Casimir force on amplifying bodies

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Outline

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Forces on ground-state systems (atoms, bodies):

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- Creation of repulsive forces ? ⇒ overcome stiction, guidance of atomic beams, trapping mechanisms

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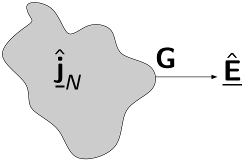
Quantization in linear absorbing media

- General

$$\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) = i\omega\mu_0 \int \mathrm{d}^3 r' \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_N(\mathbf{r}',\omega)$$



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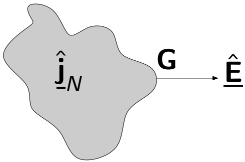

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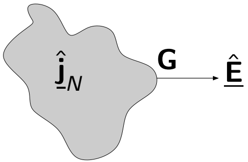

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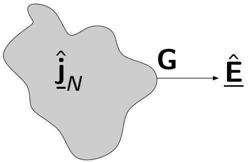

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- Hamiltonian

$$\hat{H} = \int \mathrm{d}^3 r \int_0^\infty \mathrm{d}\omega\hbar\omega\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega) \cdot \hat{\mathbf{f}}(\mathbf{r},\omega)$$

What is meant by an amplifying body?

- Amplification in a limited space and frequency regime with

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- Medium-assisted electromagnetic field is pumped in an excited state $\Rightarrow$ quasi-stationary regime

$$\hat{\mathbf{f}}(\mathbf{r},\omega)|\{0\}\rangle = \mathbf{0} \quad \forall \mathbf{r},\omega$$

Quantization in linear amplifying media

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$$\hat{\underline{\mathbf{j}}}_N(\mathbf{r},\omega) = \omega\sqrt{\hbar\varepsilon_0\pi^{-1}|\mathrm{Im}\,\varepsilon(\mathbf{r},\omega)|}$$
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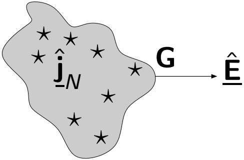

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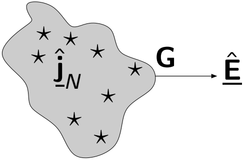

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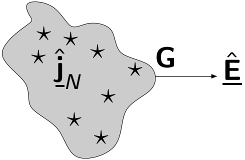

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- ⇒ explicit field quantization [1]

$$\hat{\mathbf{E}} = \hat{\mathbf{E}}[\hat{\mathbf{f}},\hat{\mathbf{f}}^\dagger],\ \hat{\mathbf{B}} = \hat{\mathbf{B}}[\hat{\mathbf{f}},\hat{\mathbf{f}}^\dagger] \text{ and } \hat{\rho} = \hat{\rho}[\hat{\mathbf{f}},\hat{\mathbf{f}}^\dagger],\ \hat{\mathbf{j}} = \hat{\mathbf{j}}[\hat{\mathbf{f}},\hat{\mathbf{f}}^\dagger]$$

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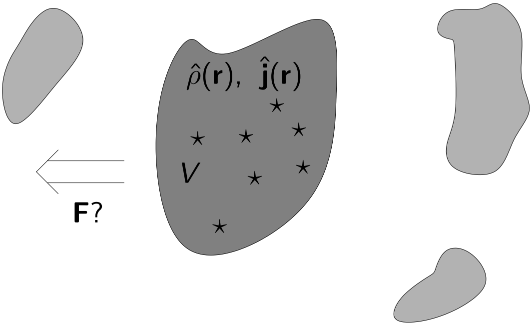
Outline

The problem



Average Lorentz force

$$\mathbf{F} = \int_V \mathrm{d}^3 r \, \langle \hat{\rho}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}') + \hat{\mathbf{j}}(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r}') \rangle_{\mathbf{r}' \to \mathbf{r}}$$

Result for absorbing media [1]

$$\mathbf{F}^{\mathrm{nr}} \equiv \mathbf{F} = -\frac{\hbar}{\pi}\int_V \mathrm{d}^3 r \int_0^\infty \mathrm{d}\xi \left\{ \frac{\xi^2}{c^2} \boldsymbol{\nabla} \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}', i\xi) \right.$$
$$\left. - \mathrm{Tr}\left[\mathbf{I} \times \left(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times + \frac{\xi^2}{c^2} \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', i\xi) \times \overleftarrow{\boldsymbol{\nabla}}' \right] \right\}_{\mathbf{r}' \to \mathbf{r}}$$

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$\Rightarrow$ emission of virtual photons

[1] Raabe and Welsch Phys. Rev. A **73**, 1 063822 (2006)

Result for amplifying media

$$\mathbf{F} = \mathbf{F}^{\mathrm{r}} + \mathbf{F}^{\mathrm{nr}}$$

$$\begin{aligned}\mathbf{F}^{\mathrm{r}} = &-\frac{2\hbar}{\pi c^2}\int_V \mathrm{d}^3 r \int_0^\infty \mathrm{d}\omega\,\omega^2 \int \mathrm{d}^3 s \,\mathrm{Im}\,\varepsilon(\mathbf{s},\omega)\Theta[-\mathrm{Im}\,\varepsilon(\mathbf{s},\omega)] \\ &\times \mathrm{Re}\left\{\omega^2/c^2 \boldsymbol{\nabla}\cdot \mathbf{G}(\mathbf{r},\mathbf{s},\omega)\cdot \mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega)\right. \\ &\left.+\mathrm{Tr}\left[\mathbf{I}\times\left(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times -\omega^2/c^2\right)\mathbf{G}(\mathbf{r},\mathbf{s},\omega)\cdot \mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega)\times \overleftarrow{\boldsymbol{\nabla}}'\right]\right\}_{\mathbf{r}'\to\mathbf{r}}\end{aligned}$$

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[1] Sambale, Welsch, Buhmann, Ho, Phys. Rev A **80** (5), 051801(R) (2009)

Force on dilute amplifying bodies

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$$\varepsilon(\omega)-1=\frac{\eta\alpha_n(\omega)}{\varepsilon_0},\ \alpha_n(\omega)=\lim_{\epsilon\to 0}\frac{1}{3\hbar}\sum_k\left[\frac{|\mathbf{d}_{nk}|^2}{\omega+\omega_{kn}+i\epsilon}-\frac{|\mathbf{d}_{nk}|^2}{\omega-\omega_{kn}+i\epsilon}\right]$$

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$$\boxed{\mathbf{F}=\int \mathrm{d}^3r\,\eta\boldsymbol{\nabla}U_n(\mathbf{r})}$$

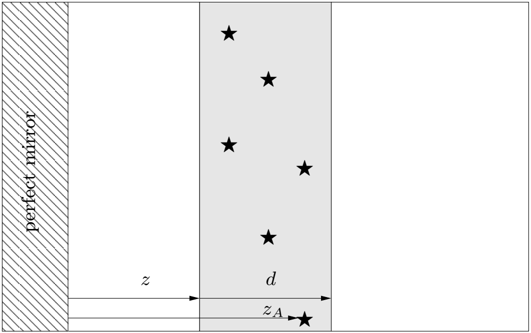
Casimir–Polder potential: $U_n(\mathbf{r})=U_n^{\mathrm{nr}}(\mathbf{r})+U_n^{\mathrm{r}}(\mathbf{r})$

$$U_n^{\mathrm{nr}}(\mathbf{r})=\frac{\hbar\mu_0}{2\pi}\int_0^\infty \mathrm{d}\xi\,\xi^2\alpha_n(i\xi)\mathrm{Tr}\overline{\mathbf{G}}^{(1)}(\mathbf{r},\mathbf{r},i\xi)$$

$$U_n^{\mathrm{r}}(\mathbf{r})=-\frac{\mu_0}{3}\sum_k\Theta(\omega_{nk})\omega_{nk}^2|\mathbf{d}_{nk}|^2\mathrm{Tr}\mathrm{Re}\overline{\mathbf{G}}^{(1)}(\mathbf{r},\mathbf{r},\omega_{nk})$$

$\overline{\mathbf{G}}$: Green tensor in the absence of the amplifying body

Example: Sample of excited gas atoms near a perfect mirror



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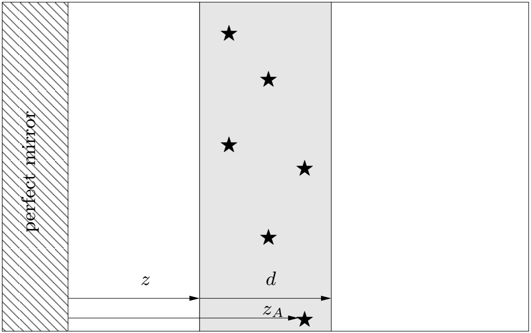
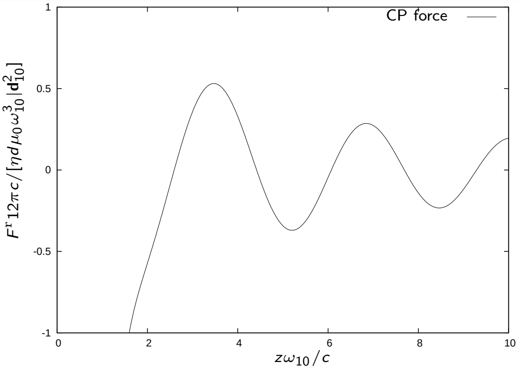


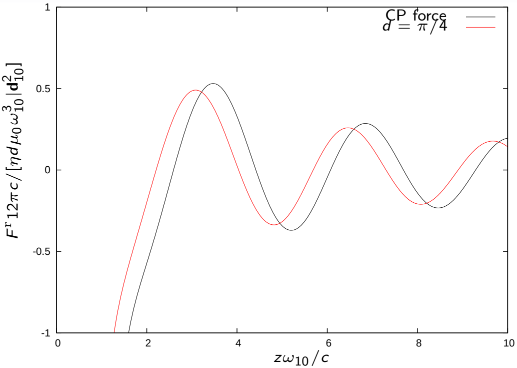
$$\mathbf{F}(z) \approx \mathbf{F}^{\mathrm{r}}(z) = \frac{\mu_0}{3}\eta\omega_{10}^2|\mathbf{d}_{10}|^2\frac{c^2}{32\pi\omega_{10}^2 z_A^3}$$

$$\times\left[(2-4\omega_{10}^2/c^2z_A^2)\cos(2\omega_{10}z_A/c)+4\omega_{10}z_A/c\,\sin(2\omega_{10}z_A/c)\right]_{z_A=z}^{z_A=z+d}\mathbf{e}_z$$



$$\mathbf{F}(z) = -\eta \int_z^{z+d} \mathrm{d}z_A \, \frac{\mathrm{d}}{\mathrm{d}z_A} U_n(z_A)$$

- Oscillations in retarded regime
- Attractive behaviour in nonretarded regime for metals



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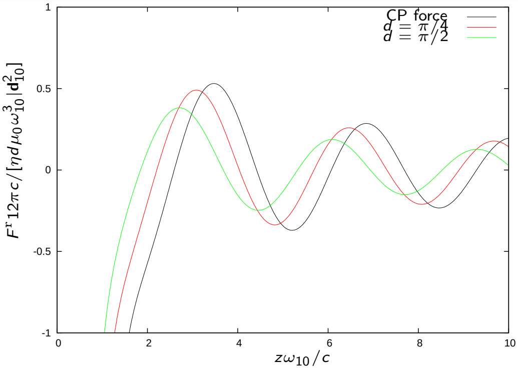

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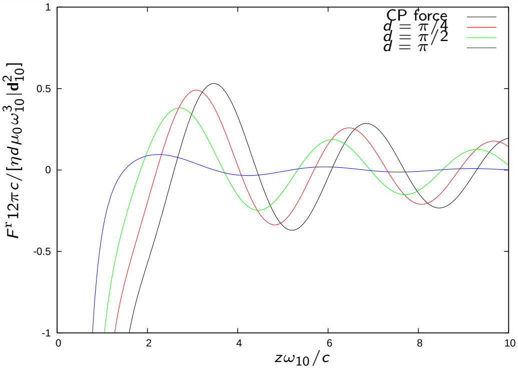

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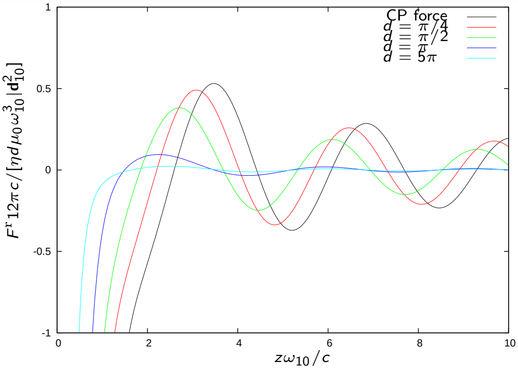

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Summary: Optical dilute amplifying body

- Nonresonant component always attractive but dominated by
- Resonant component:
  - nonretarded regime power law $1/d^3$: metals $\rightarrow$ attraction; but for dielectrica repulsion possible $F_{res} \propto \mathrm{Re}\,\frac{|\varepsilon|^2-1}{|\varepsilon+1|^2}$
  - retarded regime: force oscillates

Now: Going beyond optically dilute limit

Outline

Stress tensor approach

Velocity-independent system: Casimir force = surface integral over the outer boundaries of the body (volume V)

$$\mathbf{F} = \int_{\partial V} \mathrm{d}\mathbf{a} \cdot \mathbf{T}(\mathbf{r})$$

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with stress tensor

$$\begin{aligned}\mathbf{T}(\mathbf{r}) &= \lim_{\mathbf{r}' \to \mathbf{r}} \mathbf{T}(\mathbf{r}, \mathbf{r}') \\ &= \varepsilon_0 \langle\{0\}| \hat{\mathbf{E}}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}') |\{0\}\rangle + \mu_0^{-1} \langle\{0\}| \hat{\mathbf{B}}(\mathbf{r})\hat{\mathbf{B}}(\mathbf{r}') |\{0\}\rangle \\ &- \frac{1}{2}\left(\varepsilon_0 \langle\{0\}| \hat{\mathbf{E}}(\mathbf{r}) \cdot \hat{\mathbf{E}}(\mathbf{r}') |\{0\}\rangle + \mu_0^{-1} \langle\{0\}| \hat{\mathbf{B}}(\mathbf{r}) \cdot \hat{\mathbf{B}}(\mathbf{r}') |\{0\}\rangle\right)\mathbf{I}\end{aligned}$$

Field correlation functions

$$
\begin{aligned}
\langle 0|\hat{\mathbf{E}}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}')|0\rangle = {}& \frac{\hbar}{\pi\varepsilon_0}\int_0^\infty \mathrm{d}\omega \frac{\omega^2}{c^2}\mathrm{Im}\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \\
& - 2\frac{\hbar}{\pi\varepsilon_0}\int \mathrm{d}^3s \int_0^\infty \mathrm{d}\omega \frac{\omega^4}{c^4}\mathrm{Im}\,\varepsilon(\mathbf{s},\omega) \\
& \times \mathrm{Re}[\mathbf{G}(\mathbf{r},\mathbf{s},\omega)\cdot\mathbf{G}^*(\mathbf{s},\mathbf{r}',\omega)]\Theta[-\mathrm{Im}\,\varepsilon(\mathbf{s},\omega)]
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$$\langle 0|\hat{\mathbf{B}}(\mathbf{r})\hat{\mathbf{B}}(\mathbf{r}')|0\rangle = -\frac{\hbar}{\pi\varepsilon_0}\int_0^\infty \mathrm{d}\omega\frac{1}{c^2}\boldsymbol{\nabla}\times\mathrm{Im}\,\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}'$$
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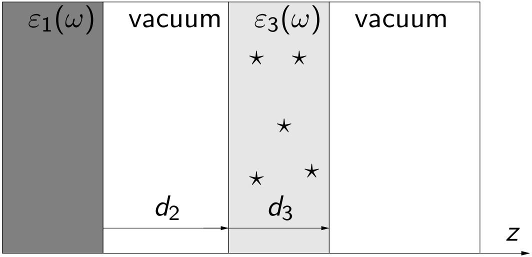
Setup



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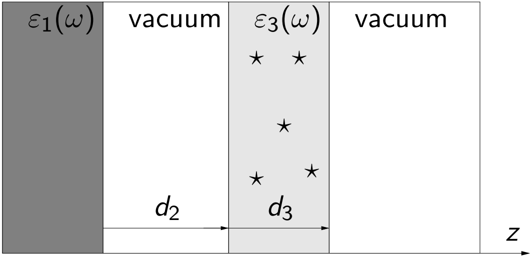

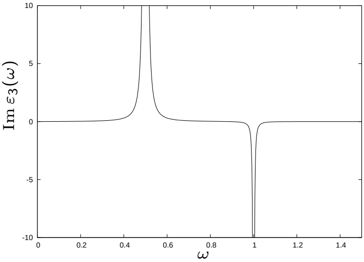

$$\varepsilon_3(\omega) = 1 - \frac{\omega_{pa}^2}{\omega_{ta}^2 - \omega^2 - i\omega\gamma_a} + \frac{\omega_{pb}^2}{\omega_{tb}^2 - \omega^2 - i\omega\gamma_b}$$

Nonresonant contribution

$$\mathbf{f} = -\frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}k^\parallel k^\parallel \kappa^\perp \sum_{\sigma=s,p} \frac{r_{2+}^\sigma r_{2-}^\sigma e^{-2\kappa^\perp d_2}}{1 - r_{2-}^\sigma r_{2+}^\sigma e^{-2\kappa^\perp d_2}}$$

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Ideal case: Amplification for all frequencies: $0 < \varepsilon_3(i\xi) < 1 \; \forall \xi \Rightarrow$ $\kappa^\perp(i\xi) = \sqrt{\varepsilon(i\xi)\xi^2/c^2 + k^{\parallel 2}}$ no ambiguity

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$\kappa^\perp(i\xi) = \sqrt{\varepsilon(i\xi)\xi^2/c^2 + k^{\parallel 2}}$ no ambiguity
Nonretarded limit: $d_3 \to \infty$

$$\mathbf{f}^{nret} = \frac{\hbar}{8\pi^2 d_2^3}\int_0^\infty \mathrm{d}\xi \mathrm{Li}_3\Big[\frac{1-\varepsilon_3(i\xi)}{\varepsilon_3(i\xi)+1}\frac{\varepsilon_1(i\xi)-1}{\varepsilon_1(i\xi)+1}\Big]\mathbf{e}_z$$

Retarded limit ($d_3 \to \infty$): $\varepsilon_{1,3}$ static values

$$\mathbf{f}^{ret} = \frac{3\hbar c}{16\pi^2 d_2^4} \int_1^\infty \frac{\mathrm{d}v}{v^2} \Big\{ \mathrm{Li}_4 \Big[\frac{v - \sqrt{\varepsilon_1 - 1 + v^2}}{v + \sqrt{\varepsilon_1 - 1 + v^2}} \frac{\sqrt{\varepsilon_3 - 1 + v^2} - v}{v + \sqrt{\varepsilon_3 - 1 + v^2}} \Big] + \mathrm{Li}_4 \Big[\frac{\varepsilon_1 v - \sqrt{\varepsilon_1 - 1 + v^2}}{\varepsilon_1 v + \sqrt{\varepsilon_1 - 1 + v^2}} \frac{\sqrt{\varepsilon_3 - 1 + v^2} - \varepsilon_3 v}{\varepsilon_3 v + \sqrt{\varepsilon_3 - 1 + v^2}} \Big] \Big\} \mathbf{e}_z$$

$\Rightarrow$ If amplification is present in a sufficiently large frequency regime the nonresonant component becomes repulsive.

Nonretarded limit (preliminary result)

Set $k^\perp = ik^\parallel$ in all layers ($k^\parallel \in \mathbb{R}$, single-layer reflection independent of wave vector, only p-polarization)

$$\mathbf{f} = \mathbf{f}^{nret}_{nres} + \mathbf{f}^{nret}_{res}$$

$$\mathbf{f}^{nret}_{nres} = -\frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}k^\parallel k^{\parallel 2} \frac{e^{-2k^\parallel d_2} r^p_{2-} r^p_{2+}}{1 - r^p_{2-} r^p_{2+} e^{-2k^\parallel d_2}} \mathbf{e}_z$$

$$\begin{aligned}\mathbf{f}^{nret}_{res} = -\frac{\hbar}{2\pi^2}\int \mathrm{d}\omega \Theta[-\mathrm{Im}\,\varepsilon_3(\omega)] |\mathrm{Im}\,\varepsilon_3(\omega)| \int_0^\infty \mathrm{d}k^\parallel k^{\parallel 2} \\ \times \mathrm{Re}\, r^p_{21} |t^p_{23}|^2 \frac{e^{-2k^\parallel d_2}}{|1 + r^p_{3-} r^p_{23} e^{-2k^\parallel d_3}|^2 |1 - r^p_{21} r^p_{23} e^{-2k^\parallel d_2}|^2} \\ \times \left[(1 - e^{-2k^\parallel d_3}) + |r^p_{23}|^2 (e^{-2k^\parallel d_3} - e^{-4k^\parallel d_3})\right] \mathbf{e}_z\end{aligned}$$

Approximation: $d_3 \to \infty$, neglect of multiple reflections

$$\mathbf{f}_{res}^{nret} = -\frac{\hbar}{2\pi^2 d_2^3} \int \mathrm{d}\omega \Theta[-\mathrm{Im}\,\varepsilon_3(\omega)] |\mathrm{Im}\,\varepsilon_3(\omega)| \frac{|\varepsilon_3(\omega)|(|\varepsilon_1(\omega)|^2 - 1)}{|\varepsilon_1(\omega)+1|^2 |\varepsilon_3(\omega)+1|^2}$$

$\Rightarrow$ repulsion possible

Perfect mirror:

$$\mathbf{f}_{res}^{nret} = -\frac{\hbar}{2\pi^2 d_2^3} \int \mathrm{d}\omega \Theta[-\mathrm{Im}\,\varepsilon_3(\omega)] |\mathrm{Im}\,\varepsilon_3(\omega)| \frac{|\varepsilon_3(\omega)|}{|\varepsilon_3(\omega)+1|^2}$$

$\Rightarrow$ attractive

Power law in agreement with resonant Casimir–Polder force on excited atom

- Nonresonant contribution: can be repulsive for large gain-assisted frequency regime but is expected to be dominated by
- Resonant contribution
  - Nonretarded limit: attraction for metals, for dielectrica repulsion possible
  - Open: Retarded limit $\Rightarrow$ discuss choice of the wave vector
    Expect: Oscillations in consisteny with the optically dilute case

Choice of the wavevector in amplifying media

Problem: Amplification in limited frequency regime, expect resonant force components, $\varepsilon_3(\omega)$ complex and
$k_3^{\perp}(\omega) = \sqrt{\varepsilon_3 \omega^2/c^2 - k^{\parallel 2}}$
Physical requirements:

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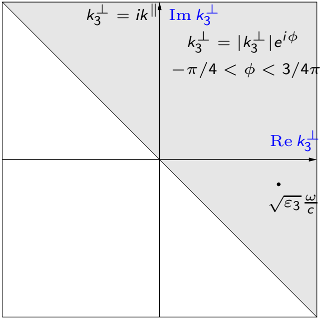

Outline

Summary and Outlook

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[1] Sambale, Welsch, Buhmann, Ho, J. Opt. Spec., **108**, 3 (2010)

Summary and Outlook

- Casimir force on a linear partially amplifying body has resonant force components $\Rightarrow$ repulsion for dielectrica possible in the nonretarded limit
- Contact to Casimir–Polder forces: force on excited atoms $\xrightarrow{\sum}$ force on dilute amplifying body
- To be done: Going beyond nonretarded limit
- Our approach can be expanded to include magnetoelectric bodies [1] $\Rightarrow$ lefthandedness, metamaterials

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