Quantum Zeno Effect and Dynamics

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What is QFT?, Benasque, 16 September 2011

Quantum Zeno effect

quantum system	ψ
Hamiltonian	H
Schrodinger equation	$\psi_t = e^{-iHt}\psi_0$

survival probability

$$p(t) = |\langle\psi_t|\psi_0\rangle|^2 = 1 - t^2/\tau_Z^2$$

$$\tau_Z^{-2} \equiv \langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2$$

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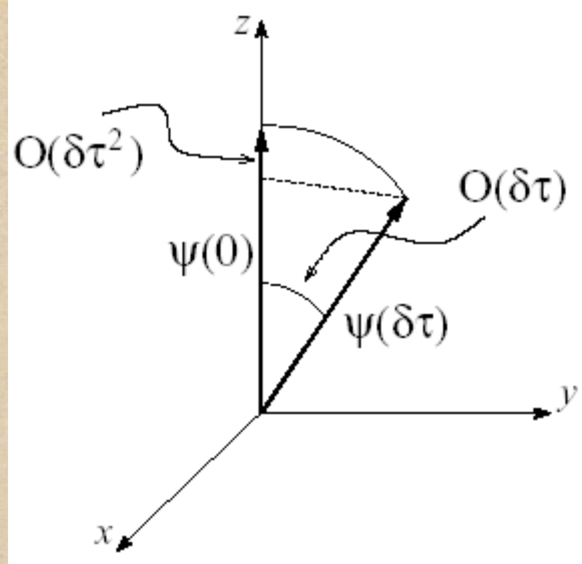

$$[p\,(t/N)]^N \simeq \left(1 - \frac{t^2}{\tau_Z^2 N^2}\right)^N \xrightarrow{N\to\infty} 1$$

Quantum Zeno effect

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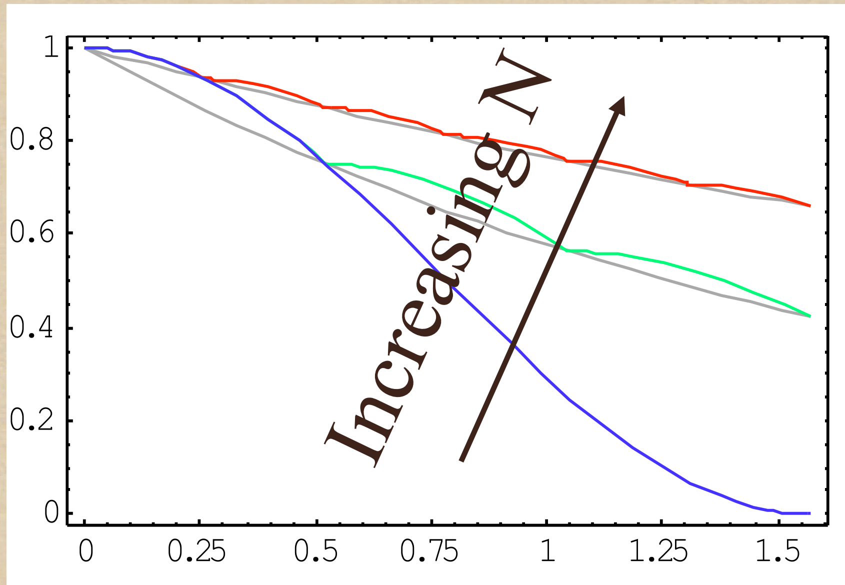


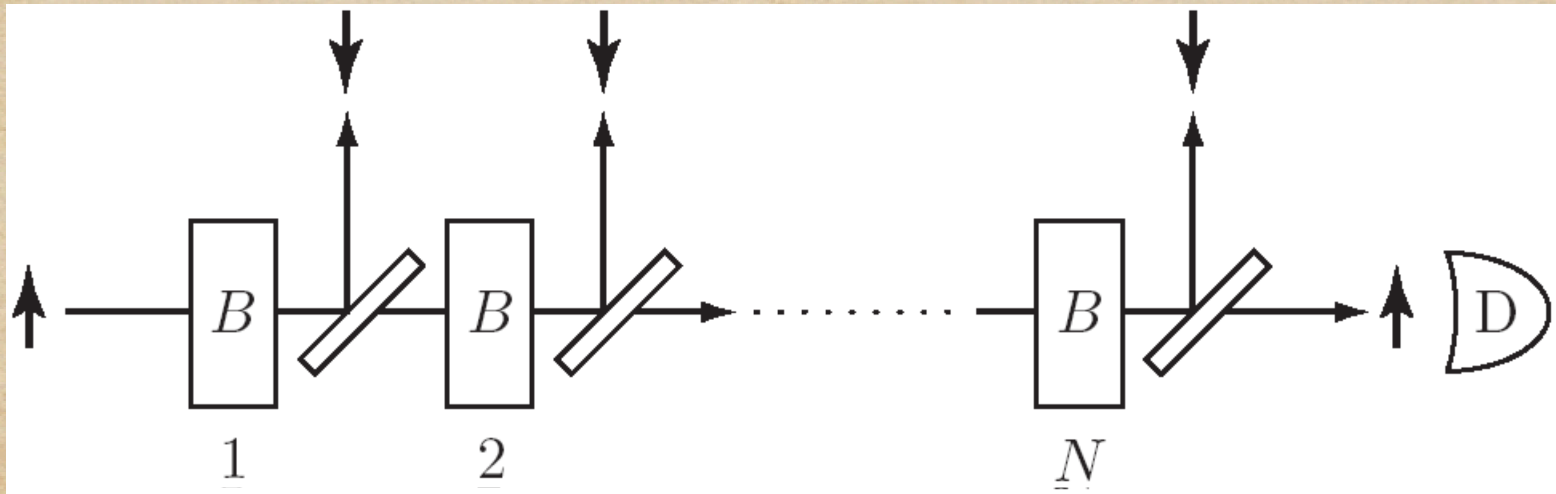
$[p\,(t/6)]^6$

$[p\,(t/3)]^3$

$[p\,(t/1)]^1$

many experiments on many physical systems

one expt: neutron spin



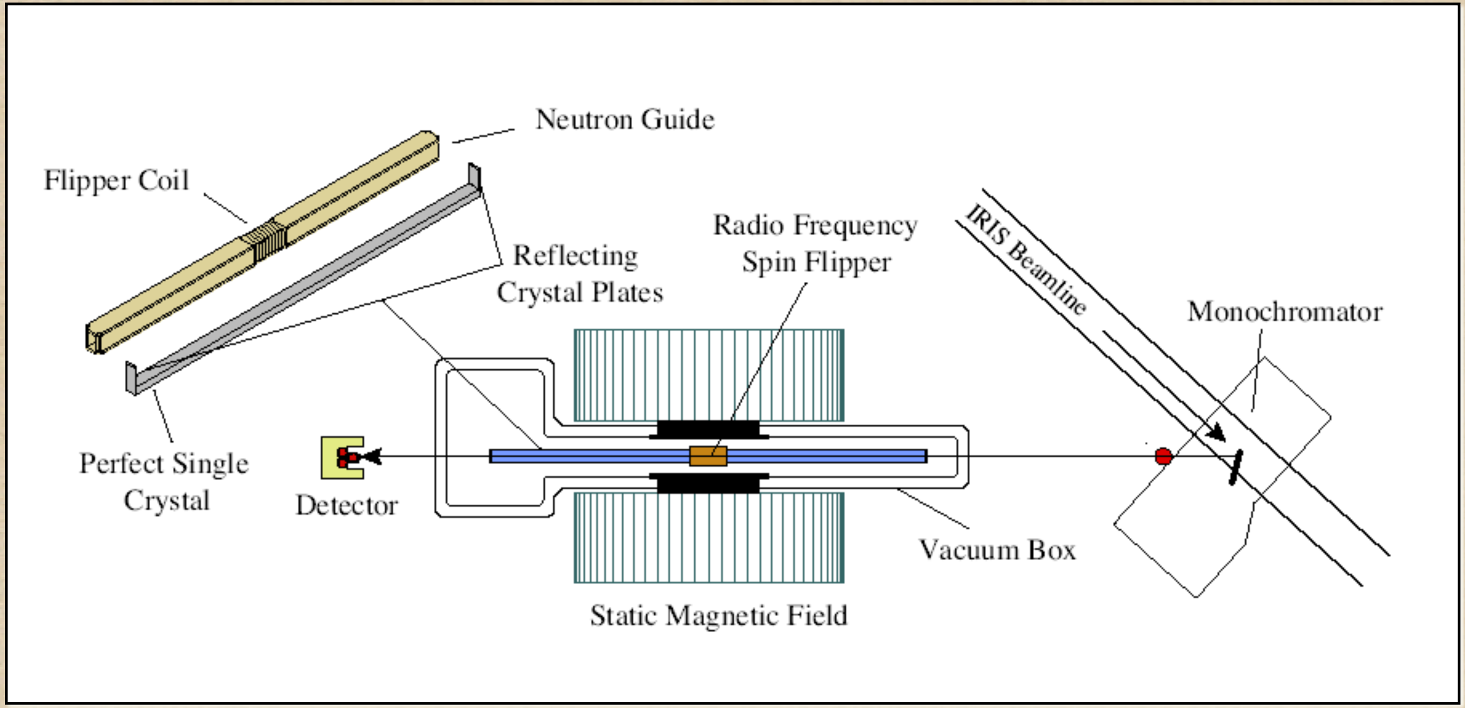
$$p(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \cos\left(\frac{t}{\tau_Z}\right)^2$$

$$p^{(N)}(t) = p\left(\frac{t}{N}\right)^N = \cos\left(\frac{t}{N\tau_Z}\right)^{2N}$$

$$p^{(N)}(t) \xrightarrow{N\to\infty} 1$$

P., Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

VESTA II @ ISIS



Jericha, Schwab, Jakel, Carlile, Rauch, Physica B 283, 414 (2000)

Rauch, Physica B 297, 299 (2001)

History

von Neumann,1932

Beskow and Nilsson,1967

Khalfin 1968

Friedman 1972

Misra and Sudarshan, 1977

Experiments

(Cook 1988)

Itano, Heinzen, Bollinger, and Wineland 1990

Nagels, Hermans, and Chapovsky 1997

Wunderlich, Balzer, and Toschek, 2001

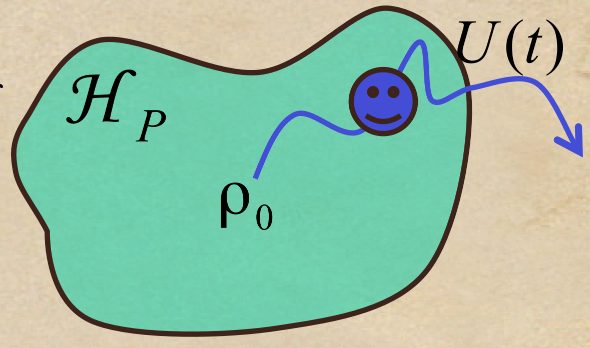
Fischer, Gutierrez-Medina, Raizen, 2001

Streed, Mun, Boyd, Campbell, Medley, Ketterle, Pritchard, 2006

Bernu, Sayrin, Kuhr, Dotsenko, Brune, Raimond, Haroche 2008

Theory and interesting Mathematics

Quantum Zeno effect



Consider a quantum system Q, whose states are in $\mathcal{H}$

Time evolution $U(t) = \exp(-iHt)$.

P a projection operator $[P, H] \neq 0$, and $P\mathcal{H} = \mathcal{H}_P$ its eigenspace.

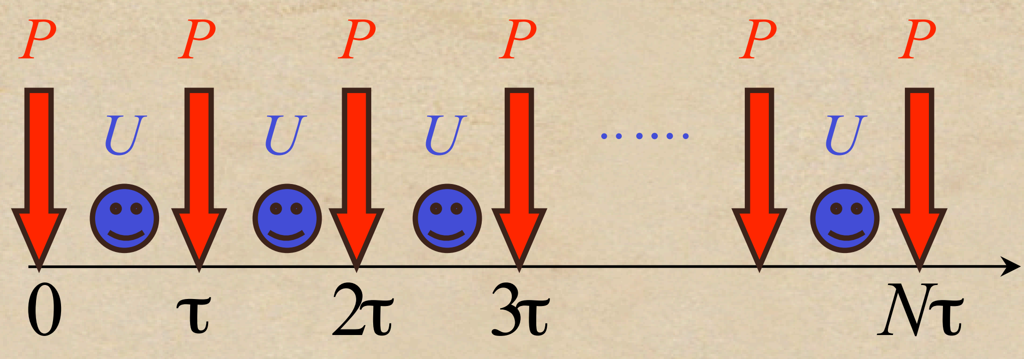
Initial density matrix ρ_0 in $\mathcal{H}_P$: $\quad \rho_0 = P\rho_0 P, \qquad \mathrm{Tr}[\rho_0 P] = 1$

Perform a measurement at time τ, in order to check whether Q has "survived".

$$\rho_0 \to PU(\tau)\rho_0 U(\tau)^\dagger P,$$

with probability

$$p(\tau) = \mathrm{Tr}[U(\tau)\rho_0 U(\tau)^\dagger P] = \mathrm{Tr}[V(\tau)\rho_0 V(\tau)^\dagger], \qquad V(\tau) = PU(\tau)P$$



$$\rho^{(N)}(t) = V_N(t)\rho_0 V_N(t)^\dagger, \qquad V_N(t) = [PU(t/N)P]^N$$

The probability to find the system in $\mathcal{H}_P$ reads

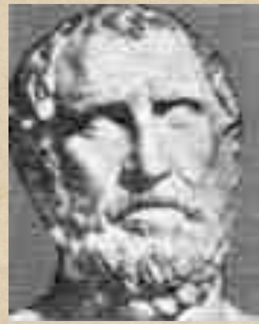
$$p^{(N)}(t) = \mathrm{Tr}[V_N(t)\rho_0 V_N(t)^\dagger] \stackrel{N\to\infty}{\longrightarrow} \mathrm{Tr}[P\rho_0] = 1$$

Frequent observation freeze system in its initial state

Formulation due to Misra and Sudarshan 1977

Zeno of Elea

Zeno was an Eleatic philosopher, a native of Elea in Italy, son of Teleutagoras, and the favorite disciple of Parmenides. He was born about 488 BC, and at the age of forty accompanied Parmenides to Athens



The flying arrow is at rest.

At any given moment it is in a space equal to its own length, and therefore is at rest at that moment. So, it is at rest at all moments. The sum of an infinite number of these positions of rest is not a motion.

Magna Graecia
Neapolis
Taras
Posidonia
(Paestum)
Elea
Metapontion
Sybaris
Croton
Zankle
Sicily
Selinus
Catana
Acragas
Syracuse
Carthage
Gela
San Rufo
NAPOLI
AVELLINO
Salerno
BASILICATA
San Rufo
Zeno
Beppe

back to physics: system need not be 1D...

example:

measure → $|1\rangle$, $|0\rangle = |\mathrm{in}\rangle$

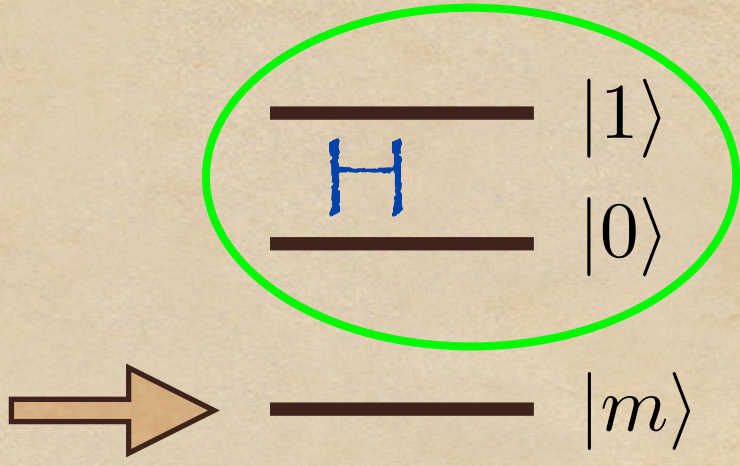
system remains in initial state

more interesting example:

$|1\rangle$, $|0\rangle$

measure → $|m\rangle$

system remains in subspace defined by (negative result, nonselective) measurement



system remains in subspace defined by (negative result, nonselective) measurement

what if there is a Hamiltonian?

Quantum Zeno DYNAMICS

Quantum Zeno Dynamics (the "simple" case :-)

Multidimensional $\mathcal{H}_P = P\mathcal{H}$, $\mathrm{Tr}\, P > 1$,

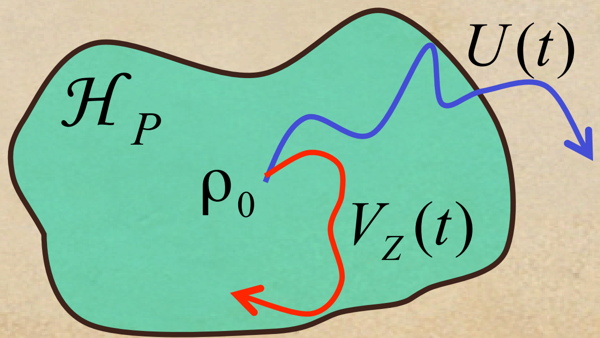
bounded Hamiltonian, $\|H\| < \infty$,

Limiting time evolution

$$V_Z(t) = \lim_{N\to\infty} V_N(t) = \lim_{N\to\infty} [PU(t/N)P]^N = P\exp(-iPHPt)$$

unitary in H_P, i.e. PHP self-adjoint in H_P.

The state of the system Q is not *completely* specified but can *evolve* in a suitable subspace $\mathcal{H}_P$ (instead of evolving "naturally" in the total Hilbert space $\mathcal{H}$).



P. Exner 1985

P. Facchi 2010

Nonselective measurements

Partition $\mathcal{H} = \bigoplus_n \mathcal{H}_{P_n}, \qquad \sum_n P_n = 1$

Free evolution $\hat{U}_t \rho = U(t)\rho U(t)^\dagger, \quad U(t) = \exp(-iHt)$

Nonselective measurement

Schwinger, Proc. Nat. Acad. Sc. **45**, 1552 (1959)

$$\hat{P}_c \rho = \sum_n P_n \rho P_n$$

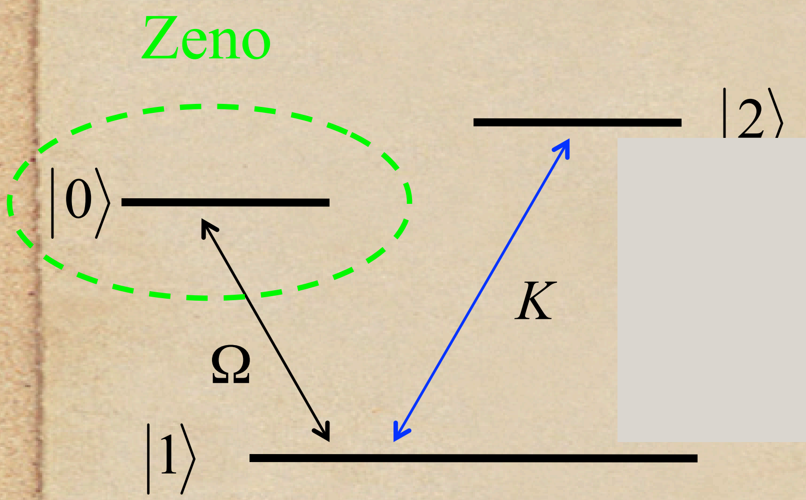
Zeno evolution

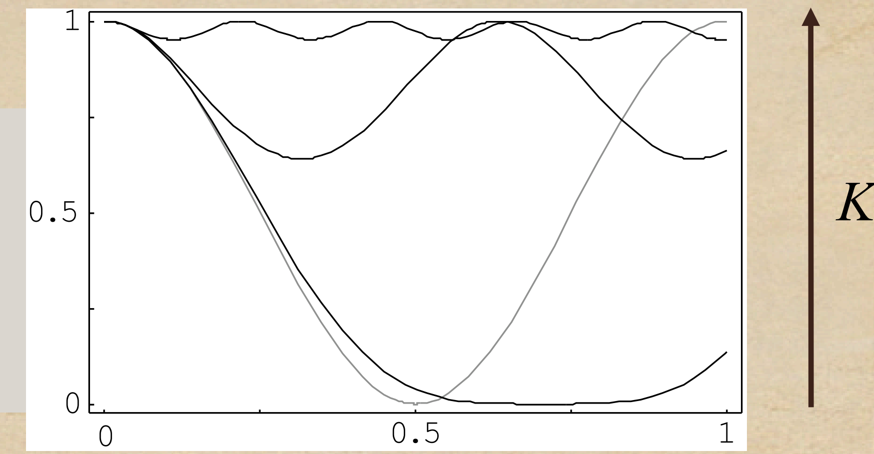
$$\hat{V}_Z(t) = \lim_{N\to\infty} \hat{V}^{(N)}(t) = \lim_{N\to\infty} \left(\hat{P}_c \hat{U}_{t/N}\right)^N$$

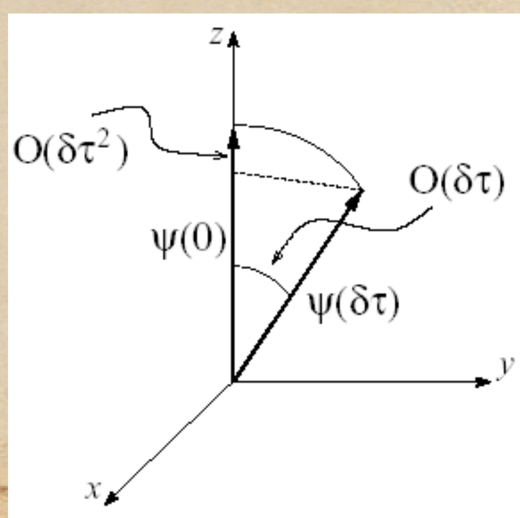
$$\hat{V}_Z(t)\rho = \sum_n V_n(t)\rho V_n(t)^\dagger, \quad V_n(t) = P_n \exp(-iP_n H P_n t)$$

Quantum Zeno subspaces and dynamics:
Facchi and P., Phys. Rev. Lett. **89**, 080401 (2002)

What really provokes Zeno





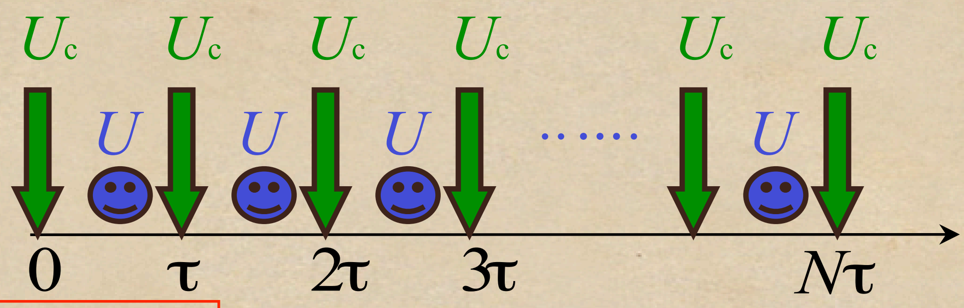


Unitary kicks

Q undergoes N kicks U_c (instantaneous unitary transformations) in a time interval t $(P \to U_c)$

$$V_N(t) = \left[U_c U\left(\frac{t}{N}\right) \right]^N ,$$

$$U(t) = \exp(-iHt)$$



Quantum maps

Berry, Balazs, Tabor, Voros (1979)

Casati, Chirikov, Ford and Izrailev (1979)

Limiting evolution $N \to \infty$

$$V_Z(t) = \lim_{N\to\infty} U_c^{\dagger N} V_N(t) = \exp(-iH_Z t)$$

$$H_Z = \sum_n P_n H P_n, \qquad U_c = \sum_n e^{-i\lambda_n} P_n$$

von Neumann's ergodic theorem

$$\text{s} - \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N} U^N = P_{\text{inv}}, \quad UP_{\text{inv}} = P_{\text{inv}}$$

$$V_Z(t) = \sum_n P_n \exp(-iP_n H P_n t)$$

Dynamical decoupling

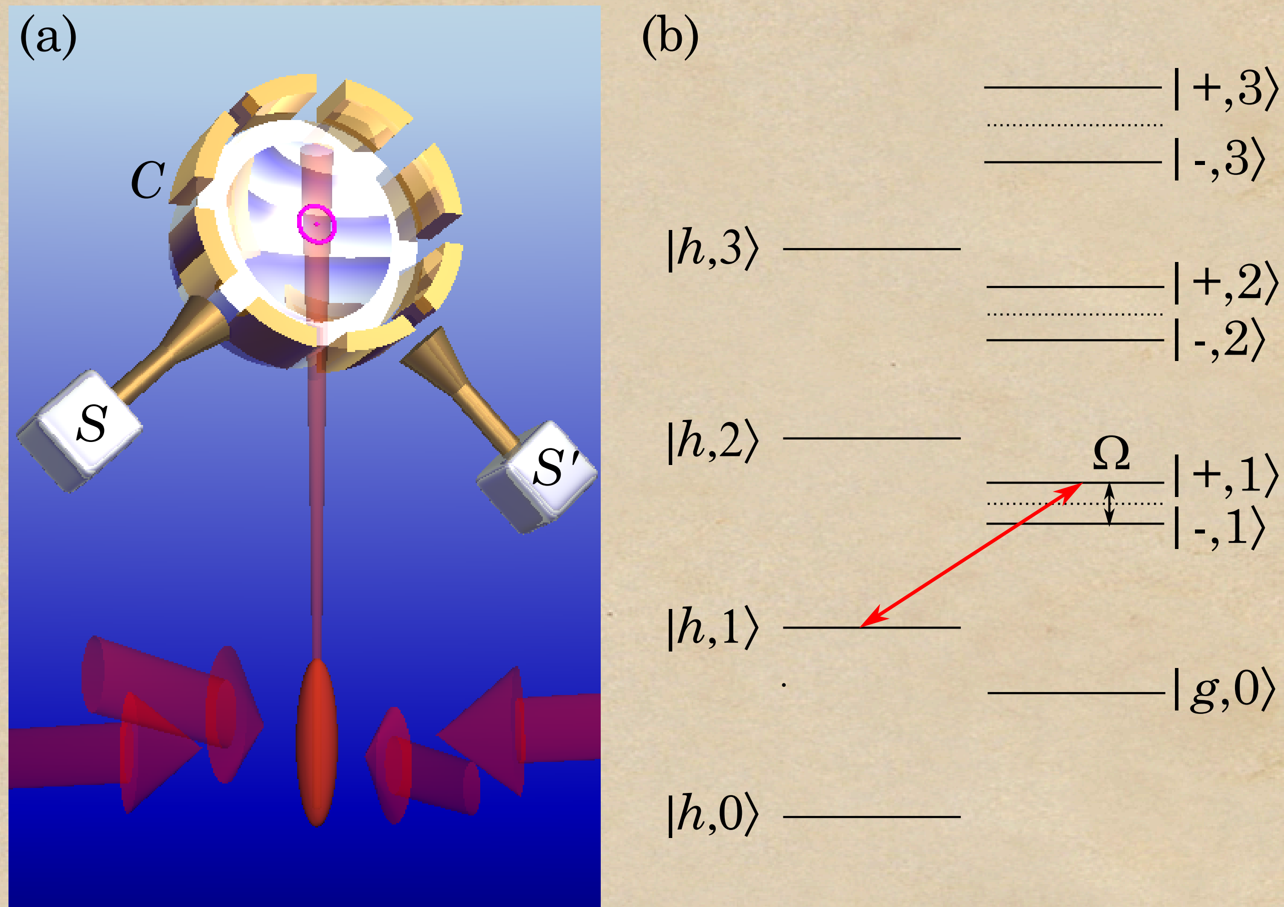
"Bang-bang" control

Viola and Lloyd, Phys. Rev. A **58**, 2733 (1998)

Facchi, Lidar, Pascazio, Phys. Rev. A **69**, 032314 (2004)

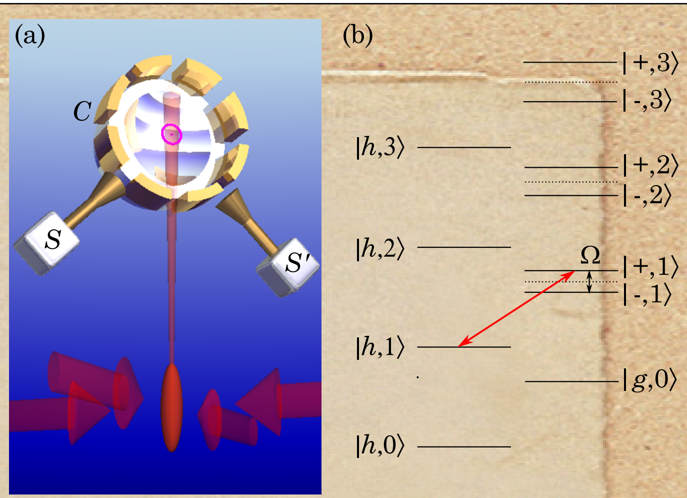
QZD in Cavity QED

atomic fountain



$$|\pm, n\rangle = \frac{1}{\sqrt{2}}(|e, n-1\rangle \pm |g, n\rangle), \quad n \geq 1$$

Hamiltonians



laser $$H = \alpha a^\dagger + \alpha^* a$$

cavity $$V = \frac{\hbar\Omega}{2}(|e\rangle\langle g|a + |g\rangle\langle e|a^\dagger)$$

kick/ measaurement
$$U_s|h,n\rangle = |h,n\rangle$$
$$U_s|+,n\rangle = |+,n\rangle \qquad (n \neq s)$$
$$U_s|-,s\rangle = |-,s\rangle$$

then measure s ⇨ $$P = 1 - |s\rangle\langle s|$$

Zeno Hamiltonian

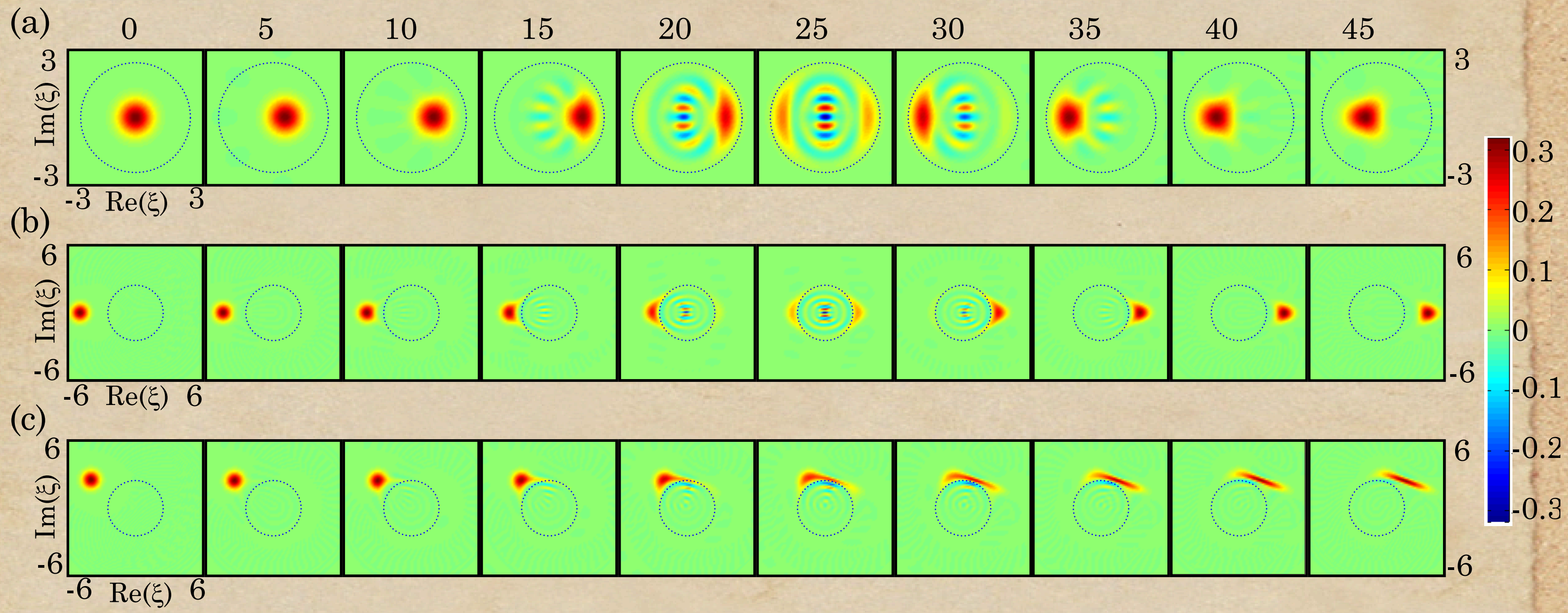
$$H_Z = P_{<s} H P_{<s} + P_{>s} H P_{>s} = H_{<s} + H_{>s}$$

$$H = \begin{pmatrix} 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \cdots \\ \alpha^* & 0 & \sqrt{2}\alpha & 0 & 0 & 0 & 0 & \\ 0 & \sqrt{2}\alpha^* & 0 & \sqrt{3}\alpha & 0 & 0 & 0 & \\ 0 & 0 & \sqrt{3}\alpha^* & 0 & \sqrt{4}\alpha & 0 & 0 & \\ 0 & 0 & 0 & \sqrt{4}\alpha^* & 0 & \sqrt{5}\alpha & 0 & \\ 0 & 0 & 0 & 0 & \sqrt{5}\alpha^* & 0 & \sqrt{6}\alpha & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha^* & 0 & \\ \vdots & & & & & & & \ddots \end{pmatrix}$$

s=4

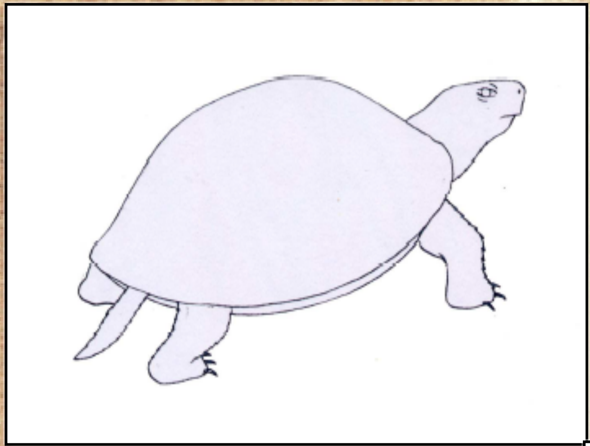
$$\xrightarrow{\text{Zeno}} \begin{pmatrix} 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \cdots \\ \alpha^* & 0 & \sqrt{2}\alpha & 0 & 0 & 0 & 0 & \\ 0 & \sqrt{2}\alpha^* & 0 & \sqrt{3}\alpha & 0 & 0 & 0 & \\ 0 & 0 & \sqrt{3}\alpha^* & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha^* & 0 & \\ \vdots & & & & & & & \ddots \end{pmatrix} = H_Z$$

s=6: phase space view

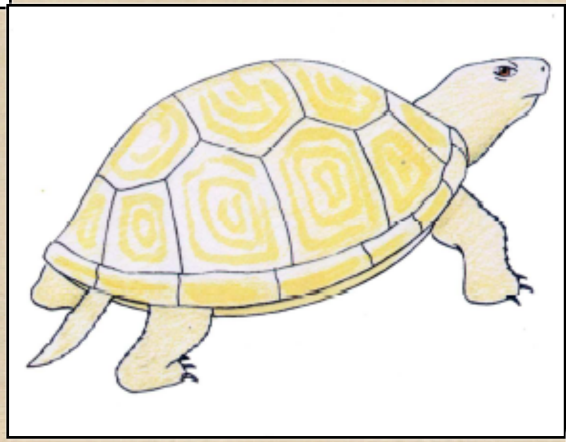


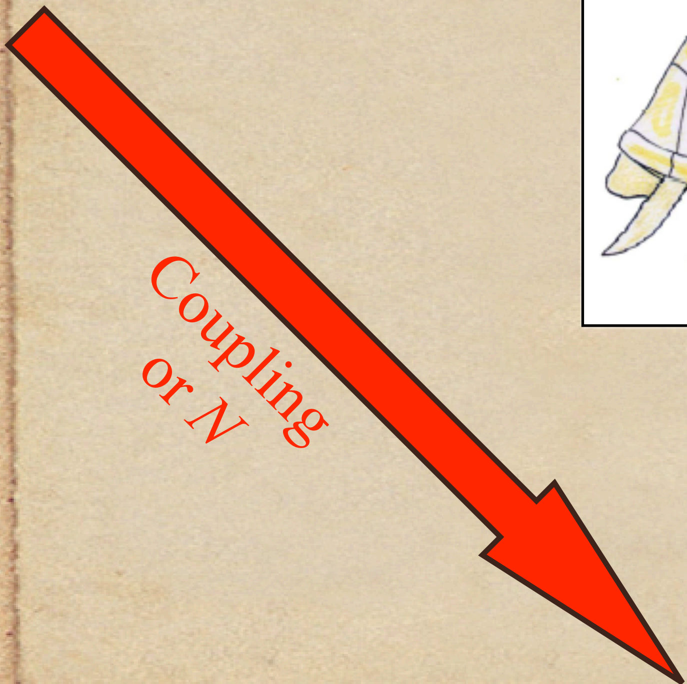
"exclusion circle"

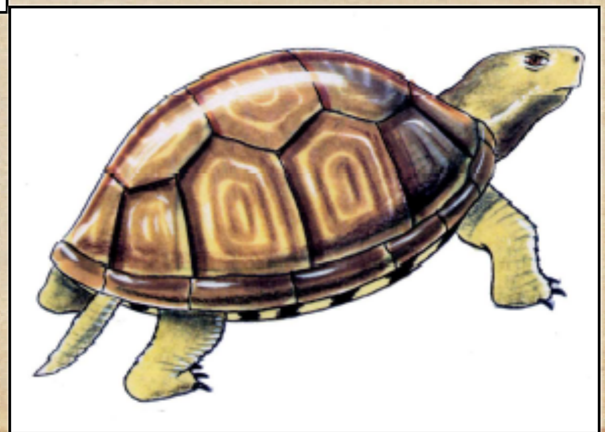
Quantum Zeno subspaces



Dynamical superselection sectors







Free particle in n dimensions

$$H = \frac{p^2}{2M} = -\frac{\hbar^2\Delta}{2M}, \quad U(t) = \exp(-iHt/\hbar) \quad \text{in } L^2(\mathbb{R}^n)$$

$\Omega \subset \mathbb{R}^n$ compact domain, $P = \chi_\Omega(x)$ spatial projection

Zeno dynamics $V_Z(t) = \lim_{N\to\infty}\left[V\left(\frac{t}{N}\right)\right]^N, \quad V(s) = PU(s)P$

How does the particle move inside Ω? Does it leak out?

The weak limit $V_Z(t)$ exists and yields

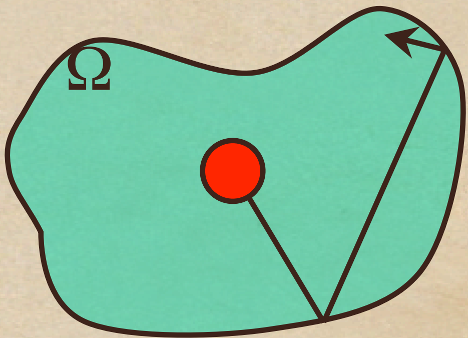
$V_Z(t) = \exp(-iH_\Omega t/\hbar)P,$

a unitary group in $L^2(\Omega)$

with Zeno Hamiltonian

$$H_\Omega = -\frac{\hbar^2\Delta}{2M}, \quad D(H_\Omega) = H^2(\Omega) \cap H^1_0(\Omega)$$

Dirichlet boundary conditions



Free particle in a box with perfectly reflecting hard walls

Facchi, Marmo, Pascazio, Scardicchio, Sudarshan 2003

Exner and Ichinose, 2005

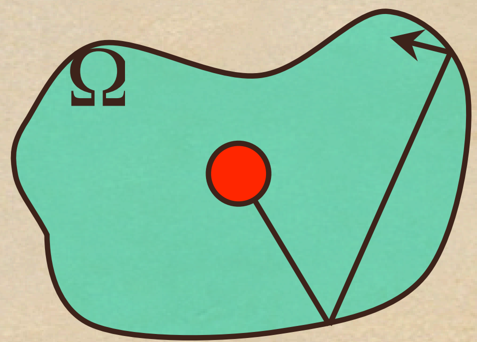
…although there is NO wall!

more on free particle

$$H = \frac{p^2}{2M} = -\frac{\Delta^2}{2M}, \quad U(t) = \exp(-iHt) \quad \text{in } L^2(\mathbf{R}^D)$$

$\Omega \subset \mathbf{R}^D$ a compact domain, $P = \chi_\Omega(x)$ spatial projection

$$H_Z = (H^{1/2}P)^*(H^{1/2}P) = -\frac{\Delta_{\text{Dirichlet}}}{2M}$$



a short note

$$U^{(N)}(t) = \underbrace{\left(e^{-iTt/N}e^{-iVt/N}\right)\left(e^{-iTt/N}e^{-iVt/N}\right)\cdots\left(e^{-iTt/N}e^{-iVt/N}\right)}_{N \text{ times}}$$

$$= \left(e^{-iTt/N}e^{-iVt/N}\right)^N$$

Feynman/Trotter

$$\sim e^{-i(T+V)t} = e^{-iHt} \qquad H = T + V$$

$$V^{(N)}(t) = \underbrace{\left(Pe^{-iHt/N}\right)\left(Pe^{-iHt/N}\right)\cdots\left(Pe^{-iHt/N}\right)}_{N \text{ times}}$$

$$= \left(Pe^{-iHt/N}\right)^N$$

Zeno (Faddeev)

$$\sim e^{-iPHPt}P$$

$$W^{(N)}(t) = \underbrace{\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\cdots\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)}_{N \text{ times}}$$

$$= \left(e^{-iH_1t/N}e^{-iH_2t/N}\right)^N$$

q. chaos/control

$$\sim e^{-i(\overline{H}_1+\overline{H}_2)t} \quad ?$$

message to Manolo

$$W^{(N)}(t) = \underbrace{\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\cdots\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)}_{N \text{ times}}$$

$$= \left(e^{-iH_1t/N}e^{-iH_2t/N}\right)^N$$

$$\sim e^{-i(\overline{H}_1+\overline{H}_2)t} \; ?$$

X !

Hopefully before you next

(Happy) Birthday!