QUANTUM FIELD THEORY in GRAPHENE
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The Dirac model for graphene was proposed in 1984 by G. Semenoff and by DiVincenzo and Mele — 20 years before the experimental discovery of graphene by Geim, Novoselov and others! The Dirac model can be derived from the Tight binding model. Graphene is a one-atom-thick (planar) system with a hexagonal lattice:
Tight binding model

\[ H = -t \sum_{\alpha \in A} \sum_{i=1}^{3} (a^\dagger(r_{\alpha}) b(r_{\alpha} + u_i) + b^\dagger(r_{\alpha} + u_i) a(r_{\alpha})) , \]
Eigenvectors (numbered by the momentum $k$)

$$|\psi\rangle = C_A \sum_{\alpha \in A} e^{ikr_\alpha} a^\dagger(r_\alpha) |0\rangle + C_B \sum_{\alpha \in B} e^{ikr_\alpha} b^\dagger(r_\alpha) |0\rangle.$$ 

Eigenvalues:

$$E = \pm t \sqrt{1 + 4 \cos(\frac{\sqrt{3}}{2} k_y d)[\cos(\frac{3}{2} k_x d) + \cos(\frac{\sqrt{3}}{2} k_y d)]}.$$ 

(with $d$ being the lattice spacing).

Spectrum: two surfaces, $E > 0$ and $E < 0$, which touch each other at 6 Fermi points where $E = 0$. Among these 6 points only two are inequivalent:

$$K_\pm = (0, \pm 4\pi/(3\sqrt{3}d))$$
Next: take each one of these Fermi points, expand in momenta in the limit $d \to 0$:

$$H_{\pm} = \frac{3}{2} td \begin{pmatrix} 0 & iq_x \pm q_y \\ -iq_x \pm q_y & 0 \end{pmatrix} = v_F (-q_x \sigma_2 \pm q_y \sigma_1),$$

where $v_F \approx 1/300$ is the Fermi velocity. Summing up two Fermi point contributions:

$$H = -iv_F (\gamma^x \partial_x + \gamma^y \partial_y), \quad \gamma^x = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \quad \gamma^y = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}.$$ 

This is the Dirac Hamiltonian. Taking into account spin variables makes the spinors 8-component.
Generalizations:
– Add an electromagnetic field by $\partial \rightarrow \partial + ieA$, that can be (i) an external electromagnetic radiation, (b) an external magnetic field, (ii) a fluctuating electromagnetic field.
– Add a temperature.
– Add a chemical potential and a mass.
– Impurities.

The Dirac model is expected to be valid up to the energies $\sim 2\text{eV}$. “Characteristic” energies of (most) current experiments are of order of fractions of $\text{eV}$ (or less).
QFT with planar fermions:
in 1980’s: Appelquist, Chodos, Semenoff, Niemi, Reddlich, Jackiw, Deser, ..... 
in XXI Century: Gusynin, Sharapov, Miransky, Gorbar, Shovkovy, Pyatkovskiy, Khveshchenko,.....  
The most relevant quantity one can calculate here by the QFT methods is the polarization tensor $\Pi$ defined through the effective action for planar fermions in the presence of an external magnetic field:
\[ S_{\text{eff}}(A) = A \Rightarrow A \Rightarrow A \]

\[ = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3}A_j(p)\Pi^{jl}(p)A_l(p). \]

Example: two-component massive fermions.

\[ \Pi^{mn} = \frac{\alpha}{v_F^2} \eta^m_j \left[ \psi(\tilde{p}) \left( g^{jl} - \frac{\tilde{p}^j \tilde{p}^l}{\tilde{p}^2} \right) + i\phi(\tilde{p})\epsilon^{jkl}\tilde{p}_k \right] \eta^n_l \]

where \( \eta = \text{diag}(1, v_F, v_F) \), \( \tilde{p}^m \equiv \eta^m_n p^n \).
Suppose, the graphene sample is flat, occupying the plane $x^3 = a$. The effective equations of motion for the electromagnetic field read

$$\partial_\mu F^{\mu\nu} + \delta(x^3 - a)\Pi^{\nu\rho}A_\rho = 0,$$

which is equivalent to a free propagation of light outside the surface and the matching conditions on the surface

$$A_\mu |_{x^3 = a+0} = A_\mu |_{x^3 = a-0},$$

$$\left(\partial_3 A_\mu\right) |_{x^3 = a+0} - \left(\partial_3 A_\mu\right) |_{x^3 = a-0} = \Pi^\nu_\mu A_\nu |_{x^3 = a}.$$

(Here $\Pi^{3\mu} = 0$)
External conditions: a strong magnetic field (1 – 10 Tesla) perpendicular to the surface of graphene; varying chemical potential $\mu$ controlled by the gate potential; zero temperature.

Quantity of interest: zero-frequency off-diagonal real part of the polarization tensor (=dc Hall conductivity).

Calculations: Beneventano, Santangelo, ....

Big success of the Dirac model: prediction/explanation of the anomalous Hall conductivity in graphene:

$$\sigma_{xy} \sim (n + \frac{1}{2})$$

(that is observed on experiment).
Still to be done:
– role of the phase of quantum determinant;
– more sophisticated external conditions.
Results: Optical absorption

**Setup:** absorption of light by suspended graphene, no magnetic field, arbitrary temperature, arbitrary, but small mass in and $\mu$.  
**Quantity of interest:** imaginary part of the diagonal polarization tensor at non-zero frequencies.  
**Theory:** universal absorption rate of about 2% – Enormous!  
Setup: Polarization rotation of EM radiation passing through graphene in a strong external magnetic field perpendicular to the surface of graphene.

On the Dirac model side: polarization tensor for non-zero frequencies, external magnetic field, and impurities (!). Fortunately, at zero temperature.

General theoretical discussion: Volkov & Mikhailov (1985); Fialkovsky & D.V. (2009) ...

Experiment: Kuzmenko (2010): Giant Faraday rotation in graphene. (up to 0,1 rad!).

IF & DV (2011): Dirac model is in an agreement with the experiment. It also predicts other peaks at different frequencies. The Faraday effect instrumental for measuring parameters of the Dirac model.
Setup: Graphene layer parallel to another graphene/metal/dielectric. No magnetic field, but other parameters are variable.

Dirac model – gives the polarization tensor which defines reflection coefficients to be substituted in the Lifshitz formula. Most spectacular prediction: temperature enhancement of the Casimir interaction (Fialkovsky, Marachevsky and DV (2011); in agreement with Gomez-Santos (2009)).

Experiment: NO EXPERIMENT.
Conclusions

The Dirac model of graphene
- has solid theoretical grounds
- confirmed by experiments whenever tested
- deserves more attention from both theoretical and experimental sides