}')} \right\rangle =$$



It is possible to re-organize the series by re-summing (infite) terms.
A new set of object appears,
(bernardeau crocce & sccocimarro 2008)

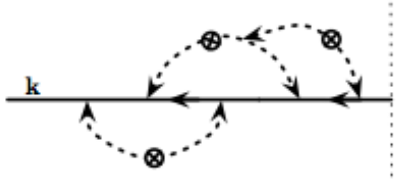
final den or vel field

$$\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1\cdots p})\,\Gamma^{(p)}_{ab_1\ldots b_p}(\mathbf{k}_1,\ldots,\mathbf{k}_p)=\frac{1}{p!}\left\langle\frac{\partial^p\Psi_a(\mathbf{k})}{\partial\Phi_{b_1}(\mathbf{k}_1)\ldots\partial\Phi_{b_p}(\mathbf{k}_p)}\right\rangle$$

initial field

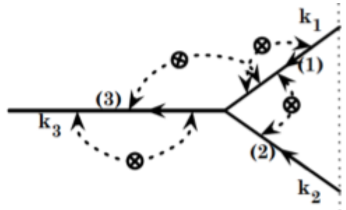
✓ Natural extension of the RPT propagator :

$$G_{ab}(k,\eta)\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}')\equiv\left\langle\frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')}\right\rangle$$



✓ "Renormalization" of the PT kernels :

$$\Gamma^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n)=\mathrm{F}^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n)+\text{“nonlinear corrections”}$$



✓ They satisfy :

$$\delta_{\rm D}(\mathbf{k}-\mathbf{q}_{1\ldots p})\Gamma^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n)\;\left\{\begin{array}{l}\delta_{\rm D}[.]\,\mathrm{F}^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n)\quad\text{for low }k\\[2ex] \boxed{\delta_{\rm D}[.]\,\exp\left(-k^2\sigma_v^2/2\right)\mathrm{F}^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n)\quad\text{for high }k}\\[2ex] \sim\langle\Psi(\mathbf{k})\Phi(-\mathbf{q}_1)\ldots\Phi(-\mathbf{q}_p)\rangle\end{array}\right.$$

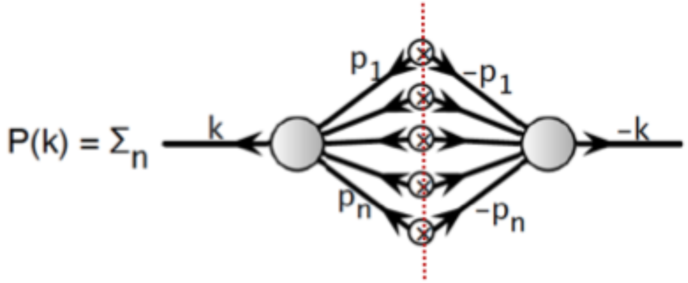
$$\sigma_v^2=\frac{1}{3}\int_0^\infty\frac{d^3\mathbf{k}}{k^2}P_{\rm Lin}(k)$$

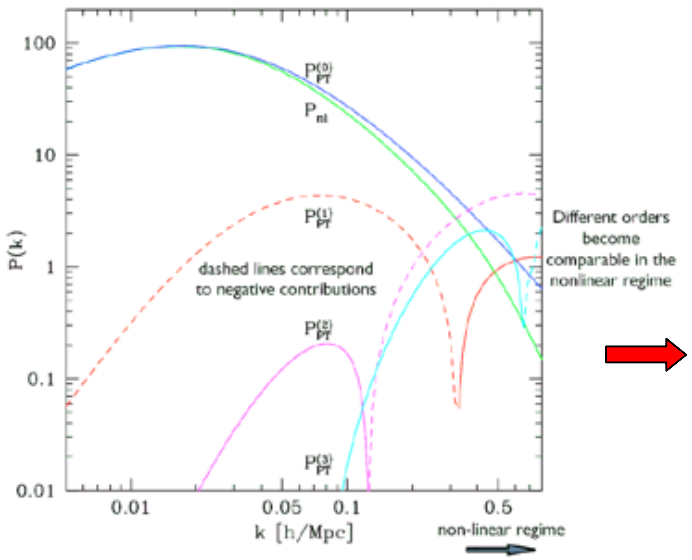
Reconstruction of P(k) from Multi-point propagators

$$P_{ab}(\mathbf{k},\eta) = \sum_t t! \int \mathrm{d}^3\mathbf{q}_1 \ldots \mathrm{d}^3\mathbf{q}_t\, \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{q}_{1\ldots t})\, \Gamma_a^{(t)}(\mathbf{q}_1,\ldots,\mathbf{q}_t;\eta)\, \Gamma_b^{(t)}(\mathbf{q}_1,\ldots,\mathbf{q}_t;\eta)\, P_0(q_1)\ldots P_0(q_t),$$

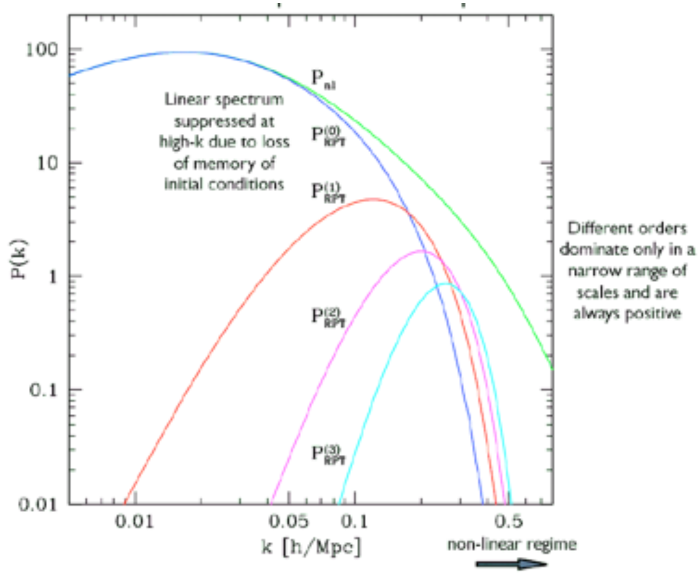
- Sum of positive contributions

$$P_{\delta\delta}(k) = \left[\Gamma_\delta^{(1)}(k,z)\right]^2 P_0(k) + 2\int d^3q \left[\Gamma_\delta^{(2)}(\mathbf{k}-\mathbf{q},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(q) + \ldots$$





ZA power spectrum in standard PT



ZA power spectrum in multi-point expansion

Multi-point Propagator Expansion : Density P(k)

We implemented the expansion up to 2 loops (2D and 5D integrations) :

$$
\begin{aligned}
P_{\delta\delta}(k) &= \left[\Gamma_\delta^{(1)}(k,z)\right]^2 P_0(k) \\
&+ 2\int d^3q \left[\Gamma_\delta^{(2)}(\mathbf{k}-\mathbf{q},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(q) \\
&+ 6\int d^3p\, d^3q \left[\Gamma_\delta^{(3)}(\mathbf{k}-\mathbf{p}-\mathbf{q},\mathbf{p},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(p)P_0(q)
\end{aligned}
$$

From PT we know $\Gamma_\delta^{(1)}(k,z) = D(z) - f(k)D^3(z) + \ldots$ so we take this interpolation between large and small scales :

$$\checkmark\ \Gamma_\delta^{(1)}(k,z) = G(k,z) = D(z)\exp\left[f(k)D^2(z)\right]$$

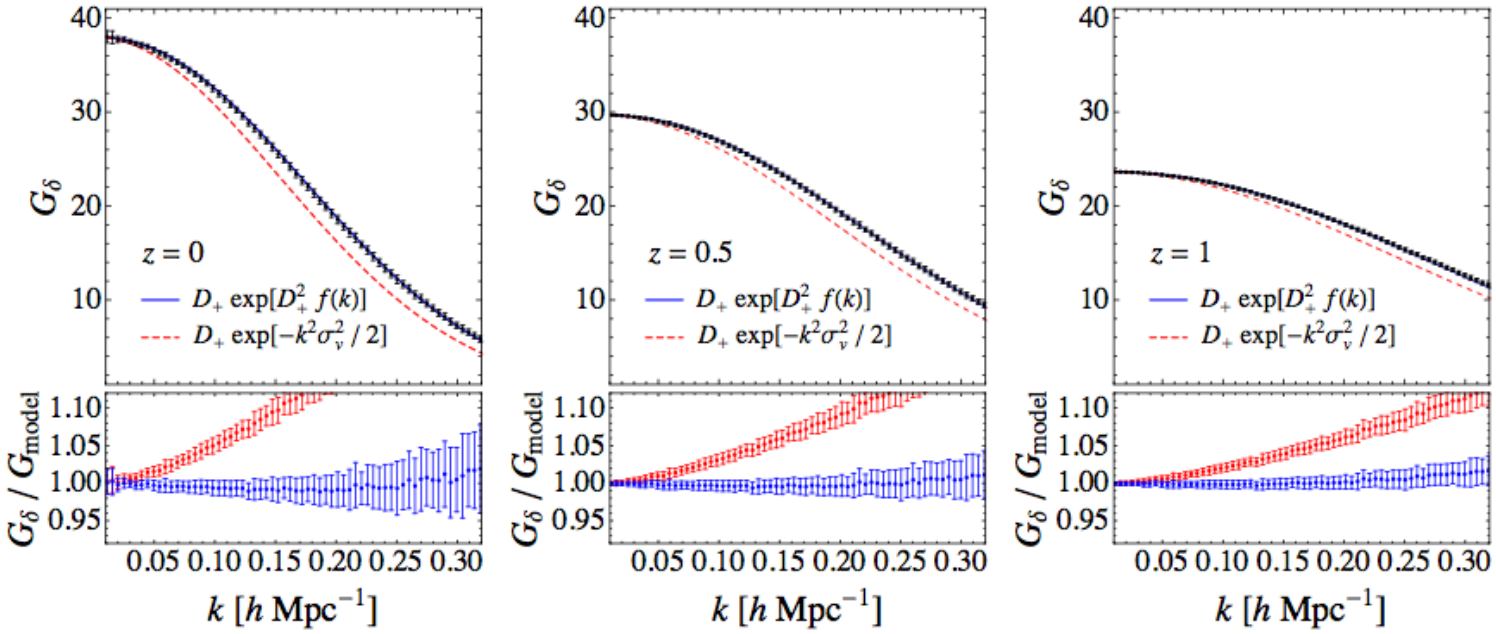
And the following ansatze for the MP propagators :
(based on N-body results)

$$
\left\{
\begin{aligned}
&\checkmark\ \Gamma_\delta^{(2)} = G(k,z) \times F_2 \\
&\checkmark\ \Gamma_\delta^{(3)} = G(k,z) \times F_3
\end{aligned}
\right.
$$

with F_2 and F_3 the standard PT kernels and

$$f(k) = \int \frac{1}{504k^3q^5}\left[6k^7q - 79k^5q^3 + 50q^5k^3 - 21kq^7 + \frac{3}{4}(k^2-q^2)^3(2k^2+7q^2)\ln\frac{|k-q|^2}{|k+q|^2}\right] P_0(q)\, d^3q,$$

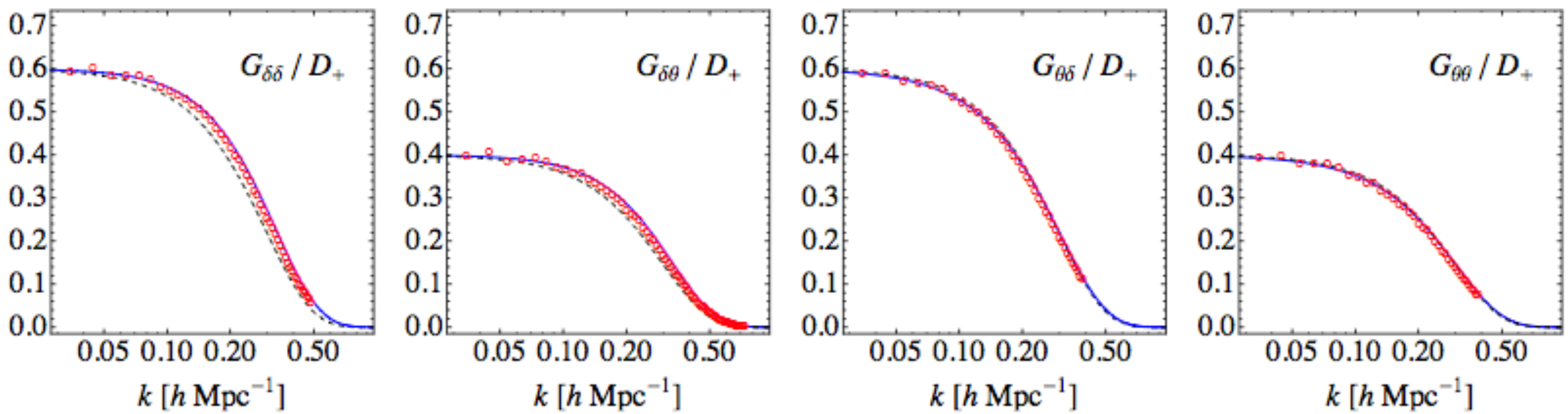
Testing the ansatz for the 2pt propagator



Simulations with independent initial positions and velocities

- We could test the full matrix structure of the propagator -

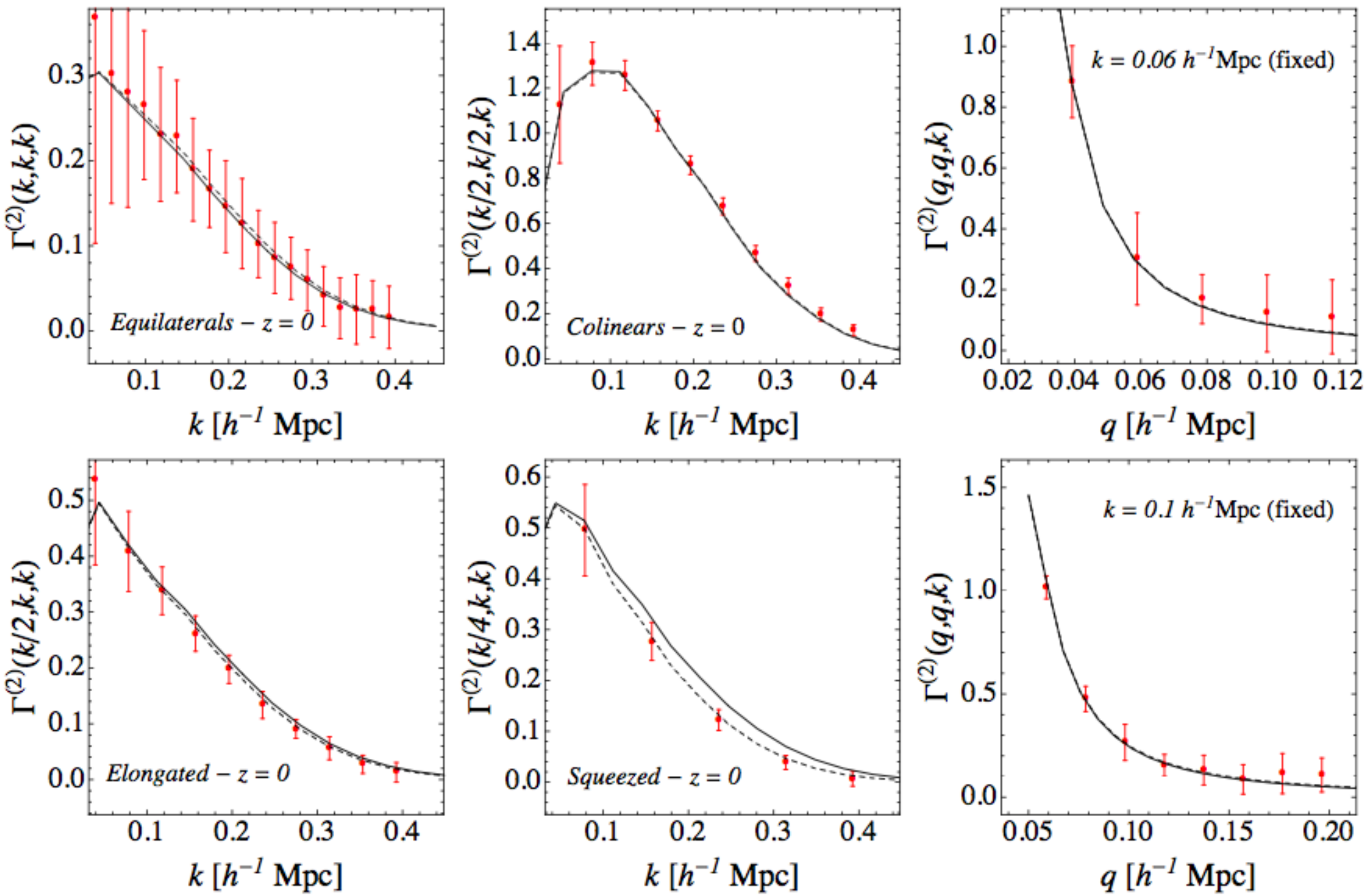
$$\{f(k), g(k)\} \to \frac{13}{25}\{f(k), g(k)\}$$



Testing the three point propagator

$\Gamma_\delta^{(2)} = G(k,z) \times F_2$

RegPT

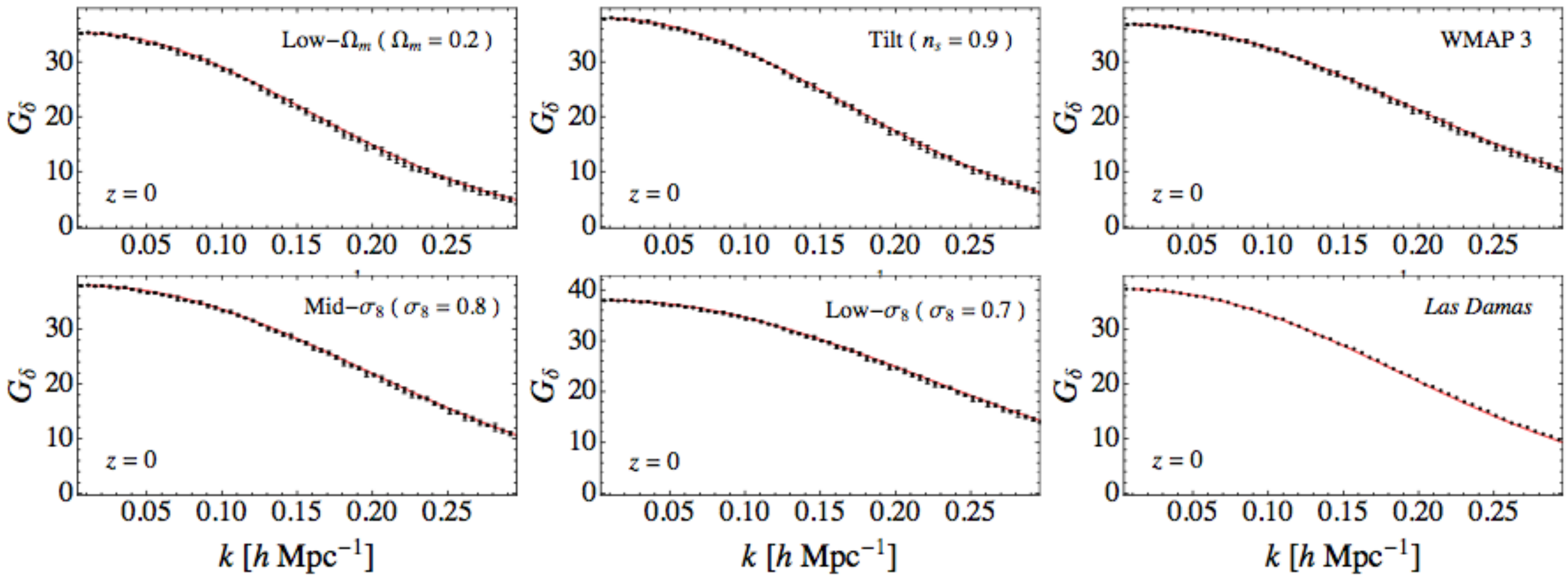


General Configurations

Configurations most relevant for one loop power spectrum at k

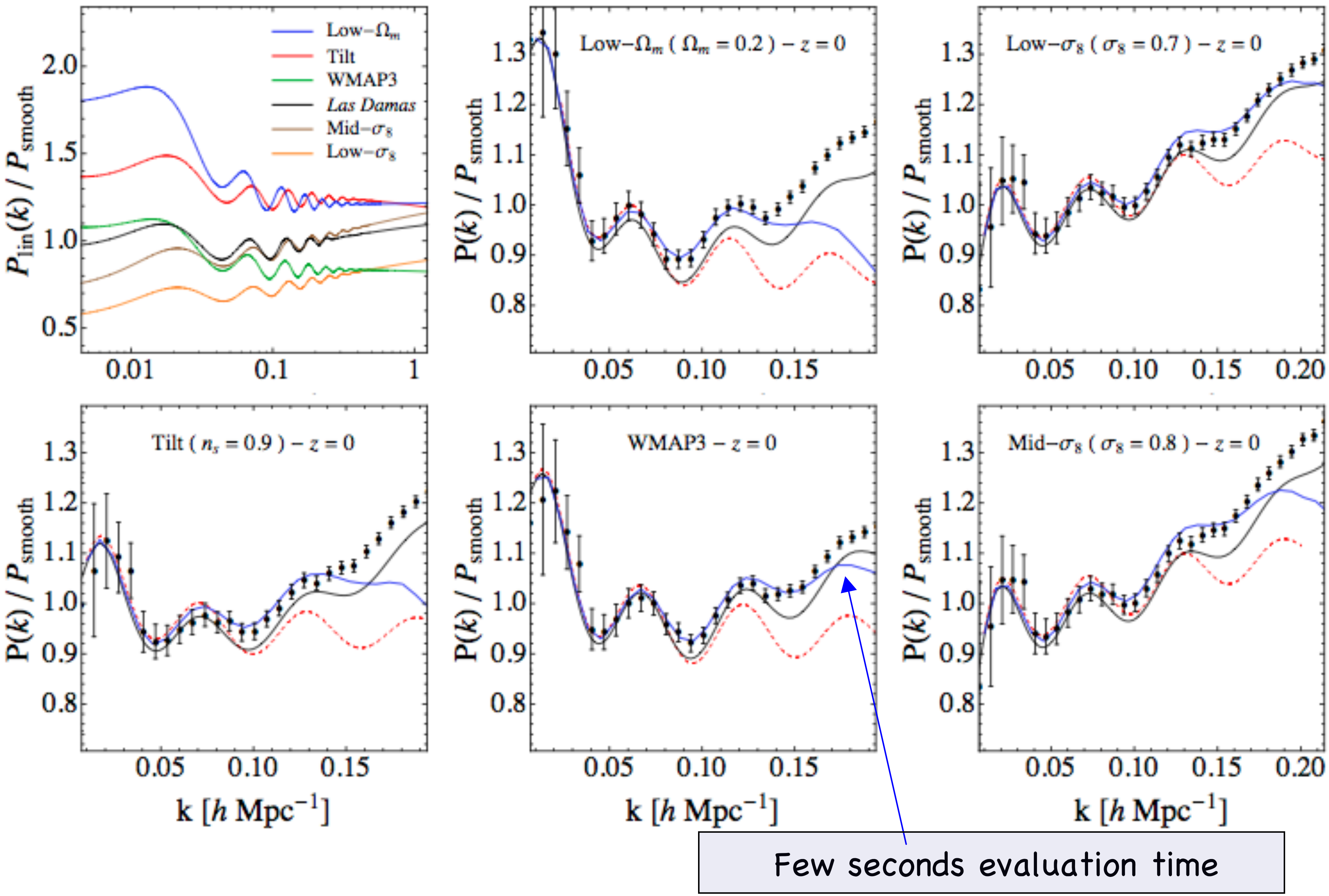
Suite of Cosmological Simulations

Run	Ω_m	Ω_b	h	σ_8	n_s	$L_{box}(h^{-1}$ Mpc)	N_{runs}	k_{nl}(z=0,0.5,1)
FID	0.27	0.04	0.7	0.9	1	1280	50	0.15 - 0.2 - 0.25
tilt					0.9	1250	4	
WMAP3	0.2383	0.0418	0.73	0.74	0.95	1250	4	
Low-Ω_m	0.20					1250	4	
Mid-σ_8				0.8		1250	4	
Low-σ_8				0.7		1250	4	
Las Damas	0.25	0.044	0.7	0.8	1	2400	4	

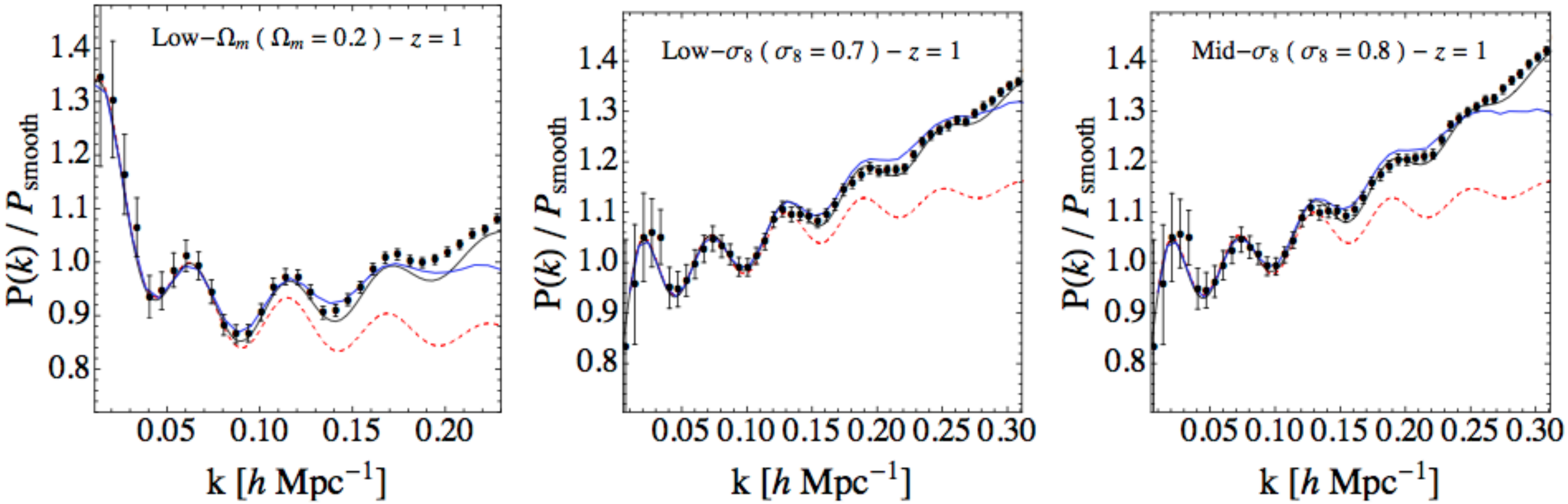


Power spectrum (MPTbreeze)

Performance for different cosmological models - z = 0



Performance for different cosmological models - z = 1

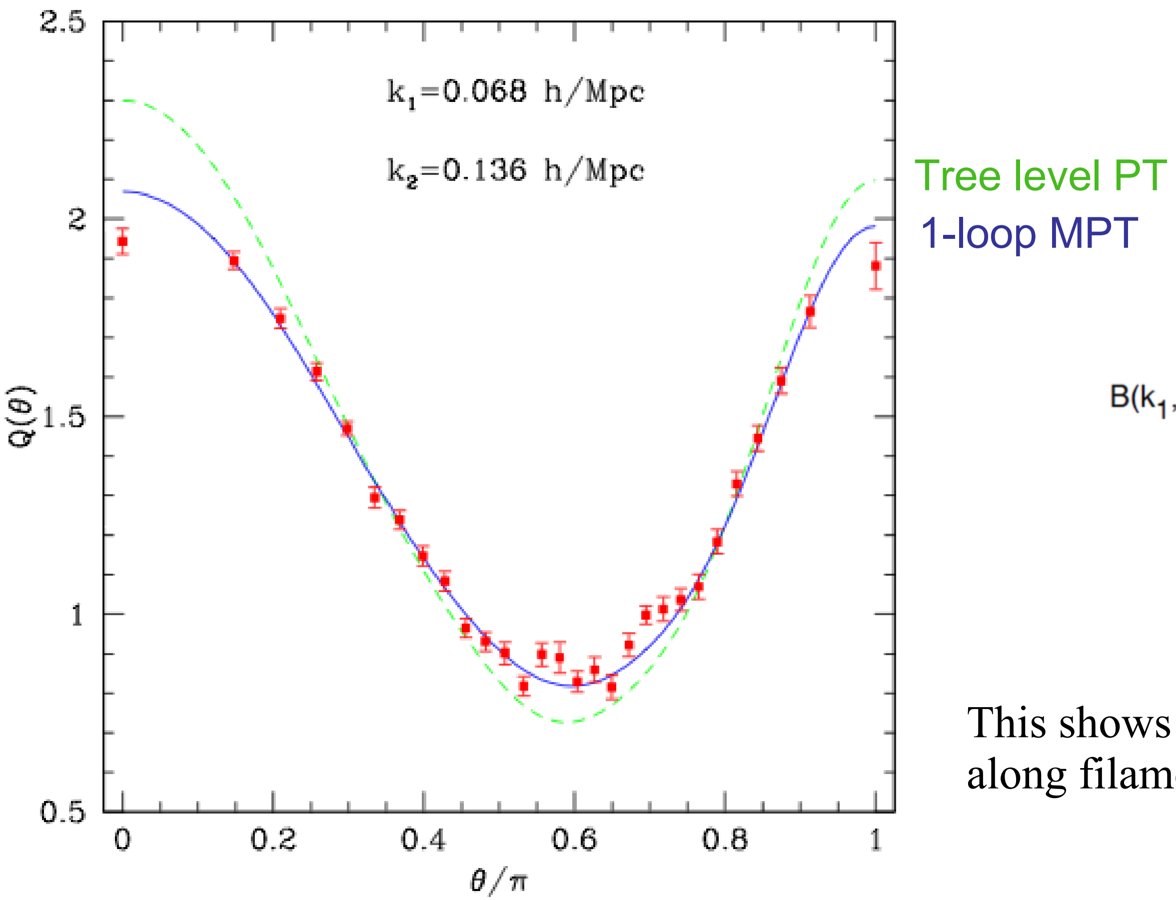


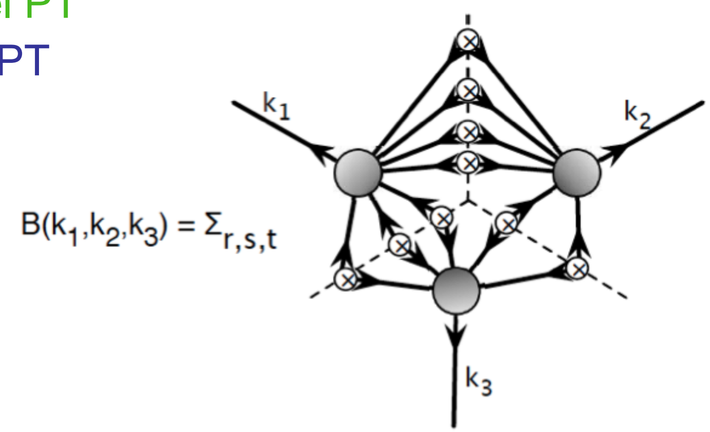
MPTbreeze
code is publicly available at http://maia.ice.cat/crocce/mptbreeze

See also Crocce, Scoccimarro & Bernardeau arXiv : 1207.1465

Reduced Bispectrum at 1-loop in multi-point propagator expansion

$$Q = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)},$$



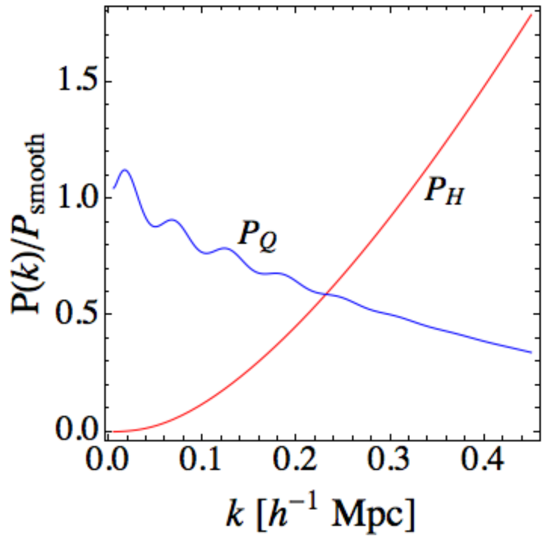


This shows that structure is distributed along filaments (maximum power at θ=0,1)

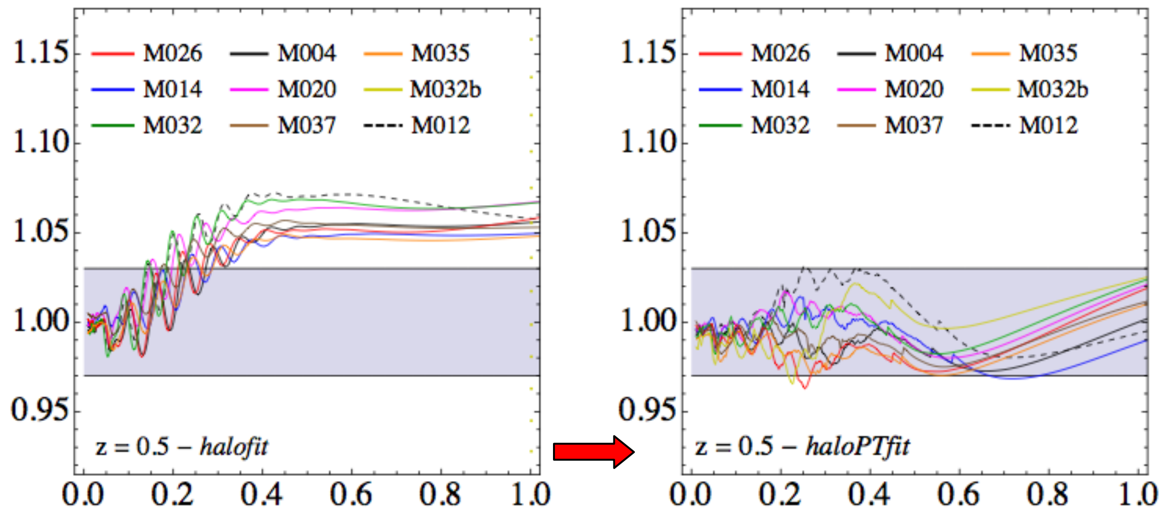
Beyond BAO - Coupling to Halofit

$$P_{NL}(k) = P_Q(k) + P_H(k)$$

Replace quasi-linear term with multi-point expansion and recalibrate the transition to the 1-halo part



Performance against Coyote Universe emulator



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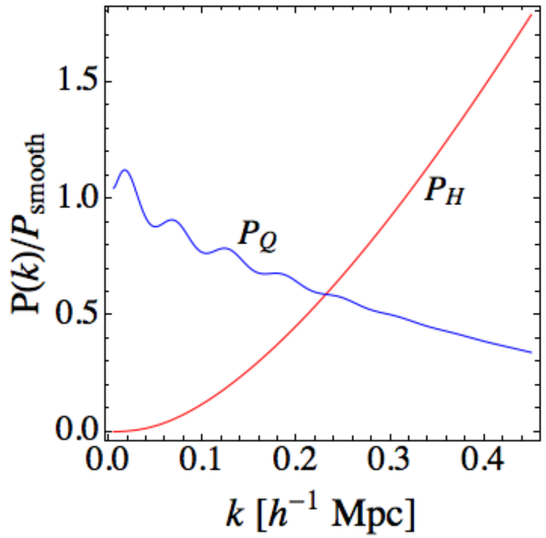
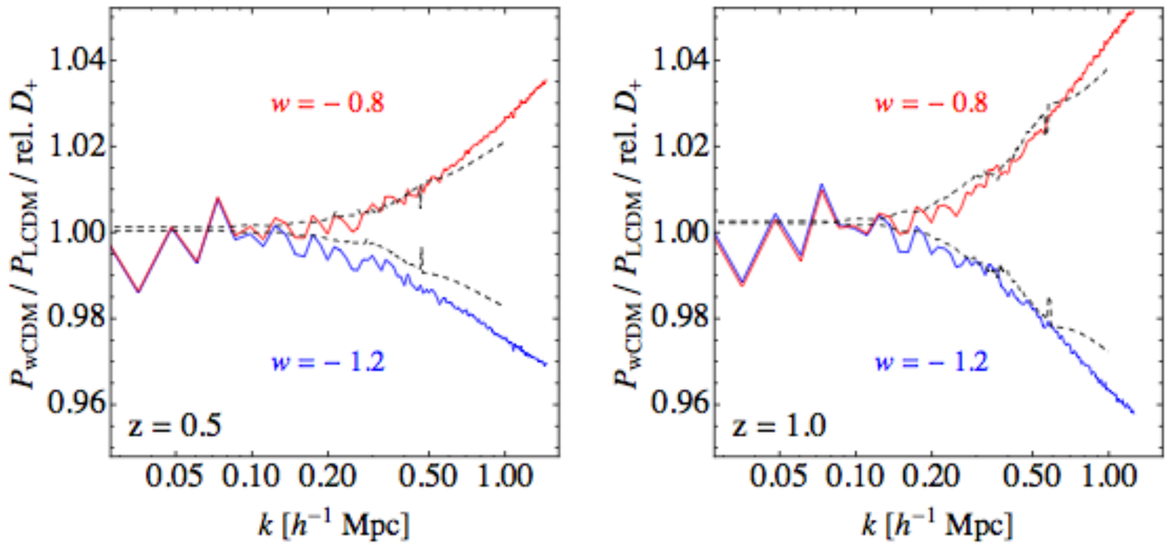


Performance against wCDM models

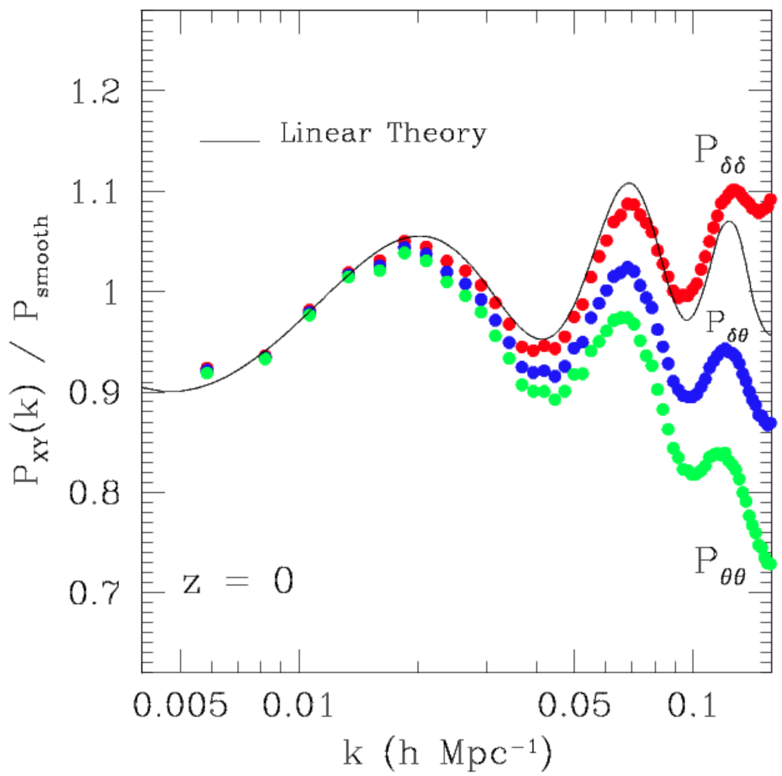


Velocity Divergence Power Spectrum

- Velocity divergence statistics is can be important to model RSD, e.g.

$$P_s(\mathbf{k}) = \left[P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)\right] \times \exp(-f^2 k_z^2 \sigma_v^2)$$

(Soccimarro 2004, Taruya et al 2010, Seljak & McDonald 2011, Vlah et al - Okumura et al 2012)



- They are about 5% (10%) below linear theory at scales 0.05 *h* Mpc^{-1} (!)

- Measuring these spectra is not trivial, but one can develop robust estimators (that converge to linear theory etc)

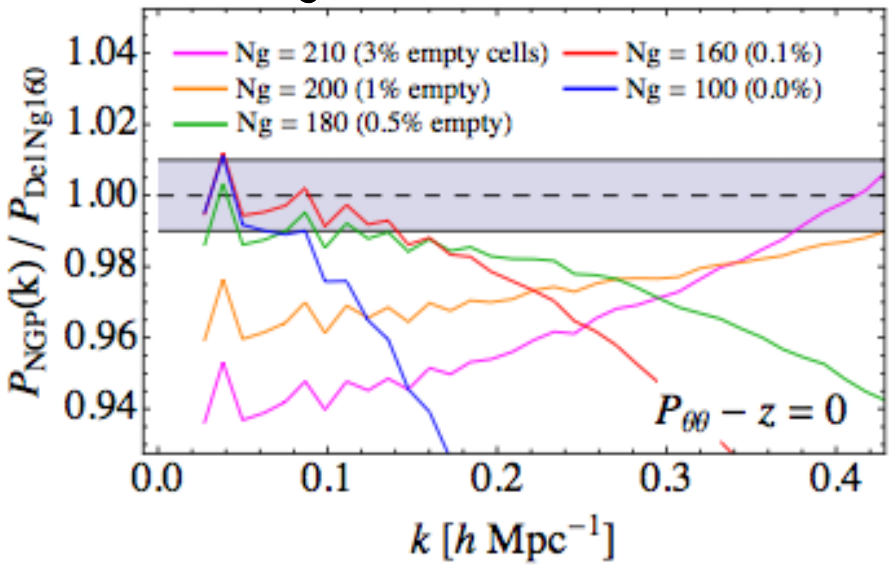
- Largest LasDamas simulation (Oriana)

(un) Convergence of velocity P(k) with naive interpolation

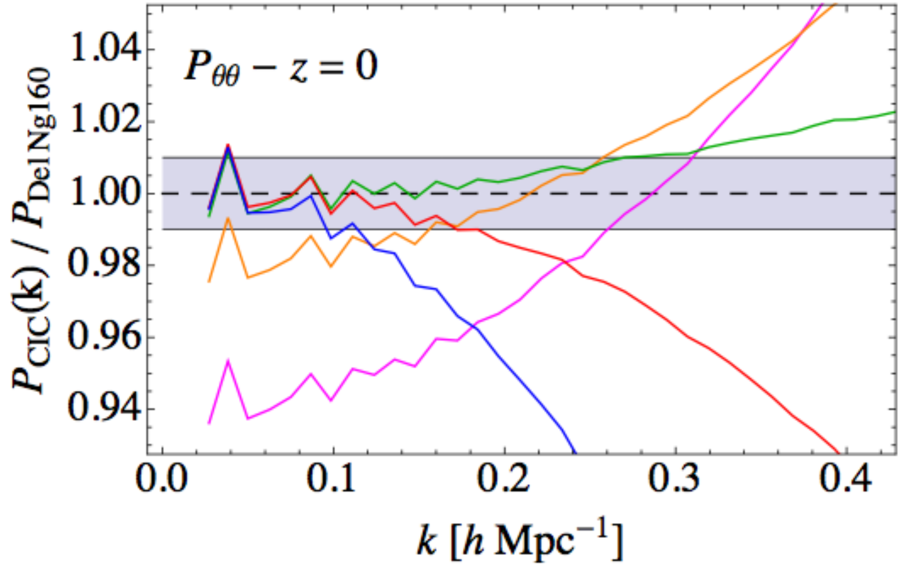
- Velocity divergence is a volume weighted quantity, it's hard to estimate because a priori we only know the velocity wherever there is a particle, what leads to a mass weighting scheme (i.e. momentum).

- Quick and dirty approach $$\mathbf{v_k} \approx \frac{\mathrm{FT}\left[\delta \mathbf{v} \otimes W_{NGP}\right]}{\mathrm{FT}\left[\delta \otimes W_{NGP}\right]} \rightarrow \theta_{\mathbf{k}} = -i\,\frac{\mathbf{k}\cdot\mathbf{v}}{k}$$

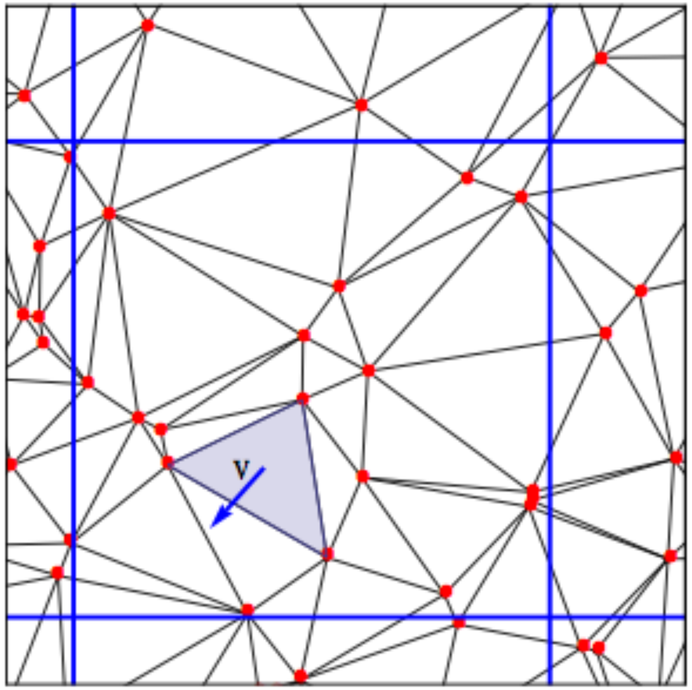
- using Nearest Grid Point (NGP)



- using Cloud in Cell (CIC)

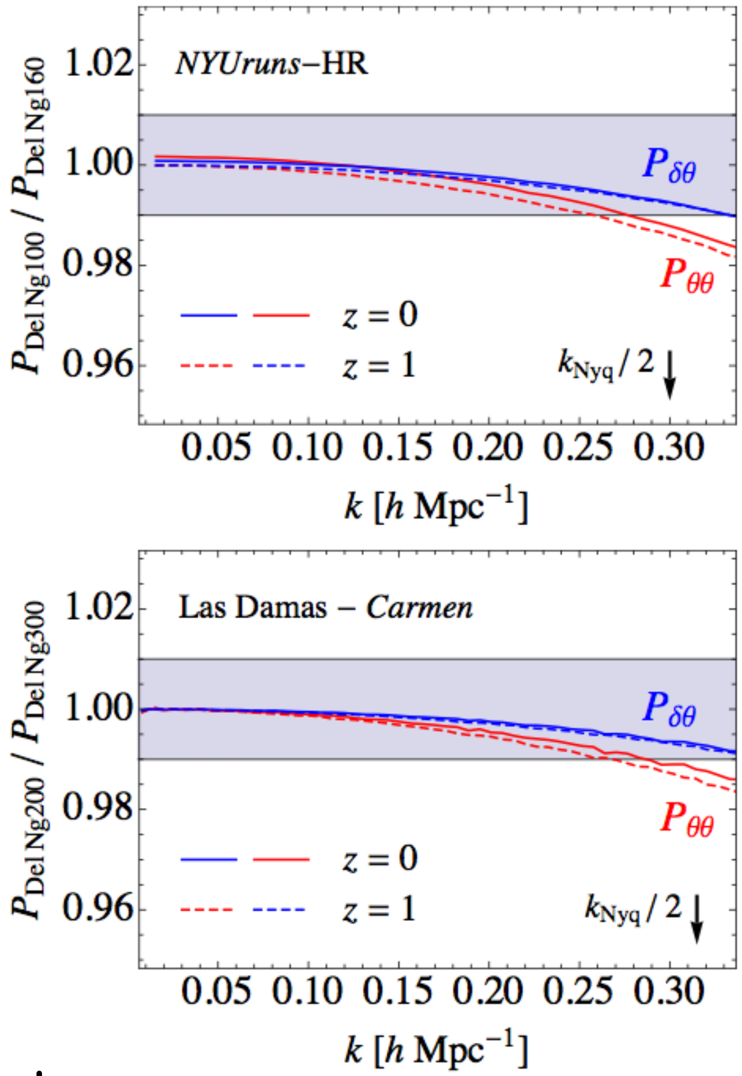


Use of Delaunay Tesselation



$$\mathbf{v}_i = \frac{\sum_{j=1}^{N_{Del}} \mathbf{v}_j vol_j}{\sum_{j=1}^{N_{Del}} vol_j}$$

Once the tesselation is finished velocity is computed in each Delaunay volume by linear combination of the velocity at the vertices, then interpolated onto a grid (in blue) weighting by the volume



Multi-point Propagator Expansion : Velocity div P(k)

We implemented the expansion up to 2 loops (2D and 5D integrations) :

$$\begin{aligned} P_{\theta\theta}(k) &= \left[\Gamma_\theta^{(1)}(k,z)\right]^2 P_0(k) \\ &+ 2\int d^3q \left[\Gamma_\theta^{(2)}(\mathbf{k}-\mathbf{q},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(q) \\ &+ 6\int d^3p\, d^3q \left[\Gamma_\theta^{(3)}(\mathbf{k}-\mathbf{p}-\mathbf{q},\mathbf{p},\mathbf{q},z)\right]^2 P_0(|\mathbf{k}-\mathbf{q}|)P_0(p)P_0(q) \end{aligned}$$

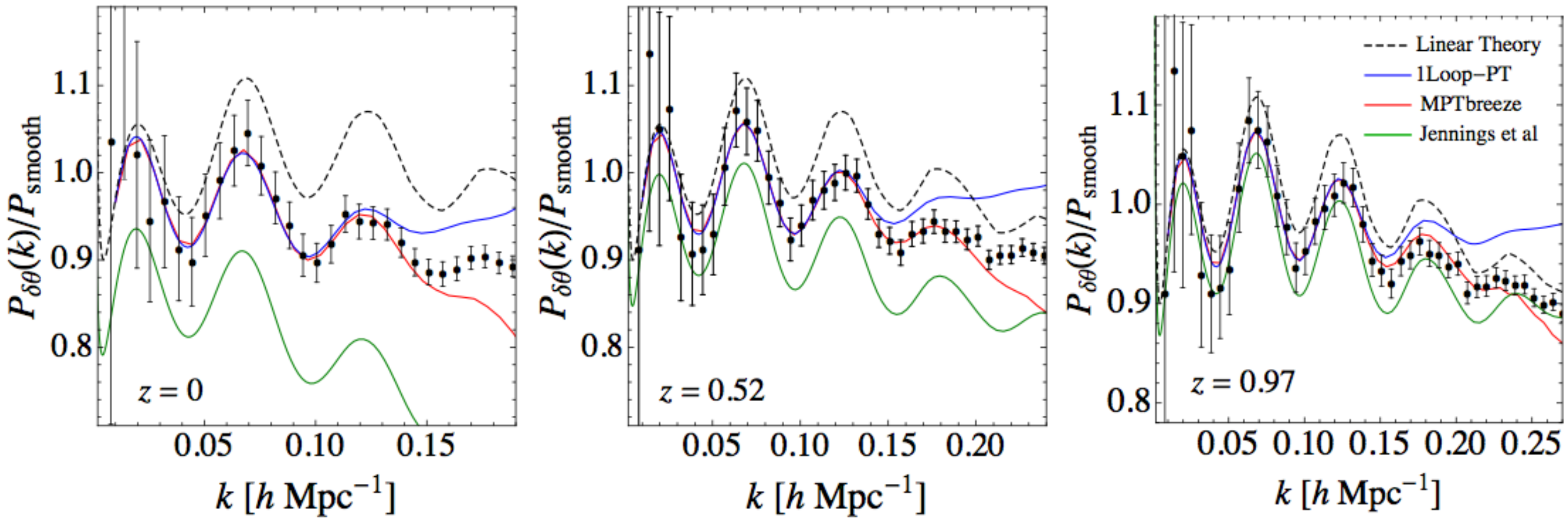
And something similar for $P_{\delta\theta}(k)$

The MP are now given by :

- ✓ $\Gamma_\theta^{(1)} = G_\theta(k,z) = D(z)\exp\left[g(k)D^2(z)\right]$
- ✓ $\Gamma_\theta^{(2)} = G_\theta(k,z) \times G_2$
- ✓ $\Gamma_\theta^{(3)} = G_\theta(k,z) \times G_3$

with G_2 and G_3 the standard PT kernels

Cross Velocity-Density power spectrum



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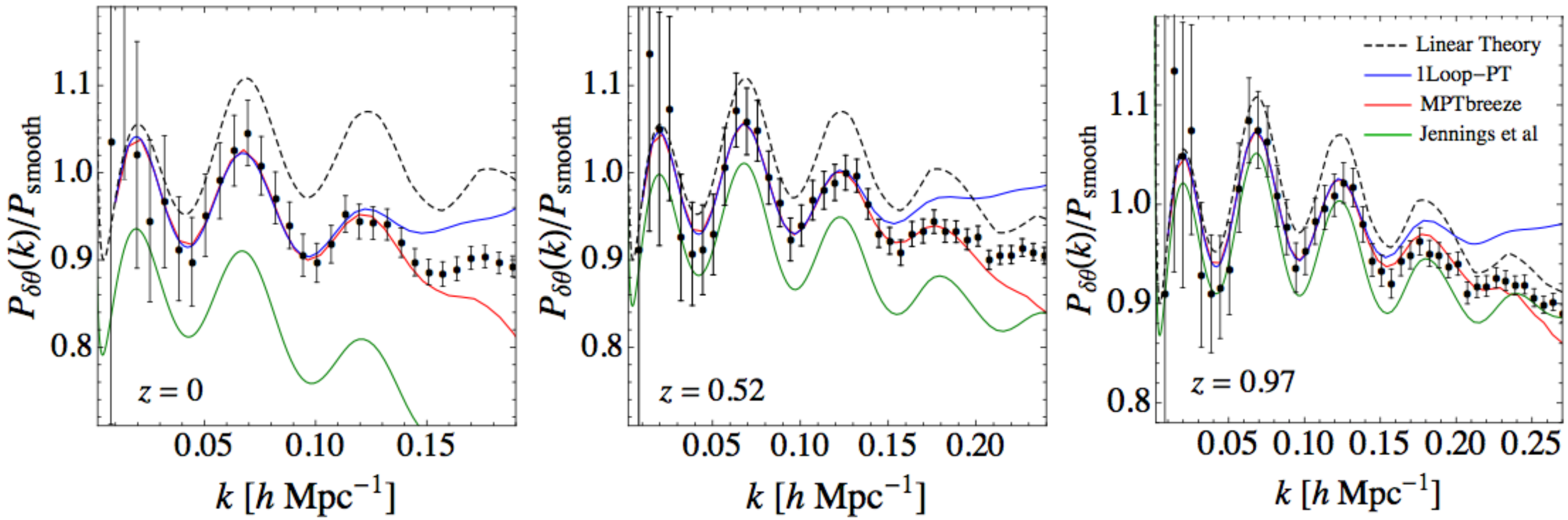
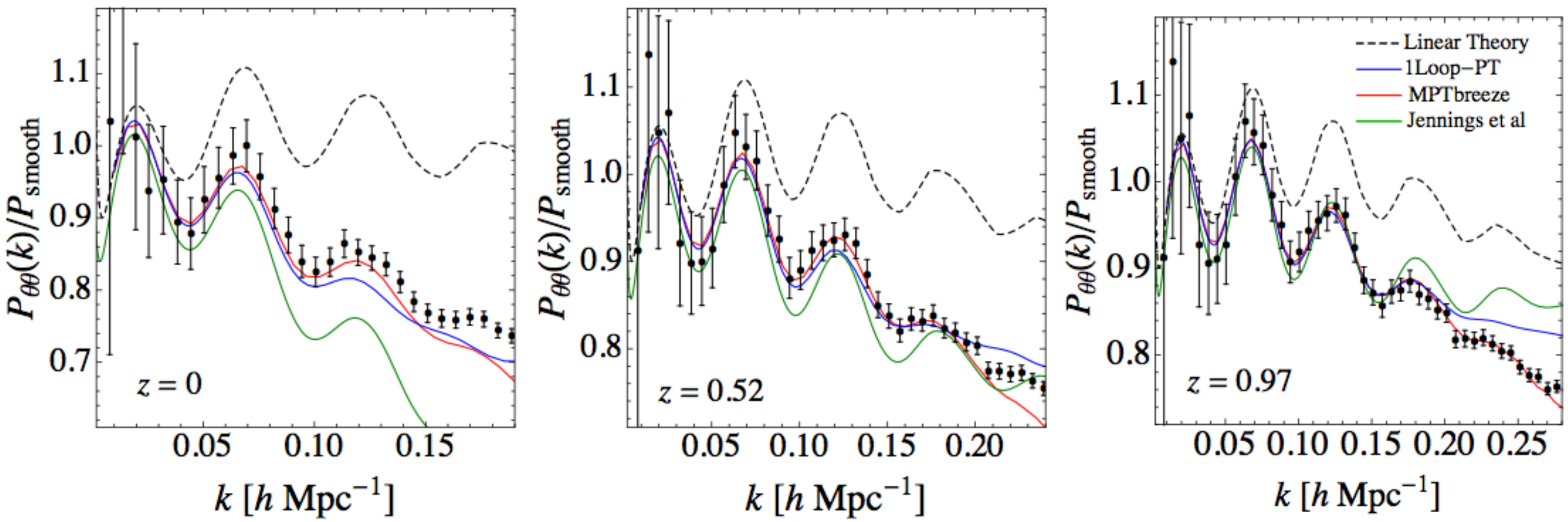


Velocity-Velocity power spectrum



A bit of Red-shift Space Distortions

$$\boldsymbol{s} = \boldsymbol{r} + \frac{v_z(\boldsymbol{r})}{a\,H(z)}\,\widehat{\boldsymbol{z}},$$

$$\{1+\delta^{(\mathrm{S})}(\boldsymbol{s})\}d^3\boldsymbol{s} = \{1+\delta(\boldsymbol{r})\}d^3\boldsymbol{r}$$

$$P^{(\mathrm{S})}(\boldsymbol{k}) = \int d^3\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\big\langle e^{-ik\mu\, f\Delta u_z} \times \{\delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r})\}\{\delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}')\}\big\rangle,$$

Ensemble average mixing density, velocity divergence and velocity difference along l.o.s

Different ways of braking down this average (or keeping perturbative orders) lead to different models of red-shift space distortions.

- ✓ Linear order and no velocity dispersion leads to Kaiser : $(1+f\mu^2)^2 P_{\delta\delta}(k)$
- ✓ Scoccimarro (2004) : $P_s(\mathbf{k}) = \left[P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)\right] \times \exp(-f^2 k_z^2 \sigma_v^2)$
- ✓ Taruya et al (2010) : $$P^{(\mathrm{S})}(k,\mu) = D_{\mathrm{FoG}}[k\mu\, f\,\sigma_{\mathrm{v}}]\Big\{P_{\delta\delta}(k) + 2\,f\,\mu^2\,P_{\delta\theta}(k) + f^2\,\mu^4\,P_{\theta\theta}(k) + A(k,\mu) + B(k,\mu)\Big\}.$$

A bit of RSD

$$A(k,\mu) = (k\mu\, f) \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, \frac{p_z}{p^2} \times \{B_\sigma(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}, -\boldsymbol{k}) - B_\sigma(\boldsymbol{p}, \boldsymbol{k}, -\boldsymbol{k}-\boldsymbol{p})\}\,,$$

$$\left\langle \theta(\boldsymbol{k}_1) \left\{ \delta(\boldsymbol{k}_2) + f\, \frac{k_{2z}^2}{k_2^2} \theta(\boldsymbol{k}_2) \right\} \left\{ \delta(\boldsymbol{k}_3) + f\, \frac{k_{3z}^2}{k_3^2} \theta(\boldsymbol{k}_3) \right\} \right\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\, B_\sigma(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3). \quad (21)$$

$$B(k,\mu) = (k\mu\, f)^2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} F(\boldsymbol{p}) F(\boldsymbol{k}-\boldsymbol{p})\,;$$

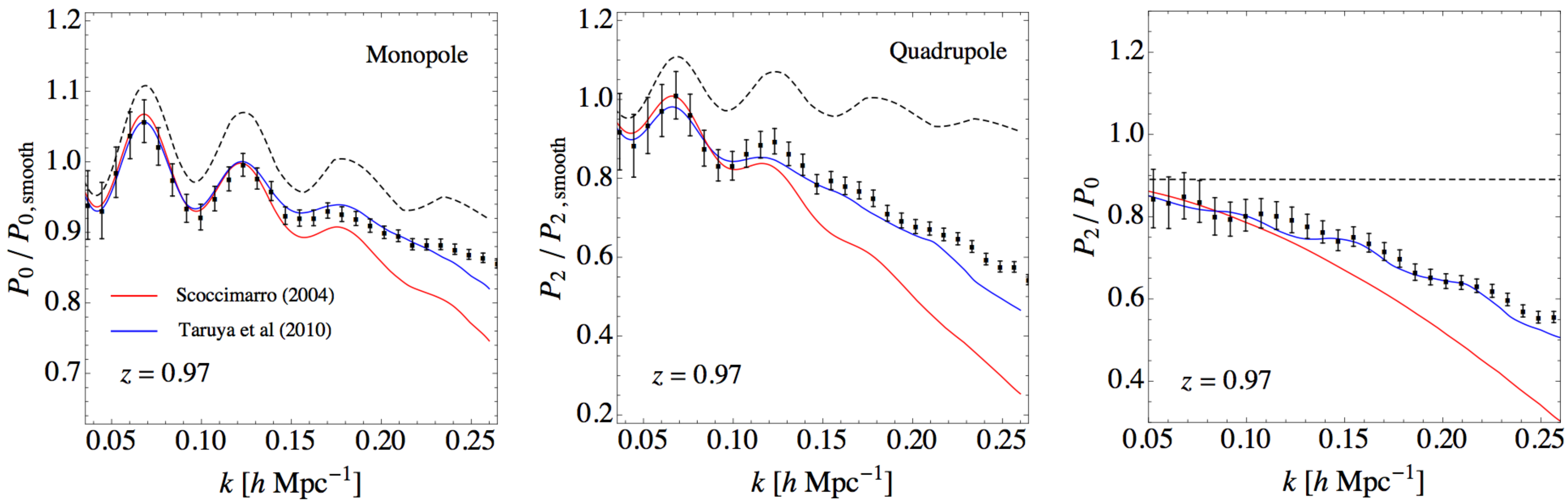
$$F(\boldsymbol{p}) = \frac{p_z}{p^2} \left\{ P_{\delta\theta}(p) + f\, \frac{p_z^2}{p^2}\, P_{\theta\theta}(p) \right\},$$

$A(k,\mu)$ has contributions in μ^2 (as $P_{\delta\theta}$), μ^4 (as $P_{\theta\theta}$) and μ^6

$B(k,\mu)$ has contributions in μ^2, μ^4, μ^6 and μ^8

A bit of RSD

Using these simple models and fitting for velocity dispersion we get



- The regime of validity is narrower than velocity - density correlations

- The need for the extra A and B terms is clear, but might not be enough for full anisotropic shape

Conclusions

- ✓ The multi-point expansion is accurate at < 2% on BAO scales for both Densities and velocities.
- ✓ Tested with dedicated simulations of different cosmological models and robust measurements of velocity field (Delaunay). Good for RSD.
- ✓ It can be executed in 3-10 seconds, fast enough for typical parameter sampling requirements.

Code publicly available at http://maia.ice.cat/crocce/mptbreeze

- ✓ Description of smaller scales can be done by coupling multi-point expansion with Halofit-like expressions

Halo Biasing

Using MICE Grand Challenge simulation with 4000^3 particles