Quantum dissipative effects in moving imperfect mirrors

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QFEXT 2011 - Benasque - Spain
September 2011

Plan of the talk

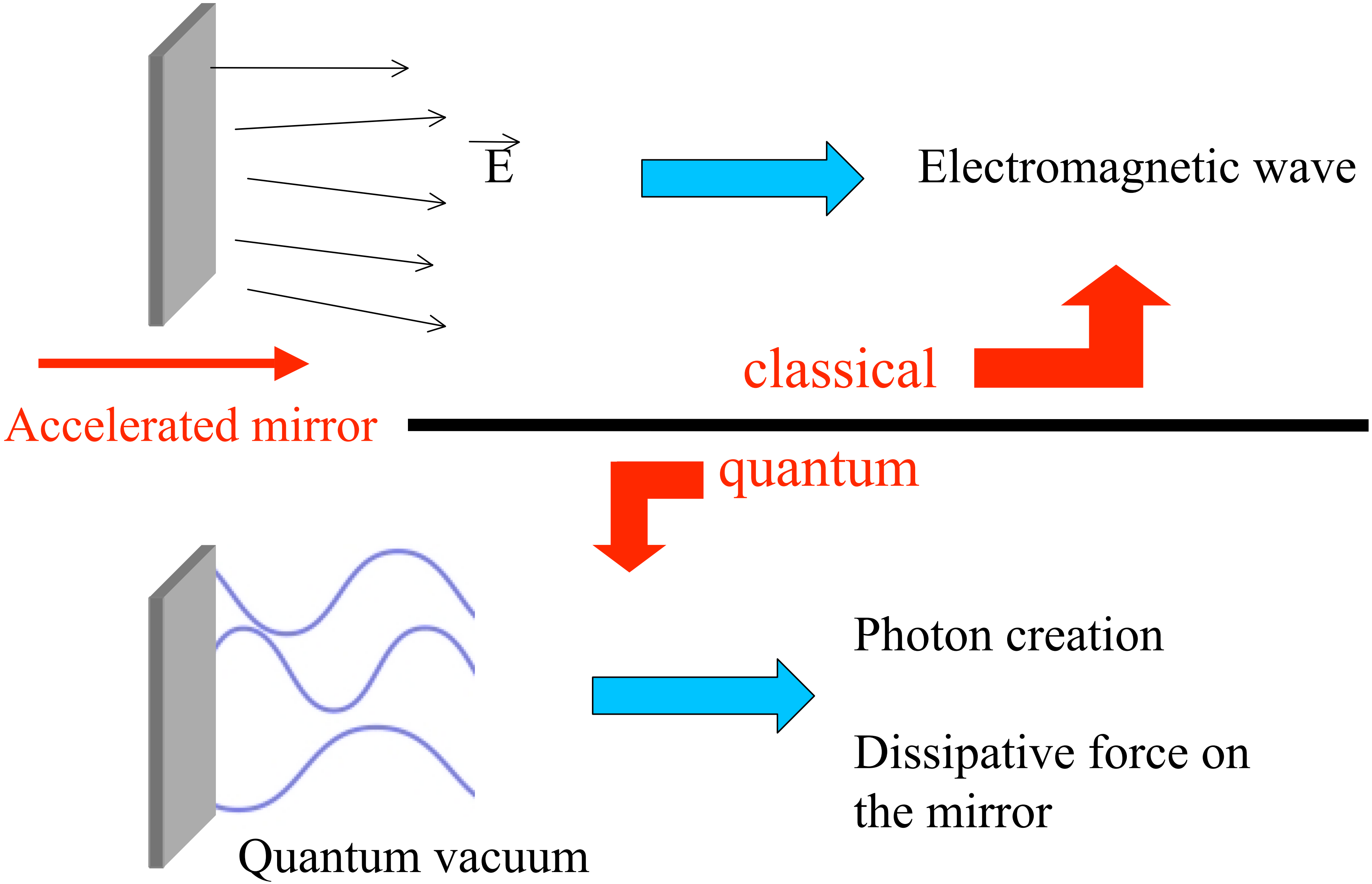
- Dynamical Casimir effect for perfect mirrors
- Functional approach, dissipation, and dynamical Casimir effect for imperfect mirrors
- Forces on accelerated mirrors due to excitation of internal degrees of freedom
- Dissipative effects in imperfect moving mirrors

  I. Normal motion

  II. Vacuum friction

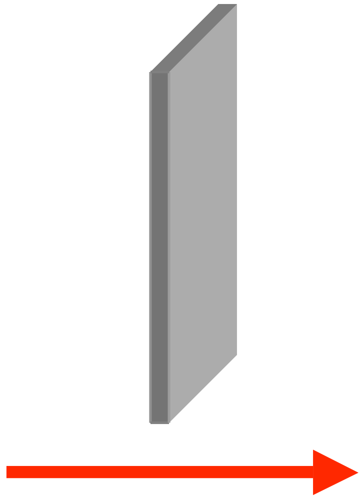
- Conclusions

Dynamical Casimir effect: intuitive idea

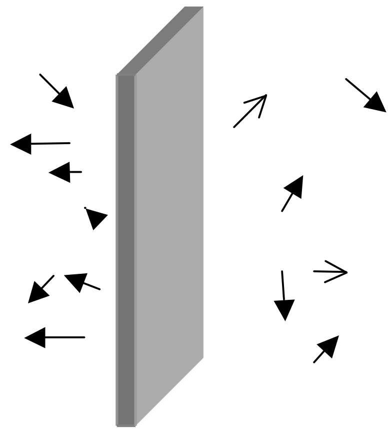


Simplest case: perfect mirrors

Boundary conditions



Accelerated mirror
(non relativistic)



Motion induced radiation

Dissipative force on the mirror. Dimensional analysis:

$$f(t) \propto \frac{\hbar A q^{(5)}(t)}{c^4}$$

Performing the calculation: $f(t) = -\frac{\hbar A q^{(5)}(t)}{30\pi^2 c^4}$

$$E = -\int_{-\infty}^{\infty} f(t) q^{(1)}(t) dt.$$

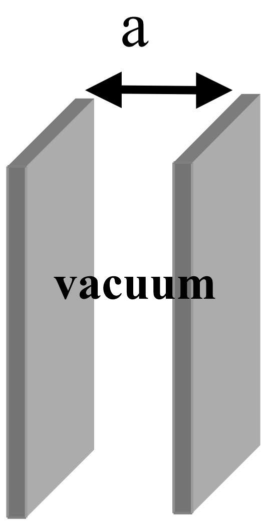
Oscillating mirror:

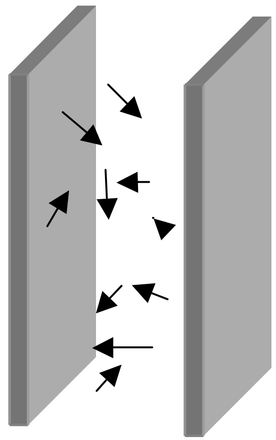
$$E = N\hbar\Omega/2$$

$v_{\max}/c \sim 10^{-7}$ $\quad A \approx 10 \ cm^2$ $\quad N/T \approx 10^{-5} \ seg^{-1}$

Less than a photon/day…..

Parametric amplification in resonant cavities





Oscillating mirror

$$q(t) = a \;\; \varepsilon \;\; Sin(\Omega t)$$

The number of created photons grows exponentially when

$$\Omega = 2 \;\; \omega_m$$

$$\boxed{\langle \mathcal{N}_{\mathbf{m}} \rangle = \sinh^2 \left[\frac{1}{\Omega} \left(\frac{m_z \pi}{L_z} \right)^2 \varepsilon t_f \right]}$$

The number of created photons is limited by the Q-factor of the cavity

- Similar calculations for TE and TM modes of the electromagnetic field, for cavities with different geometries

- Intermode coupling if $|\omega_{\mathbf{m}} \pm \omega_{\mathbf{j}}| = \Omega$

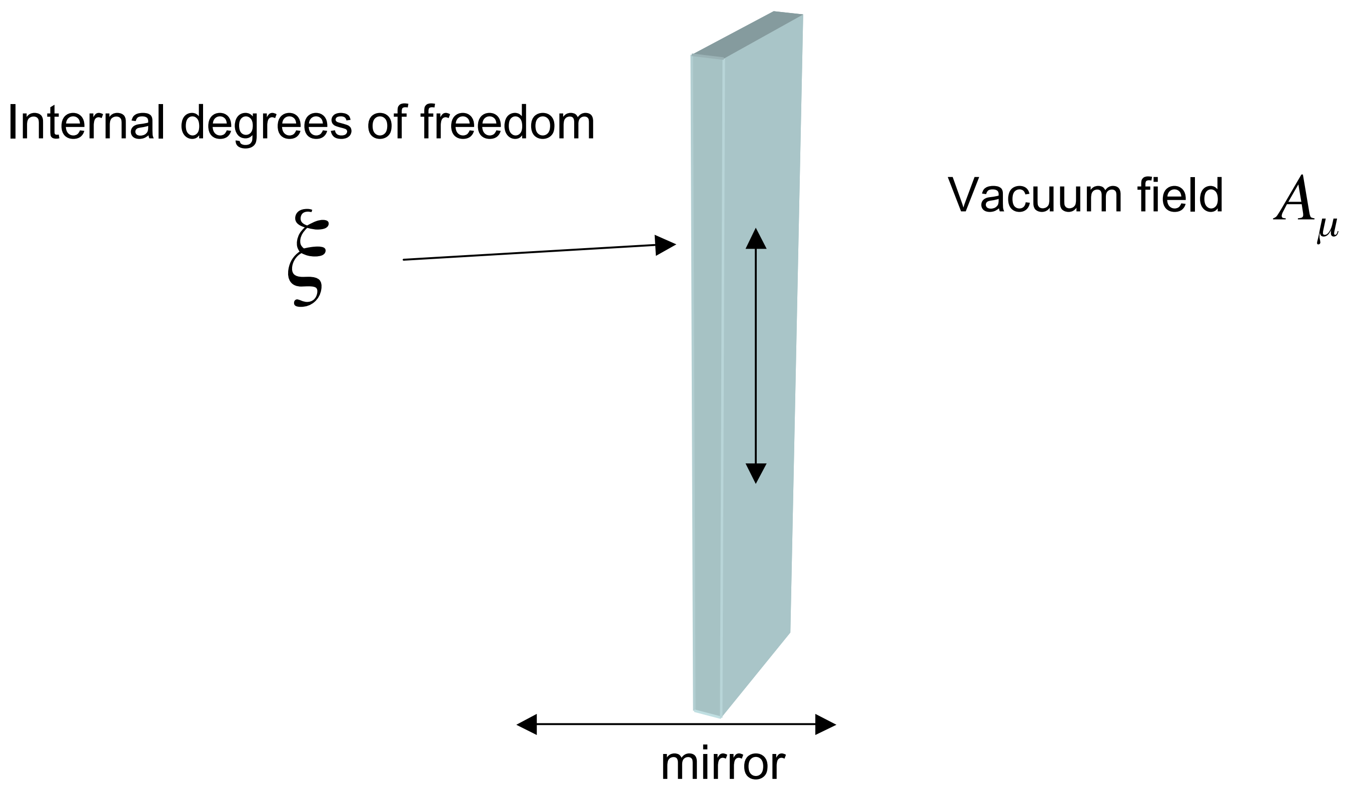
- All modes coupled in 1+1 (or TEM in 3+1)

- Time dependent electromagnetic properties: Padova experiment, Gothenburgh experiment….

Review article: D.Dalvit, P. Maia Neto, FDM (2011)

In this talk: imperfect moving (and deforming) mirrors

Analysis of the Dynamical Casimir effect taking into account the microscopic degrees of freedom of the moving mirrors



Functional approach, dissipation, and dynamical Casimir effect

Main idea:

Vacuum field + microscopic degrees of freedom on the moving mirrors

$$\boxed{\text{Dissipative effects} \quad \Longleftrightarrow \quad \left|\text{Vacuum persistence amplitude}\right| < 1}$$

Functional approach, dissipation, and dynamical Casimir effect

Main idea:

Vacuum field + microscopic degrees of freedom on the moving mirrors

$$\boxed{\text{Dissipative effects} \longleftrightarrow \left|\text{Vacuum persistence amplitude}\right| < 1}$$

$$< 0_{in} | 0_{out} > \Big|_{q(t)} = \int \underbrace{DA_\mu}_{\text{em field}} \underbrace{D\xi}_{\text{Internal degrees of freedom}} \; e^{iS} \equiv e^{i\Gamma_{in-out}[q(t)]}$$

The motion of the mirrors can produce excitations of the electromagnetic field and/or of the microscopic degrees of freedom in the mirrors

Different sources of dissipation:

- photon creation
- excitation of internal degrees of freedom due to exchange of virtual photons (vacuum friction)
- excitation of internal degrees of freedom due to inertial forces

Different sources of dissipation:

- photon creation
- excitation of internal degrees of freedom due to exchange of virtual photons (vacuum friction)
- excitation of internal degrees of freedom due to inertial forces

Technical points:

For simplicity we will work with a **scalar** vacuum field and **thin** mirrors

We will compute the **Euclidean** effective action and then obtain the vacuum persistence amplitude using a **Wick rotation**, and the force on the mirror using a **retarded prescription**

$$\Gamma_E[q(t)] \rightarrow \Gamma_{in-out}[q(t)]$$

$$\frac{\delta \Gamma_E}{\delta q} \rightarrow \left.\frac{\delta \Gamma}{\delta q}\right|_{retarded} = F_{dis}$$

The Euclidean effective action

$$S = S_f + S_m + S_I$$

Vacuum field

$$S_f(\varphi) = \frac{1}{2}\int d^{d+1}x\, \partial_\mu\varphi(x)\partial_\mu\varphi(x)$$

Internal d.o. f.

$$S_m(\xi;\mathcal{M}) = \frac{1}{2}\int d^d\sigma\sqrt{g(\sigma)}\big[g^{\alpha\beta}(\sigma)\partial_\alpha\xi(\sigma)\partial_\beta\xi(\sigma) + \mu^2\xi^2(\sigma)\big]\,,$$

$\mathcal{M}$ is the spacetime volume swept by a deforming mirror

$g_{\alpha\beta}$ Induced metric

Internal degrees of freedom = quantum field theory in curved spacetime

Integrating the internal degrees of freedom

$$e^{-\Gamma_m(\varphi;\mathcal{M})} \equiv \int \mathcal{D}\xi \, e^{-S_m(\xi;\mathcal{M})-S_I(\varphi,\xi;\mathcal{M})}$$

$$\Gamma_m(\varphi;\mathcal{M}) = \Gamma_i(\mathcal{M}) + \Gamma_b(\varphi;\mathcal{M})$$

$$\Gamma_i(\mathcal{M}) \equiv \Gamma_m(\varphi;\mathcal{M})|_{\varphi=0},$$

Inertial effects.
Do not depend on the coupling to the vacuum field

$$\Gamma_b(\varphi;\mathcal{M}) = \Gamma_m(\varphi;\mathcal{M}) - \Gamma_m(\varphi;\mathcal{M})|_{\varphi=0}$$

In some particular limits provides a boundary condition for the vacuum field

The acceleration of the mirror excites the internal degrees of freedom

$$\Gamma_i(\mathcal{M}) = \frac{1}{2}\,\mathrm{Tr}\ln\mathcal{K}$$

$$\mathcal{K} = -\partial_\alpha\left[g^{1/2}g^{\alpha\beta}\partial_\beta\right] + g^{1/2}\mu^2 = g^{1/2}\left(-\Delta_{\mathcal{M}} + \mu^2\right)$$

Known result from Quantum Field Theory in Curved Spacetimes: for massless internal d.o.f:

$$\Gamma_i(\mathcal{M}) \simeq -\frac{1}{64\times 2^{3/2}}\int d^3\sigma\sqrt{g(\sigma)}\Big[a_1 R_{\alpha\beta}(-\Delta)^{-\frac{1}{2}}R_{\alpha\beta} + a_2 R(-\Delta)^{-\frac{1}{2}}R\Big] + \Gamma_{local} \quad + \ldots\ldots$$

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Known result from Quantum Field Theory in Curved Spacetimes In 2+1 dimensions. For massless internal d.o.f:

$$\Gamma_i(\mathcal{M}) \simeq -\frac{1}{64 \times 2^{3/2}} \int d^3\sigma \sqrt{g(\sigma)} \Big[a_1 R_{\alpha\beta} (-\Delta)^{-\frac{1}{2}} R_{\alpha\beta} + a_2 R (-\Delta)^{-\frac{1}{2}} R \Big] + \Gamma_{local} \, , \qquad (35)$$

Nonlocal effective action

curvature associated to the mirror

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Effective action fro graphene: massless Dirac field

For standing waves on the mirror

$$y(\sigma^0, \sigma^1, \sigma^2) = y_0 \cos(\Omega\sigma^0) \cos\left(\sigma^1/L\right)$$

$$\boxed{\frac{\mathrm{Im}\left[\Gamma_i\right]}{T\Sigma} \sim \frac{\hbar y_0^4 \Omega^3}{v_F^2 L^4}}$$

Comparison with the usual DCE (perfect mirrors)

$$\frac{\mathrm{Im}\left[\Gamma_i\right]}{T\Sigma} \sim \frac{\hbar y_0^4 \Omega^3}{v_F^2 L^4}$$

$$\frac{\mathrm{Im}\left[\Gamma^{\mathrm{DCE}}\right]}{T\Sigma} \sim \frac{\hbar y_0^2 \Omega^5 (1 - \frac{c^2}{L^2\Omega^2})^{5/2}}{c^4}.$$

(Golestanian & Kardar, Saharian)

$$\frac{\mathrm{Im}\left[\Gamma_i\right]}{\mathrm{Im}\left[\Gamma^{\mathrm{DCE}}\right]} \sim \left(\frac{y_0}{L}\right)^2 \left(\frac{c}{v_F}\right)^2 \left(\frac{c}{\Omega L}\right)^2 \left(1 - \frac{c^2}{L^2\Omega^2}\right)^{-5/2}$$

Comparison with the usual DCE (perfect mirrors)

$$\frac{\mathrm{Im}\left[\Gamma_i\right]}{T\Sigma} \sim \frac{\hbar y_0^4 \Omega^3}{v_F^2 L^4}$$

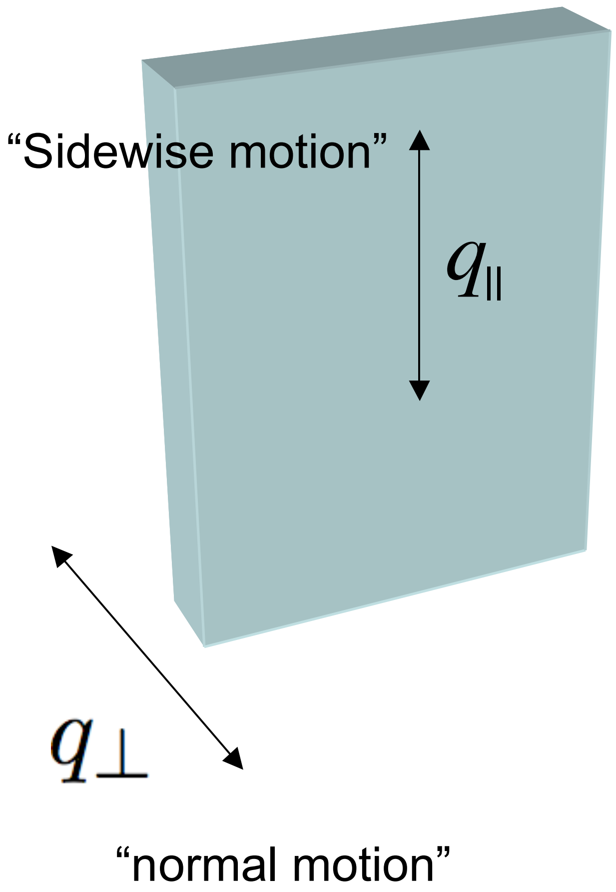
threshold

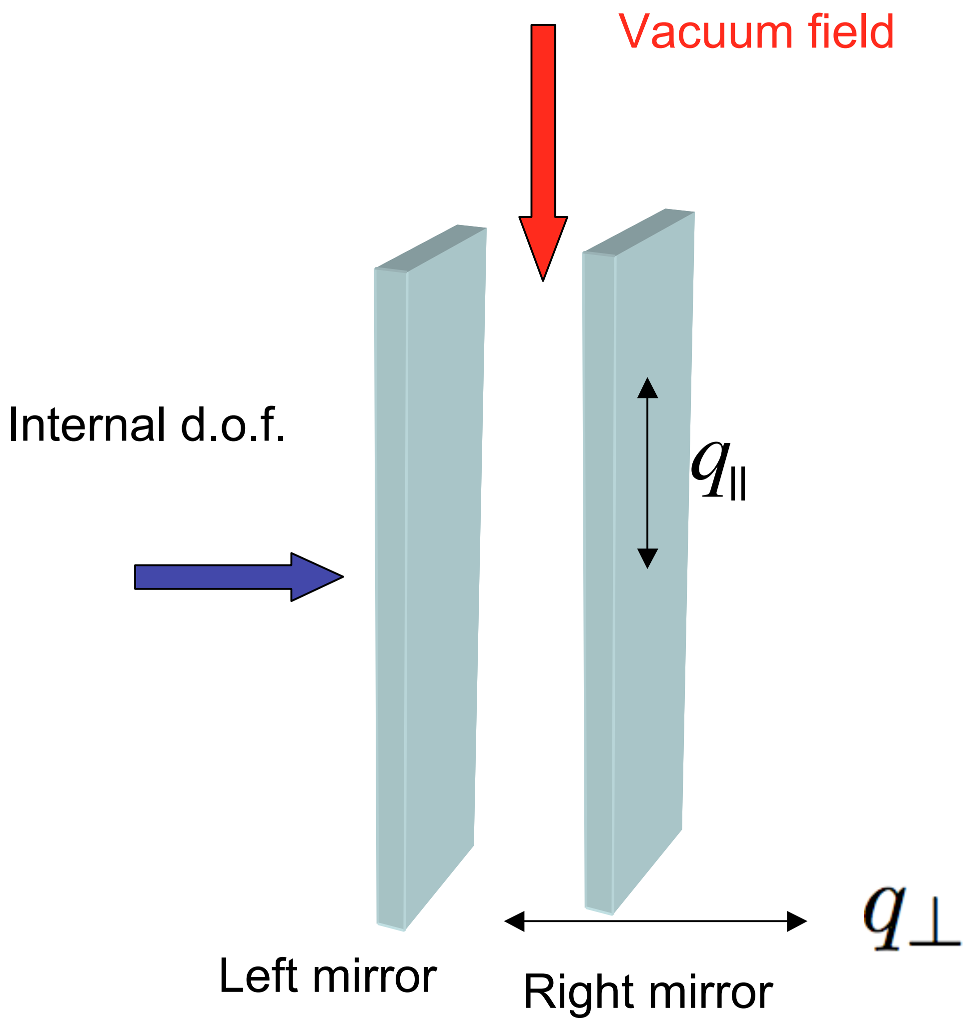
$$\frac{\mathrm{Im}\left[\Gamma^{\mathrm{DCE}}\right]}{T\Sigma} \sim \frac{\hbar y_0^2 \Omega^5 (1 - \frac{c^2}{L^2\Omega^2})^{5/2}}{c^4}.$$

(Golestanian & Kardar, Saharian)

$$\frac{\mathrm{Im}\left[\Gamma_i\right]}{\mathrm{Im}\left[\Gamma^{\mathrm{DCE}}\right]} \sim \left(\frac{y_0}{L}\right)^2 \left(\frac{c}{v_F}\right)^2 \left(\frac{c}{\Omega L}\right)^2 \left(1 - \frac{c^2}{L^2\Omega^2}\right)^{-5/2}$$

Coupling to the vacuum field: sidewise and normal motions





The effective action is of the form:

$$e^{-\Gamma(q_\perp, q_\parallel)} = \int \mathcal{D}\varphi \mathcal{D}\psi \, e^{-S_0(\varphi) - S_m^{(0)}(\psi) - S_m^{(\mathrm{int})}(\varphi, \psi)}$$

For a scalar vacuum field $$S_0 \; = \; \frac{1}{2} \int d^4x \, [(\partial\varphi)^2 \; + \; m^2\varphi^2]$$

The effective action is of the form:

$$e^{-\Gamma(q_\perp,q_\parallel)} = \int \mathcal{D}\varphi \underline{\mathcal{D}\psi}\, e^{-S_0(\varphi)-S_m^{(0)}(\psi)-\underline{S_m^{(\mathrm{int})}(\varphi,\psi)}}$$

For a scalar vacuum field $$S_0 = \frac{1}{2}\int d^4x\, \left[(\partial\varphi)^2 + m^2\varphi^2\right]$$

After integration of the internal degrees of freedom (linear response theory)

$$e^{-\Gamma(q_\perp,q_\parallel)} = \int \mathcal{D}\varphi\, e^{-S_0(\varphi)-S_I(\varphi)}$$

$$S_I(\varphi) = \frac{1}{2}\int d^4x d^4x'\, \varphi(x')V(x,x')\varphi(x)$$

$$V(x,x') = \delta\big(x_3 - q_\perp(x_0)\big)\, \Lambda(x_0,x_\parallel;x_0',x_\parallel')\, \delta\big(x_3' - q_\perp(x_0')\big)$$

Here we are neglecting the term independent of the vacuum field.

$$V(x,x') \;=\; \delta\big(x_3 - q_\perp(x_0)\big)\,\Lambda(x_0,x_\parallel;x_0',x_\parallel')\,\delta\big(x_3' - q_\perp(x_0')\big)$$

Example 1: a set of harmonic oscillators on a static mirror generate a delta-potential for the vacuum field

$$S_m^{(0)} = \frac{1}{2}\int dx_0 \int d^2x_\parallel \left[\dot{Q}(x_\parallel,x_0)^2 + \Omega^2 Q(x_\parallel,x_0)^2\right]$$

$$S_m^{\rm int} = ig\int d^4x\, Q(x_\parallel,x_0)\delta(x_3)\varphi(x_0,x_\parallel,x_3)\,,$$

$$\Lambda(x_0,x_\parallel;x_0',x_\parallel') \;=\; \lambda(x_0-x_0')\,\delta^{(2)}(x_\parallel - x_\parallel')$$

$$\lambda(x_0-x_0') \to \left(\frac{g}{\Omega}\right)^2\delta(x_0-x_0') \qquad g,\Omega\to\infty$$

Example 2: relativistic massless fermions coupled to the electromagnetic field

$$S_I \approx \int d^3y \;\; d^3y' \;\; A_a(y,0)\Pi_{ab}(y,y')A_b(y',0)$$

$$\tilde{\Pi}_{ab}(k) = e^2\delta_{ab}(k)\,|\,k\,|$$

2+1 dimensional momentum

No dimensionful constants

Static Casimir force proportional to $\dfrac{1}{a^4}$

Fosco, Lombardo, FDM, PLB (2008)

Graphene sheet if $c \leftrightarrow v_{F}$, Bordag et al PRB (2009)

Effective action for a single mirror:

$$e^{-\Gamma(q_\perp, q_\parallel)} = \int \mathcal{D}\varphi \, e^{-S_0(\varphi) - S_I(\varphi)}$$

$$\Gamma(q_\perp, q_\parallel) \; = \; \frac{1}{2} \log\det(-\partial^2 + V) \; = \; \frac{1}{2} \mathrm{Tr} \log(-\partial^2 + V)$$

Effective action for two mirrors:

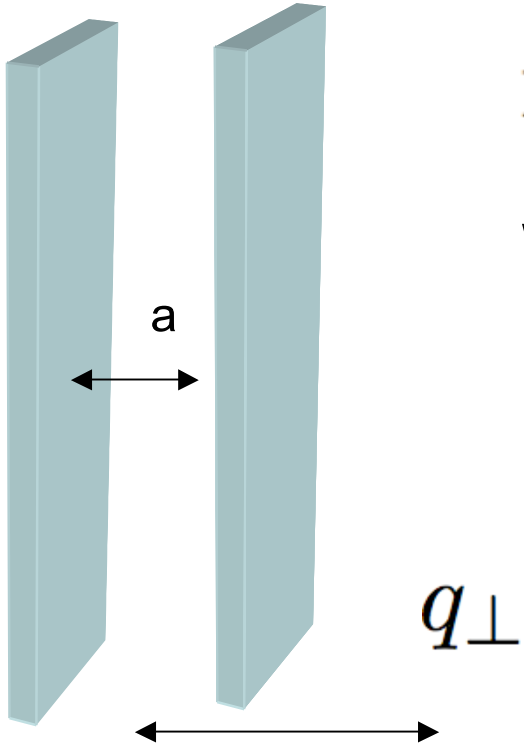
$$\Gamma(q_L, q_R) = \frac{1}{2} \log\det(-\partial^2 + V_L + V_R) = \frac{1}{2} \mathrm{Tr} \log(-\partial^2 + V_L + V_R)$$

Each one can have normal or sidewise motion

We will assume that V is spatially local in the rest frame of the mirror

$$\Lambda(x_0, x_\parallel; x_0', x_\parallel') \; = \; \lambda(x_0 - x_0') \, \delta^{(2)}(x_\parallel - x_\parallel')$$

Normal motion



$$\Gamma_I(q_L, q_R) \;=\; \frac{\Sigma}{2}\int \frac{d^2 k_\parallel}{(2\pi)^2}\, \mathrm{Tr}\big(\log \widetilde{\mathcal{K}}\big)$$

We perform an expansion in powers of $q_\perp$

$$\widetilde{\mathcal{K}} = \widetilde{\mathcal{K}}_0 + \widetilde{\mathcal{K}}_1 + \widetilde{\mathcal{K}}_2 + \ldots.$$

$$\Gamma\;(q_\perp) \;=\; \frac{1}{2}\int \frac{d^2 k_\parallel}{(2\pi)^2}\, \mathrm{Tr}\Big[\widetilde{\mathcal{K}}_0^{-1}(\widetilde{\mathcal{K}}_1 + \widetilde{\mathcal{K}}_2) - \frac{1}{2}(\widetilde{\mathcal{K}}_0^{-1}\widetilde{\mathcal{K}}_1)^2\Big]$$

Linear term

Quadratic terms

On general grounds we expect: linear term

$$\Gamma_1(q_\perp) = \int dx_0\, q_\perp(x_0) F_C$$

Usual static Casimir force between thin mirrors

Explicitly:

$$\Gamma_1(q_\perp) = -\frac{1}{2}\int dx_0\, q_\perp(x_0) \int \frac{d\omega}{2\pi} \int \frac{d^2k_\parallel}{(2\pi)^2} \frac{1}{\sqrt{\omega^2 + k_\parallel^2 + m^2}}$$

$$\times \quad \frac{e^{-2a\sqrt{\omega^2+k_\parallel^2+m^2}}}{\left(\frac{1}{\tilde\lambda(\omega)} + \frac{1}{2\sqrt{\omega^2+k_\parallel^2+m^2}}\right)^2 - \frac{e^{-2a\sqrt{\omega^2+k_\parallel^2+m^2}}}{4(\omega^2+k_\parallel^2+m^2)}}.$$

The quadratic term:

$$\Gamma_2(q_\perp) = \frac{1}{2} \int dx_0 \int dx_0'\, q_\perp(x_0) F(x_0 - x_0') q_\perp(x_0')$$

$$F_{dis}(x_0) = \int dx'_0\, F_{ret}(x_0 - x'_0) q(x'_0)$$

$$\Gamma_{2,\mathrm{in-out}}(q_\perp) = \frac{1}{2} \int dx_0 \int dx_0'\, q_\perp(x_0) F_{\mathrm{in-out}}(x_0 - x_0') q_\perp(x_0')$$

Fourier transform of the form factor $F = F^{(1)} + F^{(2)}$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{4\pi}\int d\nu \int \frac{d^2k_{\|}}{(2\pi)^2}\frac{c(\nu+\omega)\sqrt{\nu^2+k_{\|}^2+m^2}}{c^2(\nu+\omega,k_{\|}) - b^2(\nu+\omega,k_{\|})}$$

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{8\pi}\int \frac{d^2k_{\|}}{(2\pi)^2}\int d\nu \frac{1}{c^2(\omega+\nu,k_{\|}) - b^2(\omega+\nu,k_{\|})}\frac{1}{c^2(\nu,k_{\|}) - b^2(\nu,k_{\|})}$$
$$\times \Big[c(\omega+\nu,k_{\|})c(\nu,k_{\|})e^{-2a\sqrt{(\omega+\nu)^2+k_{\|}^2+m^2}} +$$
$$b(\omega+\nu,k_{\|})b(\nu,k_{\|})e^{-a\sqrt{(\omega+\nu)^2+k_{\|}^2+m^2}}e^{-a\sqrt{\nu^2+k_{\|}^2+m^2}}\Big]$$

With:

$$b(\omega,k_{\|}) = \frac{e^{-a\sqrt{\omega^2+k_{\|}^2+m^2}}}{2\sqrt{\omega^2+k_{\|}^2+m^2}}$$

$$c(\omega,k_{\|}) = \frac{1}{\widetilde{\lambda}(\omega)} + \frac{1}{2\sqrt{\omega^2+k_{\|}^2+m^2}}$$

Fourier transform of the form factor $F = F^{(1)} + F^{(2)}$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{4\pi}\int d\nu \int \frac{d^2k_{\|}}{(2\pi)^2}\frac{c(\nu+\omega)\sqrt{\nu^2+k_{\|}^2+m^2}}{c^2(\nu+\omega,k_{\|})-b^2(\nu+\omega,k_{\|})}$$

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$$\times\Big[c(\omega+\nu,k_{\|})c(\nu,k_{\|})e^{-2a\sqrt{(\omega+\nu)^2+k_{\|}^2+m^2}} +$$
$$b(\omega+\nu,k_{\|})b(\nu,k_{\|})e^{-a\sqrt{(\omega+\nu)^2+k_{\|}^2+m^2}}e^{-a\sqrt{\nu^2+k_{\|}^2+m^2}}\Big]$$

Poles at resonant frequencies for perfect mirrors. Breakdown of perturbative calculations

1+1 dimensions, "graphene-like" coupling

$$\tilde{\lambda}(\omega) = \zeta|\omega|$$

(no additional dimensionful constants)

The vacuum field propagates at a velocity $1/\sqrt{1+\zeta}$

Electromagnetic analogy: $\zeta = \varepsilon - 1$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{2\pi}\int d\nu\, |\nu+\omega|\, |\nu|\, \frac{\chi}{\chi^2 - e^{-2a|\nu+\omega|}}$$

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{2\pi}\int d\nu\, e^{-2a|\omega+\nu|} \frac{|\omega+\nu||\nu|}{\left(\chi^2 - e^{-2a|\omega+\nu|}\right)} \frac{\left(\chi^2 + e^{-2a|\nu|}\right)}{\left(\chi^2 - e^{-2a|\nu|}\right)}$$

$$\chi = (2+\zeta)/\zeta$$

Limit of perfect mirrors: $\zeta \to \infty \quad or \quad \chi \to 1$ **(strong coupling)**

$$\widetilde{F_\infty}(\omega) = \widetilde{F_\infty^{(1)}}(\omega) + \widetilde{F_\infty^{(2)}}(\omega)$$
$$= \frac{|\omega|^3}{12\pi} - \frac{\omega^2\pi}{6a^3}(1 + \frac{\omega^2 a^2}{\pi^2}) \sum_{n\geq 1} \frac{1}{\omega^2 + \frac{n^2\pi^2}{a^2}}$$

$$F_{\rm ret}(x_0) = \frac{\delta'''(x_0)}{12\pi} - \frac{\pi}{6a^2}\theta(t) \sum_{n\geq 0} \delta'(x_0 - 2na)$$

$$- \frac{1}{6\pi}\theta(t) \sum_{n\geq 0} \delta'''(x_0 - 2na)$$

Well known result: Jaeckel & Reynaud, Maia Neto & Mundarain

Weak coupling $\zeta \to 0 \quad or \quad \chi \to \infty$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{2\pi\chi}\int d\nu\, |\nu+\omega|\, |\nu|\, (1 + \frac{e^{-2a|\nu+\omega|}}{\chi^2}) + \ldots$$

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{2\pi\chi^2}\int d\nu\, |\omega+\nu||\nu| e^{-2a|\omega+\nu|} + \ldots .$$

$$\widetilde{F}(\omega) = -\frac{1}{6\pi\chi}|\omega|^3 - \frac{1}{4\pi a^3\chi^2}[e^{-2a|\omega|}(1+a|\omega|) + a|\omega|] + \ldots$$

Weak coupling $\zeta \to 0 \quad or \quad \chi \to \infty$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{2\pi\chi}\int d\nu\, |\nu+\omega|\, |\nu|\, (1 + \frac{e^{-2a|\nu+\omega|}}{\chi^2}) + \ldots$$

$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{2\pi\chi^2}\int d\nu\, |\omega+\nu||\nu| e^{-2a|\omega+\nu|} + \ldots .$$

$$\widetilde{F}(\omega) = -\underline{\frac{1}{6\pi\chi}|\omega|^3} - \frac{1}{4\pi a^3\chi^2}[e^{-2a|\omega|}(1+a|\omega|)+a|\omega|] + \ldots$$

Leading term independent of a and similar to perfect conductor

Weak coupling

$\zeta \to 0 \quad or \quad \chi \to \infty$

$$\widetilde{F^{(1)}}(\omega) = -\frac{1}{2\pi\chi}\int d\nu\,|\nu+\omega|\,|\nu|\,(1+\frac{e^{-2a|\nu+\omega|}}{\chi^2})+\dots$$

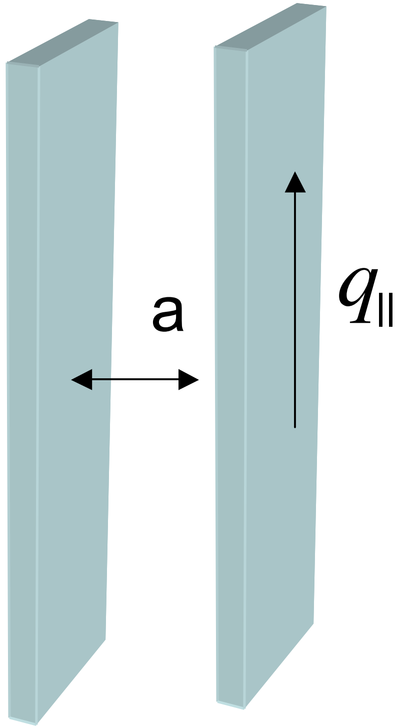
$$\widetilde{F^{(2)}}(\omega) = -\frac{1}{2\pi\chi^2}\int d\nu\,|\omega+\nu||\nu|e^{-2a|\omega+\nu|}+\dots\,.$$

$$\widetilde{F}(\omega) = -\frac{1}{6\pi\chi}|\omega|^3 - \frac{1}{4\pi a^3\chi^2}[e^{-2a|\omega|}(1+a|\omega|)+a|\omega|]+\dots$$

$1/\chi^2$-retarded contributions are proportional to $\delta'(x_0)$, $\delta(x_0-2a)$, and $\delta'(x_0-2a)$

(the time of flight between the two mirrors does not depend on χ)

Sidewise motion (Barton 1996)



We consider two parallel mirrors:

$$V_L(x,x') = \delta(x_3)\,\lambda(x_0 - x_0')\delta^{(2)}(x_{||} - x_{||}')\,\delta(x_3')$$

$$V_R(x,x') = \delta(x_3 - a)\,\lambda(x_0 - x_0')\,\delta[x_1 - x_1' - q_{||}(x_0) + q_{||}(x_0')]\,\delta(x_2 - x_2')\,\delta(x_3' - a)$$

For constant velocity

$$\Gamma \approx \frac{T\Sigma}{64\pi^3} \int d^3p \frac{e^{-2a\sqrt{p_0^2+p_1^2+p_2^2}}}{p_0^2+p_1^2+p_2^2} \widetilde{\lambda}(p_0)\widetilde{\lambda}(p_0+p_1 v)$$

The usual static Casimir force depend on the velocity

For constant velocity

$$\Gamma \approx \frac{T\Sigma}{64\pi^3}\int d^3p\,\frac{e^{-2a\sqrt{p_0^2+p_1^2+p_2^2}}}{p_0^2+p_1^2+p_2^2}\widetilde{\lambda}(p_0)\widetilde{\lambda}(p_0+p_1 v)$$

For the particular case: $\tilde{\lambda}(\omega) = \zeta|\omega|$

$$\boxed{\mathrm{Im}\Gamma_{\mathrm{in-out}} \approx \frac{T\Sigma\zeta^2}{576\pi^3}\frac{|v|^3}{a^3} + O(v^4)}$$

And we expect: $F_{dis} = \dfrac{\Sigma\zeta^2}{a^4} f(\mathrm{v})$

Summary

Interaction between internal degrees of freedom and vacuum field

$|\text{Vacuum persistence amplitude}| < 1$

Dissipative effects

- Reaction force due to photon creation
- Vacuum friction
- Excitation of internal d.o.f.

Analogue of QFTCS

In 2+1 dimensions

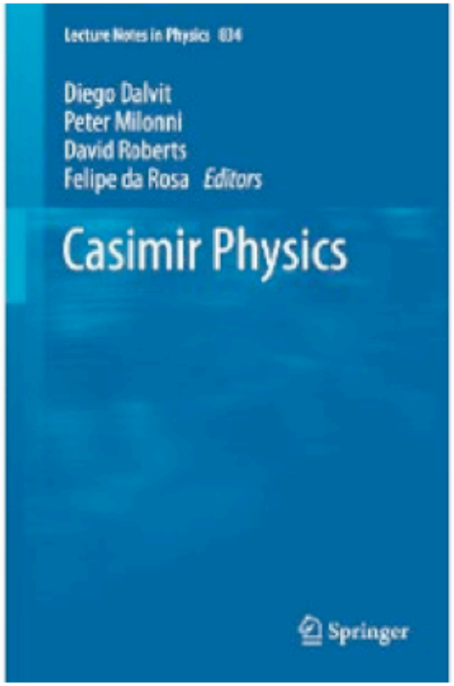
We are studying the generalization to the electromagnetic field and to realistic internal degrees of freedom, using the Schwinger-Keldysh formalism

Non perturbative?

References:

• C.D. Fosco, F.C. Lombardo and F.D. Mazzitelli, Phys. Rev. D (2010)

• C.D. Fosco, F.C. Lombardo and F.D. Mazzitelli, Phys. Rev. D (2011)



Chapter 13
Fluctuations, Dissipation
and the Dynamical Casimir Effect

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(2011)

Conclusions

- We have studied dissipative effects on imperfect moving mirrors using the functional approach. The analysis of the vacuum persistence amplitude allowed us to consider on the same footing different kinds of dissipative effects
- For "normal motion" we obtained general expressions for the effective action in terms of the (analogous of the) polarization tensor that decribes the interaction between the vacuum field and the internal degrees of freedom of the mirror
- Explicit examples in 1+1 dimensions
- For "sidewise motion" we found that in general there is "vacuum friction" between thin mirrors even for constant velocity. The interaction must be non local
- We described a new dissipative effect related to the excitation of the internal degrees of freedom of the mirrors due to the acceleration.
- For simplicity we worked with a scalar vacuum field. We are studying the generalization to the electromagnetic field and to realistic internal degrees of freedom, using the Schwinger-Keldysh formalism

For constant velocity

$$\Gamma \approx \frac{T\Sigma}{64\pi^3} \int d^3p \frac{e^{-2a\sqrt{p_0^2+p_1^2+p_2^2}}}{p_0^2+p_1^2+p_2^2} \boxed{\widetilde{\lambda}(p_0)\widetilde{\lambda}(p_0+p_1 v)}$$

The structure is similar to Pendry's result:

$$F_x = \frac{\hbar}{\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x e^{-2|\mathbf{k}|d} \int_0^{k_x v} d\omega \boxed{\mathrm{Im}\left[\frac{\epsilon(\omega)-1}{\epsilon(\omega)+1}\right] \mathrm{Im}\left[\frac{\epsilon(k_x v-\omega)-1}{\epsilon(k_x v-\omega)+1}\right]}$$