The dynamical Casimir effect (DCE) is a quantum vacuum effect which consists, basically, of two closely related phenomena, namely, particle creation due to moving mirrors and radiation reaction forces acting on moving boundaries. Analogously to what happens in ordinary quantum mechanics, where a system initially in its fundamental state can jump into an excited state due to the interaction with an external time-dependent potential, a quantized field can also leave its vacuum state and jump into an excited state due to the interaction with an external time-dependent potential.

In the DCE, moving boundaries can be considered as external time-dependent potentials (for example, a moving boundary described by a electric permittivity and a magnetic permeability changing in time). For this reason, the interaction between quantized fields and moving mirrors induces the field to go out of its vacuum state. In other words, moving boundaries are responsible by particle creation.

In the theoretical aspect of the DCE, Moore, DeWitt and Fulling-Davies [1] are considered the pioneers to discuss the DCE in the context of a real massless scalar field in two-dimensional space-time. Some years later, Ford and Vilenkin [2] proposed a perturbative approach to DCE. One of the advantages of the Ford-Vilenkin’s approach is the possibility of generation of radiation by moving mirrors in more realistic situations, such as in 3+1 dimensions. 

Exploring the method presented by Ford and Vilenkin, we found a series of papers devoted to the DCE from different boundary conditions (BC) and initial quantum states (See Refs. [3, 4]) to a generalization for the electromagnetic field [5]. In connection with the last ideas, the problem of Casimir forces and particle creation due to a moving mirror with Robin BC was discussed by Mintz et al [4].

For a scalar field, the Robin BC is defined as:
\[ \phi - \gamma \frac{\partial \phi}{\partial n} = 0, \]
where \( \gamma \) is the Robin parameter. The Robin BC has interesting properties (See, for instance, [4] for references therein). The parameter \( \gamma \) interpolates continuously Dirichlet (\( \gamma = 0 \)) and Neumann BC (\( \gamma \to \infty \)). Robin BC can simulate the plasma model in real media for low frequencies. For \( \omega < \omega_0 \), the parameter \( \gamma \) plays the role of the plasma wavelength which is directly related to the penetration depth of the field.

In the present work, we consider a real massless scalar field in 3+1 dimensions satisfying a time-independent Robin boundary condition at a moving mirror:
\[ \partial_t \phi + \gamma \phi = 0, \]
where \( \gamma \) is the Robin parameter and \( \gamma = \gamma_0 \) or \( \gamma = \gamma_1 \).

The angular and spectral distributions for the created particles are computed, generalizing previous results obtained by Mintz et al [4]. We show that the suppression in the total number of created particles is still present in 3+1 dimensions for particular values of the Robin parameter and of the mechanical frequency.

The Bogoliubov transformations

Considering the Ford and Vilenkin approach [2], the scalar field can be written as:
\[ \phi(t, \vec{r}) = \phi_0(t, \vec{r}) + \phi(t, \vec{r}). \]

The field perturbation obeys the klein-Gordon equation \[ \partial_t^2 \phi(t, \vec{r}) - \gamma^2 \phi(t, \vec{r}) = 0 \]
with the following BC:
\[ \partial_t \phi(t, \vec{r}) \big|_{t = 0} = \partial_t \phi(t, \vec{r}) \big|_{t = 0} = 0. \]

It is convenient to express the field in the Fourier domain, such that \( \Phi(\omega, \vec{K}, z) = \Phi_0(\omega, \vec{K}, z) + \Phi(\omega, \vec{K}, z) \) represents the Fourier transformation of (3). We can show that:
\[ \Phi_0(\omega, \vec{K}, z) = \left( \frac{1}{1 + i \gamma K_z^2} \right) \sin(k_z z) + i \gamma k_z \cos(k_z z). \]

In eq (11), after an integration over solid angle, we obtain
\[ \frac{dN}{d\omega} = \frac{\epsilon(\omega - \omega_0) \tan^2(\gamma_0 \omega - 2\omega_0 \tan^{-1}(\gamma_0 \omega - \omega))}{\gamma_0(\omega - 2\omega_0)} \]
as being the particle distribution per unit frequency. In the same way, after an integration over frequency, we can show that:
\[ \frac{dN}{d\omega} = \frac{\epsilon^2 \omega^2}{2 \pi} \left( 1 + \frac{\alpha}{\omega_0} \right) \left( \frac{1}{1 + (\omega_0 / \omega)^2} \right) \left[ 1 - \frac{\alpha}{\omega_0} \right] \]
where
\[ \epsilon = \epsilon(\omega - \omega_0) \tan^2(\gamma_0 \omega - 2\omega_0 \tan^{-1}(\gamma_0 \omega - \omega)). \]

References