Cosmological Simulations
Volker Springel

- Simulation predictions for the dark sector of $\Lambda$CDM
- Can we falsify $\Lambda$CDM with simulations?
- Exaflop computing
- Beyond the dark sector: Hydrodynamic simulations
- Challenges for galaxy formation simulations
The first slice in the CfA redshift survey

1100 galaxies in a wedge, 6 degrees wide and 110 degrees long

First CfA Strip

$26.5 \leq \delta < 32.5$

$m_\text{B} \leq 15.5$
Current galaxy redshift surveys map the Universe with several hundred thousand galaxies

2dF Galaxy Redshift Survey
The initial conditions for cosmic structure formation are directly observable

THE MICROWAVE SKY

The basic dynamics of structure formation in the dark matter

**BASIC EQUATIONS AND THEIR DISCRETIZATION**

- **Friedmann-Lemaitre model**
  \[ H(a) = H_0 \sqrt{a^{-3} \Omega_0 + a^{-2}(1 - \Omega_0 - \Omega_\Lambda) + \Omega_\Lambda} \]

- **Collisionless Boltzmann equation with self-gravity**
  \[ \frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{v}} = 0 \]
  \[ \nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \int f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \]

- **Hamiltonian dynamics in expanding space-time**
  \[ H = \sum_i \frac{p_i^2}{2m_i a(t)^2} + \frac{1}{2} \sum_{i,j} \frac{m_i m_j}{a(t)} \varphi(\mathbf{x}_i - \mathbf{x}_j) \]
  \[ \nabla^2 \varphi(\mathbf{x}) = 4\pi G \left[ -\frac{1}{L^3} + \sum_n \bar{\delta}(\mathbf{x} - nL) \right] \]

**Problems:**
- N is very large
- All equations are coupled with each other

---

**Gravitation**
(Newtonian approximation to GR in an expanding space-time)

- Dark matter is collisionless
- Monte-Carlo integration as N-body System

- 3N coupled, non-linear differential equations of second order
'Millennium' simulation
Springel et al. (2005)

$\Lambda$CDM

10,077,696,000 particles
$m = 8.6 \times 10^8 \ M_\odot / h$
Why are **cosmological simulations** of structure formation useful for studying the dark universe?

Simulations are the theoretical tool of choice for calculations in the non-linear regime.

They connect the (simple) cosmological initial conditions with the (complex) present-day universe.

**Predictions from N-body simulations:**
- Abundance of objects (as a function of mass and time)
- Their spatial distribution
- Internal structure of halos (e.g. density profiles, spin)
- Mean formation epochs
- Merger rates
- Detailed dark matter distribution on large and fairly small scales
- Galaxy formation models
- Gravitational lensing
- Baryonic acoustic oscillations in the matter distribution
- Integrated Sachs-Wolfe effect
- Dark matter annihilation rate
- Morphology of large-scale structure (“cosmic web”)
- ....
Simulations provide accurate measurements for halo abundance as a function of time

CONVERGENCE RESULTS FOR HALO ABUNDANCE

Simulated and observed large-scale structure in the galaxy distribution

MOCK PIE DIAGRAMS COMPARED TO SDSS, 2DFGRS, AND CFA-2

Springel et al. (2006)
The two-point correlation function of galaxies in the Millennium run is a very good power law.
The galaxy distribution is **biased** with respect to the mass distribution.

**GALAXY AND MASS CLUSTERING AT DIFFERENT EPOCHS**

- **Evolution Overview:**
  - **Galaxies:** Evolves little in time.
  - **Dark Matter:** Evolves strongly in time.

**Graphs:**
- **Galaxies:**
  - Curves for different redshifts (z = 0.00, 1.39, 5.72, 8.55).
- **Dark Matter:**
  - Curves for different redshifts (z = 0.00, 1.39, 5.72, 8.55).
The large-scale clustering pattern of halos and galaxies is already imprinted on the initial conditions.

TIME EVOLUTION OF THE MATTER AND GALAXY DISTRIBUTION
The baryonic wiggles remain visible in the galaxy distribution down to low redshift and may serve as a "standard ruler" to constrain dark energy.

DARK MATTER AND GALAXY POWER SPECTRA FROM THE MILLENIUM SIMULATION IN THE REGION OF THE WIGGLES

Springel et al. (2005)
Structure of the central cusp
Spherically averaged density profiles of dark matter halos have a nearly universal shape

DENSITY PROFILE AS A FUNCTION OF RADIUS

Fundamental importance for:
- Rotation curve of galaxies
- Internal structure of galaxy clusters
- Gravitational lensing
- DM annihilation
- Galaxy mergers
The logarithmic slope of the density profile does not show asymptotic behavior towards the core

**SLOPE OF THE DENSITY PROFILE AS A FUNCTION OF RADIUS**

\[
\frac{d \log \rho}{d \log r} = -2 \left( \frac{r}{r-2} \right)^{\alpha}
\]

Fit: \( \alpha = 0.19 \)
A consensus on the central structure of the cups seems to be emerging

**RECENT RESULTS FROM THE 'GHALO' SIMULATION OF THE ZURICH GROUP**

Stadel et al. (2009)

“The logarithmic slope of the radial density profile is close to a power law, gradually turning over to a slope of -0.8 at our innermost resolved point.”

“The Einasto profile also provides an excellent fit to the density profiles of the two simulations.”
Our simulations allow us to study the convergence of subhalo density profiles sphereically averaged density profiles in the AQ-A halo at different resolution.
Dark matter substructure
Zooming in on dark matter halos reveals a huge abundance of dark matter substructure.

DARK MATTER DISTRIBUTION IN A MILKY WAY SIZED HALO AT DIFFERENT RESOLUTIONS.
Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

**DARK MATTER IN A MILKY WAY SIZED HALO AT ULTRA-HIGH RESOLUTION**
The subhalo abundance per unit halo mass is surprisingly uniform

VELOCITY FUNCTION IN OUR DIFFERENT HALOS
The cumulative mass fraction in resolved substructures reaches about 12-13%, we expect up to ~18% down to the thermal limit.
The radial distribution of substructures is strongly antibiased relative to all dark matter, and independent of subhalo mass.

**RADIAL SUBSTRUCTURE DISTRIBUTION IN Aq-A-1**

Most subhalos are at large radii, subhalos are more effectively destroyed near the centre.

Subhalos are far from the Sun.

see also Diemand et al. (2007, 2008)
The local mass fraction in substructures is a strong function of radius.

**MASS FRACTION IN SUBSTRUCTURES AS A FUNCTION OF RADIUS IN HALO AQ-A**

![Graph showing the mass fraction in substructures as a function of radius in halo AQ-A. The graph displays multiple curves representing Aq-A-1 to Aq-A-5, with the solar circle marked at 1 kpc.]
Dark matter
annihilation predictions
Is the dark matter annihilation flux boosted significantly by dark matter substructures?

\[ L \propto \rho^2 \, dV \]
The annihilation luminosity from main halo and subhalos has a very different radial distribution.

The relative distribution of mass, main halo, and subhalo luminosity.

Lower mass limits of $10^5$ $M_\odot$, $10^6$ $M_\odot$, $10^7$ $M_\odot$, $10^8$ $M_\odot$.

Can use this to predict total luminosity down to the thermal limit.
Surface brightness profile of a typical subhalo with $V_{\text{max}} = 10$ km/s at different distances from the galactic center

SURFACE BRIGHTNESS PROFILE OF DIFFERENT SUBHALO COMPONENTS

The sub-sub component appears as a (extended) “disk” on the sky

The central surface brightness of the smooth component actually increases with smaller distance (because the concentration increases)
Dark matter annihilation can be best discovered with an optimal filter against a bright background.

**THE SIGNAL-TO-NOISE FOR DETECTION WITH AN OPTIMAL FILTER**

The optimal filter is proportional to the signal:

\[ S/N = \sqrt{\tau A_{\text{eff}}} \left[ \int \frac{n_\gamma^2(\theta, \phi)}{n_\gamma(\theta, \phi) + b_\gamma(\theta, \phi)} \, d\Omega \right]^{1/2} \]

The background dominates, then:

- **Main halo's smooth component:**
  \[ (S/N)_{\text{MainSm}} = f_{\text{MainSm}} \left( \frac{\tau A_{\text{eff}}}{b_\gamma} \right)^{1/2} \frac{F}{\theta_h} \]

- **Subhalo's smooth component:**
  \[ (S/N)_{\text{SubSm}} = f_{\text{SubSm}} \left( \frac{\tau A_{\text{eff}}}{b_\gamma(\bar{\alpha})} \right)^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}} \]

- **Sub-substructure of a subhalo:**
  \[ (S/N)_{\text{SubSub}} = f_{\text{SubSub}} \left( \frac{\tau A_{\text{eff}}}{b_\gamma(\bar{\alpha})} \right)^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}} \]

**S/N ~ F / \theta**

**S/N ~ L / r_h d**
Detectability of different annihilation emission components in the Milky Way

S/N for detecting subhalos in units of that for the main halo

30 highest S/N objects, assuming the use of optimal filters

\[ S/N \propto CV_{\text{max}}^4 / (r_{\text{half}}^2 d) \]

Highest S/N subhalos have 1% of S/N of main halo
Highest S/N subhalos have 10 times S/N of known satellites
Substructure of subhalos has no influence on detectability
The velocity distribution of dark matter at the Sun's position shows residual structure.

**DISTRIBUTION OF VELOCITY COMPONENTS AND VELOCITY MODULUS**

Vogelsberger et al. (2009)
Wiggles in the distribution of the modulus of the velocity point to residual structure in energy space

DISTRIBUTION OF THE VELOCITY MODULUS

\[ f(v) \times 10^{-3} \]

\[ v \text{ [km s}^{-1}\text{]} \]
The wiggles are the same in well-separated boxes at the same radial distance, and are reproduced in simulations of different resolution.
Cosmological N-body simulations have grown rapidly over the last four decades. "N" as a function of time.

- Computers double their speed every 18 months (Moore's law).
- N-body simulations have doubled their size every 16-17 months.
- Recently, growth has accelerated further. The Millennium Run should have become possible in 2010 – we have done it in 2004!
Millennium-XXL

Largest N-body simulation ever

303 billion particles

$L = 3 \text{ Gpc/h}$

$\sim 700$ million halos at $z=0$

$\sim 25$ billion (sub)halos in mergers trees

$m_p = 6.1 \times 10^9 \, M_\odot/h$

12288 cores, 30 TB RAM on Supercomputer JuRoPa in Juelich

2.7 million CPU-hours
Are the presently known high-mass clusters still consistent with $\Lambda$CDM?

Detection of one violating cluster would invalidate $\Lambda$CDM

Holz & Perlmutter (2010) argued XMMU J2235.3-2557 to be inconsistent with $\Lambda$CDM at 3$\sigma$.

Boyle, Jimenez & Verde (2010) argue that $\sigma_8$ would have to be $\sim$4$\sigma$ higher to accommodate massive clusters. Suggest non-Gaussian ICs as a solution.
Diversity in the Extreme

THE MOST MASSIVE AND RAREST CLUSTERS FOLLOW THE SCALING RELATIONS EXPECTED FROM MORE ABUNDANT SMALLER SYSTEMS

So far reported massive clusters not in conflict with $\Lambda$CDM (yet)
The existence of a firm upper limit for the properties of clusters is a strong and falsifiable prediction of $\Lambda$CDM.

**ABUNDANCE OF CLUSTERS USING DIFFERENT OBSERVATIONAL PROBES**

- Virial Mass
- X-Ray
- Lensing
- tSZ

$z = 0$

No objects above these limits should be detectable by any technique.
Trouble ahead in the Exaflop regime
ENIAC, 1946

Zuse Z3, 1941

Performance: ~ 1000 Flops
Currently the fastest supercomputers carry out about ~1 Petaflop, which are one thousand billion floating point operations per second.

### Jaguar: Department of Energy Leadership computer
**Designed for science from the ground up**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak performance</td>
<td>1.645 petaflops</td>
</tr>
<tr>
<td>System memory</td>
<td>362 terabytes</td>
</tr>
<tr>
<td>Disk space</td>
<td>10.7 petabytes</td>
</tr>
<tr>
<td>Disk bandwidth</td>
<td>240 gigabytes/second</td>
</tr>
<tr>
<td>Interconnect BW</td>
<td>532 terabytes/second</td>
</tr>
<tr>
<td>Number cores</td>
<td>181504</td>
</tr>
</tbody>
</table>

60 years later - \(10^{12}\) times faster
How long would the Millennium-XXL take on a Exaflop Supercomputer at peak performance? 15 min

One of the main problems: 
*Power Consumption*

Petaflop Computer:  6 MW

Exaflop Computer:  \( \sim \text{GW} \) ?

Need to get this down to 20-40 MW.
The number of cores on the top supercomputers grows exponentially

EXTREME GROWTH OF PARALLELISM

Exponential growth in parallelism for the foreseeable future
Challenges in exascale computing

CAN WE HAVE THE CAKE AND EAT IT?

- Memory per core decreasing.

- Applications need to deal with multiple hierarchies of memory.
  (especially on GPU-accelerated or hybrid systems)

- On systems with $>10^6$ cores, need fault-tolerant algorithms and codes.
  (resilience has to be built into simulation codes)

Cost of data access relative to floating point ops drastically increasing.
Typical astrophysics codes run only at $\sim10\%$ of the peak performance – and its getting worse with time.

*)None of our existing codes will survive and run on exascale platforms.*

*)Astrophysics may be left behind in using these systems.*
Hydrodynamical simulations
Dynamics of structure formation in baryonic matter

**BASIC EQUATIONS**

Astrophysical plasmas are extremely thin, with (usually) negligible viscosity

\[
\begin{align*}
\frac{\partial \rho_c}{\partial t} + \frac{1}{a} \nabla_c (\rho_c \mathbf{v}) &= 0 \\
\frac{\partial (\rho_c \mathbf{v})}{\partial t} + \frac{1}{a} \nabla_c [(\rho_c \mathbf{v} \mathbf{v}^T + P_c) \mathbf{v}] &= -H(a) \rho_c \mathbf{v} - \frac{\rho_c}{a^2} \nabla_c \Phi_c \\
\nabla_c^2 \Phi_c &= 4\pi G \left[ \rho_c(\mathbf{x}) - \bar{\rho}_c \right]
\end{align*}
\]

**Important hydrodynamical processes**
- Shock waves
- Turbulence
- Radiative transfer
- Magnetic fields
- Star formation
- Supernova explosions
- Black holes, etc...
Supersonic motion creates shock waves

SHOCK WAVES OF A BULLET TRAVELLING IN AIR
Weak lensing mass reconstructions have confirmed an offset between mass peaks and X-ray emission.

MASSE CONTOURS FROM LENSING COMPARED TO X-RAY EMISSION

Clowe et al. (2006)

Magellan Optical Image

500 ksec Chandra exposure

weak lensing mass contours overlaid
NASA Press Release Aug 21, 2006:

1E 0657-56: NASA Finds Direct Proof of Dark Matter
Fitting the density jump in the X-ray surface brightness profile allows a measurement of the shock's Mach number

Markevitch et al. (2006)

shock strength: \( M = 3.0 \pm 0.4 \)

shock velocity: \( v_s = 4700 \text{ km/s} \)

Usually, shock velocity has been identified with velocity of the bullet.
How rare is the bullet cluster?

DISTRIBUTION OF VELOCITIES OF THE MOST MASSIVE SUBSTRUCTURE IN THE MILLENNIUM RUN

Hayashi & White (2006)

Adopted mass model from Clowe et al. (2004):

NFW-Halo with:

\[ M_{200} = 2.96 \times 10^{15} \, M_\odot \]
\[ R_{200} = 2.25 \, \text{Mpc} \]
\[ V_{200} = 2380 \, \text{km/sec} \]
\[ V_{\text{shock}} = 4500 \, \text{km/sec} \]
\[ \frac{V_{\text{sub}}}{V_{\text{shock}}} = 1.9 \quad \text{chance: } 10^{-2} \]

But, revised data from Clowe et al. (2006) and Markevitch et al. (2006):

\[ M_{200} = 1.5 \times 10^{15} \, M_\odot \]
\[ V_{200} = 1680 \, \text{km/sec} \]
\[ V_{\text{shock}} = 4740 \, \text{km/sec} \]
\[ \frac{V_{\text{sub}}}{V_{\text{shock}}} = 2.8 \quad \text{chance: } 10^{-7} \]
A simple toy merger model of two NFW halos on a zero-energy collision orbit

PARAMETERS OF A BASIC TOY MODEL

Mass model from Clowe et al. (2006):

- NFW-Halos
  - $f_{\text{gas}} = 0.17$
  - $1870 \text{ km/sec}$
  - $-187 \text{ km/sec}$

$M_{200} = 1.5 \times 10^{14} M_{\odot}$
$R_{200} = 1.1 \text{ Mpc}$
$c = 7.2$
$V_{200} = 780 \text{ km/sec}$

$M_{200} = 1.5 \times 10^{15} M_{\odot}$
$R_{200} = 2.3 \text{ Mpc}$
$c = 2.0$
$V_{200} = 1680 \text{ km/sec}$
VIDEO OF THE TIME EVOLUTION OF A SIMPLE BULLET CLUSTER MODEL

- X-rays
- Temperature
- Shocks
- x-Velocity
Drawing the observed X-ray map and the simulation images with the same color-scale simplifies the comparison.

SIMULATED X-RAY MAP COMPARED TO OBSERVATION

Candra 500 ks image

bullet cluster simulation

Springel & Farrar (2007)
The model also matches the observed temperature and mass profiles.

**COMPARISON OF SIMULATED TEMPERATURE AND MASS PROFILE WITH OBSERVATIONS**

Data from Markevitch et al. (2006)

Data from Bradac et al. (2006)
Despite a shock speed of \( \sim 4500 \text{ km/s} \), the bullet moves considerably slower.

**VELOCITIES AND POSITIONS OF MAIN BULLET CLUSTER FEATURES AS A FUNCTION OF TIME**

- Shock speed: \( 4500 \text{ km/s} \)
- Pre-shock infall: \( -1100 \text{ km/s} \)
- Shock speed relative to bullet: \( -800 \text{ km/s} \)
- Speed of bullet: \( 2600 \text{ km/s} \)
Uncertainties and errors in hydrodynamical numerical techniques
A cloud moving through ambient gas shows markedly different long-term behavior in SPH and Eulerian mesh codes.

DISRUPTION OF A CLOUD BY KELVIN-HELMHOLTZ INSTABILITIES

Agertz et al. (2007)
Moving-mesh hydrodynamics with AREPO

Volker Springel

Heidelberg Institute for Theoretical Studies

UNIVERSITÄT HEIDELBERG
Voronoi and Delaunay tessellations provide unique partitions of space based on a given sample of mesh-generating points.

**BASIC PROPERTIES OF VORONOI AND DELAUNAY MESHES**

- Each Voronoi cell contains the **space closest** to its generating point.
- The Delaunay triangulation contains only triangles with an **empty circumcircle**. The Delaunay triangulation maximizes the minimum angle occurring among all triangles.
- The centres of the circumcircles of the Delaunay triangles are the vertices of the Voronoi mesh.
- In fact, the two tessellations are the topological **dual graph** to each other.
The fluxes are calculated with an exact Riemann solver in the frame of the moving cell boundary

**SKETCH OF THE FLUX CALCULATION**

The motion of the mesh generators uniquely determines the motion of all cell boundaries.
On large scales, the code produces very similar results as standard SPH techniques.
AREPO produces much better galaxy morphologies than SPH for identical initial conditions.

GAS AND TEMPERATURE FIELDS IN A COSMOLOGICAL HYDRODYNAMIC SIMULATION
AREPO produces much better galaxy morphologies than SPH for identical initial conditions.

GAS AND TEMPERATURE FIELDS IN A COSMOLOGICAL HYDRODYNAMIC SIMULATION
The moving-mesh approach can also be used to realize arbitrarily shaped, moving boundaries.

**STIRRING A COFFEE MUG**
The challenge to simulate galaxy formation
Hydrodynamical simulations aim to predict:
- Morphology of galaxies
- Fate of the diffuse gas, WHIM, metal enrichment
- X-ray atmospheres in halos
- Turbulence in halos and accretion shocks
- Large-scale regulation of star formation in galaxies through feedback processes from stars and black holes
- Transport processes (e.g. conduction)
- Radiative transfer
- Dynamical transformations (e.g. ram-pressure stripping)
- Magnetic fields
A long standing issue in galaxy formation theory: The shapes of the CDM halo mass function and the galaxy luminosity function are very different.

THE OBSERVED LF COMPARED TO THE SHAPE OF THE CDM HALO MASS FUNCTION

van den Bosch et al. (2004)
Abundance matching gives the expected halo mass – stellar mass relation in $\Lambda$CDM

** STELLAR MASSES FROM SDSS/DR7 MATCHED TO $\Lambda$CDM SIMULATION EXPECTATIONS **

**Assumption:** Stellar mass is monotonically increasing with halo mass

Current cosmological hydrodynamic simulations have trouble to explain such a low galaxy formation efficiency

GALAXY FORMATION EFFICIENCY AS A FUNCTION OF HALO MASS


Sawala & White (2010)
Summary points

- Direct numerical simulations have become indispensable for studying the non-linear growth of structures in $\Lambda$CDM cosmologies.

- Current numerical techniques allow high-resolution simulations with an unprecedented dynamic range. One presently reaches $N > 10^{11}$, with a dynamic range of $10^5 – 10^7$ in 3D.

- The future observation of a sufficiently massive cluster may easily rule out the $\Lambda$CDM model. The predicted satellite population may still be in tension with the observations.

- Understanding galaxy formation physics remains a serious challenge in $\Lambda$CDM, both at the faint and the bright end.

- Radiative magneto-hydrodynamics codes that follow structure formation still in their infancy.

- Exaflop computers arrive at the end of the decade, but it is highly questionable whether astrophysics can use them at scale.
Discussion points

Future codes cannot be written by lonely graduate students any more...

They require large, interdisciplinary teams with sustained funding.

Biggest challenges in arriving at better codes for galaxy formation:

• Cope with huge dynamic range in time and space.

• Avoid work-load imbalance losses when on $10^3$ to $10^7$ cores.

• Code validation.

• Simulation data management.

• Dragging bright physics students into computational cosmology.