Is the Universe homogeneous?

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What is the basis for homogeneity?
We cannot observe homogeneity, only isotropy

SDSS: $z \sim 0.3$

CMB: $z \sim 1100$
What do (perfect) observations tell us?

(I) Without assuming the Copernican Principle:
   (I.1) What do observations tell us directly
         - for any gravity theory?
         - for GR?
   (I.2) What can we say from isotropy of
         - matter observations?
         - the CMB?

(II) With the Copernican Principle:
   (II.1) What do isotropic matter observations tell us?
   (II.2) What do isotropic CMB observations tell us?

(III) Testing the Copernican Principle

(IV) Towards the real Universe

CP: we are not at a special position in the universe
(I) Without the Copernican Principle

(I.1) What do observations tell us directly?
Try to determine spacetime geometry from lightcone observations.  (Ellis 1975; Ellis, Nel, RM et al 1985)

*Problem:* CDM and DE cannot be directly observed.

(Gravitational lensing? – only determines matter if geometry is assumed a priori.)
Assume we the know ‘missing’ baryon distribution, and

**CDM**
\[ \rho_c \text{ known from } \rho_b, \quad u^a_b = u^a_c := u^a \]

**DE**
A known independently of cosmological observations
Observational coordinates in a general spacetime

\[ x^\mu = (w, y, x^I) \]
\[ x^I = (\theta, \phi) \]
Metric in observational coordinates

\[ ds^2 = -A^2 dw^2 + 2B dy dw + 2C_I dx^I dw + D^2 \left( d\Omega^2 + L_{IJ} dx^I dx^J \right) \]

- past lightcone: \( C^{-}(w_0): w = w_0 \) and \( y = z \)
- lightray 4-vector: \( k_\mu = \partial_\mu w, \ k^\mu = B^{-1} \delta_\mu^y \)
- angular distance: \( D = D_A \)
- lensing distortion: \( L_{IJ} \)

Matter 4-velocity

\[ u^0 = 1 + z, \ u^1 = 0, \ u^I = (1 + z)V^I, \ V^I := \frac{dx^I}{dw} \]

transverse velocities
Lensing convergence and shear in a general spacetime:

\[ \hat{\Theta} = \frac{1}{BD} \frac{\partial D}{\partial y}, \quad \hat{\sigma}_{\mu\nu} = \delta_{\mu}^{\ I} \delta_{\nu}^{\ J} \frac{D^2}{2B} \frac{\partial}{\partial y} L_{IJ}. \]

\[ \frac{d}{dv} \hat{\Theta} = -\frac{1}{2} \hat{\Theta}^2 - \hat{\sigma}_{ab} \hat{\sigma}^{ab} - R_{ab} k^a k^b \]  
(Ricci)

\[ \frac{d}{dv} \hat{\sigma}_{ab} = -\hat{\Theta} \hat{\sigma}_{ab} - C_{acbd} k^c k^d \]  
(Weyl)

Number counts in a general spacetime:

\[ dN = f d\eta D^2 (1 + z) B \, d\Omega_0 \, dz \]

\( B = dv/dz, \ n = \rho/m \)
(I.1a) What do observations tell us directly - without field equations?

In principle, for ideal observations:

* standard candles/ sirens/ rulers give $D$

* number counts give $B\rho_m$
  from galaxy surveys on lightcone
  (+ assumptions on CDM)

* lensing shear gives $L_{ij}$
  if we know intrinsic shapes

* transverse motions give $V^I$
  ?
The maximum achievable in principle

\[
\text{Idealized data } \Rightarrow \{u^\mu, B\rho_m, g_{IJ}\} \text{ on } C^-(w_0)
\]

We cannot determine our past lightcone without field equations. Thus:

- Observations cannot directly test GR on cosmological scales (or any modified gravity)
- We need to assume the spacetime geometry first
We also get an interesting test for transverse velocities:

Anisotropy in the observed Hubble parameter implies that the transverse velocities are nonzero

\[
\frac{\partial}{\partial x^I} H_0^{\text{obs}} = -\frac{1}{3} \frac{\partial}{\partial x^I} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial x^J} \left( \sin \theta V_0^J \right) \right]
\]

(RM 1980; RM, Matravers 1994)
(I.1b) What do observations tell us directly - with GR?

1. \( \{u^\mu, B\rho_m, g_{IJ}\} \) on \( C^-(w_0) \)

is exactly the data needed on the past lightcone for EFE to uniquely determine the matter distribution (i.e. \( \rho_m, u^\mu \)) and geometry (i.e. \( g_{\mu\nu} \)) on the past lightcone.

2. Then EFE propagate off the lightcone to determine the interior (past)

(Ellis, Nel, RM et al 1985)
3. EFE *cannot* propagate to the future since new data can destroy the predictions.
(I.2a) What can we say from isotropy of matter observations?

Isotropic matter observations:

\[ V^I = 0 = L_{IJ}, \quad \frac{\partial D}{\partial x^I} = 0 = \frac{\partial n}{\partial x^I} \]

This is exactly enough to produce isotropic geometry:

**Matter isotropy on lightcone gives isotropy of geometry**

- If one observer comoving with matter sees isotropic angular distances, number counts, bulk velocities and lensing, in a dust Universe with \( \Lambda \), then spacetime is isotropic, i.e. LTB

(Ellis, Nel, RM et al 1984; RM, Matravers 1994)
(1.2b) What can we say from isotropy of the CMB?

Seems obvious that this enforces isotropy of the spacetime. It is plausible: we expect isotropic decoupling surfaces, which then evolves to isotropic future. But this does not follow from Einstein-Liouville equations (at least not in any obvious way).

We cannot deduce isotropy of the geometry, without further assumptions on the matter.
(II) With the Copernican Principle

Without the CP, we cannot establish homogeneity: homogeneity cannot be directly observed in the matter or CMB

(II.1) What do isotropic matter observations tell us?

Isotropy about all observers implies homogeneity:

\[
\text{Matter isotropy on light-cones} \rightarrow \text{FLRW}
\]

In a dust region of a universe with \( \Lambda \), if all fundamental observers measure isotropic area distances, number counts, bulk velocities, and lensing, then the spacetime is FLRW in that region.

An observational basis for the Cosmological Principle
A more powerful result

Isotropy of area distances alone, and for small $z$, about all observers - implies homogeneity:

*Isotropic distances to $3^{\text{rd}}$ order in $z$ imply FLRW*

- In a dust region of a Universe with $\Lambda$, if all fundamental observers measure isotropic distances to $O(z^3)$, then spacetime is FLRW in that region

(Hasse, Perlick 1999; Clarkson 2000; Clarkson, RM 2010)
Series expansion (Kristian, Sachs 1966):

\[
z = \left[ K^a K^b \nabla_a u_b \right]_0 D + \frac{1}{2} \left[ K^a K^b K^c \nabla_a \nabla_b u_c \right]_0 D^2 \\
+ \frac{1}{6} \left[ K^a K^b K^c K^d \nabla_a \nabla_b \nabla_c u_d + \frac{1}{2} K^a K^b K^c K^d R_{cd} \nabla_a u_b \right]_0 D^3 + \ldots
\]

where

\[
\nabla_a u_b = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} - \omega_{ab} - u_a \nu_b
\]

\( O(z) \):

\[
\left( K^a K^b \nabla_a u_b \right)_o = \left[ \frac{1}{3} \Theta + \dot{u}_a e^a + \sigma_{ab} e^a e^b \right]_o = H^\text{obs}_o
\]

Isotropy: \( \dot{u}_a = 0, \quad \sigma_{ab} = 0 \) etc.
(II.2) What do isotropic CMB observations tell us?

CMB isotropy for all fundamental observers gives the strongest basis that we have for homogeneity.

History: 1968 theorem by Ehlers, Geren, Sachs (EGS)
Update: generalize to include baryons, CDM and DE
It seems obvious that we should get FLRW – but we have to show it using the general, fully nonlinear Einstein-Liouville equations.

Nonlinear perturbations are not an option – we cannot assume the FLRW background that we are trying to prove.

\[
\text{CMB isotropy + Copernican Principle} \rightarrow \text{FLRW}
\]

In a region, if
- collisionless radiation is exactly isotropic,
- the radiation four-velocity is geodesic and expanding,
- there are pressure-free baryons and CDM, and dark energy in the form of \( \Lambda \), quintessence or a perfect fluid,

then the metric is FLRW in that region.

(EGS 1968; Ellis, Treciokas 1971; Stoeger, RM, Ellis 1995; Ferrando, Morales, Portilla 1999; Clarkson, Barrett 1999; Clarkson, Coley 2001; Rasanen 2009; Clarkson, RM 2010)

Note: It follows that (1) matter and DE have the same 4-velocity as radiation (2) matter, DE anisotropic stress =0
Liouville equation in any spacetime

\[ \frac{df}{d\tau} = p^a \frac{\partial f}{\partial x^a} + \frac{dp^a}{d\tau} \frac{\partial f}{\partial p^a} = 0 \]

Covariant harmonics:

\[ f(x,p) = \sum_{\ell=0}^{\infty} F_{A_\ell}(x,E) e^{A_\ell} = F(x,E) + F_a(x,E) e^a + F_{ab}(x,E) e^a e^b + \ldots \]

Integrated multipoles:

\[ \rho_\gamma \propto \int dE E^3 F(x,E) \]
\[ q_\gamma^a \propto \int dE E^3 F^a(x,E) = 0 \]
\[ \pi_\gamma^{ab} \propto \int dE E^3 F^{ab}(x,E) = 0 \]

\[ p^a = E(u^a + e^a) \quad (= \hbar k^a) \]
General intensity multipoles:

Lowest ones

\[ I = \rho, \quad I_a = q_a, \quad I_{ab} = \pi_{ab}. \]

Then Liouville becomes:

\[
0 = \dot{I}_{\langle A_{\ell} \rangle} + \frac{4}{3} \Theta I_{\langle A_{\ell} \rangle} + \frac{\ell}{(2\ell + 1)} \nabla \langle a_{\ell} I_{A_{\ell-1}} \rangle + \nabla^b I_{bA_{\ell}} + \frac{\ell(\ell + 3)}{(2\ell + 1)} \dot{u}_{\langle a_{\ell} I_{A_{\ell-1}} \rangle} \\
- (\ell - 2) \dot{u}^b I_{bA_{\ell}} - \ell \omega^b \eta_{bc}(a_{\ell} I_{A_{\ell-1}})^c - (\ell - 1) \sigma^{bc} I_{bcA_{\ell}} \\
+ \frac{5\ell}{(2\ell + 3)} \sigma^b \langle a_{\ell} I_{A_{\ell-1}} b \rangle - \frac{(\ell - 1)\ell(\ell + 2)}{(2\ell - 1)(2\ell + 1)} \sigma_{\langle a_{\ell} a_{\ell-1} I_{A_{\ell-2}} \rangle}, \tag{11.1}
\]

(Ellis, Treciokas, Matravers 1984; RM, Gebbie, Ellis 1999)
Quadrupole evolution in a general spacetime:

\[
\begin{align*}
\dot{\pi}^{(ab)}_\gamma &+ \frac{4}{3} \Theta \pi^{ab}_\gamma + \frac{8}{15} \rho_\gamma \sigma^{ab}_\gamma + \frac{2}{5} \nabla^{(a}_\gamma q^{b)}_\gamma + 2 \dot{u}^{(a}_\gamma q^{b)}_\gamma - 2 \omega^c \eta_{cd} (a \pi^{b)}_\gamma \\
&+ \frac{10}{7} \sigma^{c}_\gamma \pi^{b)_c}_\gamma + \nabla_c I^{abc} - \sigma_{cd} I^{abcd} = 0.
\end{align*}
\]

where \( \nabla_a = (\nabla_a)_\perp \)

Then we get zero shear:

\( \sigma_{ab} = 0 \)

Momentum conservation:

Then

\[
\text{curl } \nabla_a \rho_\gamma = -2 \dot{\rho}_\gamma \omega_a \Rightarrow \Theta \rho_\gamma \omega_a = 0
\]

\( \omega_a = 0 \)

e tc
More powerful result:

\[ CMB \text{ partial isotropy } + \text{ Copernican Principle } \rightarrow \text{ FLRW} \]

In a region, if
- collisionless radiation has vanishing dipole, quadrupole and octupole, \( F_a = F_{ab} = F_{abc} = 0 \),
- the radiation four-velocity is geodesic and expanding,
- there are pressure-free baryons and CDM, and dark energy in the form of \( \Lambda \), quintessence or a perfect fluid,
then the metric is FLRW in that region.

Ellis, Treciokas, Matravers 1985 (ETM theorem – generalized in Clarkson, RM 2010)

This is the best basis we have for (exact) homogeneity
(III) Testing the Copernican Principle

The CP is the foundation of theoretical homogeneity.

Can we test it?
* Sunyaev-Zeldovich tests
  Scattered photons at distant clusters will distort the CMB blackbody spectrum if there is anisotropy at the cluster

(Goodman 1995; Caldwell, Stebbins 2008)
* Constancy of curvature

A consistency test for homogeneity (Clarkson, Bassett, Lu 2009)

Luminosity distance

\[ D_L(z) = \frac{(1 + z)}{H_0 \sqrt{-\Omega_{K0}}} \sin \left( \sqrt{-\Omega_{K0}} \int_0^z \frac{dz'}{H(z')/H_0} \right) \]

implies

\[ \Omega_{K0} = \frac{[H(z)D'_L(z)]^2 - 1}{H_0 D_L(z)^2} \]

Differentiate the constant curvature. Then the quantity

\[ C_1(z) := 1 + H^2(z) \left[ D_L(z)D''_L(z) - D'_L(z)^2 \right] \]
\[ + H(z)H'(z)D_L(z)D'_L(z) \]

vanishes identically in Robertson-Walker spacetimes (for any matter content and any field equations)
(IV) Towards the realistic situation

(IV.1) The CMB is almost isotropic

Partial result – we get almost-homogeneity if we make additional assumptions

\[ \text{CMB almost-isotropy + Copernican Principle} \rightarrow \text{almost-FLRW} \]

In a region of an expanding universe with cosmological constant, if all observers comoving with the matter measure an almost isotropic distribution of collisionless radiation, and if some of the time and spatial derivatives of the covariant multipoles are also small, then the region is almost FLRW.

(Stoeger, RM, Ellis 1995)

Open question: can we remove the assumptions on derivatives using other observations?

(Nilsson et al 1999; Clarkson et al 2003; Rasanen 2009; Clarkson, RM 2010)
(IV.2) We also need:

- A statistical approach to isotropy
- A statistical formulation of the CP
- A better understanding of light propagation in a lumpy Universe
- A better understanding of how we average over inhomogeneities
Summary

What is the basis for homogeneity in the idealized case?

(I) Without assuming the Copernican Principle:
   (I.1) What do observations tell us directly
         - for any gravity theory? VERY LITTLE
         - for GR? PAST LIGHTCONE + INTERIOR
   (I.2) What can we say from isotropy of
         - matter observations? LTB GEOMETRY
         - the CMB? VERY LITTLE

(II) With the Copernican Principle: WE CAN TEST IT
   (II.1) What do isotropic matter observations tell us? FLRW
   (II.2) What do isotropic CMB observations tell us? FLRW

Open problems of the realistic case: almost-isotropy, etc.