Quantum Darwinism in an Everyday Environment

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with

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Ref: C. J. Riedel, W. H. Zurek, Phys. Rev. Lett. 105, 020404 (2010).

Outline

- Why you should care about quantum Darwinism
  - Decoherence is only part of the story
  - Analogy with special relativity
  - What quantum Darwinism gives you
- Mathematical development (easy)
- Theoretical results
  - Previous work
  - The everyday environment
  - Generalizations

Classical observers

- In a classical universe there are few limits on observers. The universe is…
  - Definitive: All observables have definite values at all times
  - Unique: there is one reality/branch/history
  - Robust: in principle, observers can make measurements which disturb systems arbitrarily little
  - Objective: measurements by different observers agree within error

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Quantum observers

- In a quantum universe, all of this goes out the window. The universe is no longer…
  - ~~Definitive~~: systems need not be in eigenstates of observables. Further, there is no consistent way to pretend that they were.
  - ~~Unique~~: different measurement outcomes become correlated with different states of the observer. Further, a third party need not be correlated with the outcomes.
  - ~~Robust~~: if system is not diagonal in the basis of measurement, the observer disturbs the system. Further, without knowledge about the state of the system, the observer *almost always* disturbs.
  - ~~Objective~~: different observers may disagree after taking non-commuting measurements.

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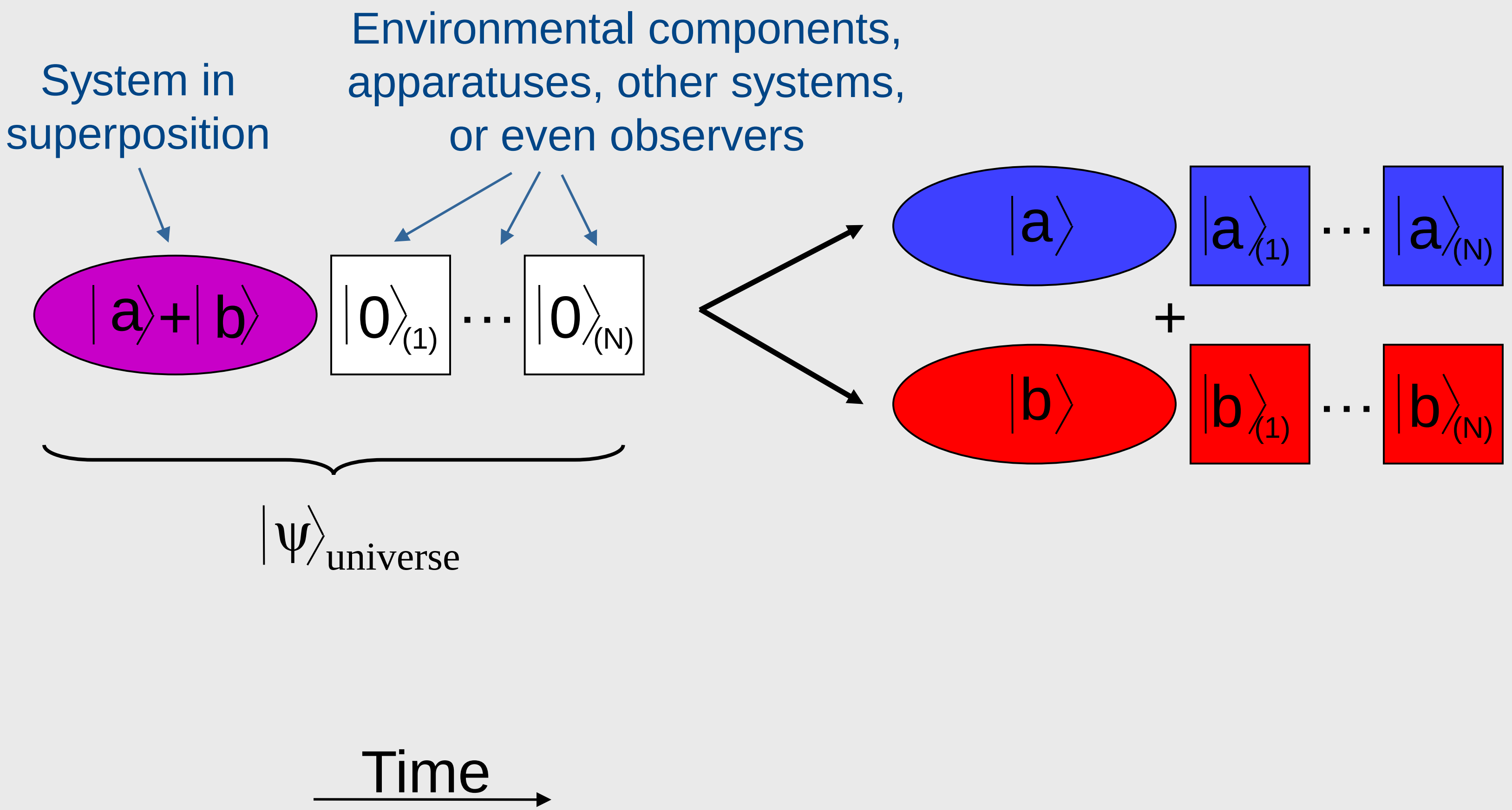
Decoherence

- Decoherence is a *partial* solution. In the limit of "good" decoherence, the universe is…
  - Definitive: a certain *preferred* set of observables have approximately definite values at all times
  - Unique: given sufficiently large environments, decoherent outcomes never interact, so outcomes *appear* unique
  - Robust (…conditionally): measurements do not disturb the system ***if*** the observer measures in the preferred basis
  - Objective (…conditionally): two observers will agree on measurement outcomes ***if*** they both measured in the preferred basis

What we want: Global branching



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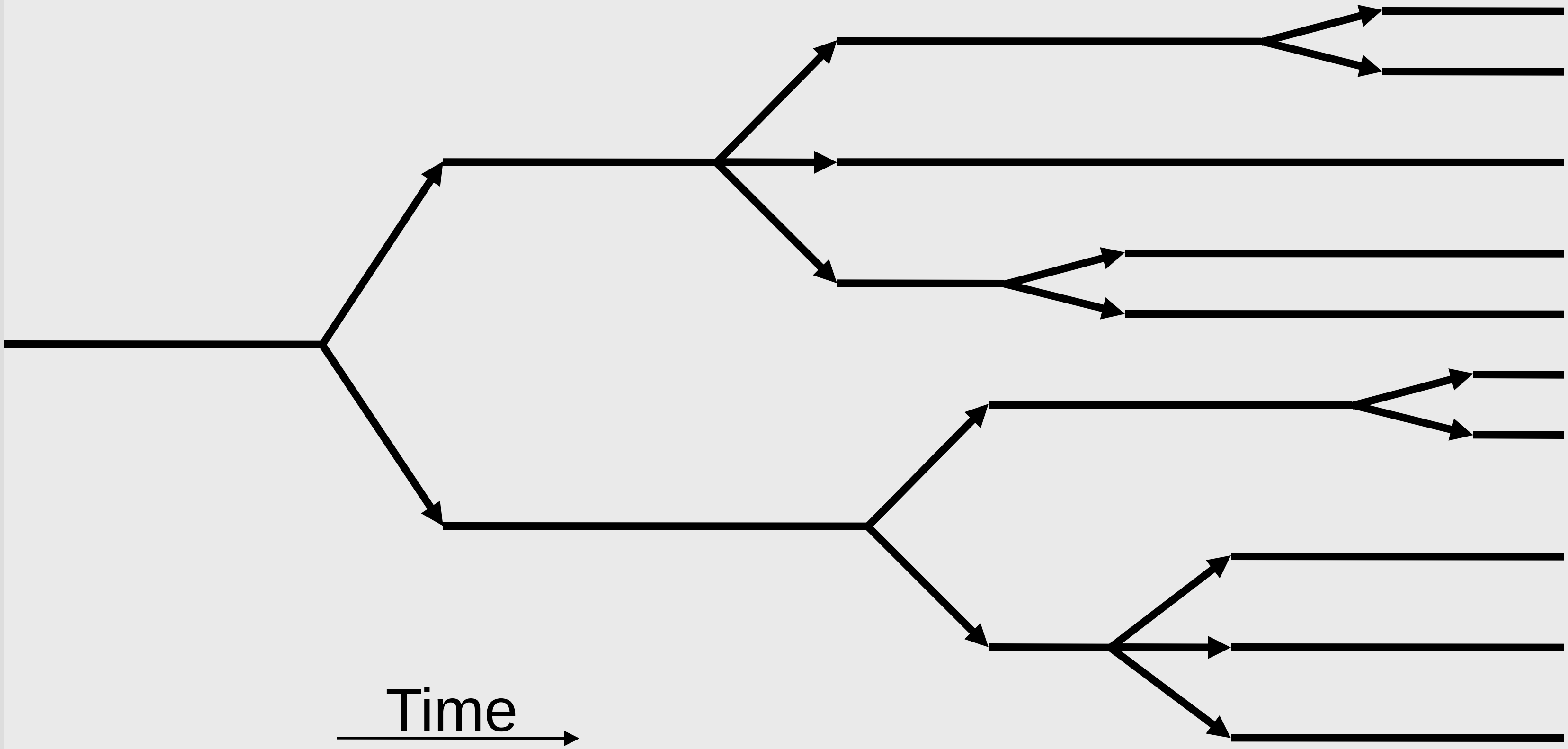
What we want: Global branching



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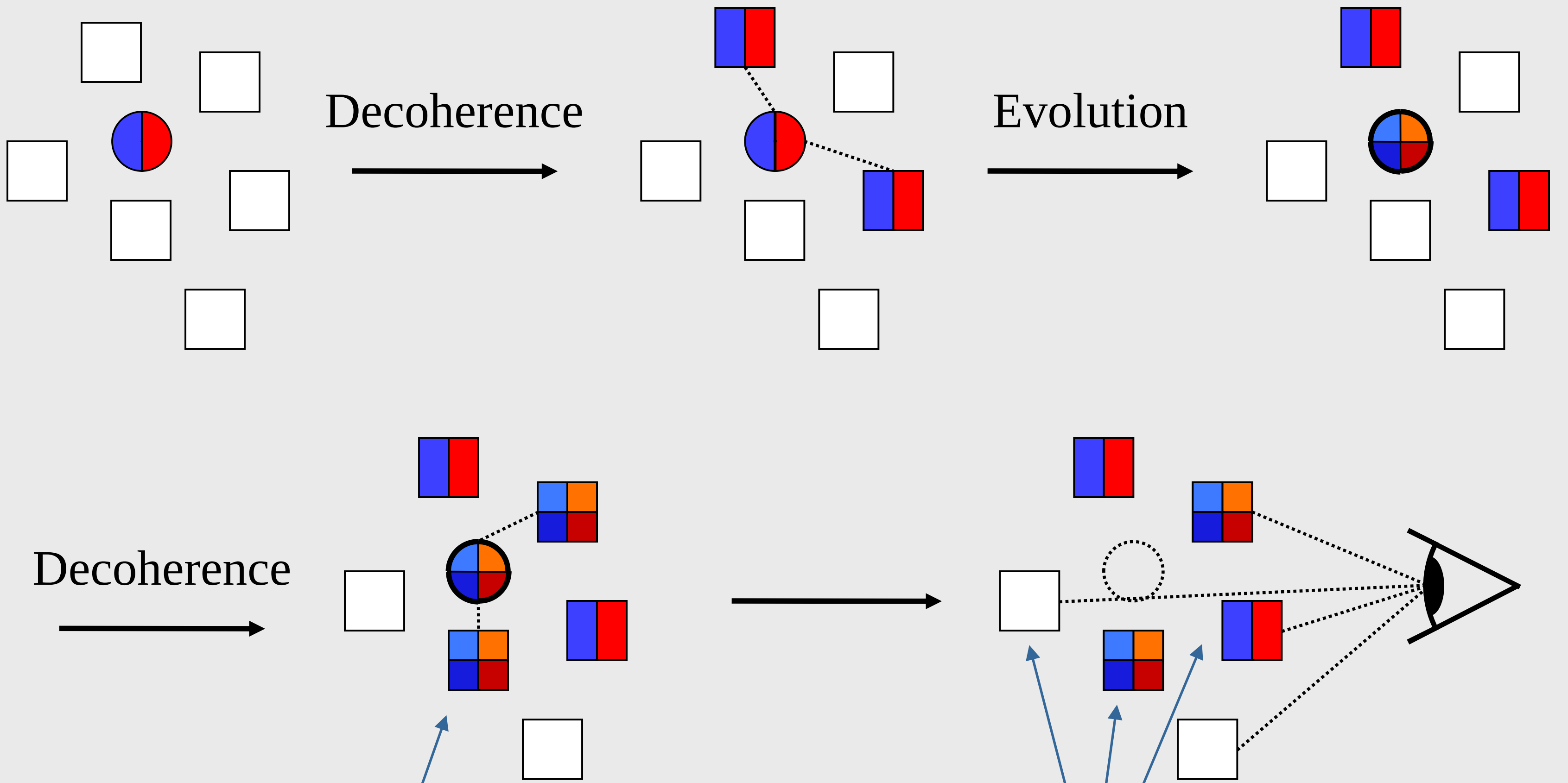
Decoherence guarantees only local branching



Just one environmental interaction is sufficient to decohere system...

...but there is no global branching.

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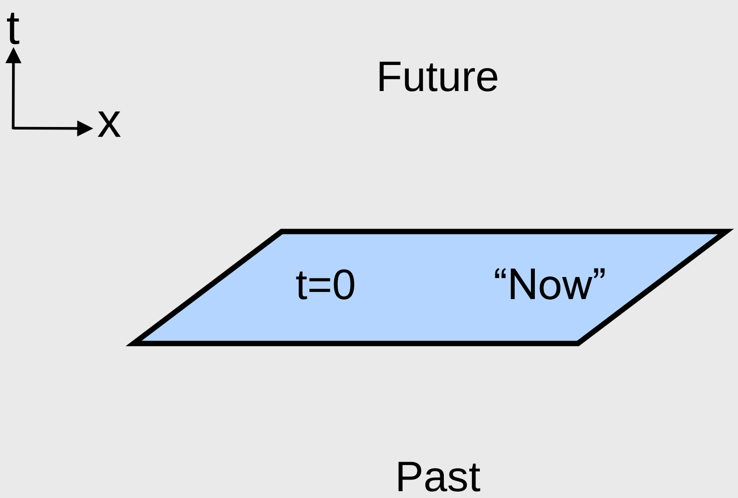
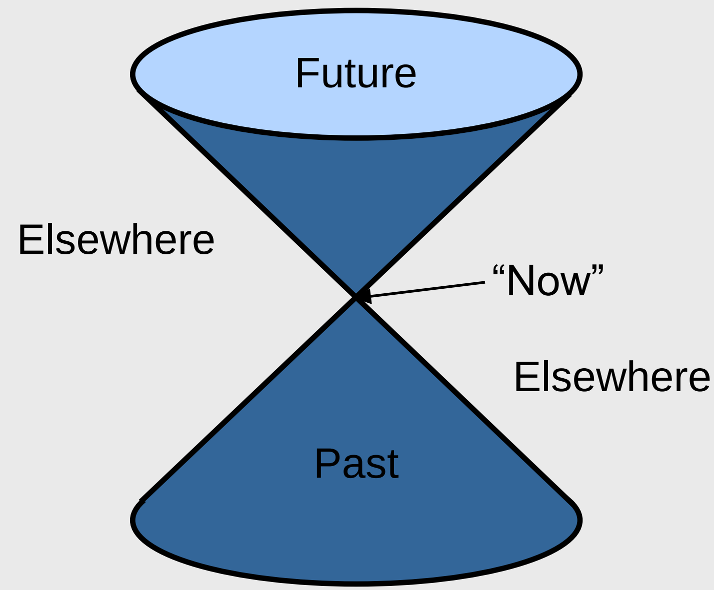
Special relativity analogy

Pre-relativity

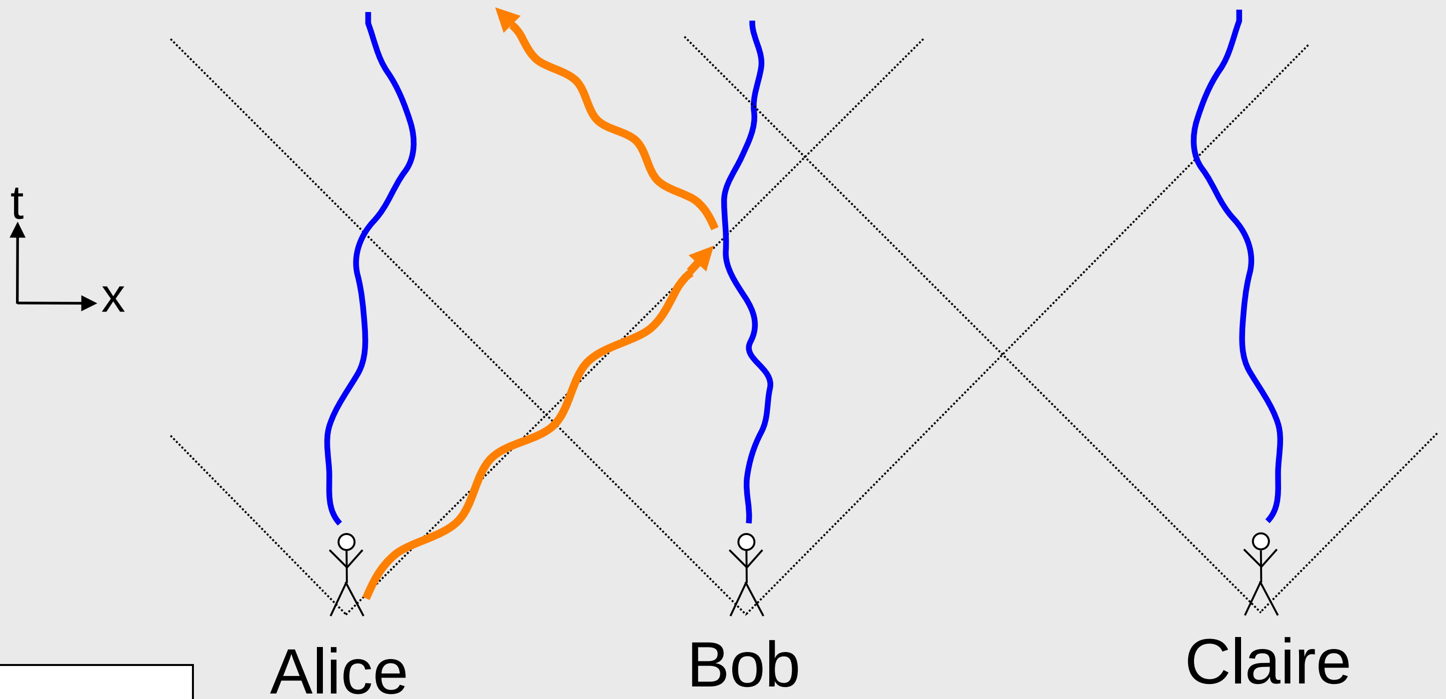


Special relativity



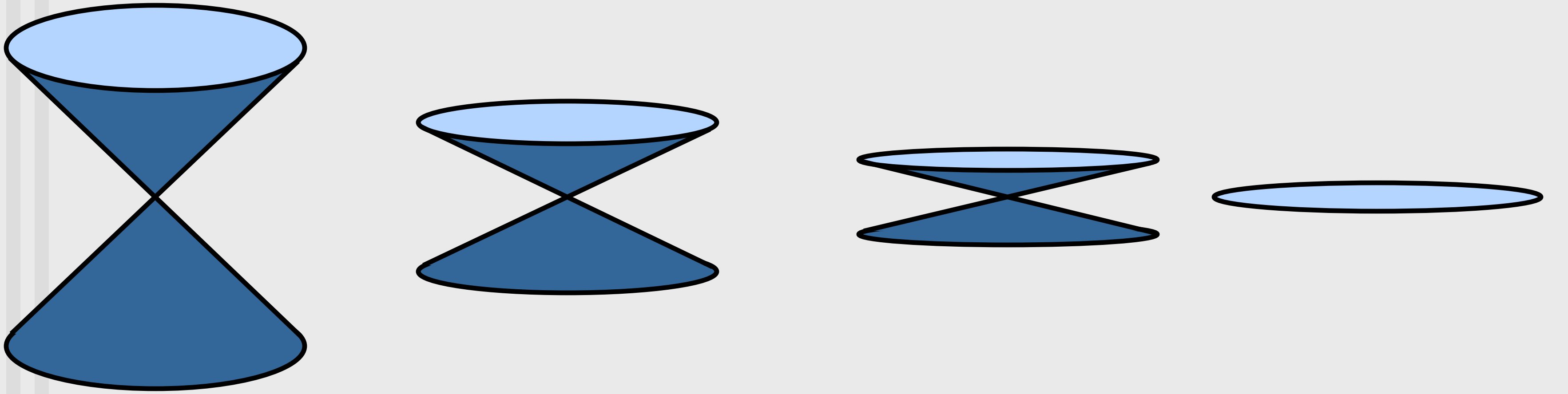
Special relativity analogy

Observers are isolated. They can exchange delayed messages, but cannot interact *continuously* with each other.

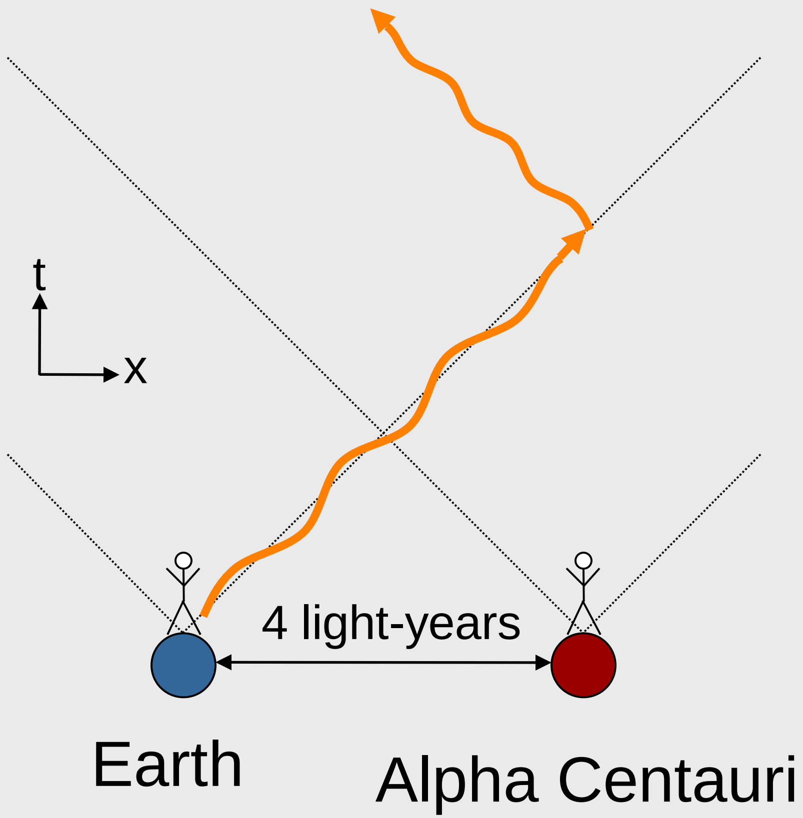


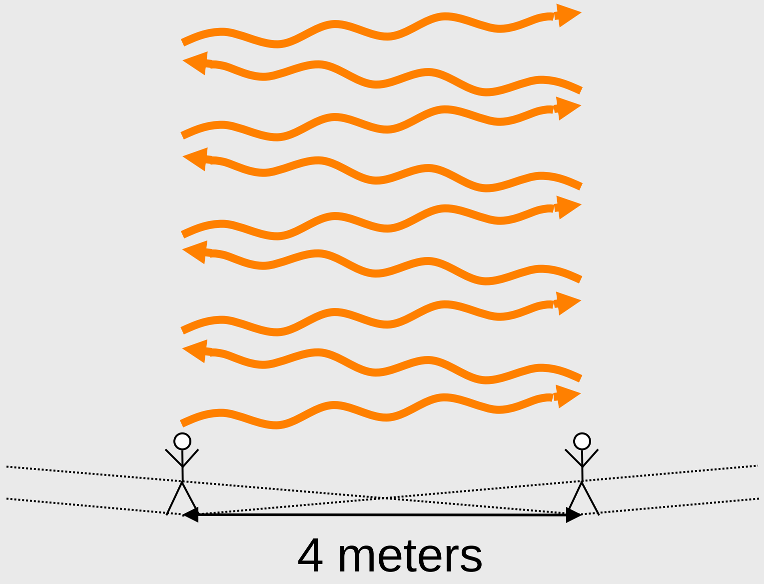
Special relativity analogy

$$c \longrightarrow \infty$$



Special relativity analogy



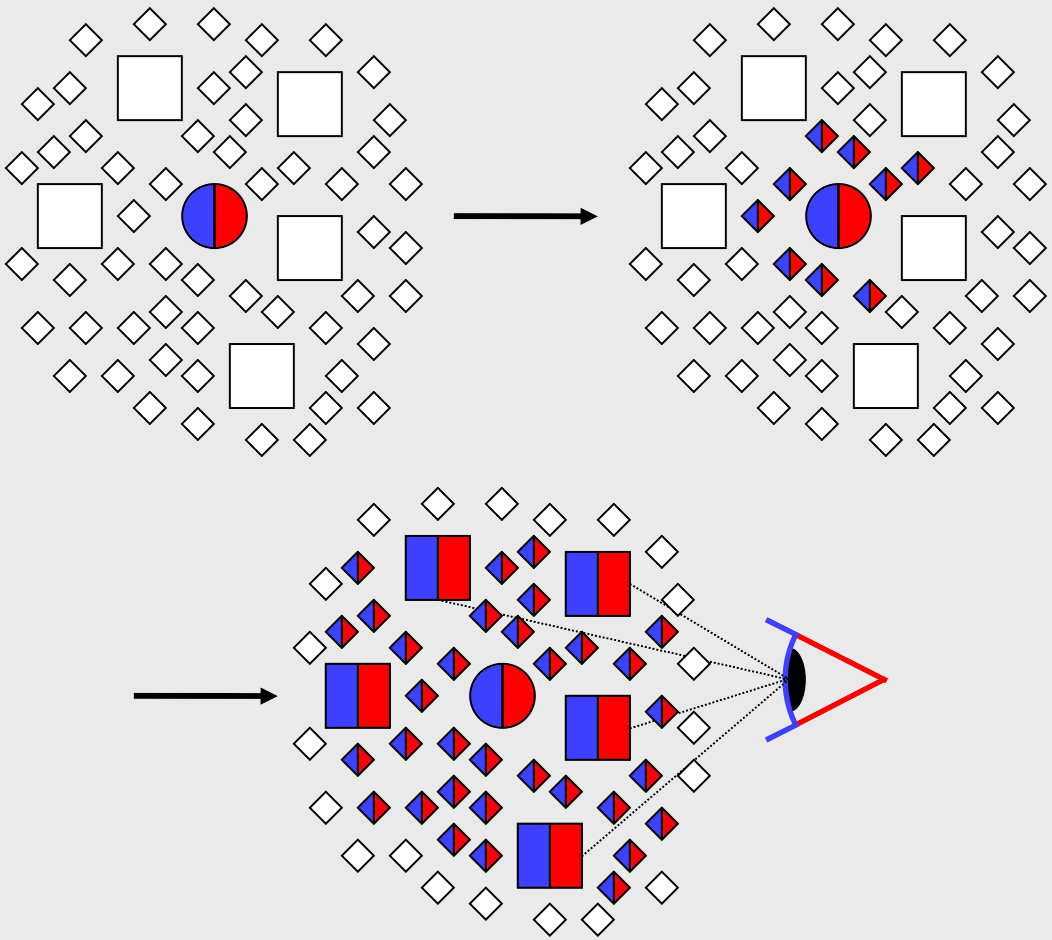


Key idea: c is very fast compared to length and time scales of everyday systems. This allows observers to exchange information back-and-forth more *rapidly than systems typically evolve*.

Quantum Darwinism

- Observers do not typically interact directly with systems
- Rather, systems and observers are bathed in an (untracked) environment
- Through decoherence, many copies of records about the state of the system are imprinted in the environment...often *rapidly*
- The environment carries these records away where they may be accessed by observers
- Many redundant copies ensure observers can agree (i.e. “objectivity”)

Information proliferation



How is this described?

$$
\begin{aligned}
\mathcal{H} &= \mathcal{S} \otimes \mathcal{E} \\
&= \mathcal{S} \otimes [\mathcal{E}_1 \otimes \cdots \otimes \mathcal{E}_N]
\end{aligned}
$$

$$
\underbrace{|\psi_{\mathcal{S}}^0\rangle\langle\psi_{\mathcal{S}}^0|}_{\rho_S^0} \otimes \underbrace{[\rho_{\mathcal{E}_1} \otimes \cdots \otimes \rho_{\mathcal{E}_N}]}_{\rho_E^0} \quad \rightarrow \quad \rho_{\mathcal{S}\mathcal{E}}
$$

How is information quantified?

Von Neumann entropy

$$H = H[\rho] = -\mathrm{Tr}[\rho \ln \rho]$$

Mutual Information

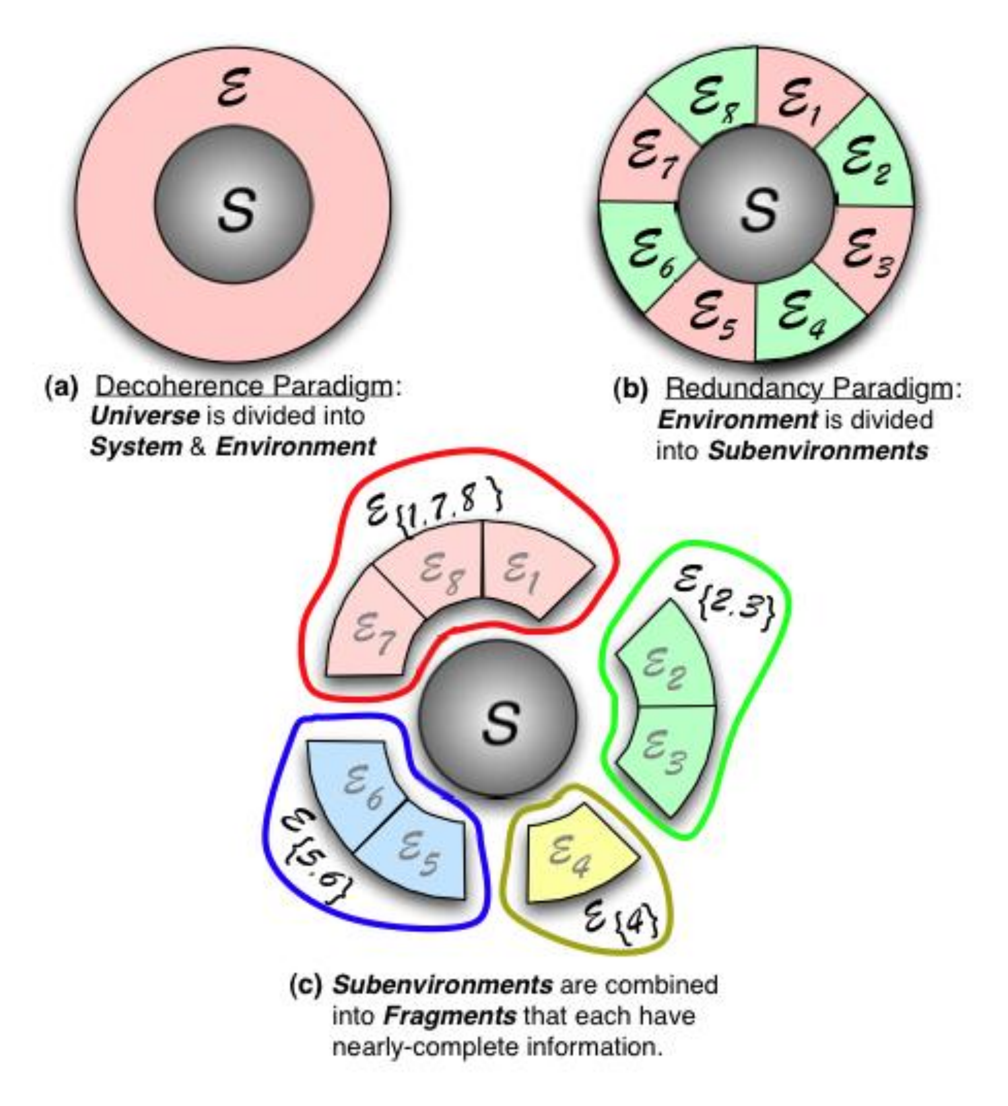
$$I_{SE} = H_S + H_E - H_{SE}$$

$$(H_{\mathcal{O}} = H[\rho_{\mathcal{O}}] \quad \text{for} \quad \mathcal{O} = \mathcal{S}, \mathcal{E}, \mathcal{S}\mathcal{E})$$

Fragments

- The mutual information I_{SE} gives the total correlation between the state of the system and the environment
- Observers do not access complete environment
- We want to know about redundant copies
- For this, we need a partitioning of the environment into fragments
- Most environments have natural, spatially local fragments, e.g.
  - The photons in this room
  - Molecules in a gas
  - Oscillating degrees of freedom in a material mechanically coupled to the system

Fragmentation



Fragmentation

Fragment size : $f \in [0,1]$ (typically, $f \ll 1$)

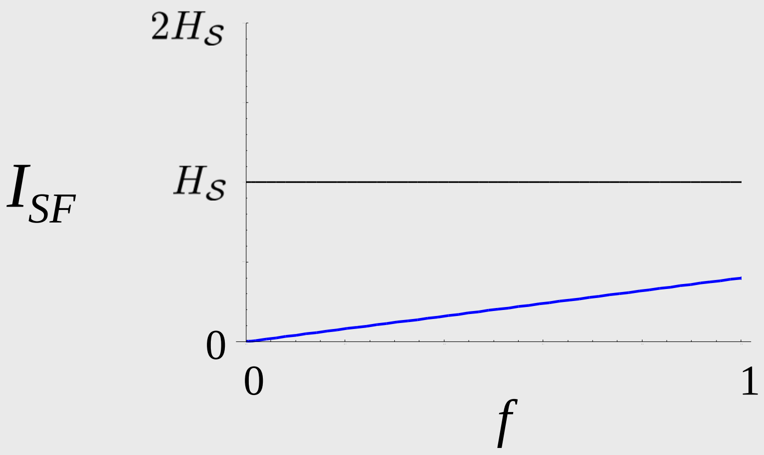
$$\begin{aligned}\mathcal{E} &= \mathcal{F} \otimes \bar{\mathcal{F}} \\ &= [\mathcal{E}_1 \otimes \cdots \otimes \mathcal{E}_{fN}] \otimes [\mathcal{E}_{fN+1} \otimes \cdots \otimes \mathcal{E}_N]\end{aligned}$$

$$I_{\mathcal{S}\mathcal{F}} = H_{\mathcal{S}} + H_{\mathcal{F}} - H_{\mathcal{S}\mathcal{F}}$$

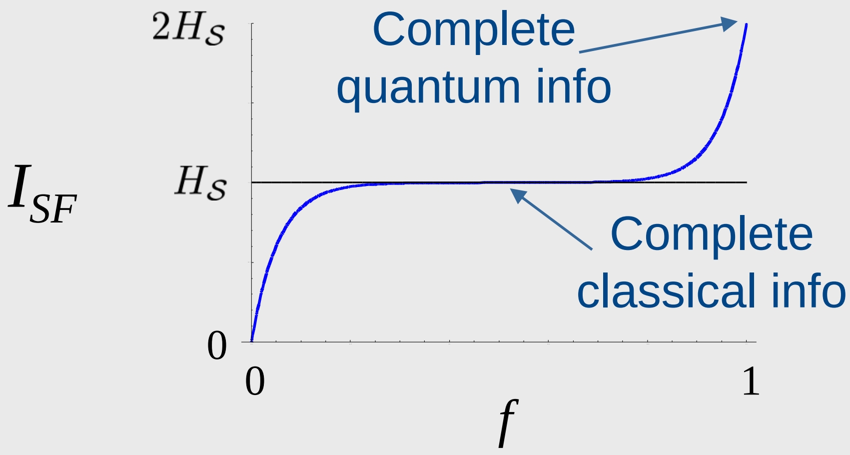
Partial information plots

- Mutual information (I_{SF}) vs. fragment size (f)
- Monotonically increasing with f
- Anti-symmetric for pure initial states

<u>Environment knows little</u>



<u>Environment knows lots</u>



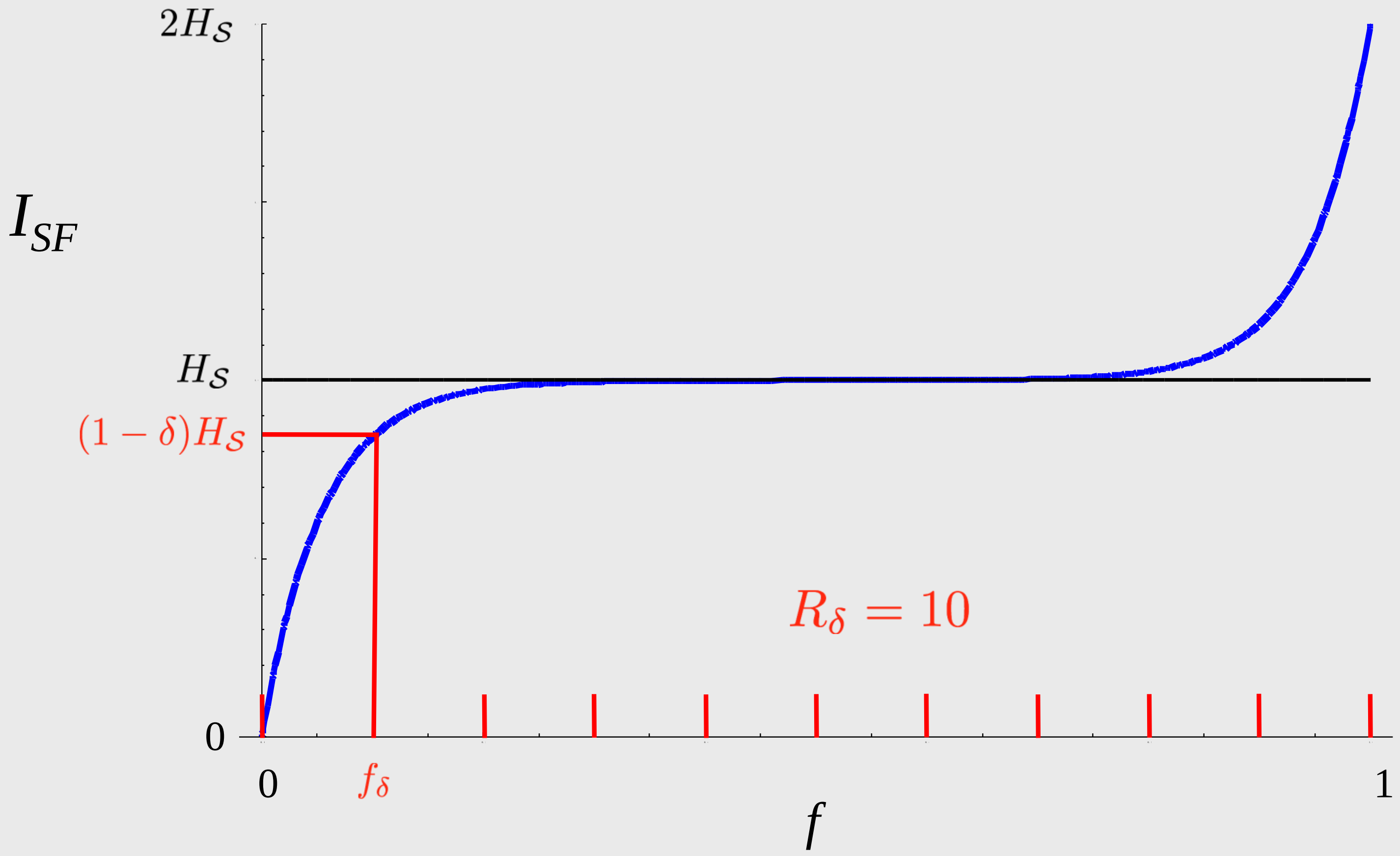
Information deficit and redundancy

- Monotonicity and antisymmetry imply no sensible fragment ($f < 0.5$) has full classical information
- Agrees with classical case: no records are perfect
- Define a fragment to be a “record” only up to some information deficit, $0 < \delta << 1$
- Define redundancy R_δ to be the total number of records in the environment:

$$R_\delta \equiv \frac{1}{f_\delta}, \qquad \text{where } f_\delta \text{ is smallest } f \text{ such that}$$

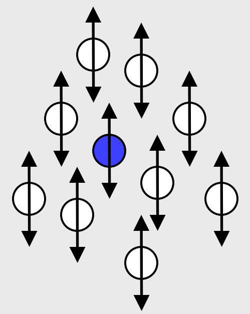
$$I_{\mathcal{S}\mathcal{F}}(f) \geq (1-\delta) H_{\mathcal{S}}$$

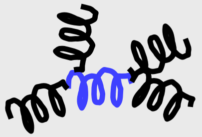
Information deficit and redundancy



Previous systems explored

- Single spin monitored by an (initially) pure spin environment[1]
- Single spin monitored by a mixed spin environment[2,3]
- Harmonic oscillator monitored by a pure environment of oscillators[4,5]





[1] R. Blume-Kohout and W. H. Zurek, Found. Phys. 35, 1857 (2005).

[2] M. Zwolak, H. T. Quan, and W. H. Zurek, Phys. Rev. Lett. 103, 110402 (2009).

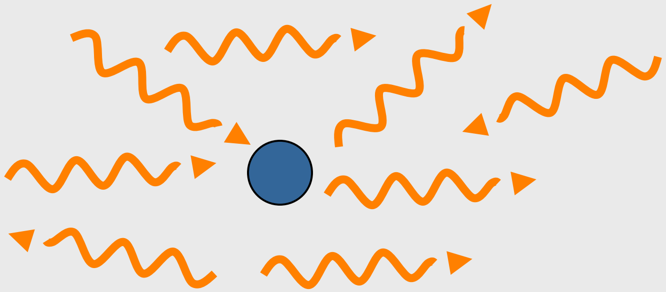
[3] M. Zwolak, H. Quan, and W. H. Zurek, Phys. Rev. A 81, 062110 (2010).

[4] R. Blume-Kohout and W. H. Zurek, Phys. Rev. Lett. 101, 240405 (2008).

[5] J. P. Paz and A. J. Roncaglia, Phys. Rev. A 80, 042111 (2009).

Everyday environment

- Seek model which is…
  - Found in real world
  - Computationally tractable
  - Not hampered by symmetries or size restrictions (for computational tractability) which prohibit the large redundancies we expect
- Collisional decoherence: an objected bathed in photons



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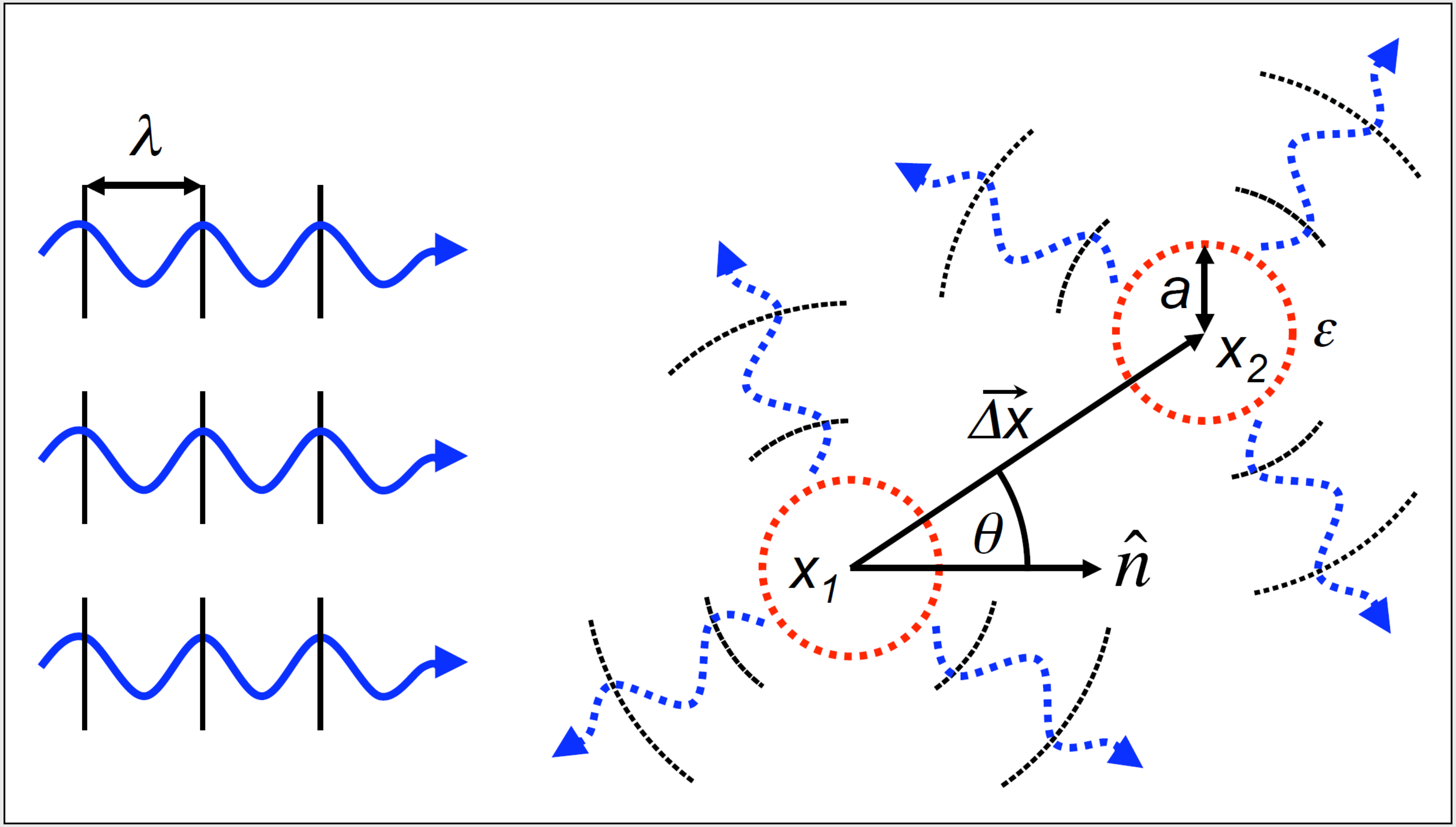
Collisional decoherence with photons



$$\Delta x, a \ll \lambda$$

S-matrix

infinite object mass

$$
\begin{aligned}
U|\vec{x}\rangle|\vec{k}\rangle &= |\vec{x}\rangle S_{\vec{x}}|\vec{k}\rangle \\
&= |\vec{x}\rangle e^{i\vec{x}(\vec{k}-\vec{\mathfrak{p}})} S_0|\vec{k}\rangle \\
&= |\vec{x}\rangle e^{i\vec{x}(\vec{k}-\vec{\mathfrak{p}})} S_0 \frac{1}{k}|k\rangle|\hat{k}\rangle \\
&= |\vec{x}\rangle e^{i\vec{x}(\vec{k}-\vec{\mathfrak{p}})} \frac{1}{k}|k\rangle S_0^{(k)}|\hat{k}\rangle \\
&\approx |\vec{x}\rangle e^{i\vec{x}(\vec{k}-\vec{\mathfrak{p}})} \frac{1}{k}|k\rangle (I + iT^{(k)})|\hat{k}\rangle
\end{aligned}
$$

system position eigenstate

photon momentum eigenstate

decompose Hilbert space

elastic scattering

long wavelength

In other words, scattering mixes photon angles, *not* energies

$$|\langle\hat{k}|T^{(k)}|\hat{k}'\rangle|^2 \propto \left(\frac{a}{\lambda}\right)^6 \left[1 + \cos^2\theta(\hat{k},\hat{k}')\right]$$

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Decoherence

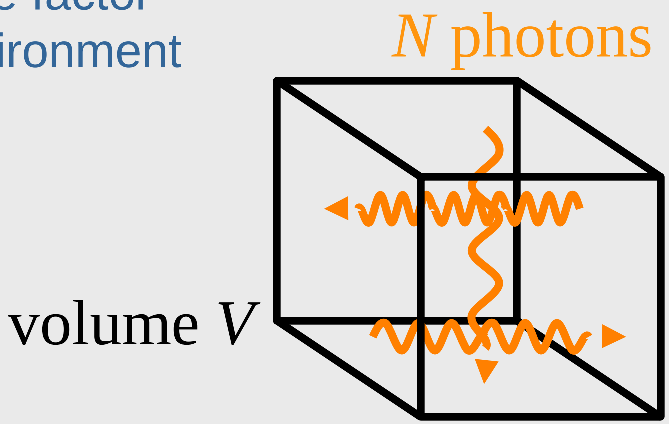
$$|\langle \vec{x_1}|\rho_S|\vec{x_2}\rangle|^2 = \gamma^N |\langle \vec{x_1}|\rho_S^0|\vec{x_2}\rangle|^2$$

decoherence factor
for single photon

Take $N, V \to \infty$ while holding photon density constant.

$\gamma \to 0$, but $\Gamma \equiv \gamma^N = e^{-t/\tau_D}$ remains finite.

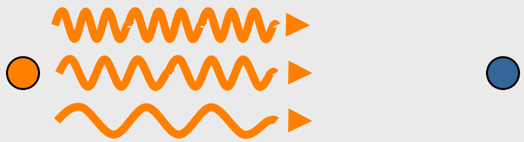
decoherence factor
for whole environment



Mixedness of environment

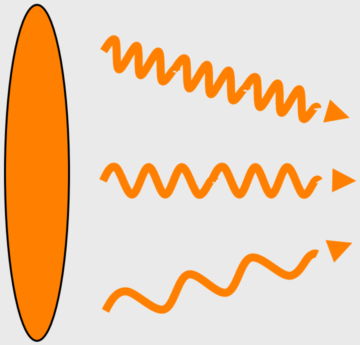
Monochromatic point sources

Thermal point sources

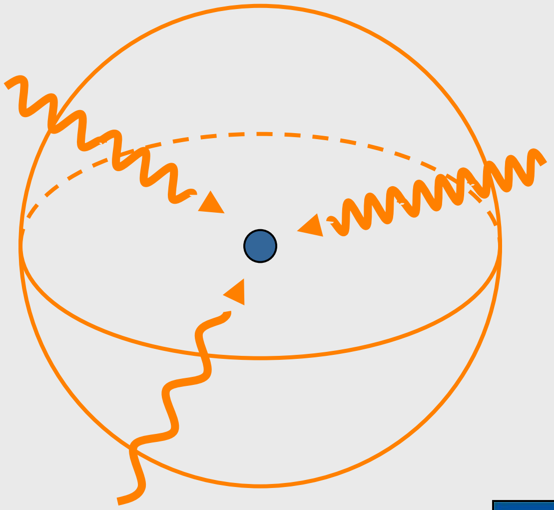


Thermal disk

new



Thermal isotropic



Initial state

- System: $|\psi_{\mathcal{S}}^0\rangle = |\vec{x}_1\rangle + |\vec{x}_2\rangle$ "cat" state
- Environment: $\rho_{\mathcal{E}}^0 = \bigotimes_{i=1}^N \rho_{\mathcal{E}_i}^0$ ← identical, incoherent photons

<u>Monochromatic point source</u>

$$\begin{aligned} \rho_{\mathcal{E}_i}^0 &= |\vec{k}\rangle\langle\vec{k}| \\ &= \frac{1}{k^2}|k\rangle\langle k| \otimes |\hat{k}\rangle\langle\hat{k}| \end{aligned}$$

<u>Thermal point source</u>

$$\begin{aligned} \rho_{\mathcal{E}_i}^0 &= |\hat{k}\rangle\langle\hat{k}| \otimes \rho_{|\mathcal{E}_i|}^0 \\ &= |\hat{k}\rangle\langle\hat{k}| \otimes \left[\int_0^\infty \mathrm{d}k\, p(k)\frac{1}{k^2}|k\rangle\langle k|\right] \end{aligned}$$

new → <u>Thermal disk / Thermal isotropic</u>

$$\begin{aligned} \rho_{\mathcal{E}_i}^0 &= \rho_{|\mathcal{E}_i|}^0 \otimes \rho_{\hat{\mathcal{E}}_i}^0 \\ &= \left[\int \mathrm{d}\hat{k}\, p(\hat{k})|\hat{k}\rangle\langle\hat{k}|\right] \otimes \left[\int_0^\infty \mathrm{d}k\, p(k)\frac{1}{k^2}|k\rangle\langle k|\right] \end{aligned}$$

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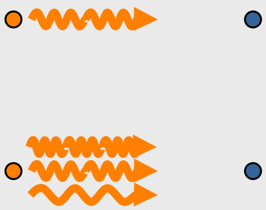
Point sources (effectively pure)



- Same form for monochromatic and thermal spectrum:

$$\rho^0_{\hat{\mathcal{E}}_i} = |\hat{k}\rangle\langle\hat{k}| \qquad\qquad \Gamma = e^{-t/\tau_d}$$

$$I_{\mathcal{S}:\mathcal{F}_f} = \ln 2 + \sum_{n=1}^{\infty} \frac{\Gamma^{(1-f)n} - \Gamma^{fn} - \Gamma^{n}}{2n(2n-1)}$$

- Decoherence rate (thermal):

temperature

radius

separation

irradiance

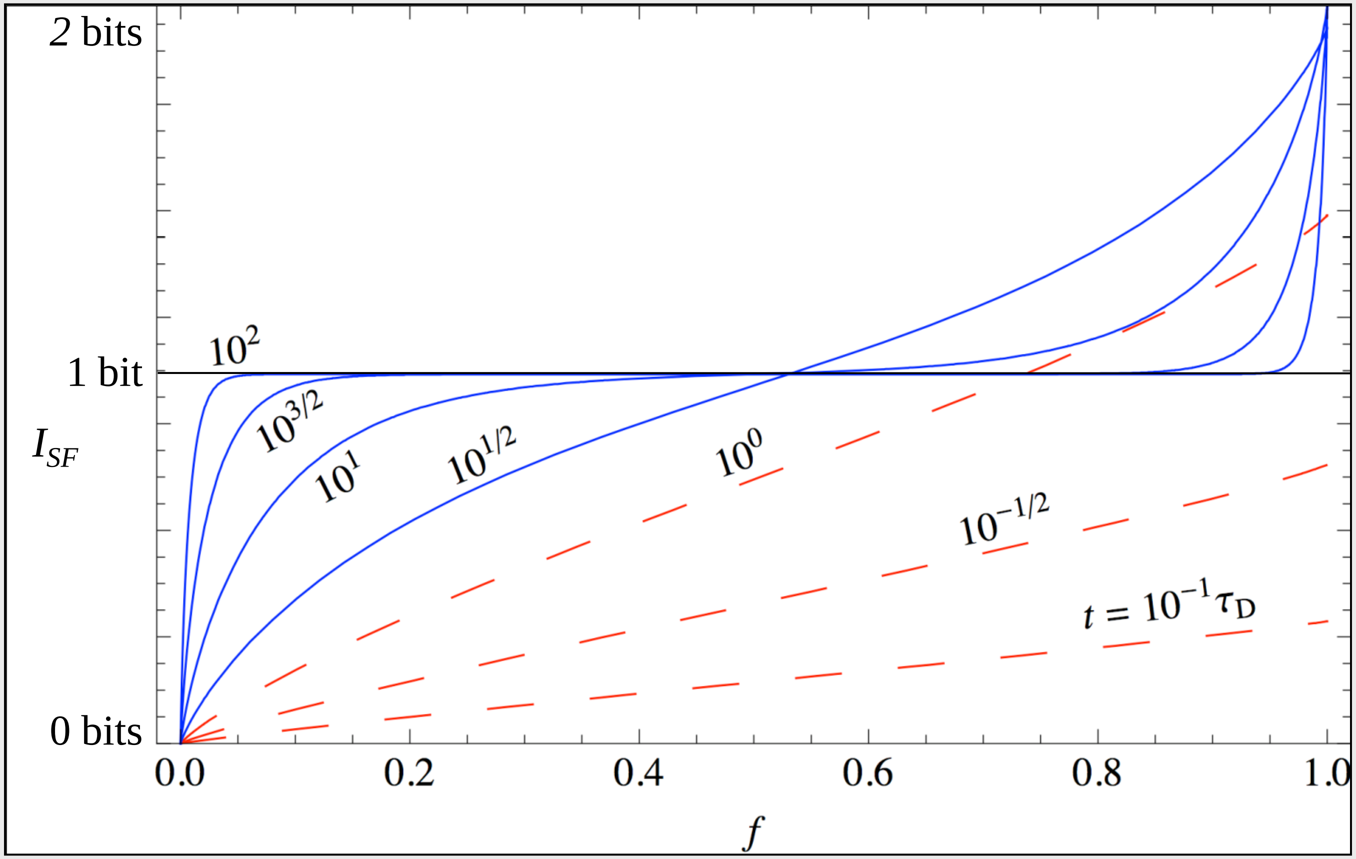
zeta function

$$\frac{1}{\tau_D} = \frac{161280\zeta(9)}{\pi^3}(3 + 11\cos^2\theta)\left(\frac{\epsilon-1}{\epsilon-2}\right)^2 \frac{I a^6 \Delta x^2 k_B^5 T^5}{c^6 \hbar^6}$$

relative permittivity of object

(monochromatic case differs only by constant)

Point source mutual information



Point source redundancy

- For large times, Γ is exponentially small, so:

$$I_{\mathcal{S}:\mathcal{F}_f} \approx \ln 2 - \frac{1}{2}\Gamma^f$$

$$R_\delta = \frac{1}{\ln[(2\delta \ln 2)^{-1}]} \frac{t}{\tau_D}$$

linear time dependence

weak dependence on δ

$$\text{Redundancy growth rate} \approx \tau_D^{-1}$$

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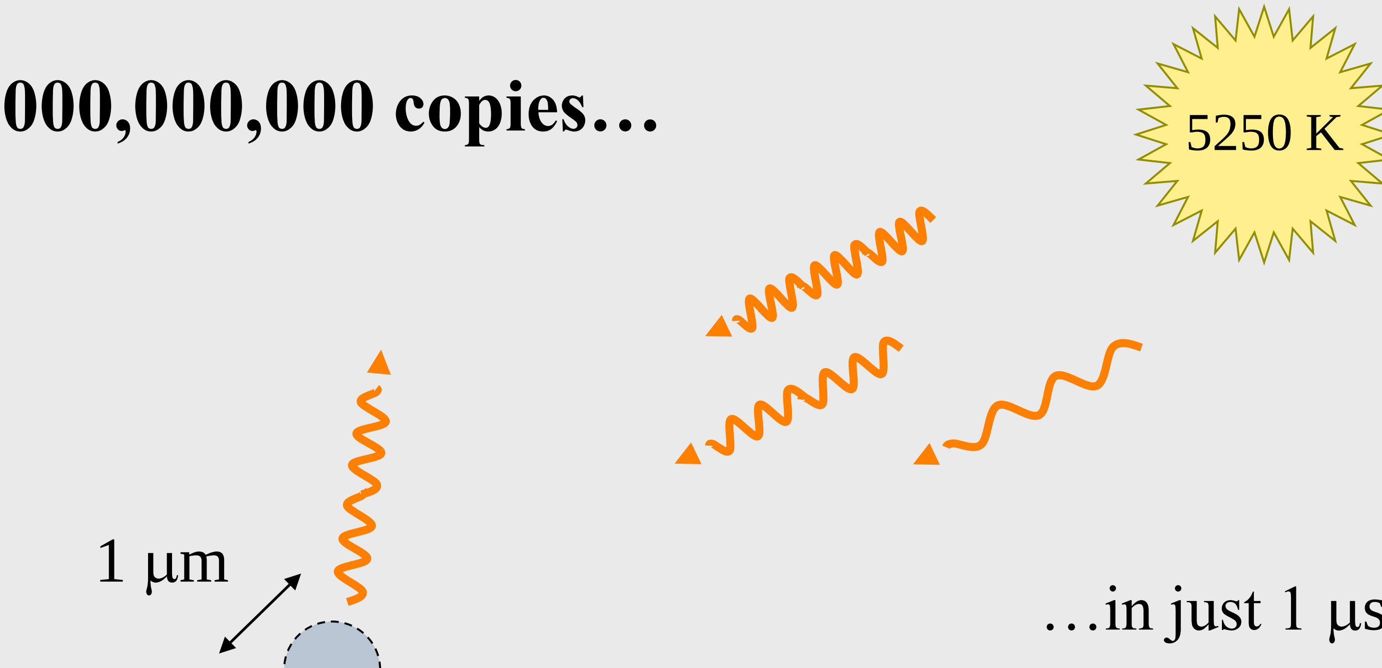
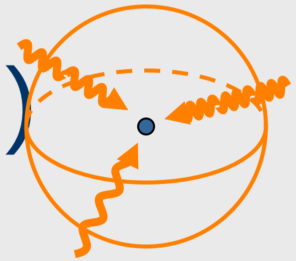
A speck of dust on the surface of the Earth…

100,000,000,000 copies…



…in just 1 µs!

Isotropic sources (angularly mixed)



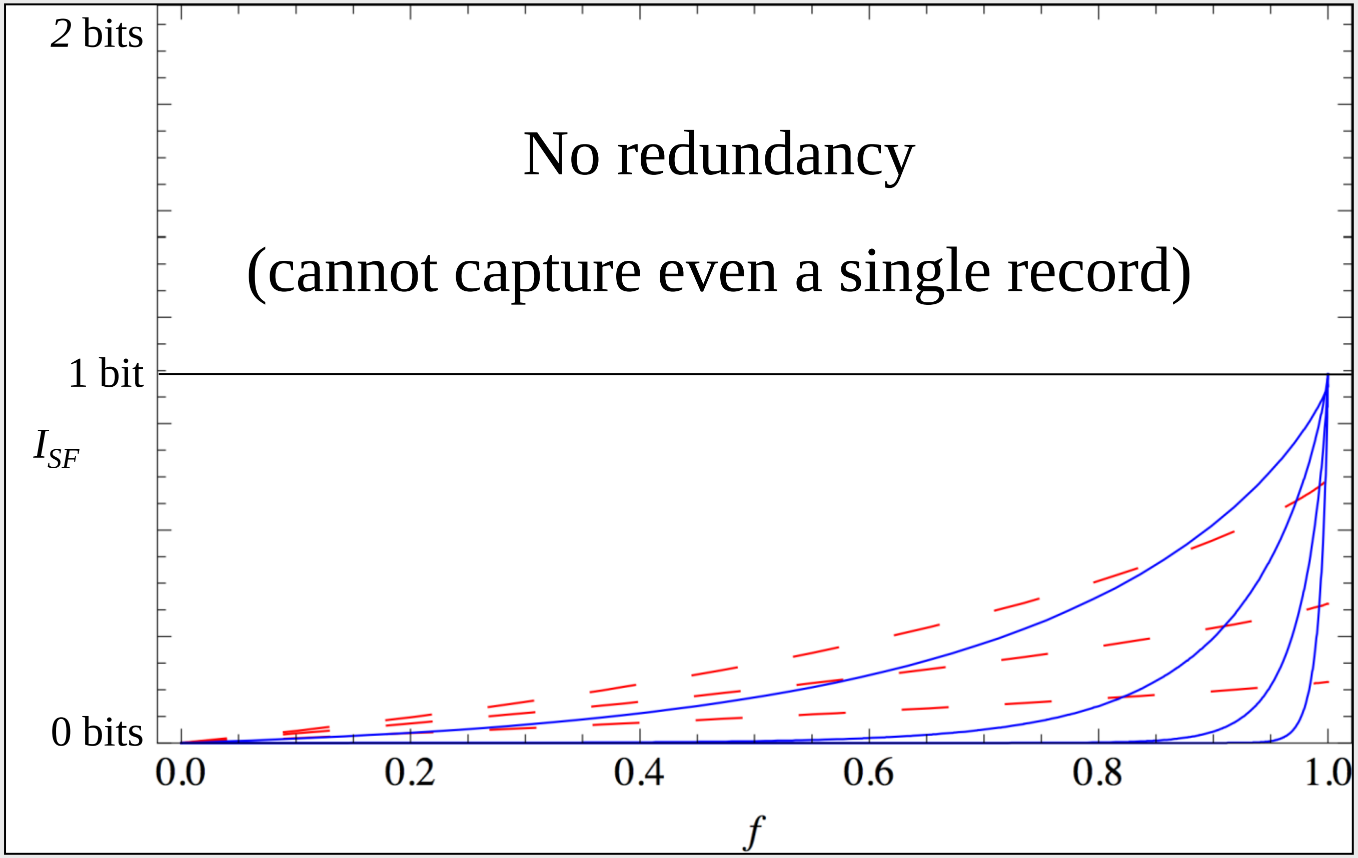
- Same form for monochromatic and thermal spectrum:

$$\rho^0_{\hat{\mathcal{E}}_i} = \int_\Omega \frac{\mathrm{d}\hat{k}}{4\pi} |\hat{k}\rangle\langle\hat{k}| \qquad \Gamma = e^{-t/\tau_d}$$

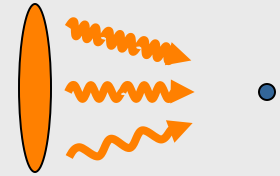
$$I_{\mathcal{S}:\mathcal{F}_f} = \sum_{n=1}^{\infty} \frac{\Gamma^{(1-f)n} - \Gamma^n}{2n(2n-1)}$$

  - Decoherence rate: <u>unchanged</u>
  - Redundacy: <u>none</u>

Isotropic source mutual information



Small disk source: partially mixed



new

- Valid for disk D with small solid angle A_D:

$$\rho^0_{\hat{\mathcal{E}}_i} = \int_{D\subset\Omega} \frac{\mathrm{d}\hat{k}}{4\pi} |\hat{k}\rangle\langle\hat{k}| \qquad \Gamma = e^{-t/\tau_d}$$

≈ 1, so little difference visible plotting mutual information

$$I_{\mathcal{S}:\mathcal{F}_f} \quad = \ln 2 + \sum_{n=1}^{\infty} \frac{\Gamma^{(1-f)n} - (\Gamma^{fn})^{\alpha} - \Gamma^{n}}{2n(2n-1)}$$

$$\alpha = 1 - A_D \frac{640\pi^5}{21}\left(\frac{a}{\lambda}\right)^6 \left[1 + O(\frac{\Delta x}{\lambda}) + O(A_D^{1/2}\frac{\lambda}{\Delta x}) + O(A_D \frac{\lambda}{\Delta x}\frac{\lambda^3}{a^3})\right]$$

solid angle of disk

Small disk redundancy

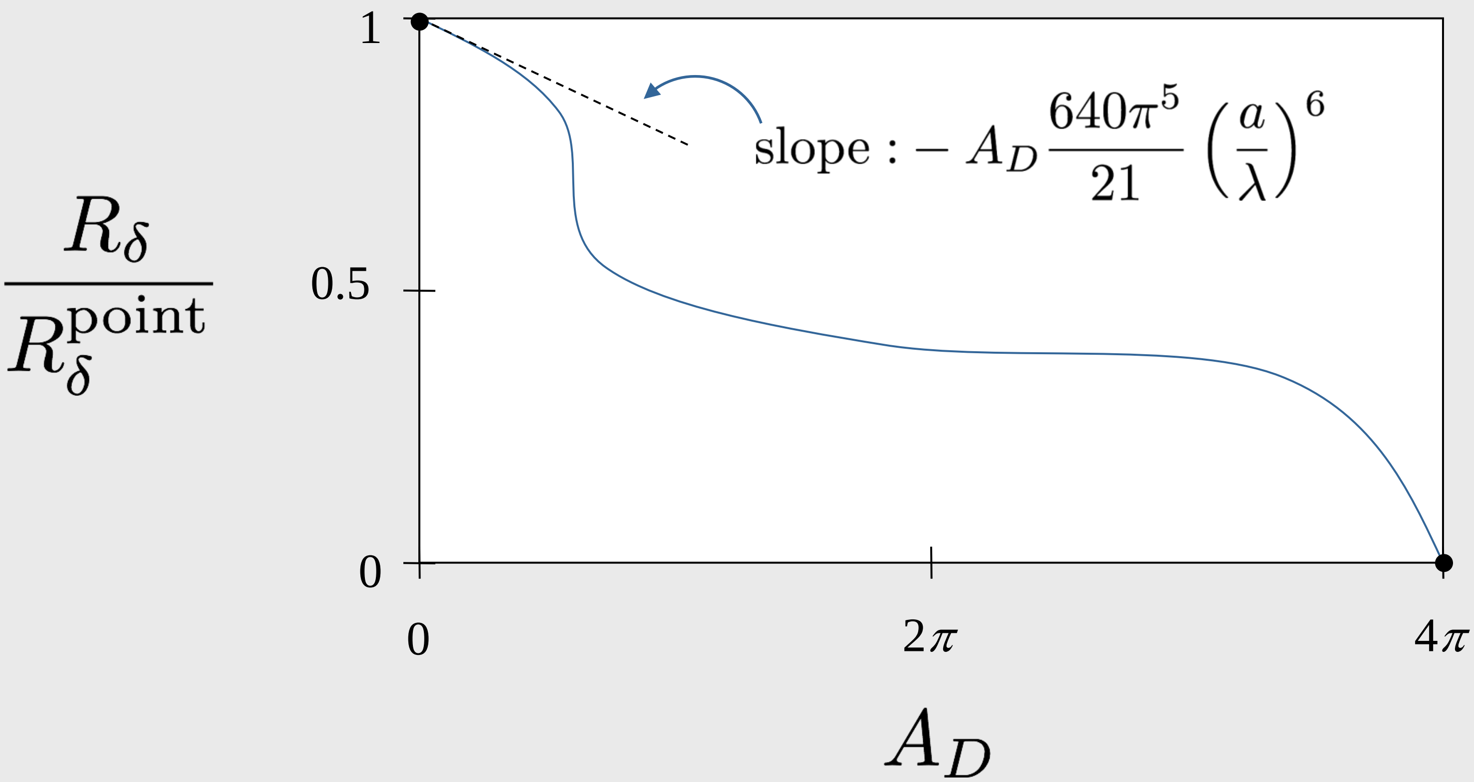
- For large times, δ is exponentially small and

$$I_{\mathcal{S}:\mathcal{F}_f} \approx \ln 2 - \frac{1}{2}(\Gamma^f)^\alpha$$

$$\alpha \approx 1 - A_D \times \mathrm{const}$$

$$\begin{aligned} R_\delta^{\mathrm{disk}} &= \frac{\alpha}{\ln[(2\delta \ln 2)^{-1}]} \frac{t}{\tau_D} \\ &= \alpha R_\delta^{\mathrm{point}} \end{aligned}$$

Redundancy vs. mixedness



Agrees with spin model

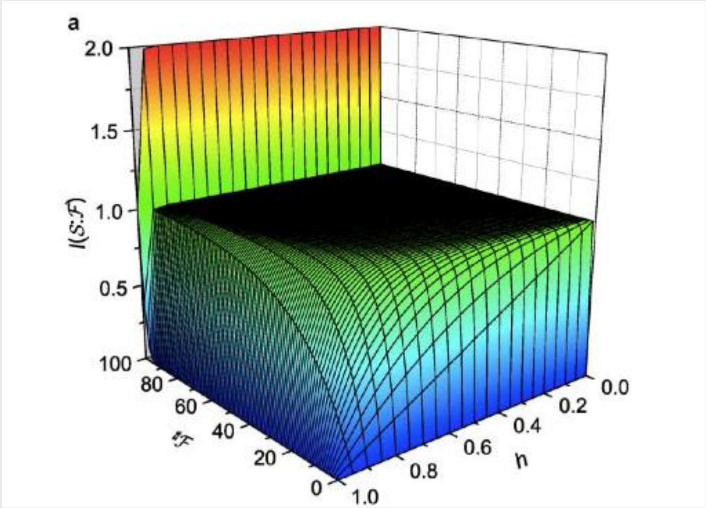
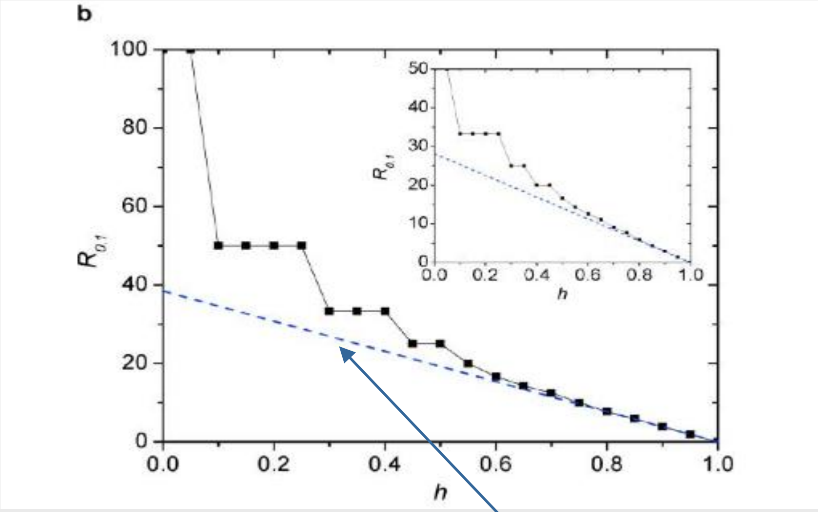


Figure 2: (a) Mutual information at $t = \pi/2$ versus $^\sharp\mathcal{F}$ and h for the same conditions as in Fig. 1. (b) Redundancy versus h for the information deficit $\delta = 0.1$ and at $t = \pi/2$ (the inset shows $t = \pi/3$). The black line (squares) is the exact data. The redundancy can only take on rational values with $^\sharp\mathcal{E}$ in the numerator because of the discrete nature of the spin environment, which is particularly visible at high redundancy. The blue dashed line is that obtained by the scaling $1 - h$, which is a good approximation when h is near one. Thus, even initially mixed $\mathcal{E}$ can store information about $\mathcal{S}$ in many copies. However, it takes larger $^\sharp\mathcal{F}$ to acquire the same information about $\mathcal{S}$.



$\propto 1 - h$

Spin model studied by Zwolak et al. first demonstrated reduction of redundancy by initial mixedness.

M. Zwolak, H. T. Quan, W. H. Zurek, Phys. Rev. Lett. 103, 110402 (2009).

M. Zwolak, H. T. Quan, W. H. Zurek, Phys. Rev. A 81, 062110 (2010).

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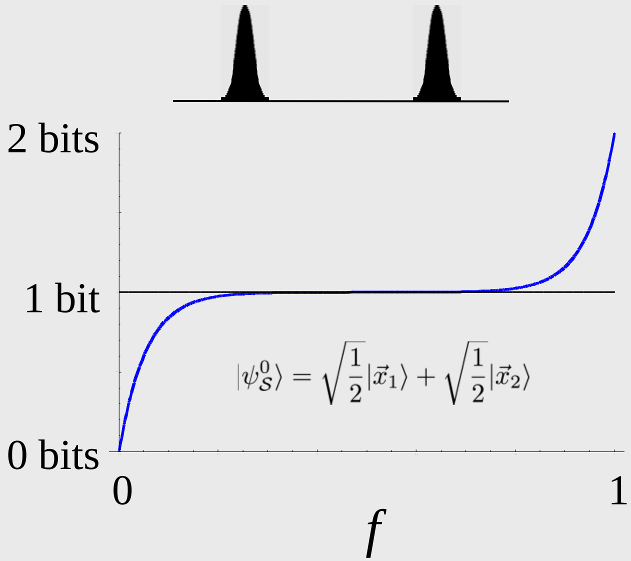
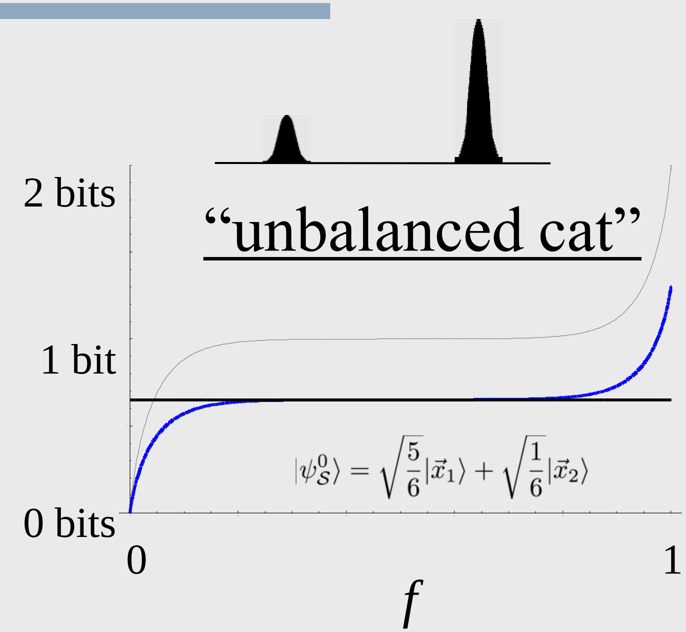
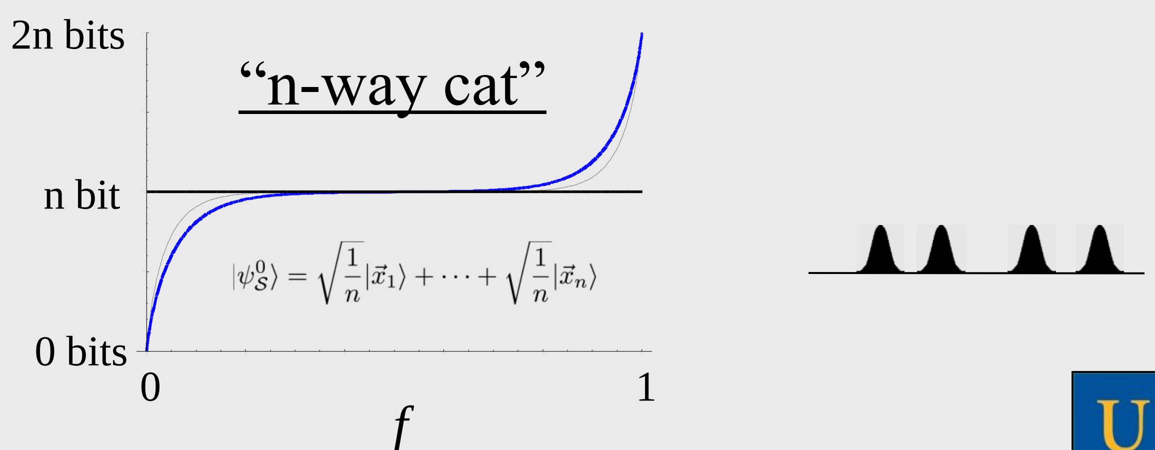
Other extensions



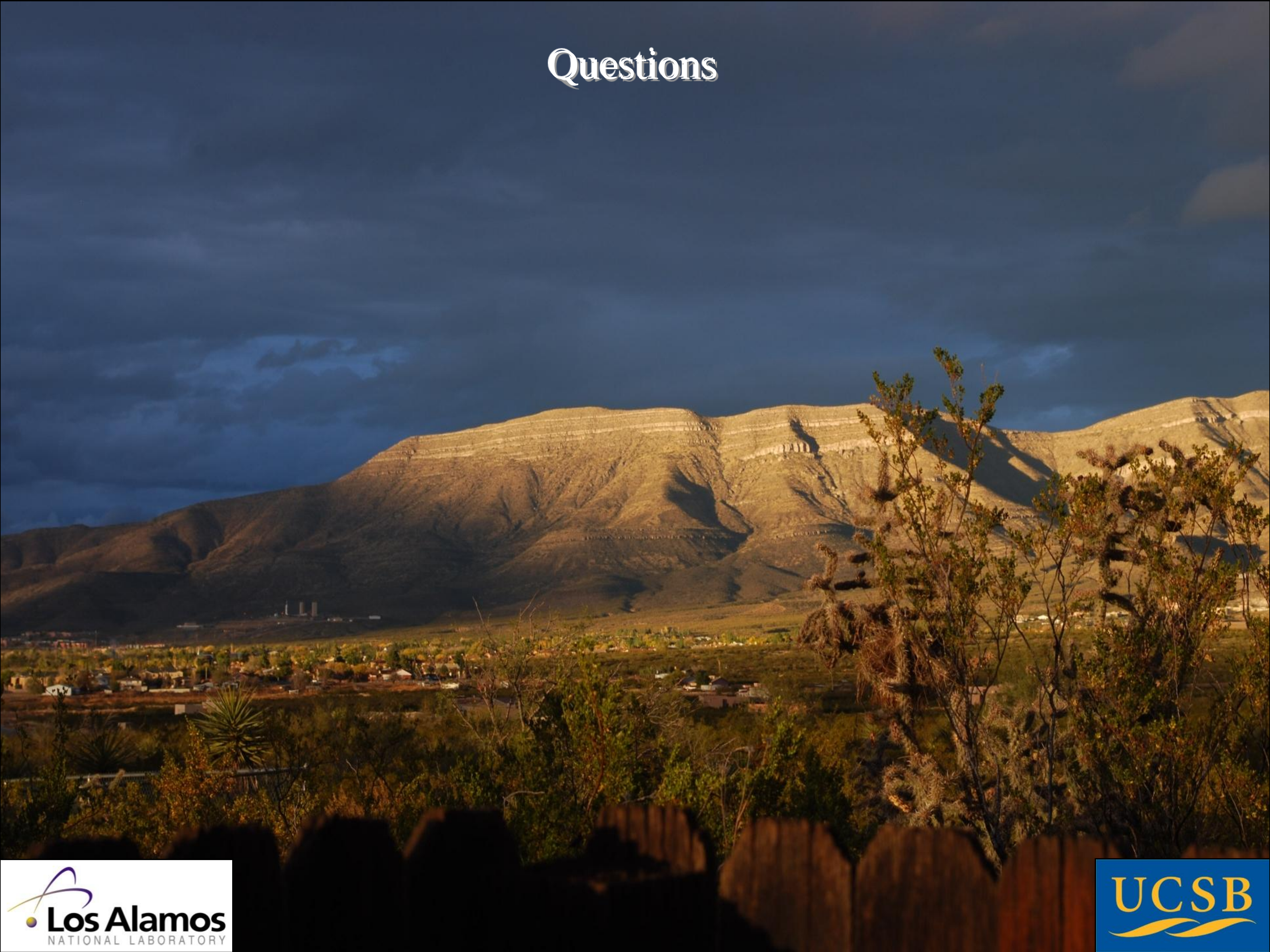




Summary

- Why you should care about quantum Darwinism
  - Decoherence is only part of the story
  - Analogy with special relativity
  - What quantum Darwinism gives you
- Mathematical development (easy)
- Theoretical Results
  - Previous work
  - The everyday environment → huge redundancies
  - Generalizations

Questions



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