Radiation Damping Effects in High Intensity Laser Fields

QFEXT11, Benasque

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Collaborators

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Outline of Talk

Introduction
- Motivation for studying radiation damping,
- Governing equations.

Radiation damping
- Overview of radiation damping effects,
- Radiation damping induced electron capture,
- Mass shift,
- Plane wave limit.

Nonlinear Compton Scattering
- Mass shift.

Conclusion
Introduction
Laser intensities increasing $\rightarrow$ new physics.

[Adapted from Tajima and Mourou (2002)]

[Diagram showing the progression of laser intensities from 1960 to 2010, with labels for Strong Field QED, Schwinger Limit, Relativistic Optics, Vulcan 10PW, ELI, CPA, and a scale for focused intensity (W/cm²) from $10^{10}$ to $10^{30}$]
Introduction
Laser intensities increasing → new physics.

Schwinger Field
Vacuum pair prod

Quantum Effects
Vacuum birefringence.... Nonlinear Compton

Radiation Reaction
\[ F = F_{\text{Lorentz}} + F_{\text{Reaction}} \]

Lorentz Force
\[ F_{\text{Lorentz}} = e(E + v \times B) \]
Euler-Heisenberg Effective Action

\[ \Gamma = - \int_0^\infty \frac{dT}{T} e^{-m^2 T} \int d^4 x \int Dx \ e^{-S[x]} . \]

Worldline instanton – semiclassical – use classical paths.
Dunne and Schubert (2005)

\[ S[x] = \int_0^T d\tau \left( \frac{1}{4} \dot{x}_\mu^2 + eA_\mu \dot{x}_\mu \right) . \]

This Talk
Classical solutions with radiation reaction

Tom Heinzl, Anton Ilderton, Felix Karbstein
Tree level, Loops
Laser Intensity Parameter – $a_0$.

Laser beam characterised by the ‘dimensionless laser amplitude’

$$a_0 = \frac{eE\lambda_L}{mc^2}.$$ 

- Ratio of the energy gain of the electron moving over a laser wavelength with the electron’s rest mass.
- Classical quantity.

(Lorentz and gauge invariance [Heinzl and Ilderton, 2009].)

With lasers can study phenomenology of high intensity $a_0 > 1$ and low energy $\omega \ll mc^2$ regime.
Classical Radiation Damping

Strong acceleration: electron’s radiation will affect its motion.

- Simulate interaction of electron with realistic pulsed Gaussian beam.
- Assess the importance of radiation damping.
- Look for regimes where radiation damping prominent: test theory.
### Governing Equations

**Lorentz Abraham Dirac**

\[ m\dot{u}^\mu = eF^{\mu\nu}u_\nu - \frac{2}{3}\frac{e^2}{4\pi}(u^\mu\ddot{u}^\nu - u^\nu\ddot{u}^\mu) \]

**Problem**

Runaway solutions: unphysical

Well established solution: approximate \( \ddot{u} \) terms using Lorentz force

Landau Lifshitz equation

\[ \dot{u}^{\mu} = e F^{\mu\nu} u_\nu + \frac{2}{3}\frac{e^2}{4\pi} \left\{ e_m F^{\mu\nu} u_\nu + e_2 m^3 F^{\mu\alpha} F^{\nu\alpha} u_\nu - e_2 m^3 u_\alpha F^{\alpha\nu} F^{\beta\nu} u_\beta u^{\mu} \right\} \]
Governing Equations

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Landau Lifshitz equation

\[ \dot{u}^\mu = \frac{e}{m} F^{\mu \nu} u_\nu + \frac{2}{3} \frac{e^2}{4\pi} \left\{ \frac{e}{m^2} \dot{F}^{\mu \nu} u_\nu + \frac{e^2}{m^3} F_{\mu \alpha} F_{\alpha \nu} u_\nu - \frac{e^2}{m^3} u_\alpha F^{\alpha \nu} F_{\nu \beta} u_\beta u^\mu \right\} \]

Perturbative expansion of LAD

- No runaway solutions.
Parameter Constraints

Two constraints on parameter values:

- Validity of Landau Lifshitz equation: radiation damping term smaller than Lorentz force term

\[ \alpha \omega a_0 \gamma^2 \ll mc^2. \]

- Classical regime: work done by laser field over a Compton wavelength

\[ \chi \equiv \frac{e \hbar \sqrt{(F_{\mu \nu} u_\nu)^2}}{m^2 c^4} \ll 1, \quad \Rightarrow \quad \hbar a_0 \gamma \omega \ll mc. \]

(Quantum effects dominate when \( \chi \sim 1 \).)
$a_0 = 150, \gamma_0 = 100.$

$\gamma_0 = 100.$

$a_0 = 250, \gamma_0 = 150.$

$x (m)$

$z (m)$

With RR

Without RR

With RR

Without RR

Chris Harvey (Umeå University, Sweden)
Electron Beam Size Effects

Radiation damping induced capture stable with respect to size of electron beam. \( a_0 = 250, \gamma_0 = 100. \)
Radiation Damping Effects
C. Harvey and M. Marklund (to appear)

Find that radiation damping causes:

- net energy loss.
- deflection/reflection of the electron.

(Significant change to trajectory and therefore to emission spectra.)

Introduce displacement measure $D$:

- longitudinal displacement of electron (compared to where it would be if no field present).

Fix $a_0$ and consider displacement as a function of $\gamma_0$. 
Regime \(2\gamma_0 > a_0, a_0 \gg 1\): damped electron displaced, undamped electron not displaced

- radiation damping induced electron capture.

Condition \(2\gamma_0 > a_0\): onset of reflection for head on collisions with plane waves.

Intensity Dependent Mass Shift

Electron in a plane wave exhibits a ‘quiver’ motion.

- Typically too small to be resolved by laser field.
- Proper time average: quasi momentum $q$.
- Square $q$ to obtain mass shift

$$m^2 \rightarrow m_*^2 \equiv q^2 = m^2(1 + a_0^2).$$

Condition $2\gamma_0 = a_0$ defines centre-of-mass frame.

Harvey, Heinzl, Ilderton (2009).
Plane Wave Approximation

- Compare Gaussian beam results with plane wave approximation.
- When $2\gamma_0 > a_0$ plane wave gives accurate estimation of net energy change.

In the plane wave approximation strong field QED calculations possible.
- Solution to Dirac equation Volkov (1935)
The mass shift also occurs in the QED calculation:

- Apply kinetic momentum operator $\hat{p} - eA = i\partial - eA$ to Volkov solution,
- Take time average: quasi momentum $q$,
- Effective electron mass: $m^2 \rightarrow m_*^2 \equiv q^2 = m^2(1 + a_0^2)$,

Sengupta (1952), Brown and Kibble (1964)
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Sengupta (1952), Brown and Kibble (1964)

This is exactly the same mass shift as we had in the classical theory!

The condition $2\gamma_0 = a_0$ defines the centre-of-mass frame.
Example: Nonlinear Compton Scattering
C. Harvey, T. Heinzl and A. Ilderton PRA 79 063407 (2009)

Most important process that can be observed with current intensities.

- Electrons in collision with high intensity laser,
- Electron absorbs $n$ laser photons $\gamma_L$ of momentum $k$,
- Emits one photon $\gamma$ of momentum $k'$,

$$q_\mu + nk_\mu \rightarrow q'_\mu + k'_\mu,$$

centre-of-mass
$$q + nk = 0 \quad \Longrightarrow \quad a_0 = 2\gamma_0.$$
Spectral Flow \((\gamma_0 = 100 \implies a_0, \text{CoM} = 200)\)

Emission harmonics collapse to line spectra in the centre-of-mass frame.
Summary

New generation of high intensity lasers: new physics.

Classical domain: RR effects will become important:
- Radiation reaction induced electron capture,
- Stable with respect to electron beam width,
- Occurs when \( 2\gamma_0 > a_0 \),
- Plane wave approximation good,
- \( \implies \) mass shift important.

Beyond classical: nonlinear Compton scattering
- Mass shift important.
Summary/Outlook

Current facilities
Classical (LF) approximation good

New facilities
Classical radiation reaction and QED effects

Questions to address:
- When does the classical theory break down?
- When do quantum effects become important?
- Better understanding of the mass shift.
Appendix: The Mass Shift

\[ \Delta m^2 / m^2 \]

Finite pulse duration effects.

Mass shift \( \Delta m = m_*^2 - m^2 \).

Number of cycles \( N \).

\[ a_0 = 1 \]