Quantum randomness ...from locality

Quantum Coherence and Decoherence
Centro de ciencias de Benasque Pedro Pascual
September 2010

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Wojciech Zurek
LANL

research supported by the
Quantum initiative @LANL

Los Alamos
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Quantum randomness

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Work in progress!!!

Please, provide feedback, and give me a hard time!

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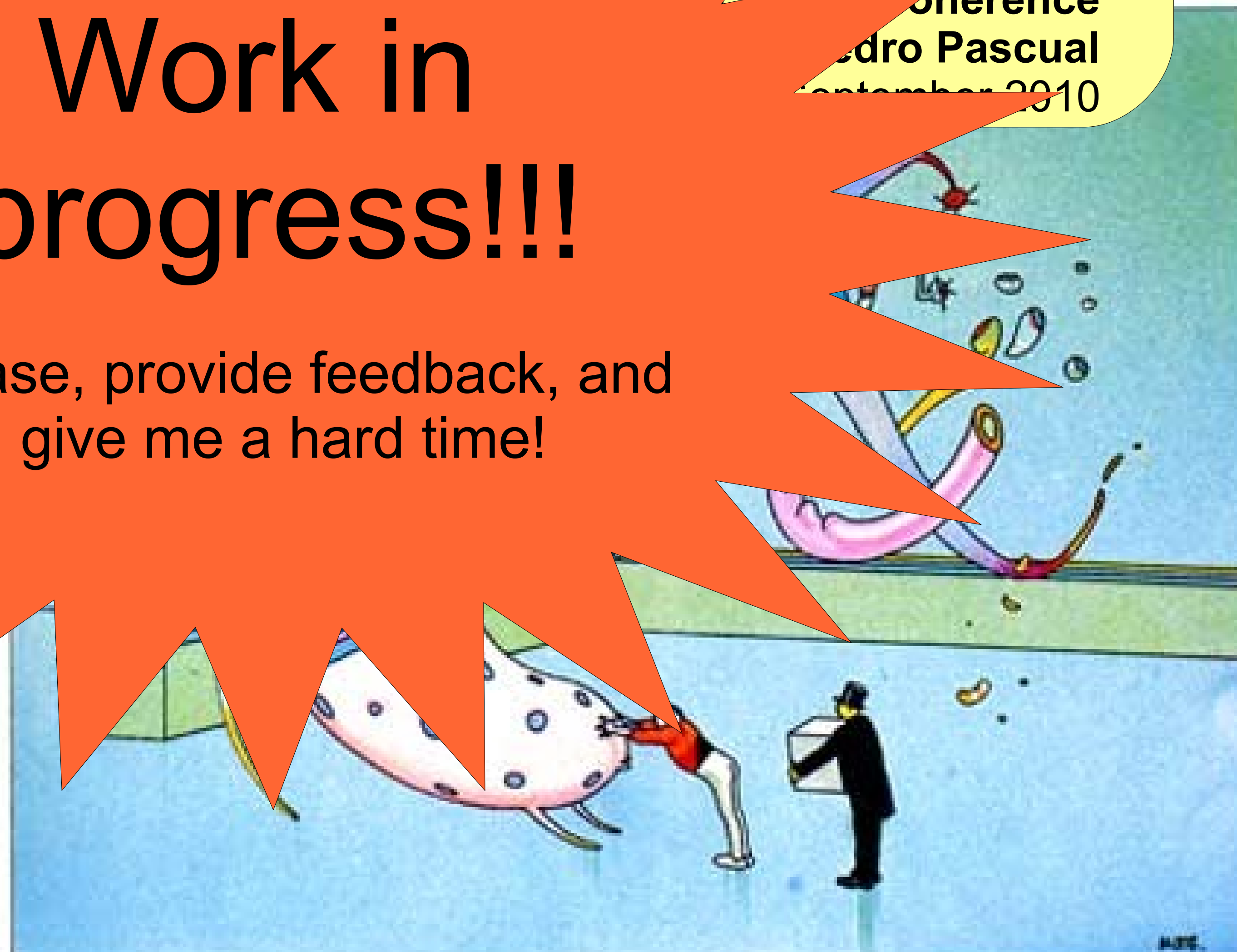
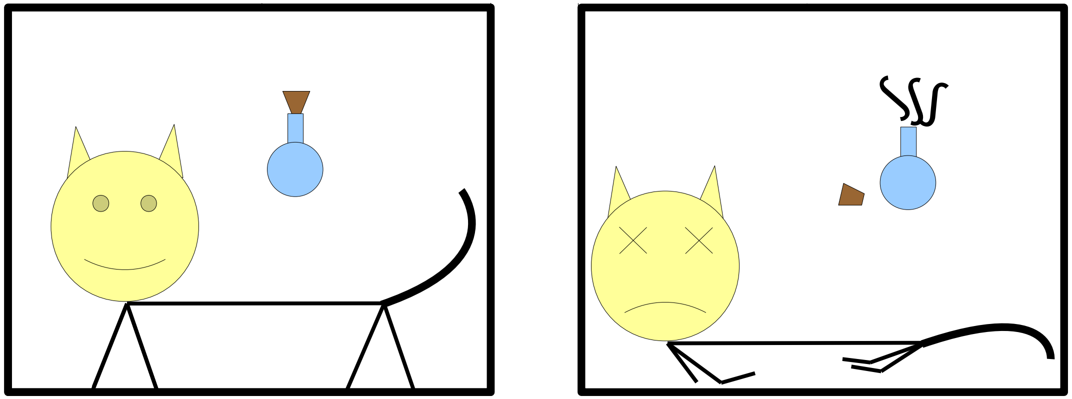


Schroedinger's cat

$$|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$



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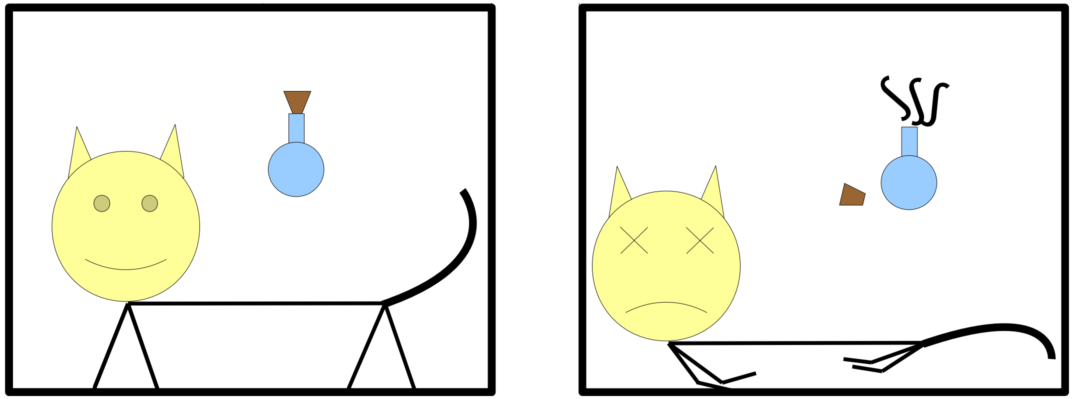
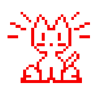


Unitary (deterministic) evolution

$\frac{1}{\sqrt{2}}\Big(\Big|$  $\Big\rangle \pm \Big|$  $\Big\rangle\Big)$

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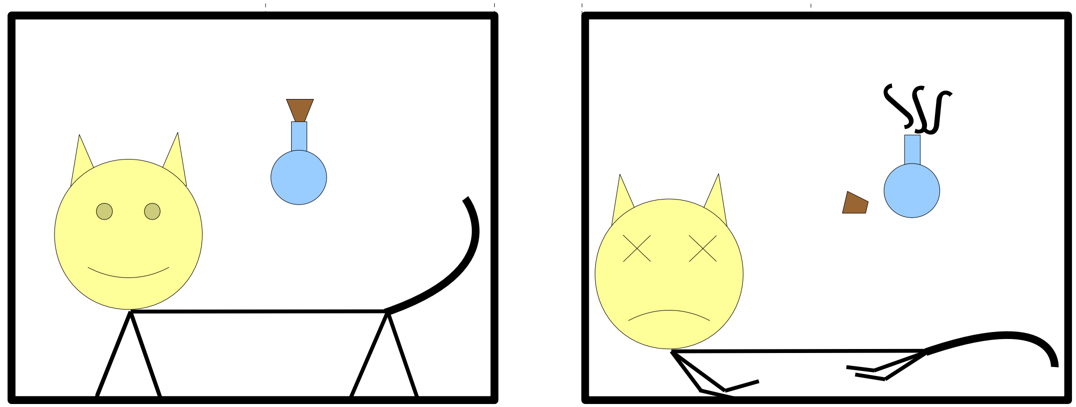

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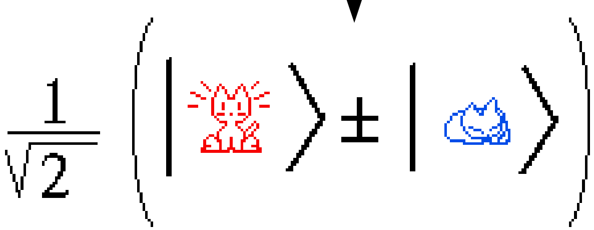
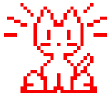
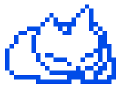


Measurement: random outcome

 p=1/2

 p=1/2

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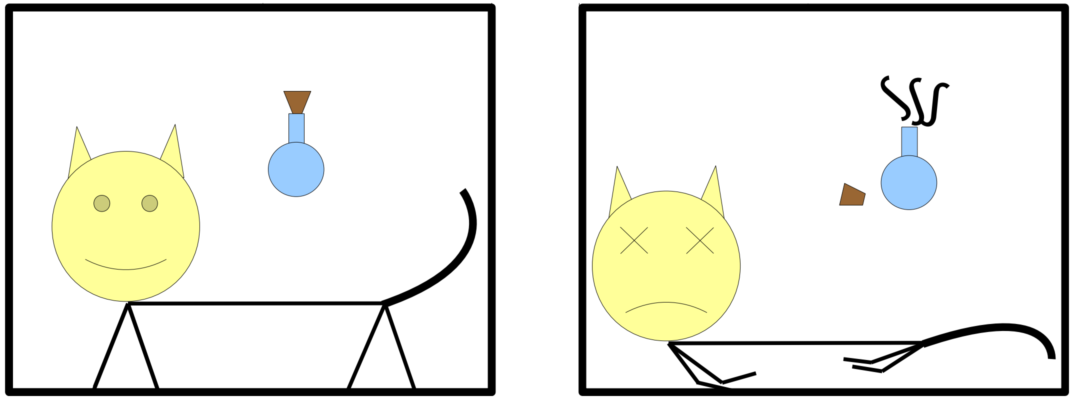

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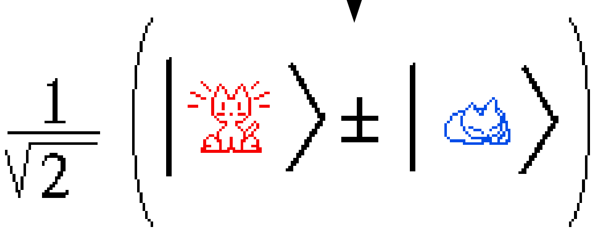

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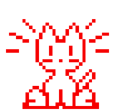
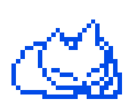
 p=1/2

 p=1/2

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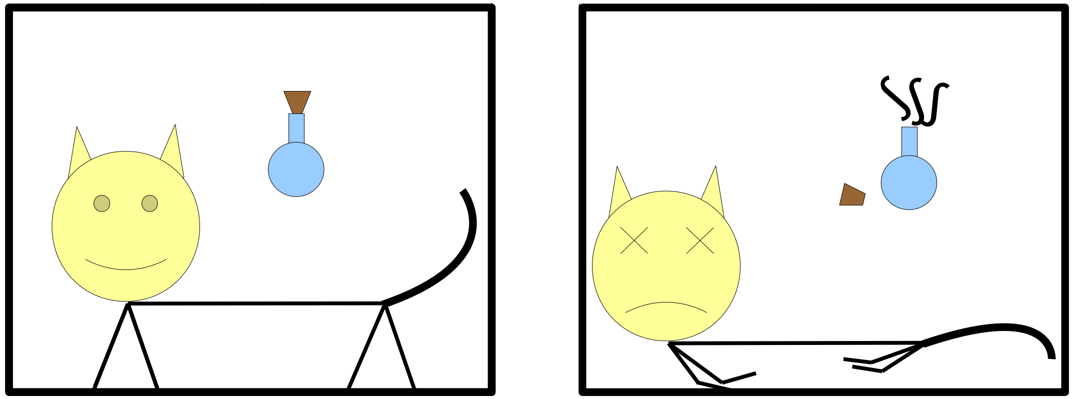

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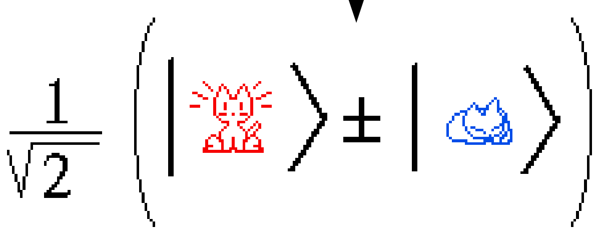

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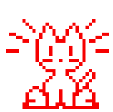
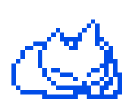
 p=1/2  p=1/2

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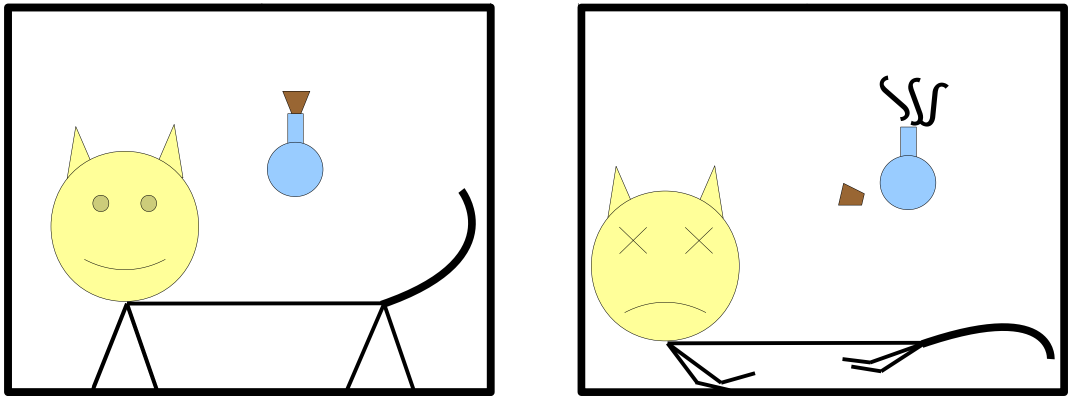

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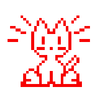
$\frac{1}{\sqrt{2}}\Big(\big|$  $\big\rangle \pm \big|$  $\big\rangle\Big)$

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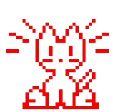
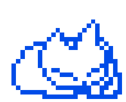
 p=1/2  p=1/2

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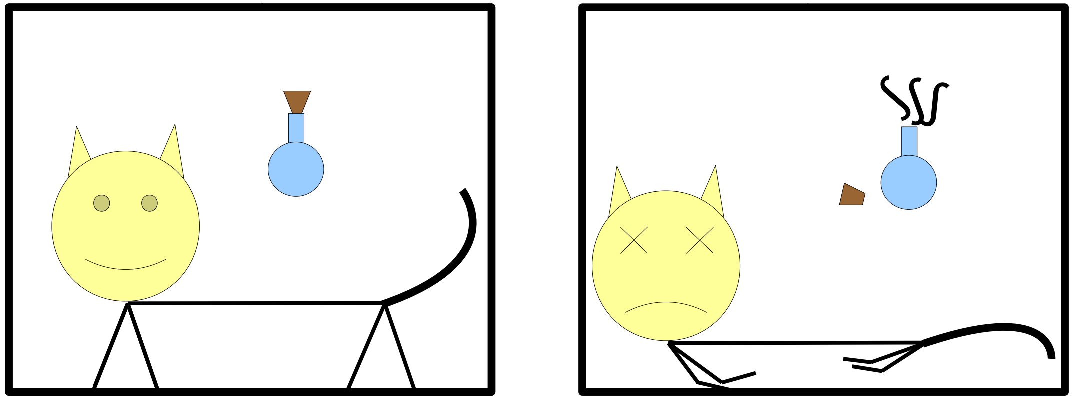

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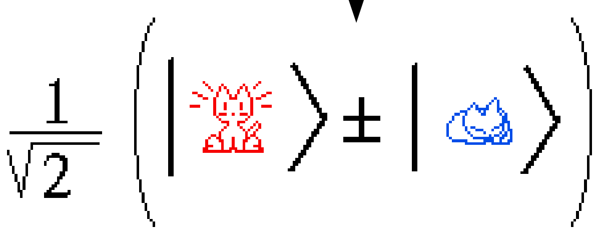

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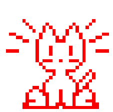
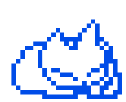
 p=1/2

 p=1/2

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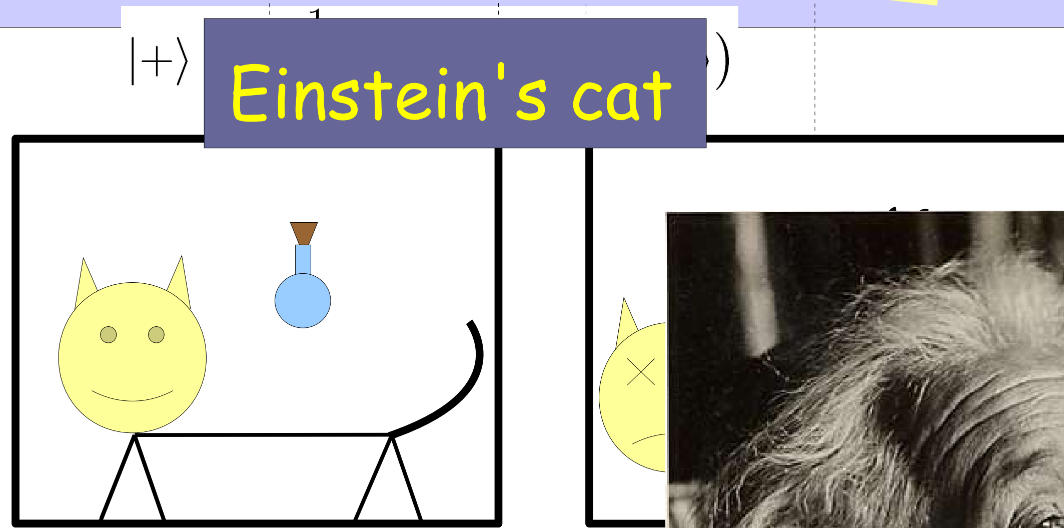
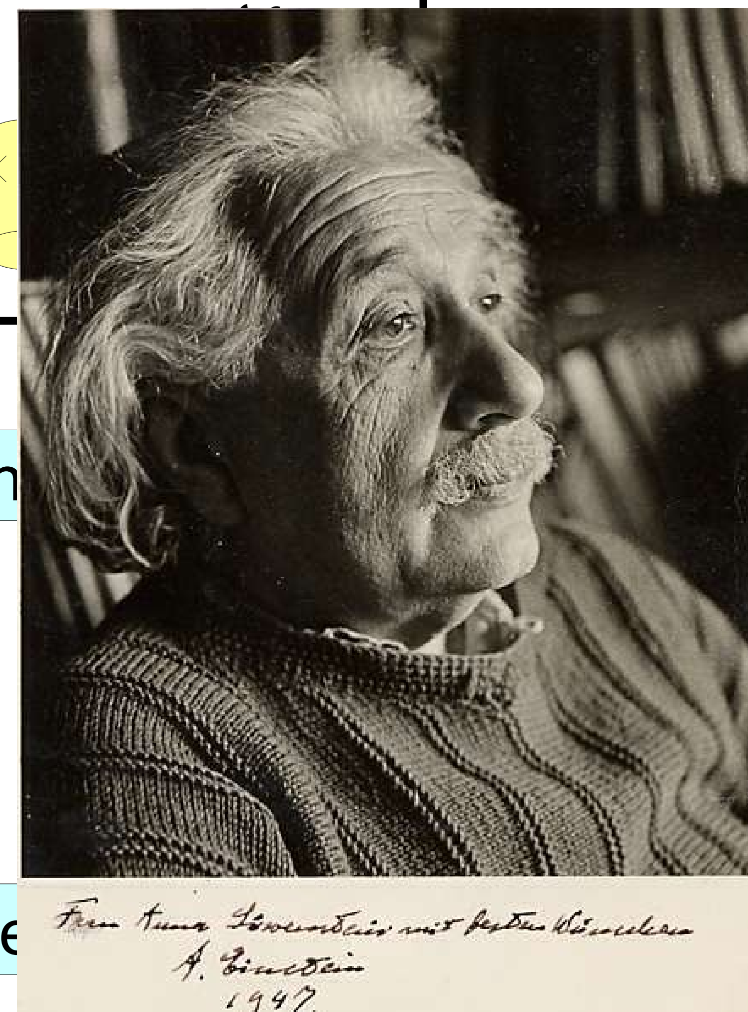
Einstein's cat





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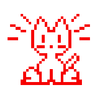
$\frac{1}{\sqrt{2}}\Big(\big|$  $\big\rangle \pm \big|$  $\big\rangle\Big)$

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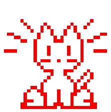
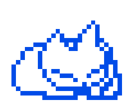
 p=1/2

 p=1/2

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  (no-signaling and hence causality)

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$\Downarrow$

- **Quantum randomness**

  (no deterministic measurement outcomes are possible)

Is quantum mechanics local or non-local?!

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...it depends!

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acting on a system cannot change the local properties of a different system (super-luminal communication would void the causal structure of spacetime)

- “causal” non-locality

acting on a system can change the **statistical correlations** with a different system (obvious), even in such a way that **global properties are changed**.

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We **think** that QM is local

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we'll consider

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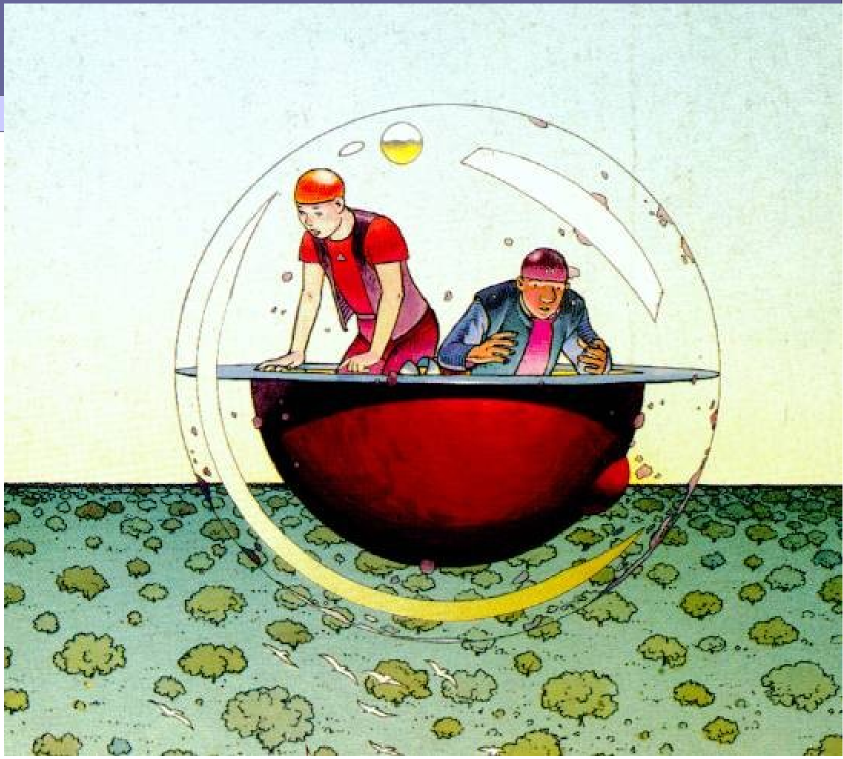
Outline

- Assumptions we use
- The tools we use
- Intuitive idea of the argument
- Details
- What does it all mean?



Abstract: Conventional textbook quantum mechanics introduces randomness of measurement outcomes, along with the Born rule, as a postulate of the theory. Nonetheless, after various attempts the Born rule was recently derived from the other postulates. A common assumption of these derivations is that different ``branches'' of the wavefunction represent alternative situations. Without this assumption there is no compelling reason for a probabilistic interpretation: alternatives are possible. Here, by using envariance and a modified Bell inequality that employs no Born rule, we show that randomness of outcomes is inevitable if one wants the theory to be local, and hence causal. In other words, we prove inevitability of randomness using locality to justify Everett's identification of random ``events'' with ``branches'', and thus show that one can obtain the Born rule replacing the wavefunction-branches assumption with a physically-motivated locality one.

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The non probabilistic postulates of quantum mechanics

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The state space is a Hilbert space: physical properties are represented by vectors $|\psi>$

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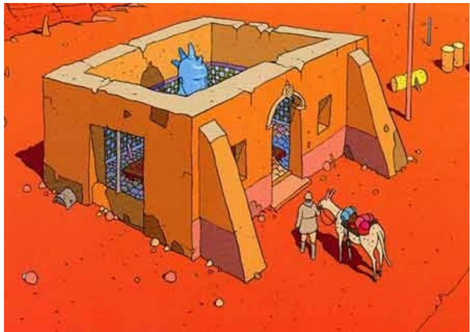
3. Tensor product structure

   The space of a composite system is the tensor product of the components: $|\psi>|\phi>$

The ENVARIANCE program

W.H.Zurek, PRL **90**,120404



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NPQM

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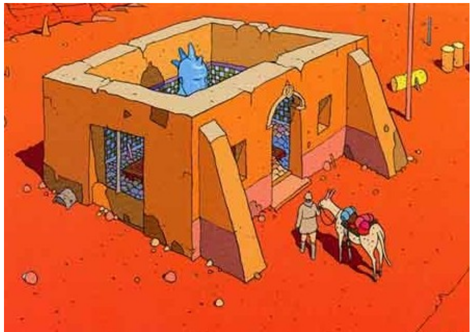

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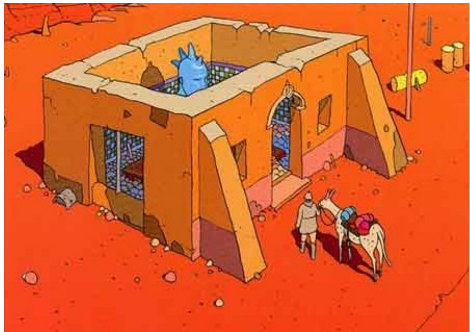
4. “Wavefunction branches” assumption

The states in different branches of the wavefunction represent alternatives



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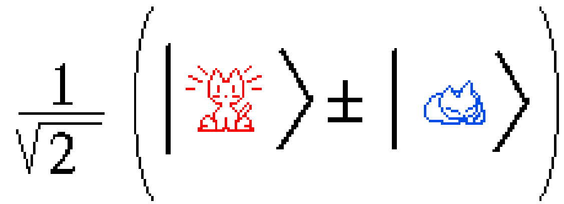
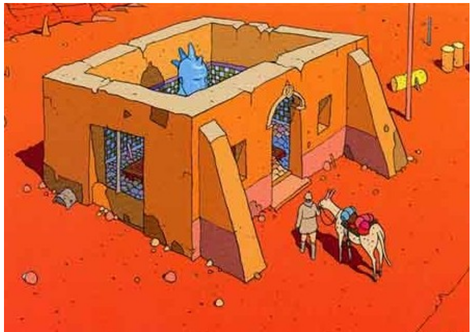


either alive or dead



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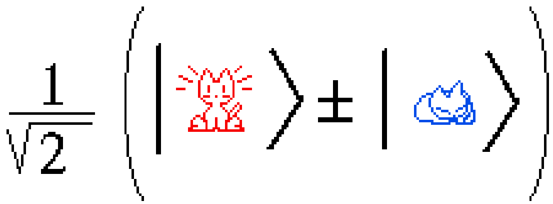

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$\Downarrow$

Born rule (the probability law of qm)

The ENVARIANCE program

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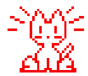
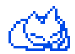
$\frac{1}{\sqrt{2}}\Bigg(\Bigg|$  $\Bigg\rangle \pm \Bigg|$  $\Bigg\rangle\Bigg)$

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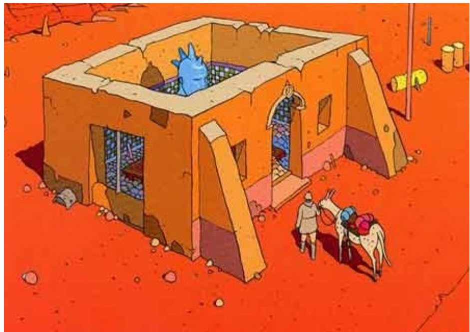
prob. of outcomes

scalar product



Our current result

NPQM

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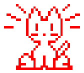
$\frac{1}{\sqrt{2}}\Big(\Big|$  $\Big\rangle \pm \Big|$  $\Big\rangle \Big)$

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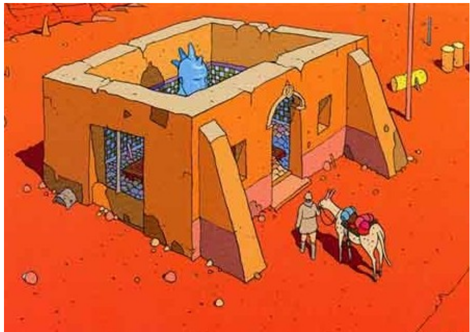

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4. Locality (no-signaling)

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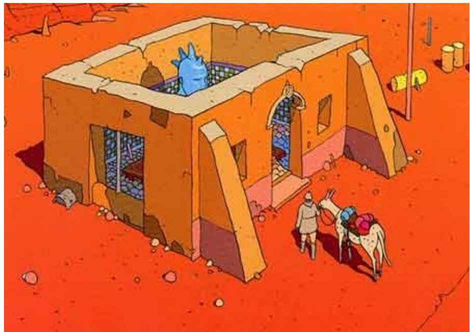

The “extended glow”

Is the “wavefunction branches” assumption **avoidable**?

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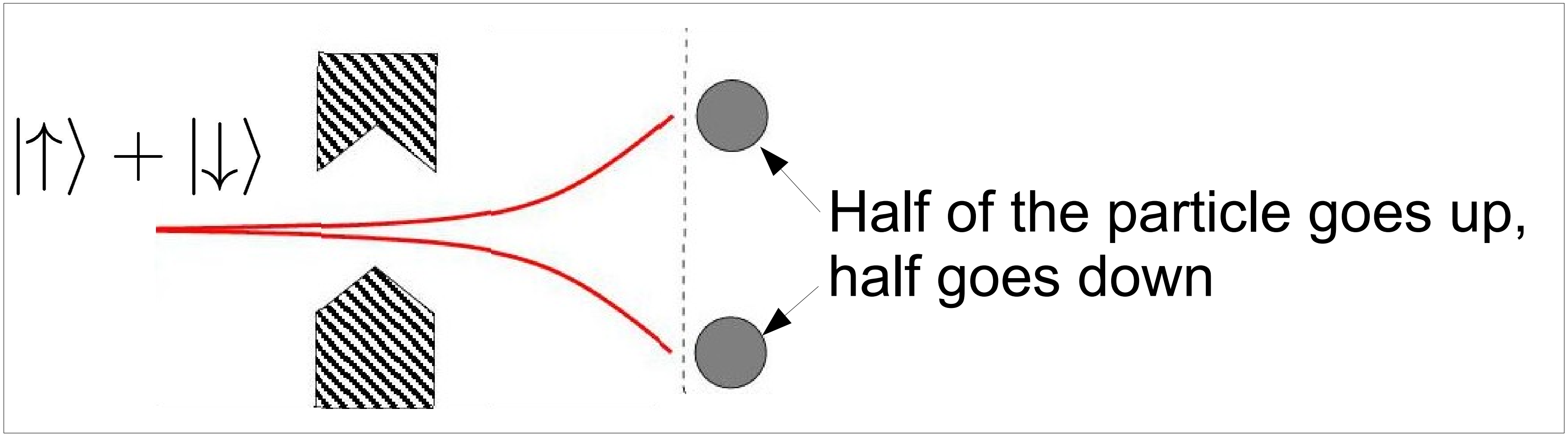

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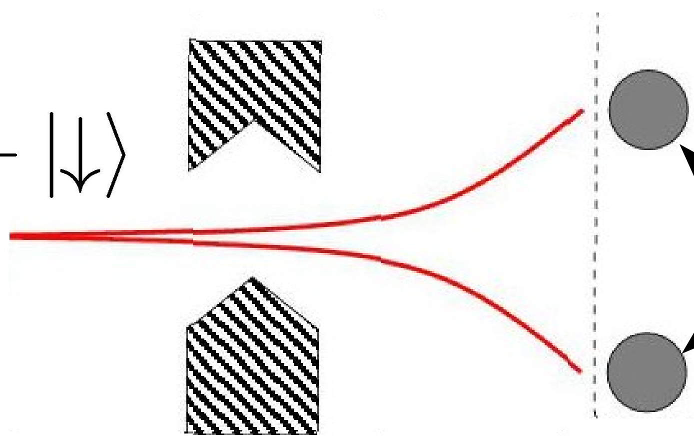
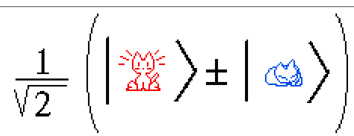
$|\uparrow\rangle + |\downarrow\rangle$



Half of the particle goes up,
half goes down



Half of the cat is alive, half is dead

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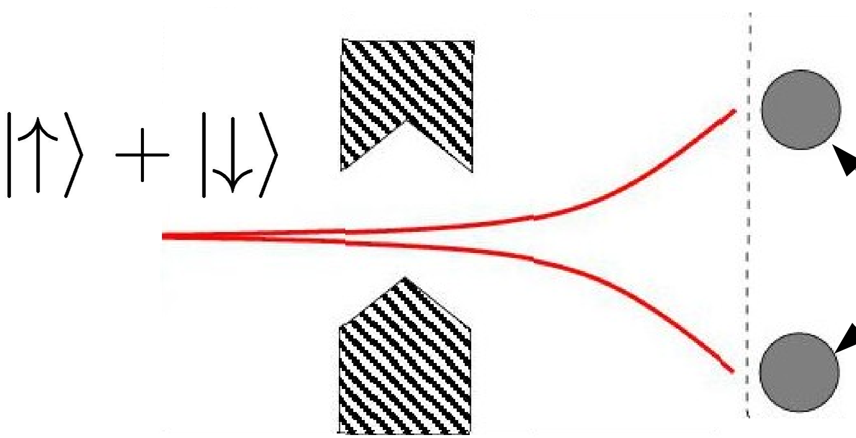

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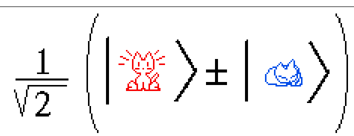


Half of the cat is alive, half is dead

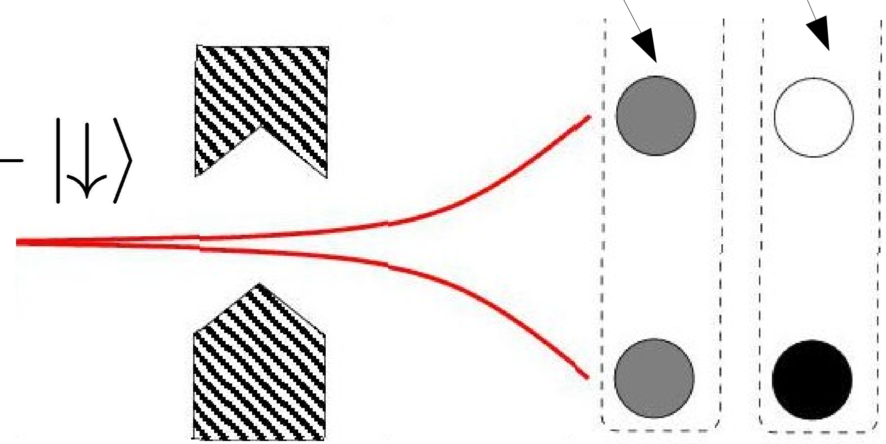
Bell called this “**extended glow**” scenario

Bell's "extended glow"

"*Inspection of the [state vector] itself gives no hint that the experienced reality is a <u>scintillation</u>... rather than, for example, an <u>extended glow</u> of unpredicted colour. That is to say, the [state] does not simply fail to specify one of the possibilities as actual... it fails to list the possibilites.*"

[J. Bell, Speakable and Unspeakable in quantum mechanics, pg.193]

$|\uparrow\rangle + |\downarrow\rangle$



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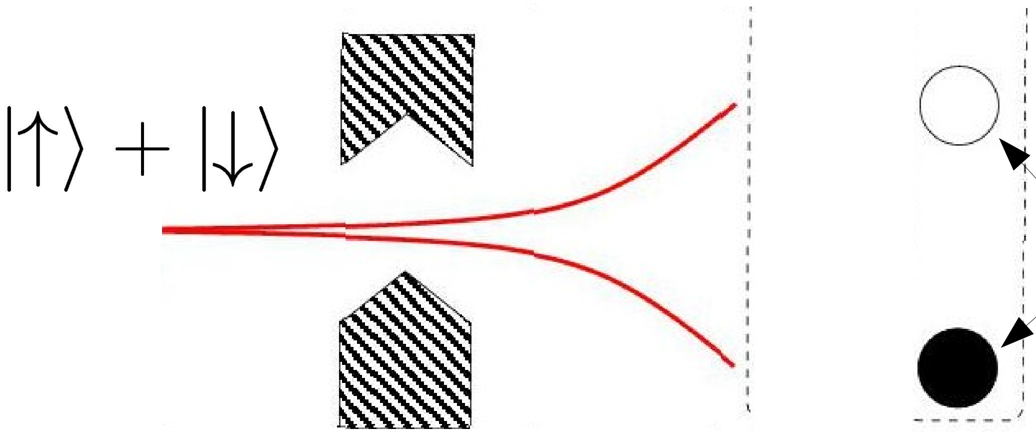

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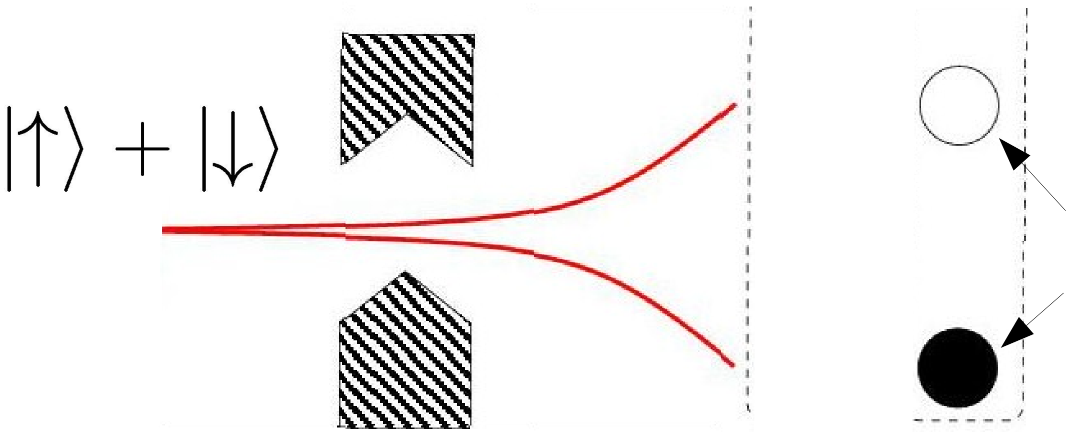

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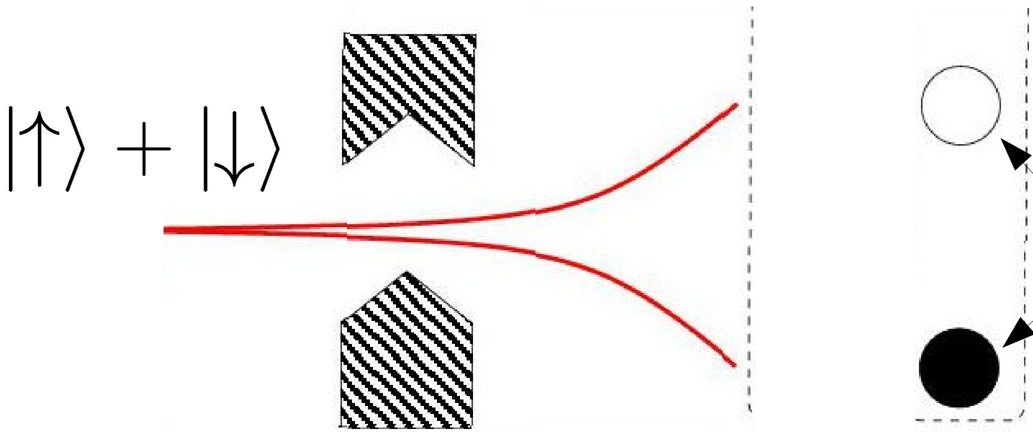

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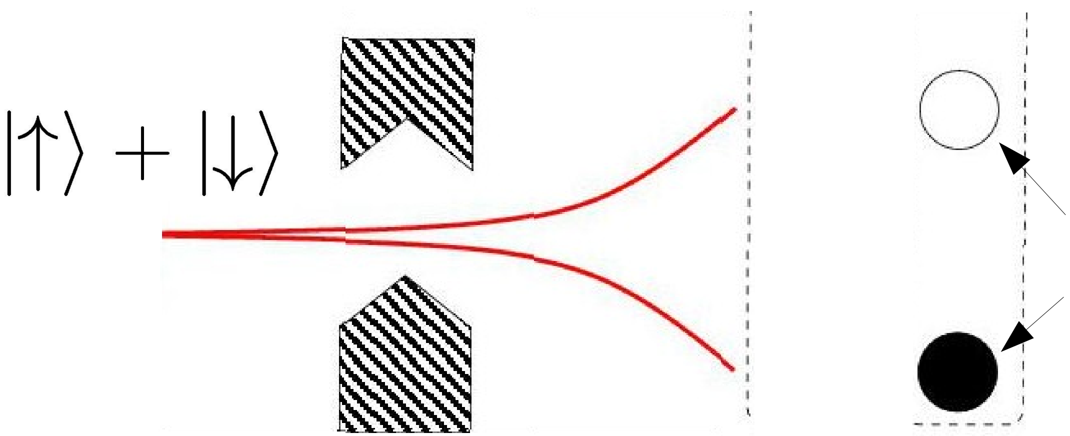

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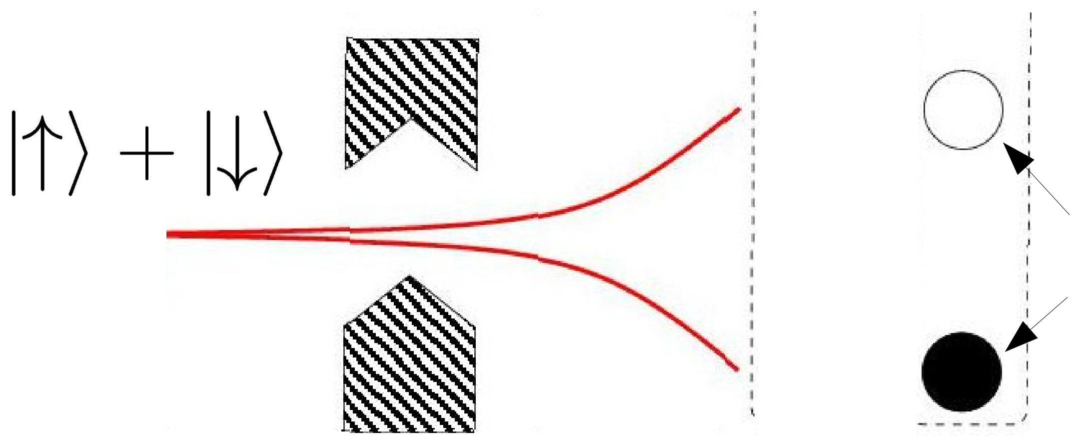

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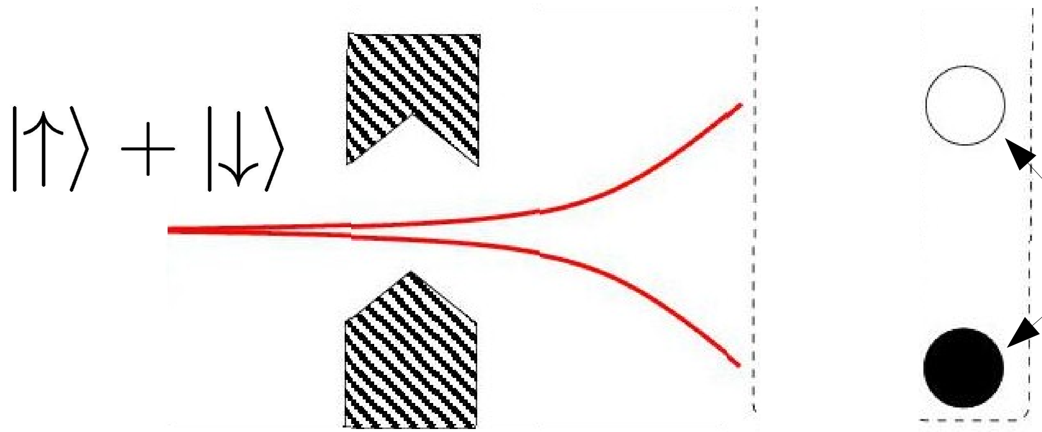

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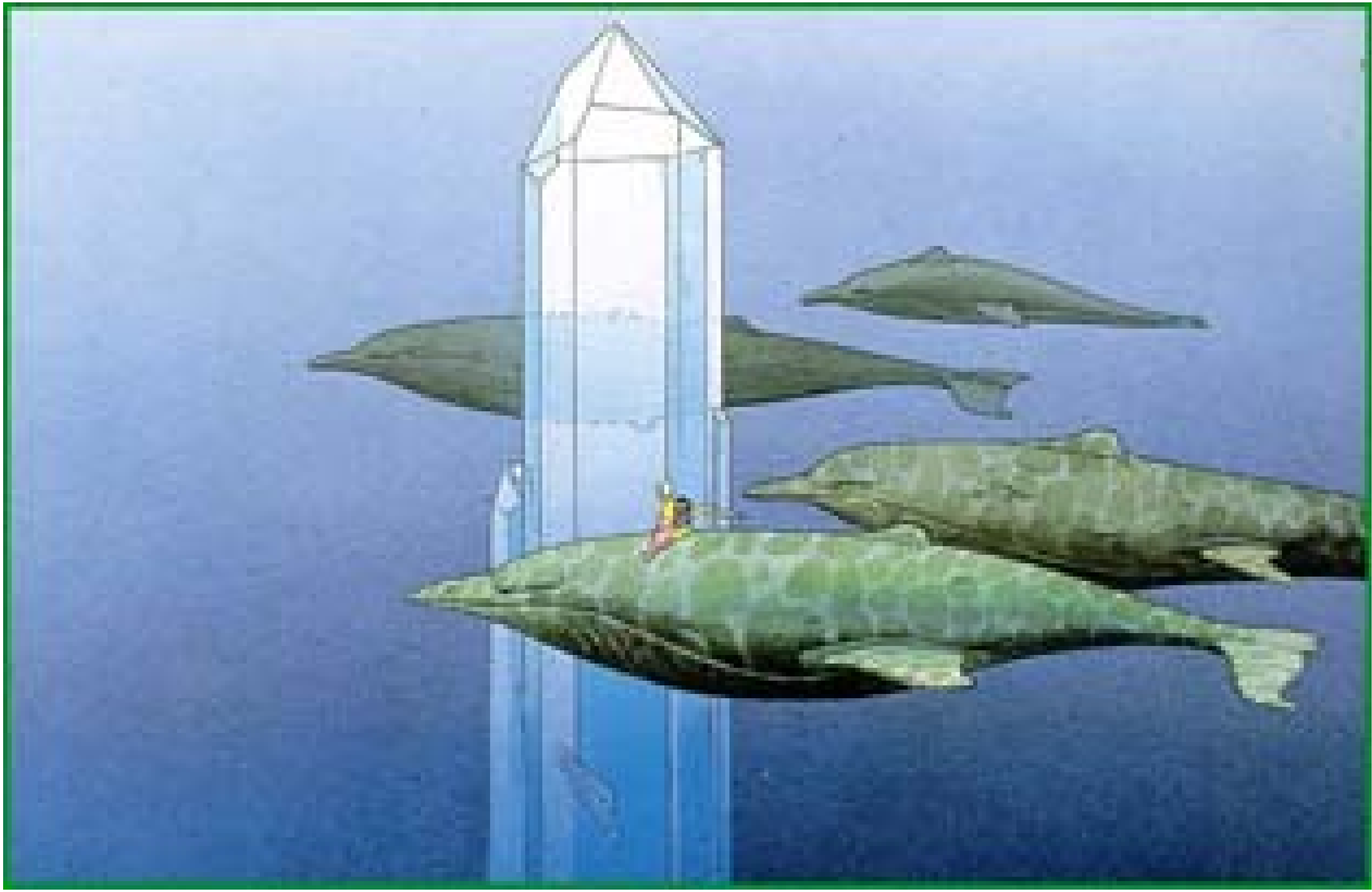
it is well known that if we add some kind of phenological input, we can obtain quantum randomness:



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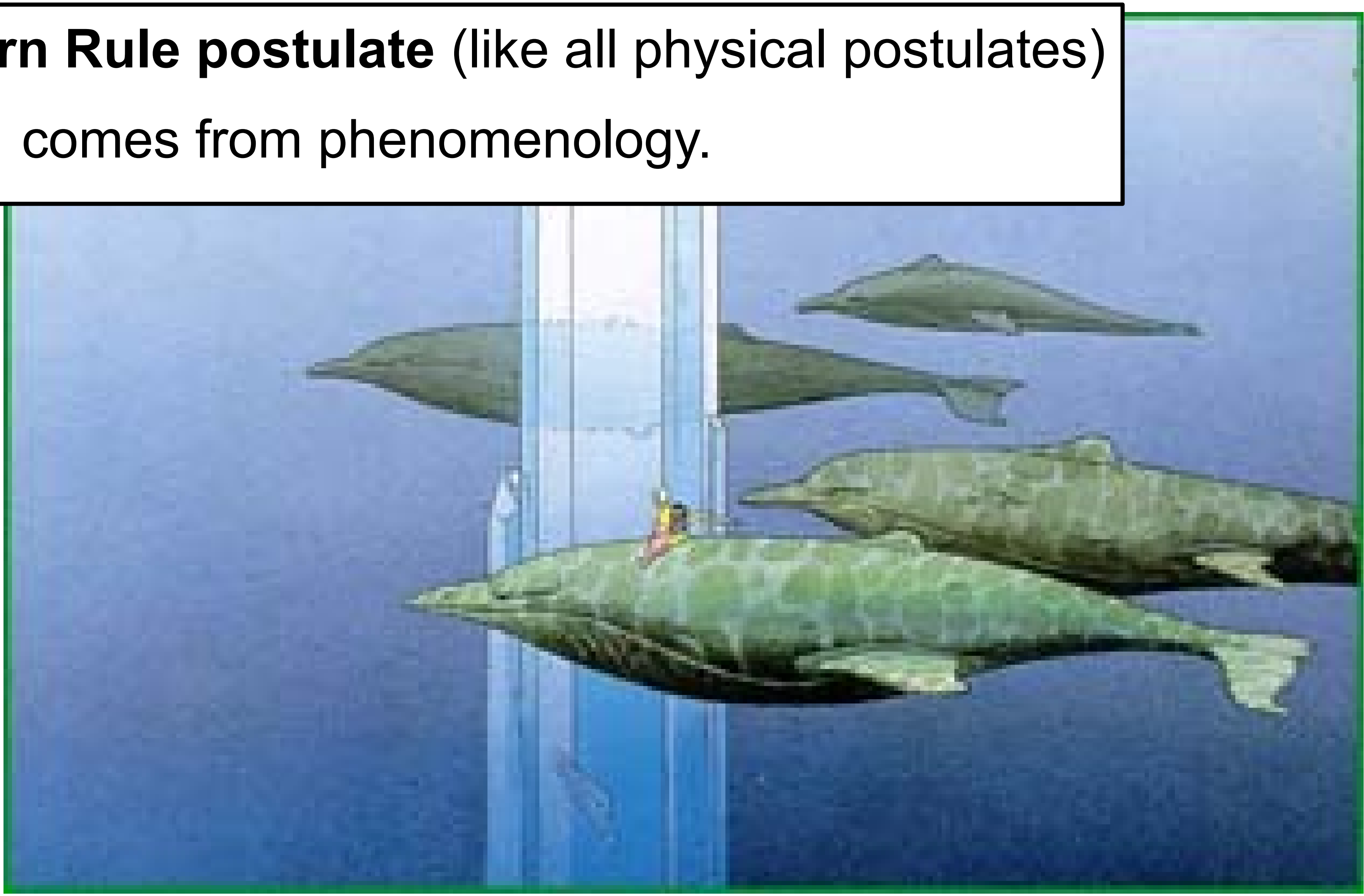

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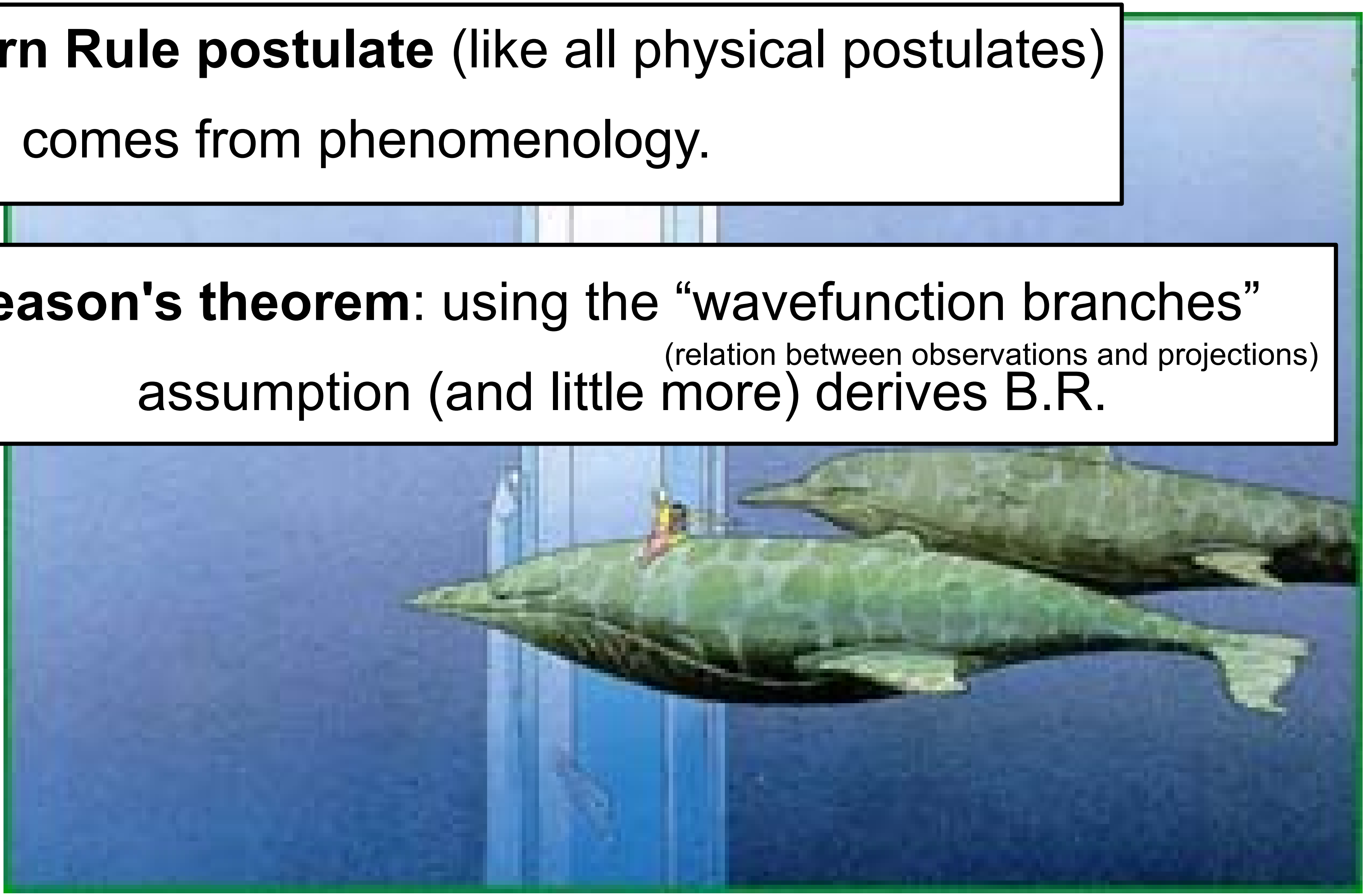

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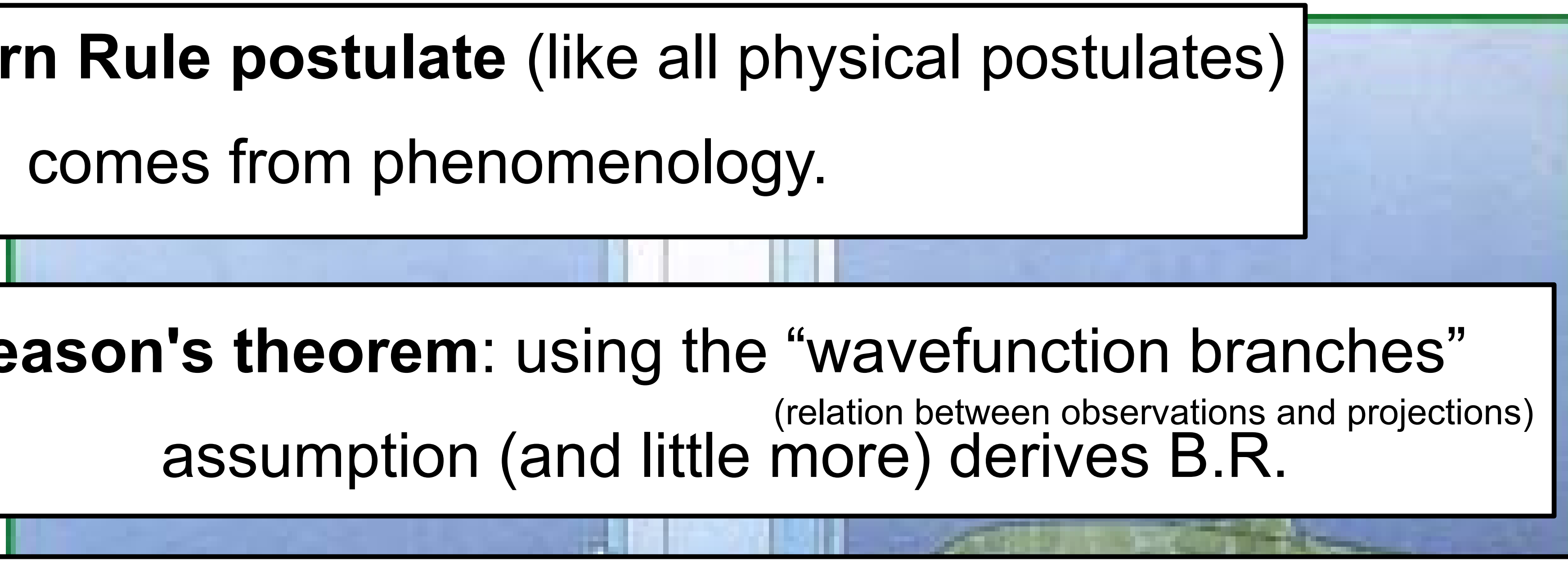

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Branciard,Brunner,Gisin, Kurtsiefer,Linares, Ling, Scarani, Nat. Phys. **4**, 681 (2008).

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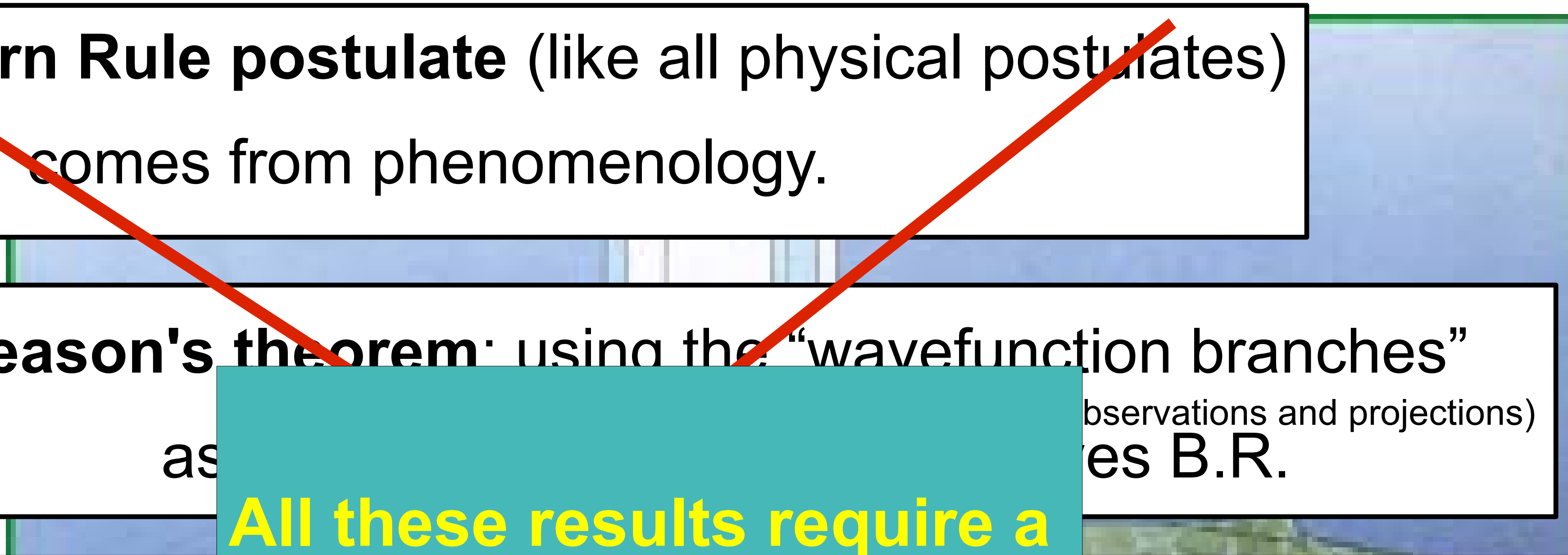

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The tools

- **Envariance** (without "branches" assumption) [W.H.Zurek, PRL **90**,120404]

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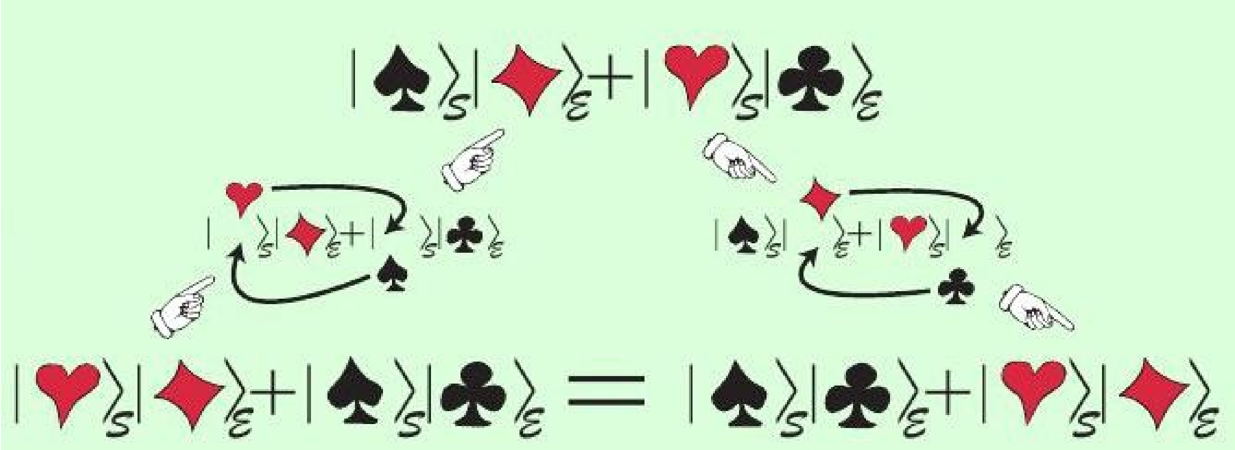

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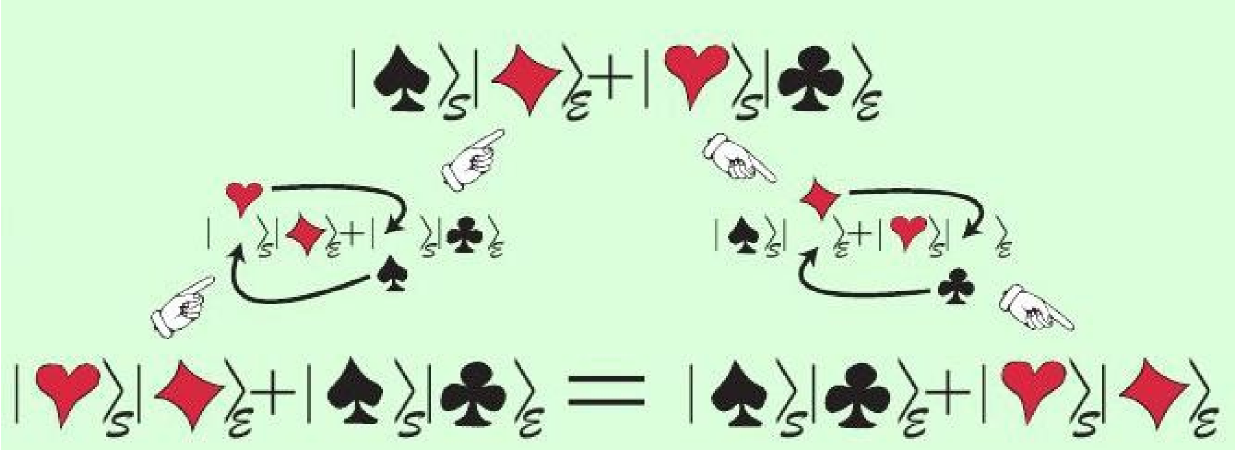

- **Bell inequalities** (modified not to use the Born rule)

  Imply that **quantum mechanical correlations** cannot be described by **local hidden variables**.

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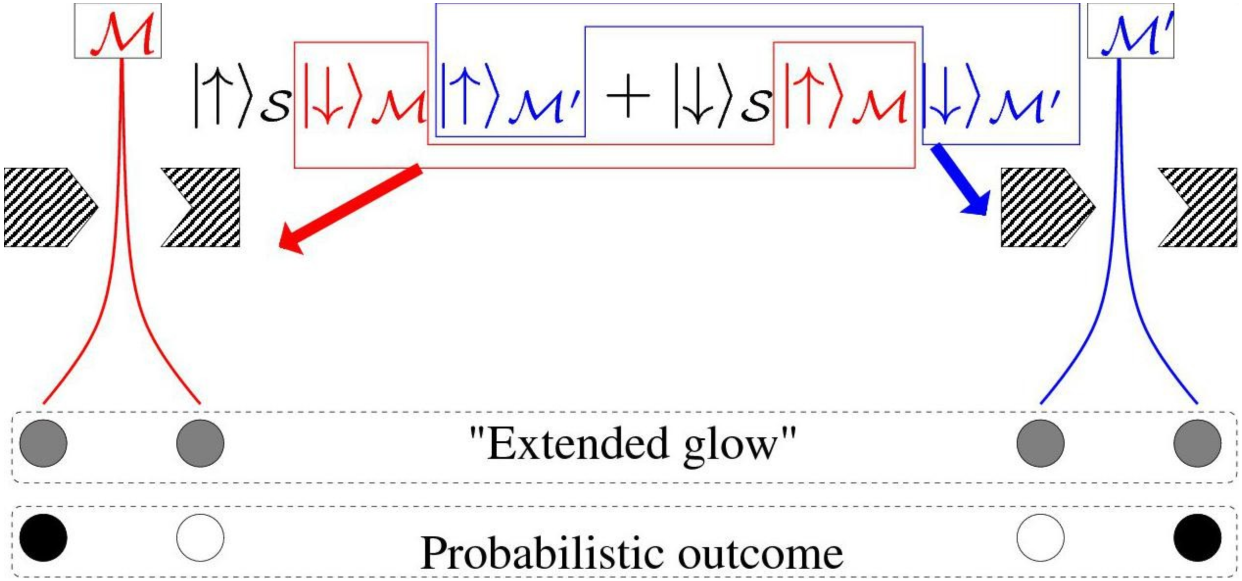

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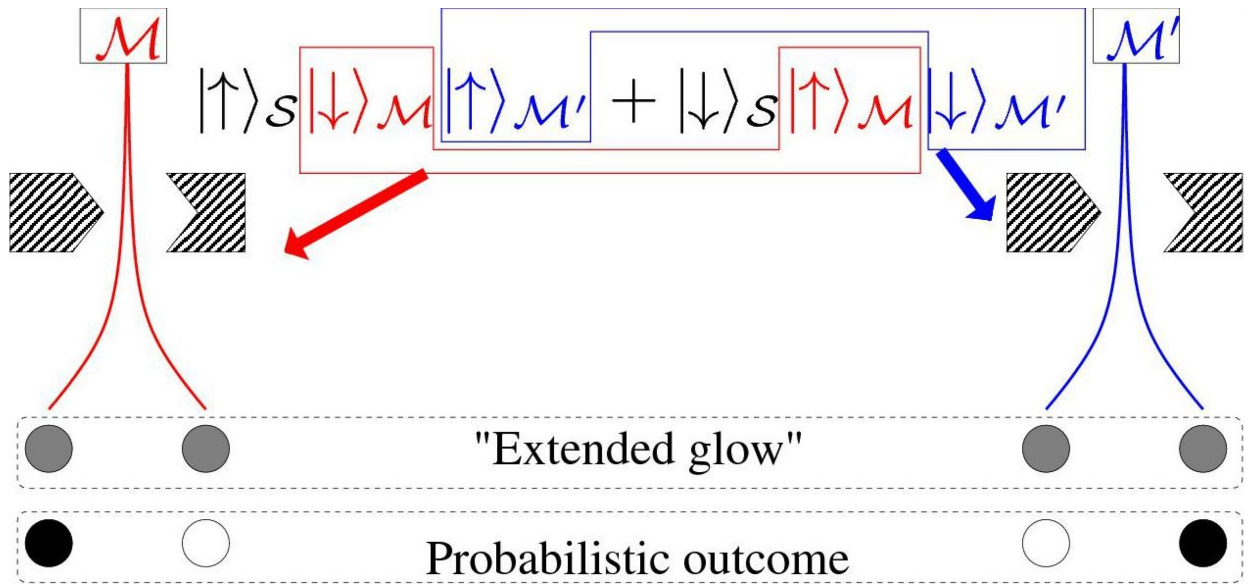

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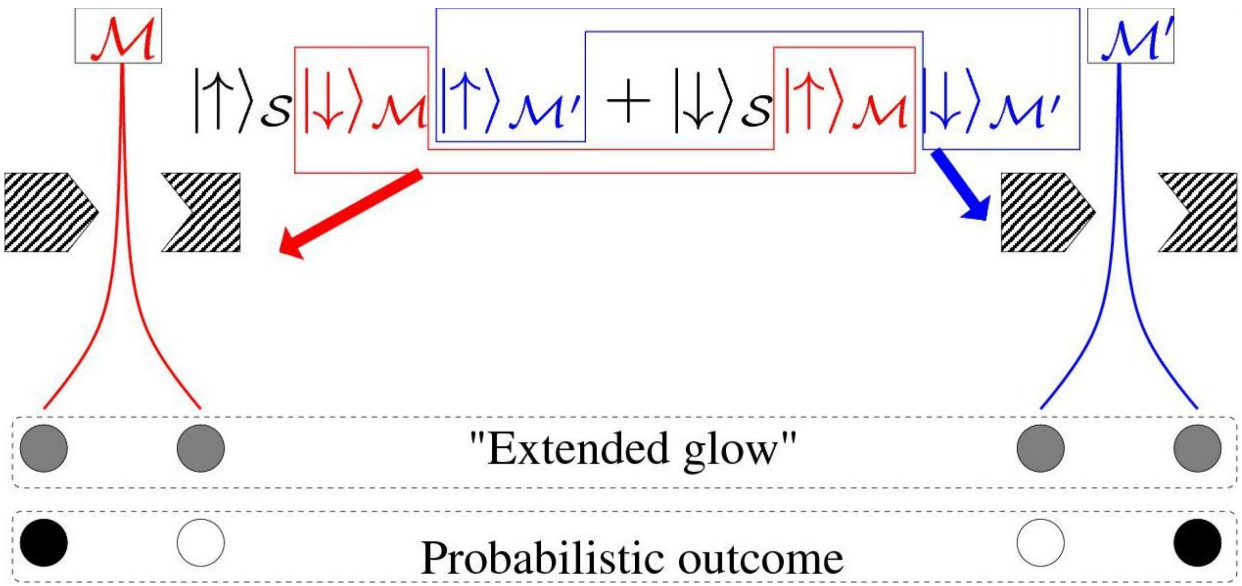

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$\Rightarrow$ **RANDOMNESS!**

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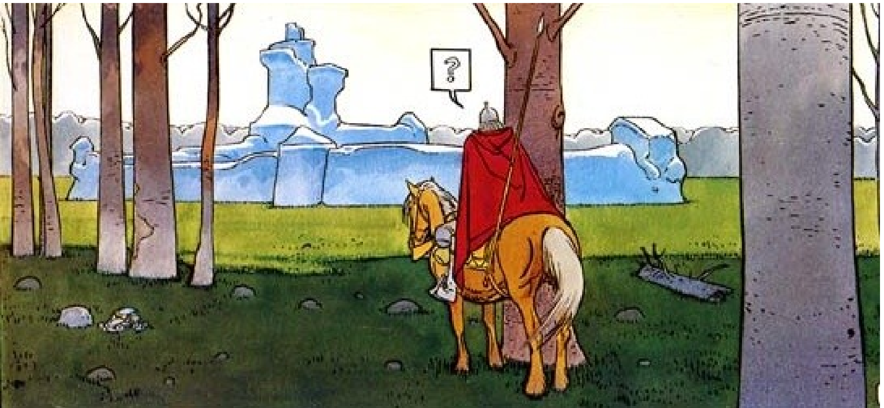
4. The symmetries from envariance constrain the randomness to **conform to the Born rule**



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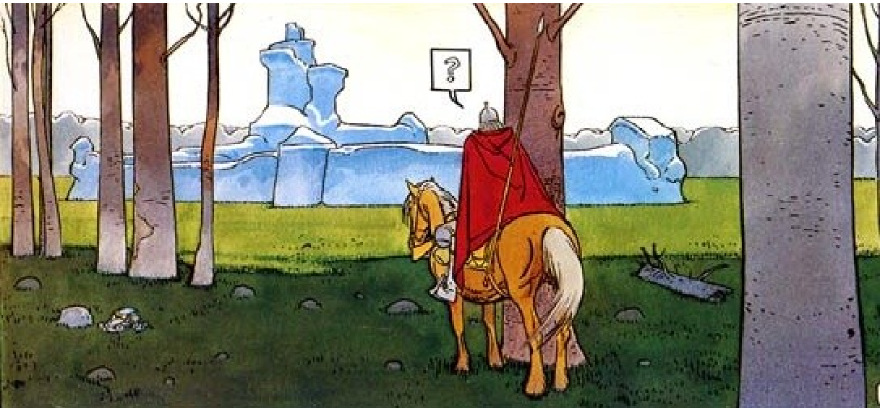

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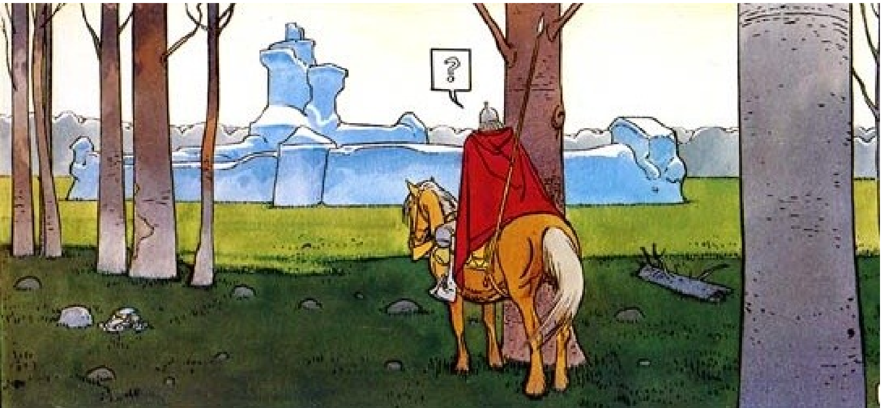


$$\boxed{\text{Born rule} \implies \text{collapse}}$$

(using only Bayesian inference)

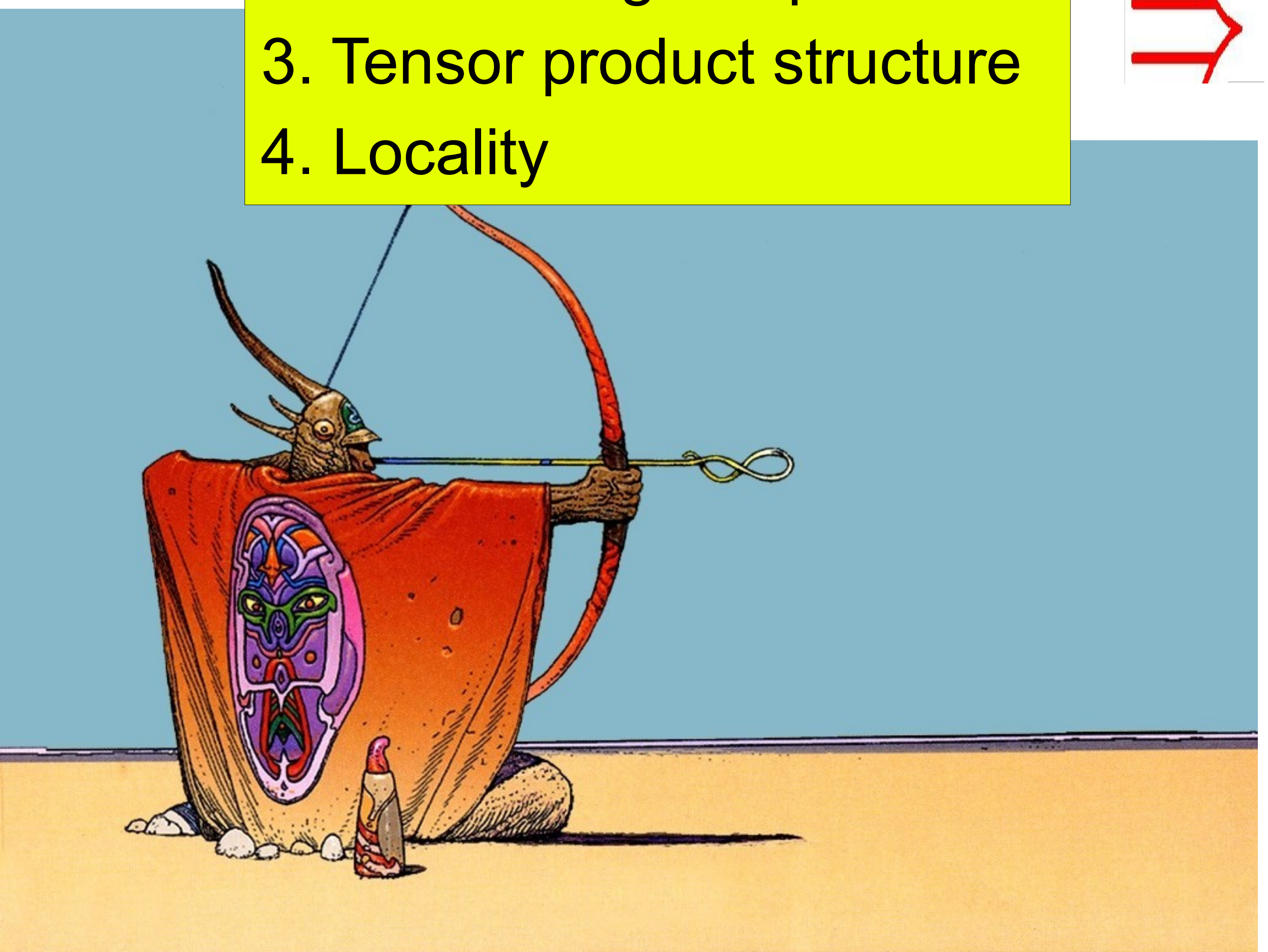
[Ozawa, quant-ph/9705030]

Intuition behind our proof

What have we done?

1. States postulate
2. Schroedinger equation
3. Tensor product structure
4. Locality

$\Rightarrow$ **QM**



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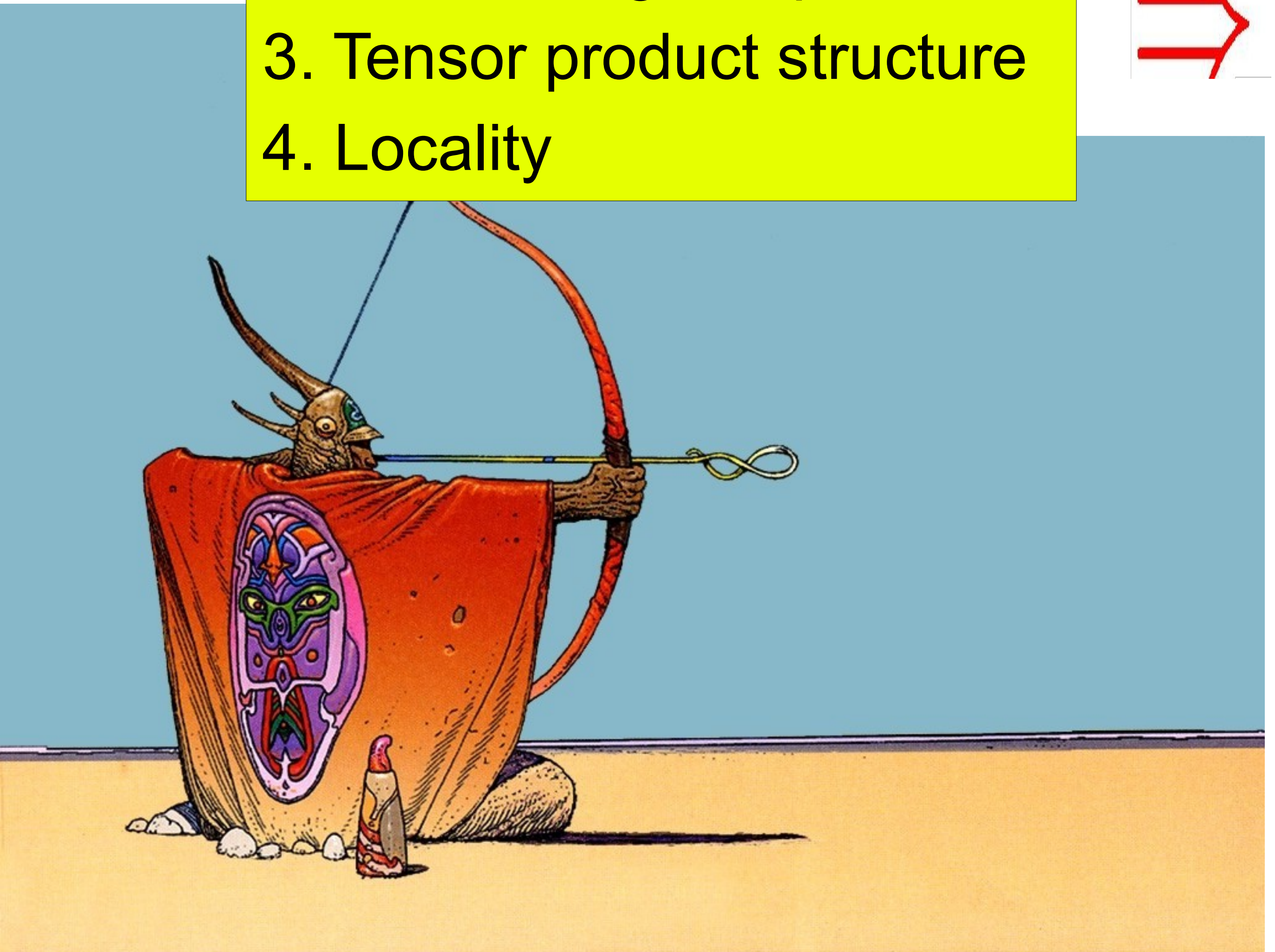

And now...
the details! →

What is a measurement?

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In a quantum universe (NPQM), a measurement is a unitary interaction between the system and some memory degree of freedom:

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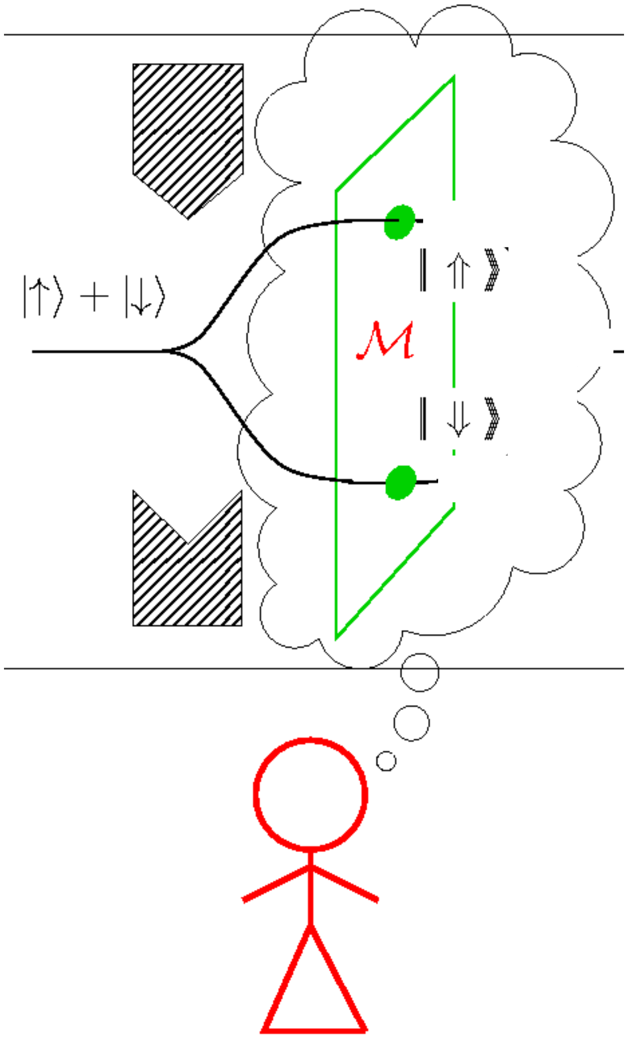

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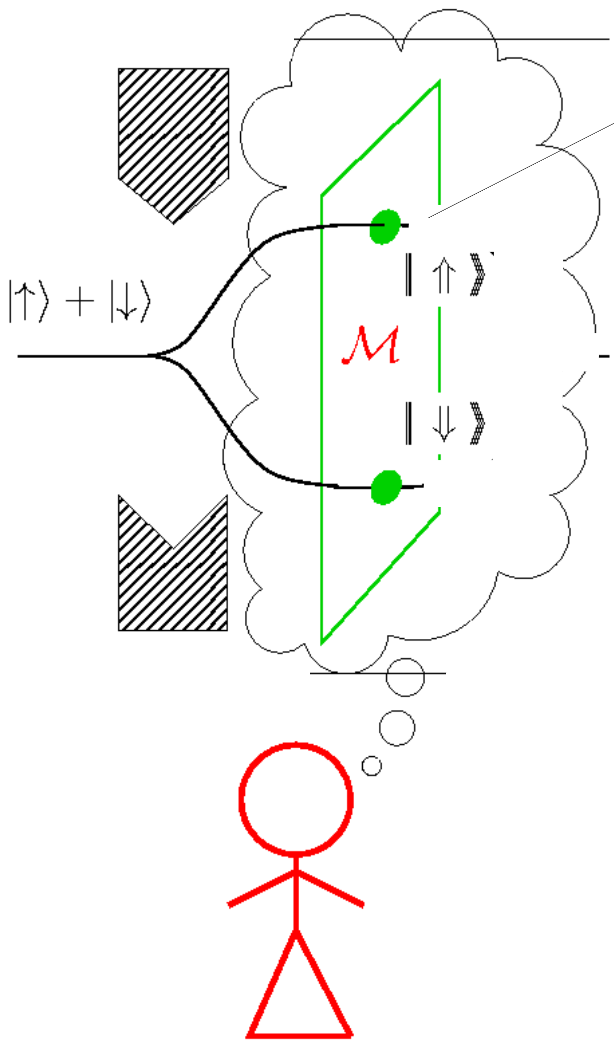

$$U_O \equiv |\uparrow\rangle\langle\uparrow| \otimes |\Uparrow\rangle\langle ?| + |\downarrow\rangle\langle\downarrow| \otimes |\Downarrow\rangle\langle ?| + C$$

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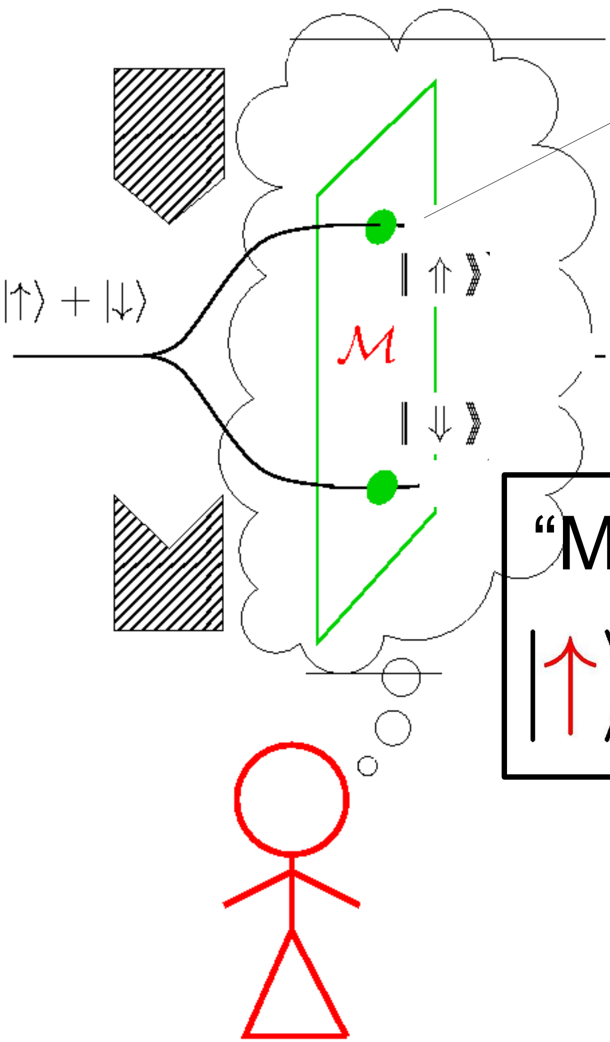

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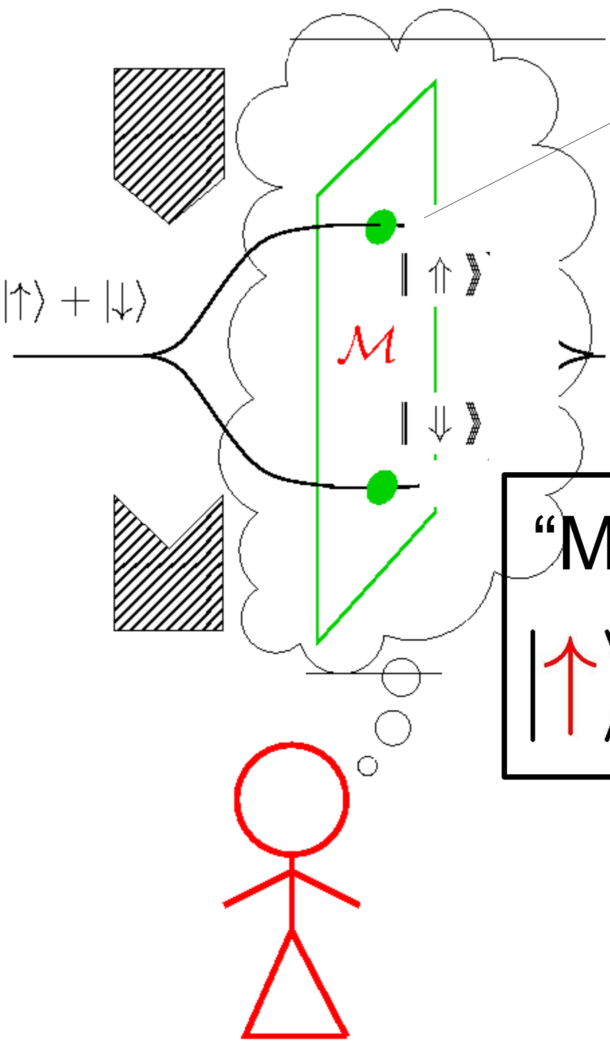

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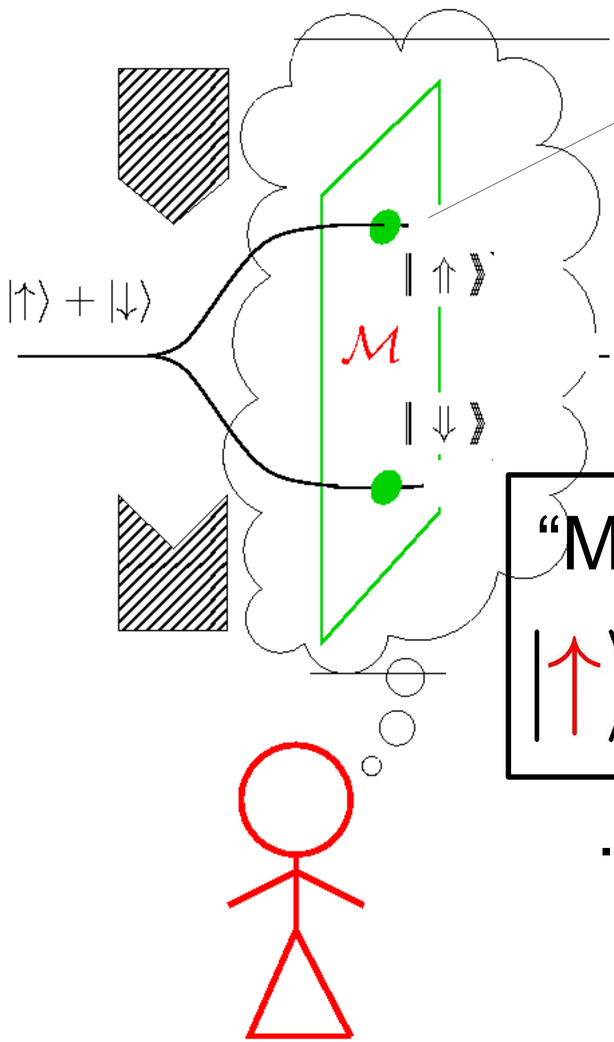

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measurement problem!

Measurement outcome

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$f_{\mathcal{M}}$ **measurement outcome function**

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$\Rightarrow$ We have to introduce perceptions $\longrightarrow$ $f_{\mathcal{M}}$ describes the perception of an entangled observer

$$|\uparrow\rangle|\Uparrow\rangle + |\downarrow\rangle|\Downarrow\rangle$$

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$$|\downarrow\rangle \Big|\Downarrow\Big\rangle + |\uparrow\rangle \Big|\Uparrow\Big\rangle$$

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we'll look only at the **symmetry properties** of $f_{\mathcal{M}}$ through envariance.

The symmetry stemming from entangled states.

FLIPS

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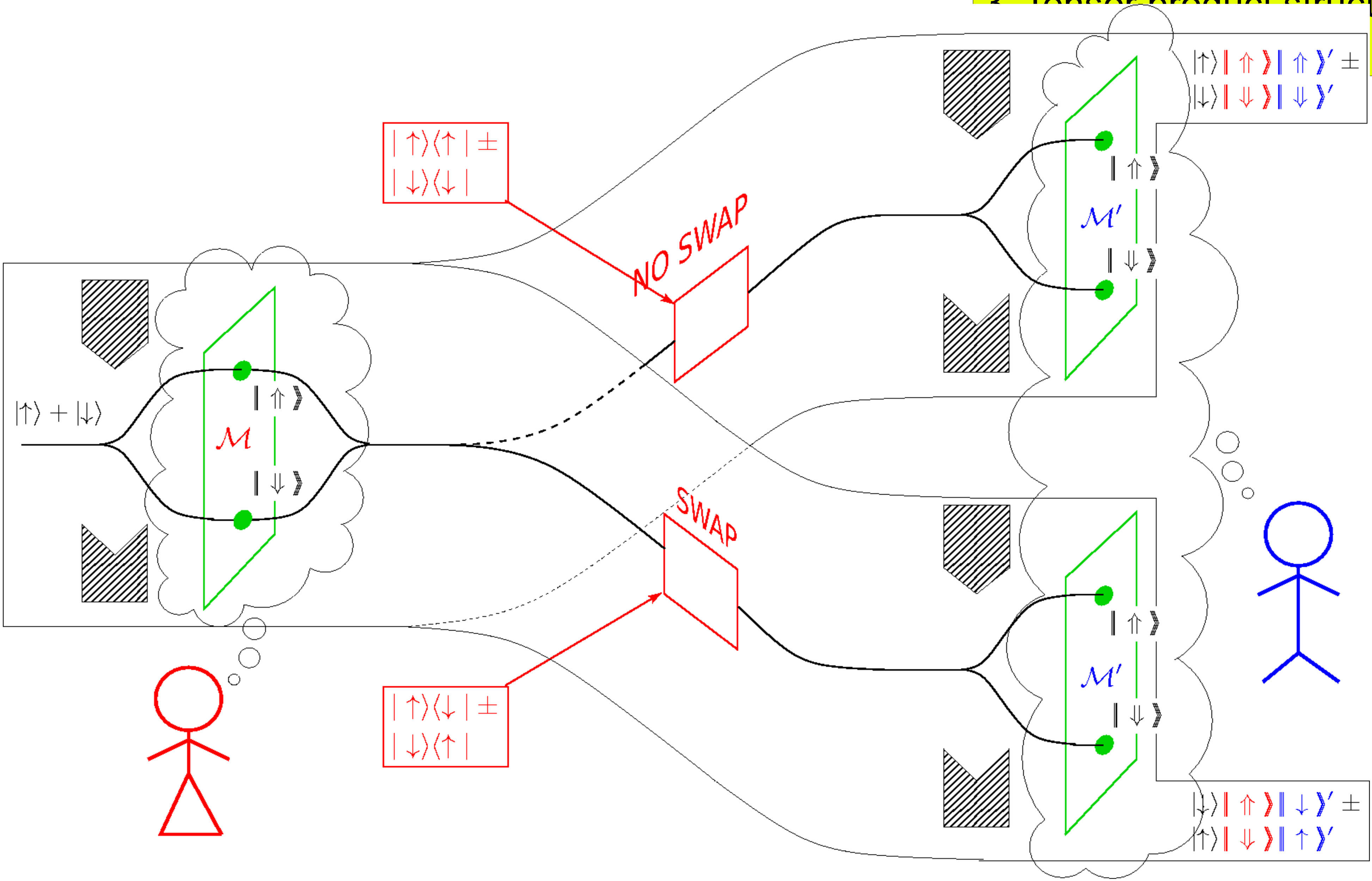

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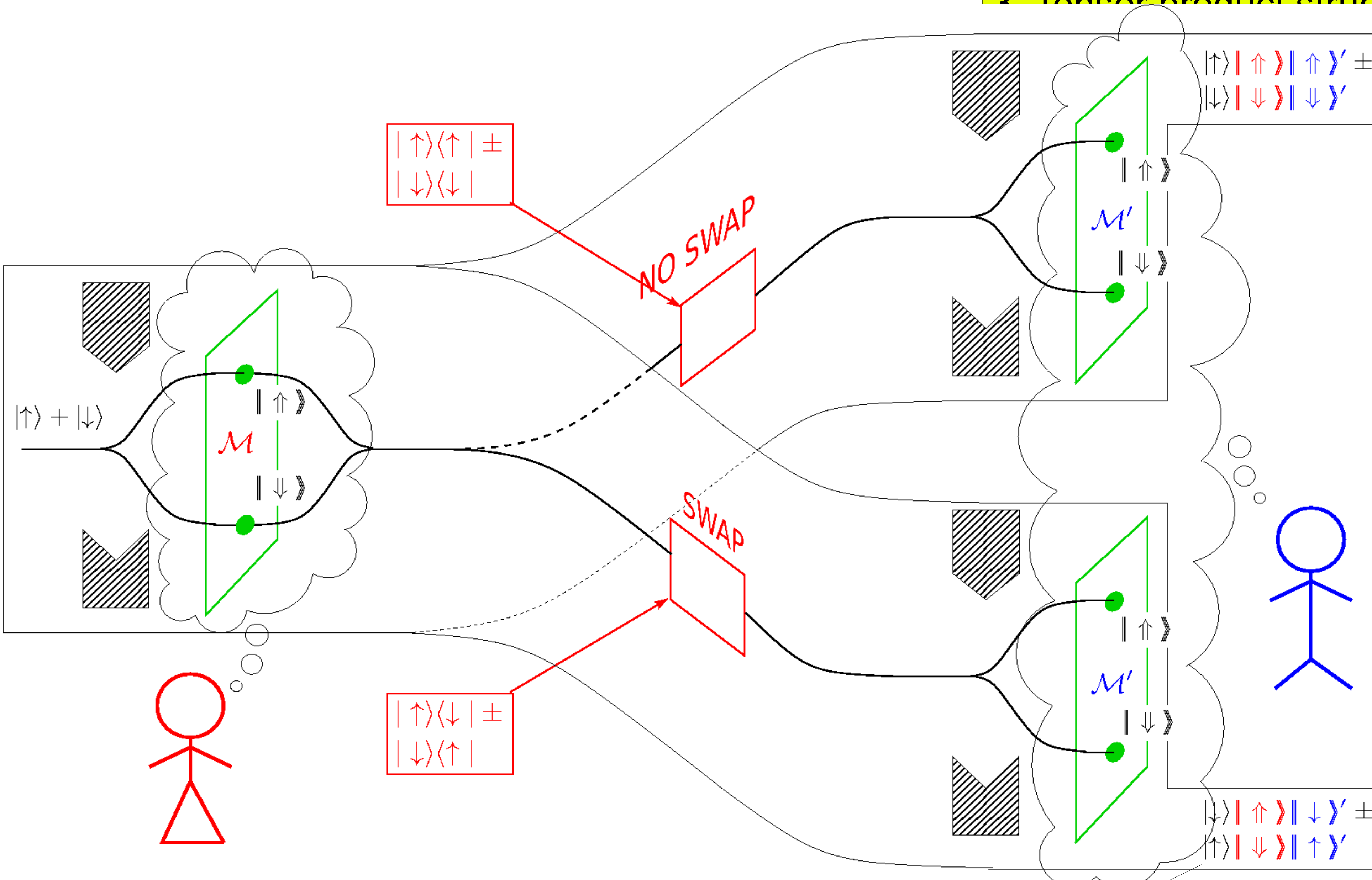

$$|\downarrow\rangle \, \| \Uparrow \rangle\rangle_{\mathcal{M}} \, \| \Downarrow \rangle\rangle_{\mathcal{M}'} \pm |\uparrow\rangle \, \| \Downarrow \rangle\rangle_{\mathcal{M}} \, \| \Uparrow \rangle\rangle_{\mathcal{M}'}$$

FLIPS (formulas)

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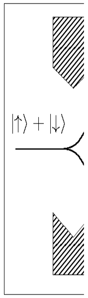

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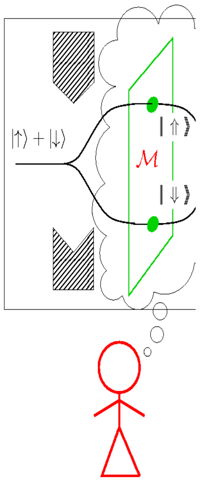
2. measure: $|{\uparrow}\rangle \boldsymbol{|} \Uparrow \boldsymbol{\rangle} + |{\downarrow}\rangle \boldsymbol{|} \Downarrow \boldsymbol{\rangle}$



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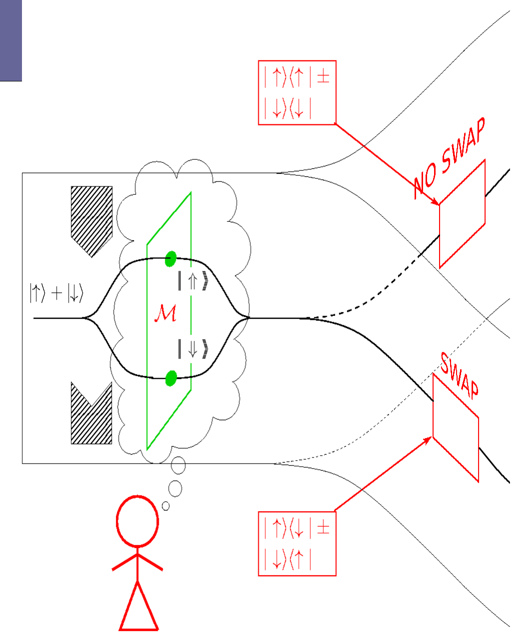

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using Pauli op. $|\uparrow\rangle\langle\downarrow| \pm |\downarrow\rangle\langle\uparrow|$ (or $|\uparrow\rangle\langle\uparrow| \pm |\downarrow\rangle\langle\downarrow|$)

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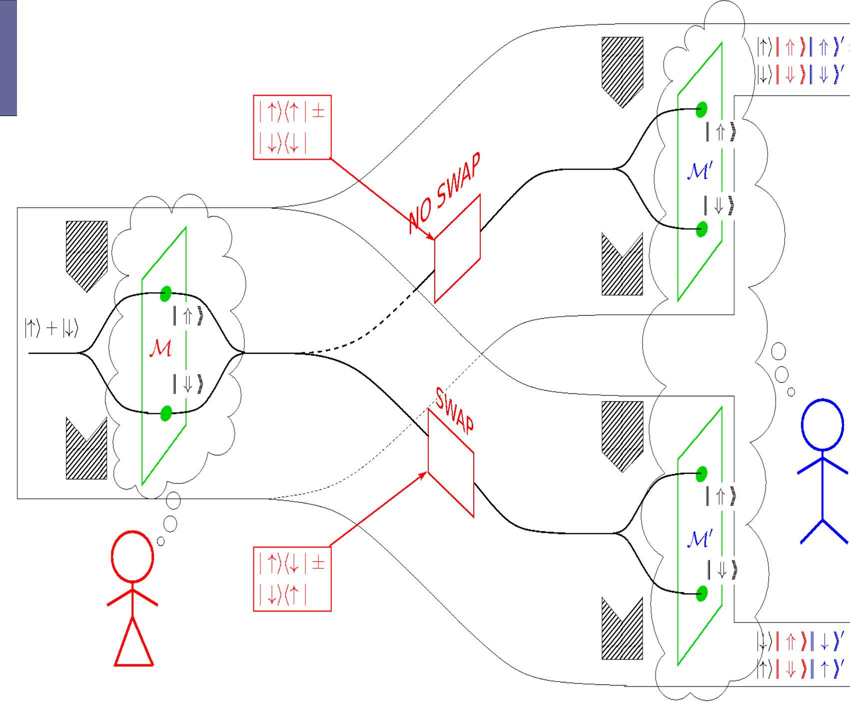

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$$|\downarrow\rangle|\Uparrow\rangle_{\mathcal{M}}|\Downarrow\rangle_{\mathcal{M}'} \pm |\uparrow\rangle|\Downarrow\rangle_{\mathcal{M}}|\Uparrow\rangle_{\mathcal{M}'}$$

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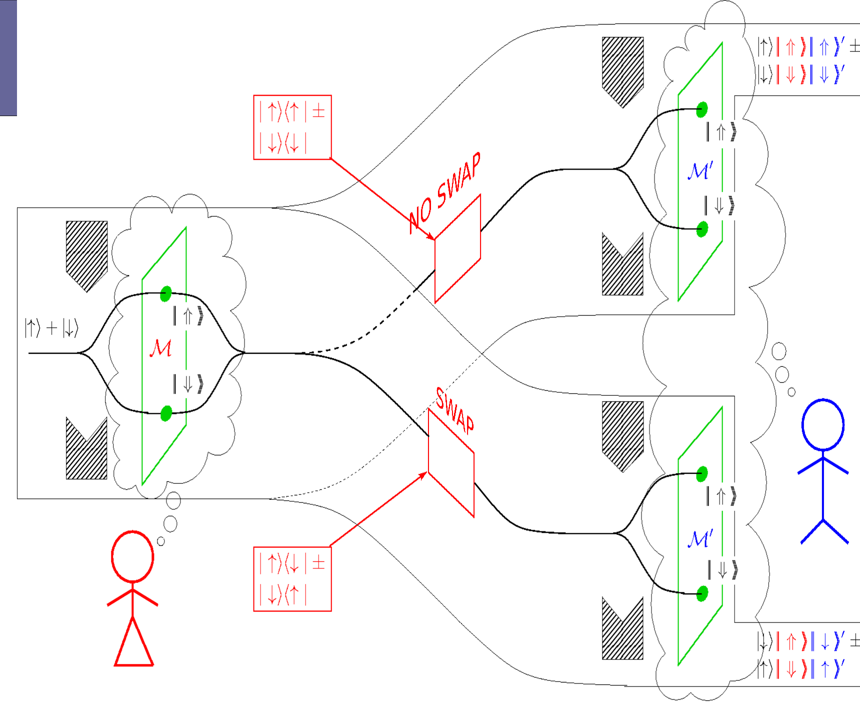

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first observer second observer

Envariance

1. States postulate **NPQM**
2. Schroedinger equation
3. Tensor product structure
4. Locality

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The local state of each observer ($\mathcal{M}$ or $\mathcal{M}'$) is the same: swaps have no influence!

(if we could use partial traces, that's obvious!)

$$\boxed{\rho_{\mathcal{M}} = \rho_{\mathcal{M}'}}$$

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$U_{\mathcal{S}}$

same state!

$$\boxed{\pm|\uparrow\rangle|\Downarrow\rangle_{\mathcal{M}}|\Uparrow\rangle_{\mathcal{M}'} + |\downarrow\rangle|\Uparrow\rangle_{\mathcal{M}}|\Downarrow\rangle_{\mathcal{M}'}}$$

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$$\pm|\uparrow\rangle|\Downarrow\rangle_{\mathcal{M}}|\Uparrow\rangle_{\mathcal{M}'} + |\downarrow\rangle|\Uparrow\rangle_{\mathcal{M}}|\Downarrow\rangle_{\mathcal{M}'}$$

same state!

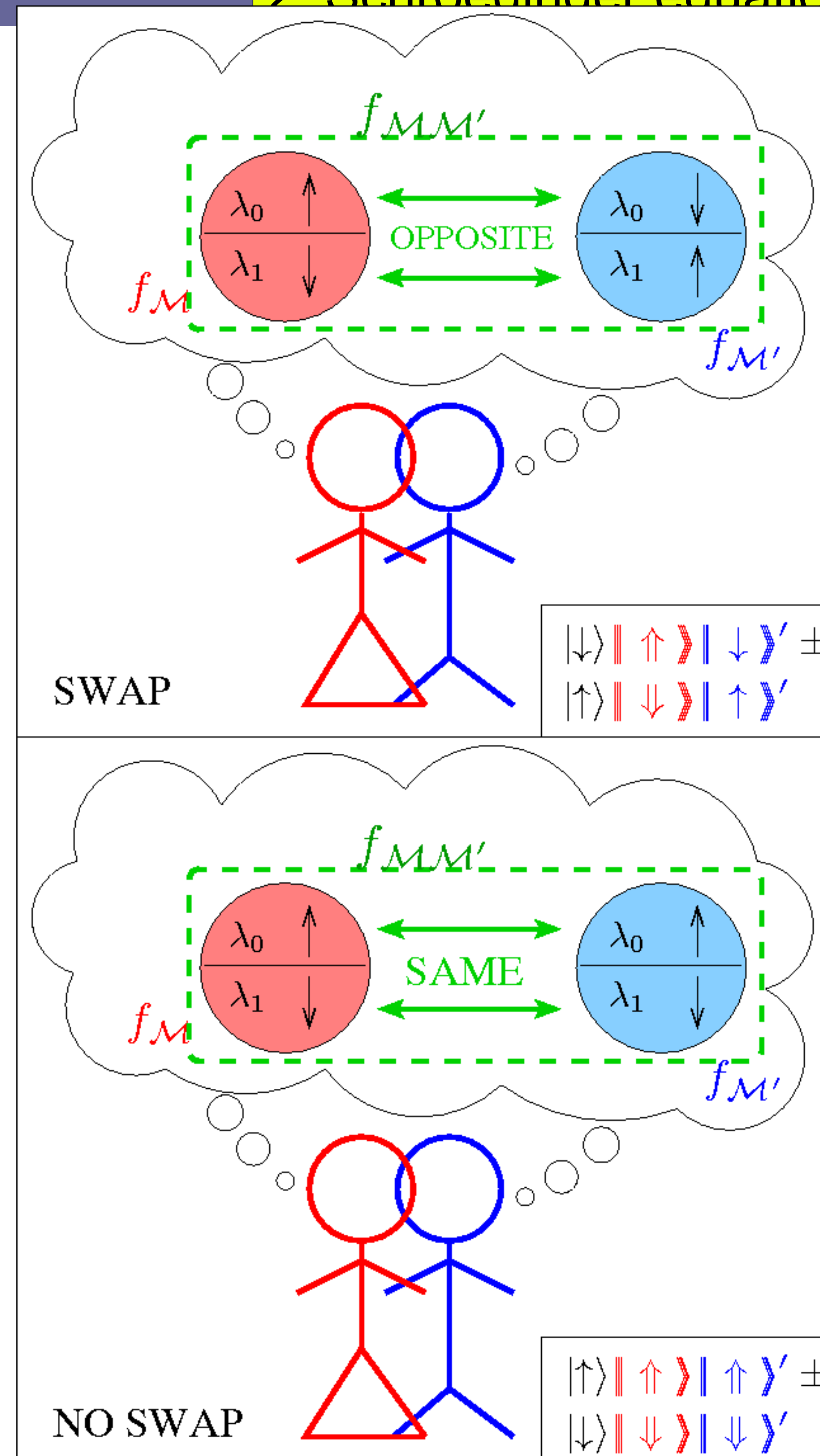
$\Rightarrow$ the measurement outcome is symmetric for swaps!

$$\boxed{f(|\uparrow\rangle, |\downarrow\rangle) = f(|\downarrow\rangle, |\uparrow\rangle)}$$

Correlations

1. States postulate **NPQM**
2. Schroedinger equation

Nonetheless, if they join forces $\mathcal{M}$ and $\mathcal{M}'$ can recover whether a swap has occurred (the correlations) using only 1, st. postulate.



Correlations

1. States postulate **NPQM**
2. Schroedinger equation
3. Tensor product structure
4. Locality

to recover whether a swap has occurred (the correlations) using only 1, st. postulate they can use the unitary:

$$(|\Uparrow\rangle\langle\Uparrow|\otimes|\Downarrow\rangle'\langle\Downarrow| + |\Downarrow\rangle\langle\Downarrow|\otimes|\Uparrow\rangle'\langle\Uparrow|)\otimes|y\rangle_t\langle n| + D$$

gives "*y*" if the two arrows are **opposite**

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gives "*y*" if the two arrows are **opposite**

applied to $|\downarrow\rangle|\Uparrow\rangle_{\mathcal{M}}|\Downarrow\rangle_{\mathcal{M}'}\pm|\uparrow\rangle|\Downarrow\rangle_{\mathcal{M}}|\Uparrow\rangle_{\mathcal{M}'}$

returns $|y\rangle_t$ **factorized** from the rest

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1. st postulate

Correlations 2

1. States postulate **NPQM**
2. Schroedinger equation
3. Tensor product structure
4. Locality

This means that, by comparing their measurement outcomes, they can **conclusively** track swaps, but they cannot track eventual phases that are introduced.

Correlations 2

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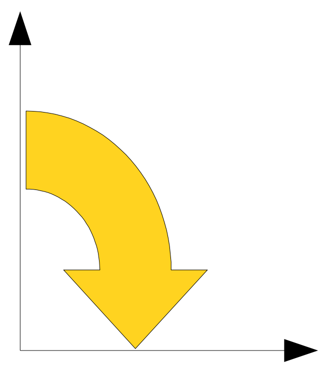
The only way this can be done is by rotating a state by 90°



$$U_R|\psi\rangle = |\psi^{\perp}\rangle$$

$$\langle\psi|\psi^{\perp}\rangle = 0$$

(in NPQM, or QM, two states can be **conclusively** distinguished **only** if they're **orthogonal**!)

... now, which are the only states that are either left

untouched or made **orthogonal** by the **Pauli operators?**

$$|\uparrow\rangle\langle\downarrow| \pm |\downarrow\rangle\langle\uparrow| \qquad |\uparrow\rangle\langle\uparrow| \pm |\downarrow\rangle\langle\downarrow|$$

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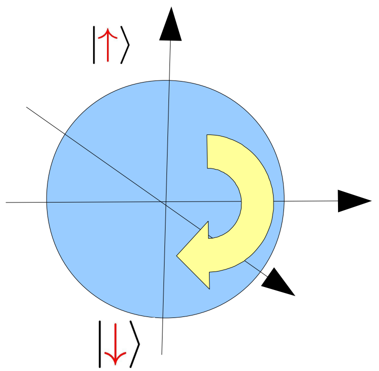
$$\{|\uparrow\rangle, |\downarrow\rangle\}$$

(the eigenstates of the observable for S-G)



Correlations 3

1. States postulate **NPQM**
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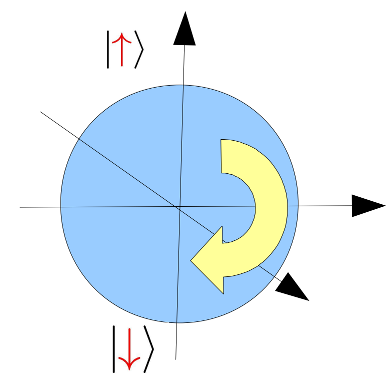
$$|\uparrow\rangle\langle\downarrow| \pm |\downarrow\rangle\langle\uparrow| \qquad |\uparrow\rangle\langle\uparrow| \pm |\downarrow\rangle\langle\downarrow|$$

$$\longrightarrow \boxed{\{|\uparrow\rangle, |\downarrow\rangle\}}$$

(the eigenstates of the observable for S-G)



$\Rightarrow$ to track the correlations **conclusively** we must have

$f_{\mathcal{M}}$ depends on at least **one** of these $\{|\uparrow\rangle, |\downarrow\rangle\}$

(so it's rotated to an orthogonal state by a Pauli op)

Correlations 4

1. States postulate **NPQM**
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... but $f_{\mathcal{M}}$ also satisfies the envariance symmetry!

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$\Rightarrow$ if $f_{\mathcal{M}}$ depends on **one** of the $\{|\uparrow\rangle, |\downarrow\rangle\}$ it must also depend (symmetrically) on **all** of them!

$$\boxed{\exists \text{ 2 values } \lambda_\ell \text{ such that } f_{\mathcal{M}}(\lambda_0) = |\uparrow\rangle,\ f_{\mathcal{M}}(\lambda_1) = |\downarrow\rangle}$$

where λ is some free variable that keeps track of the variations of $f_{\mathcal{M}}$

Bell inequalities

1. States postulate **NPQM**
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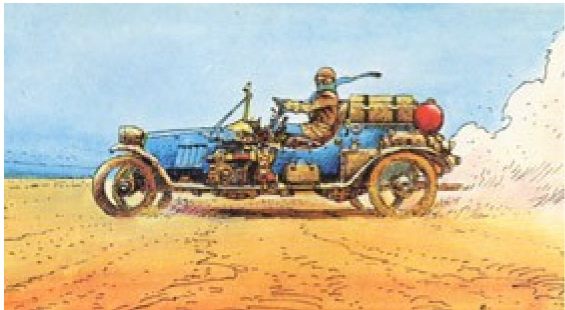
4. Locality

$\exists \lambda$ that allows them to compare outcomes



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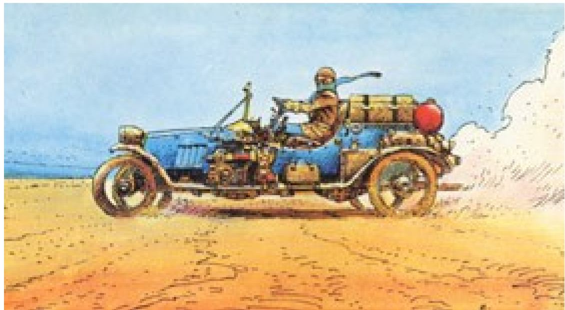

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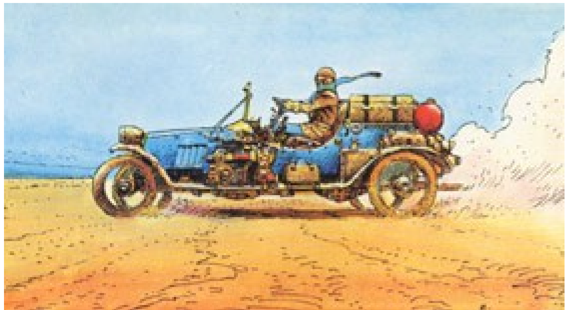
QM is incompatible with **all** three:

1) Free will or measurement independence

2) Locality

3) Counterfactual definiteness (assignement of $f_{\mathcal{M}}(\lambda)$ independently of whether the measurement is performed)



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NOT IN CONTRAST WITH DETERMINISM!
(compatibilism)

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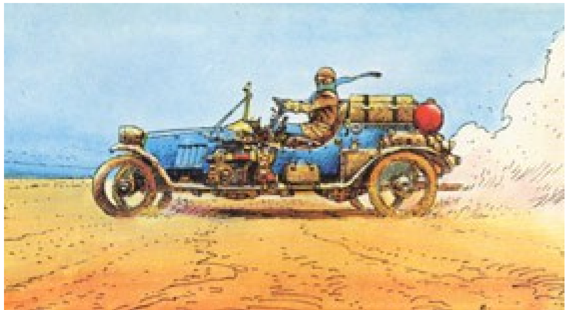

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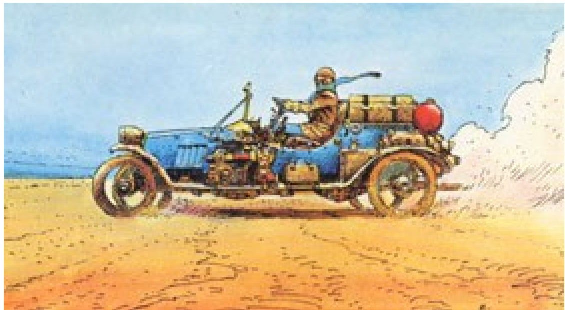

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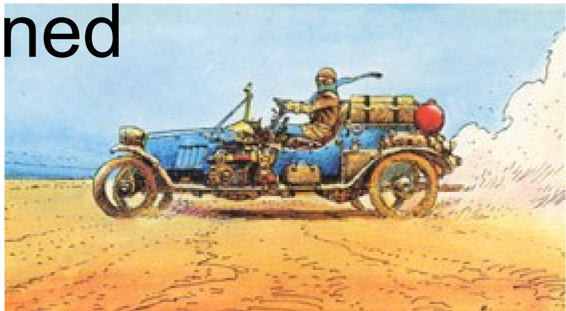

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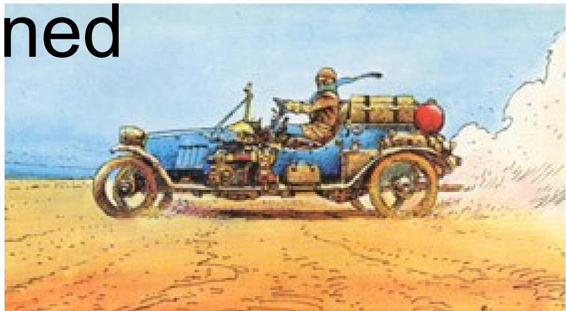
$\Rightarrow$ Randomness



Born rule

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+

- symmetry of measurement outcomes

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$\Downarrow$

equiprobability of each outcome

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- symmetry of measurement outcomes

$$f(|\uparrow\rangle, |\downarrow\rangle) = f(|\downarrow\rangle, |\uparrow\rangle)$$

$$\Downarrow$$

equiprobability of each outcome

$$\downarrow$$

i.e. the Born rule for the uniform state

$$|\uparrow\rangle + |\downarrow\rangle$$

Fine graining

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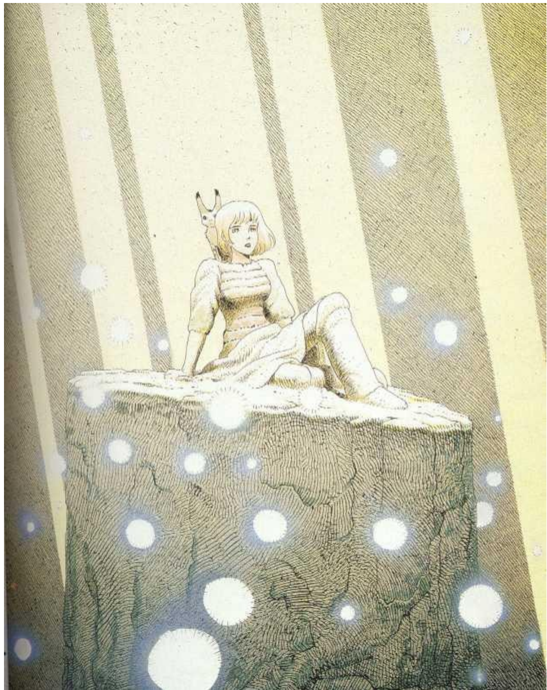
... what about the non-uniform case?



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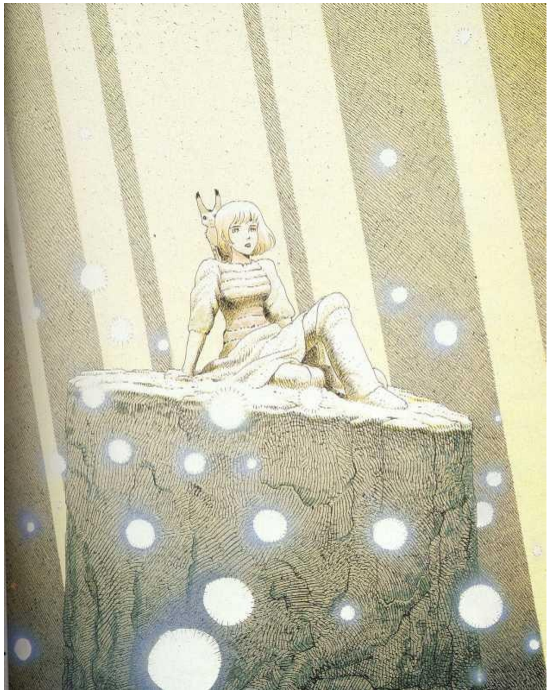

W.H.Zurek, PRL **90**,120404

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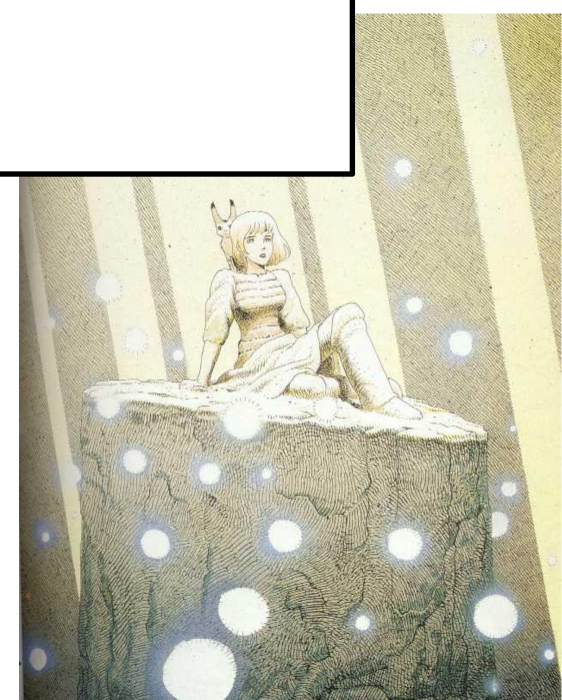
$$|0\rangle + \sqrt{2}|1\rangle$$ non uniform state



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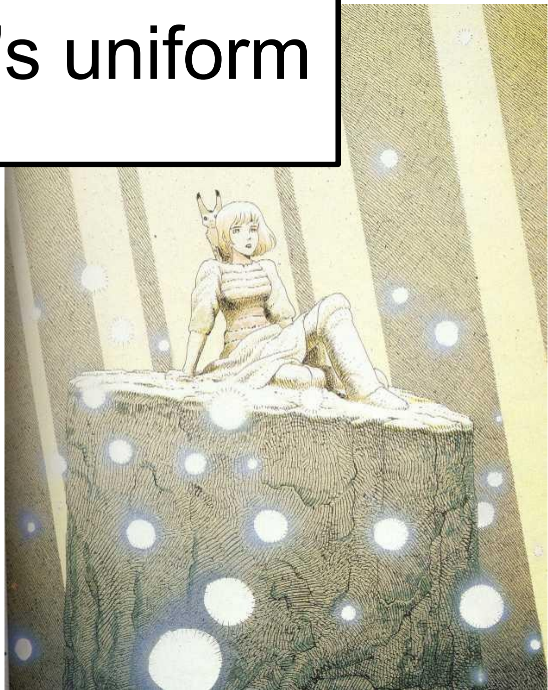
add a “fine-graining” secondary system *g*

$$|0\rangle|a\rangle_g + |1\rangle|b\rangle_g + |1\rangle|c\rangle_g$$ now it's uniform



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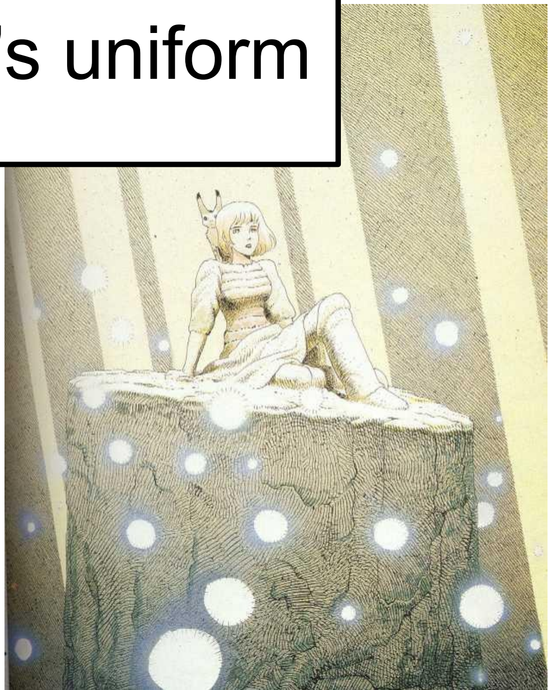
$$|0\rangle|a\rangle_g + |1\rangle|b\rangle_g + |1\rangle|c\rangle_g$$ ← now it's uniform

$$\left\{\begin{array}{l} |0\rangle|a\rangle_g \; p = 1/3 \\ |1\rangle|b\rangle_g \; p = 1/3 \\ |1\rangle|c\rangle_g \; p = 1/3 \end{array}\right.$$

$$\longrightarrow |0\rangle \; p = 1/3,$$

$$\longrightarrow |1\rangle \; p = 2/3$$

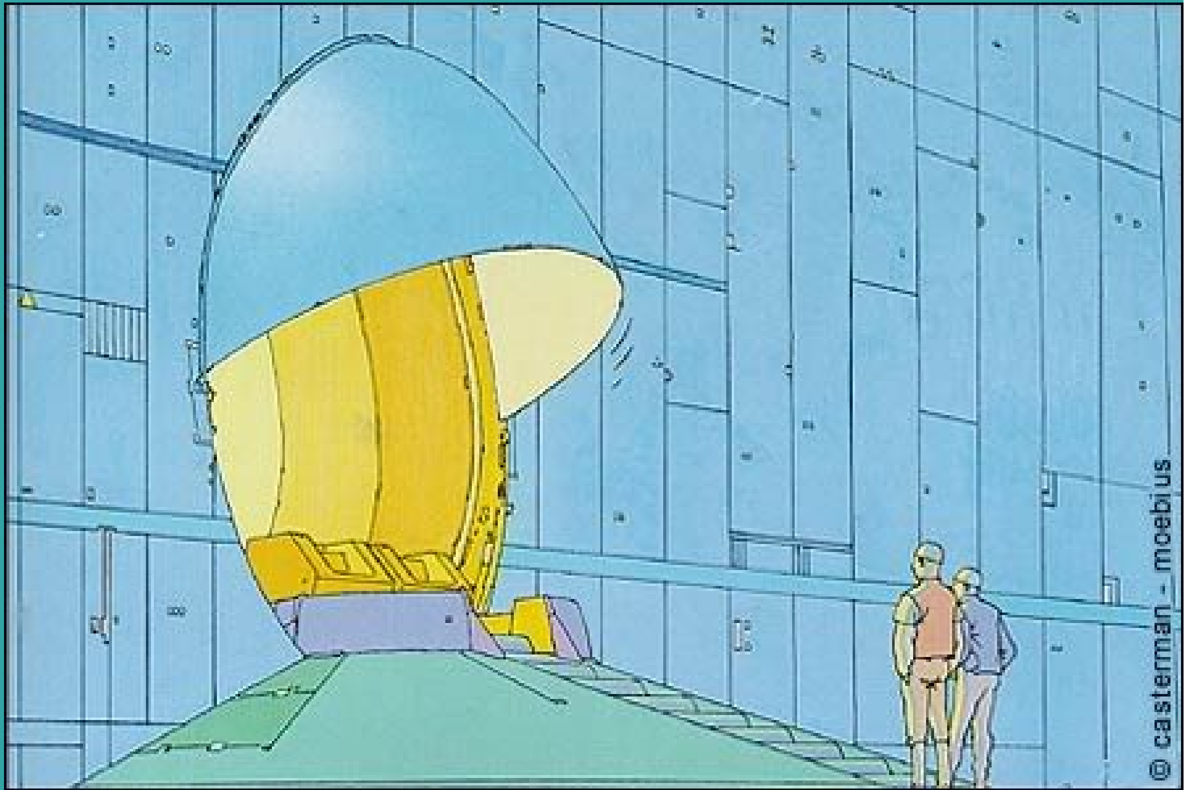


W.H.Zurek, PRL **90**,120404

... at this point the argument should be clear!!!

(If not, please ask questions!)

The rest of the talk is just **book-keeping** to finalize the details...



... and to conclude the argument...

Bell inequality without the Born rule

... and to conclude the argument...

Bell inequality without the Born rule

...but first we need to generalize the argument from spins to *d*-dimensional systems

Measurement in NPQM

1. States postulate **NPQM**
2. Schroedinger equation
3. Tensor product structure
4. Locality

Measurement of observable $\{|a_\ell\rangle\}$

system

memory

$$U_O \equiv \sum_\ell \underbrace{|a_\ell\rangle_{\mathcal{S}}\langle a_\ell|}_{} \otimes \left(|a_\ell\rangle_{\mathcal{M}}\langle ?|\right)^{\otimes N} + C$$

Copy

Redundancy (for einselection)

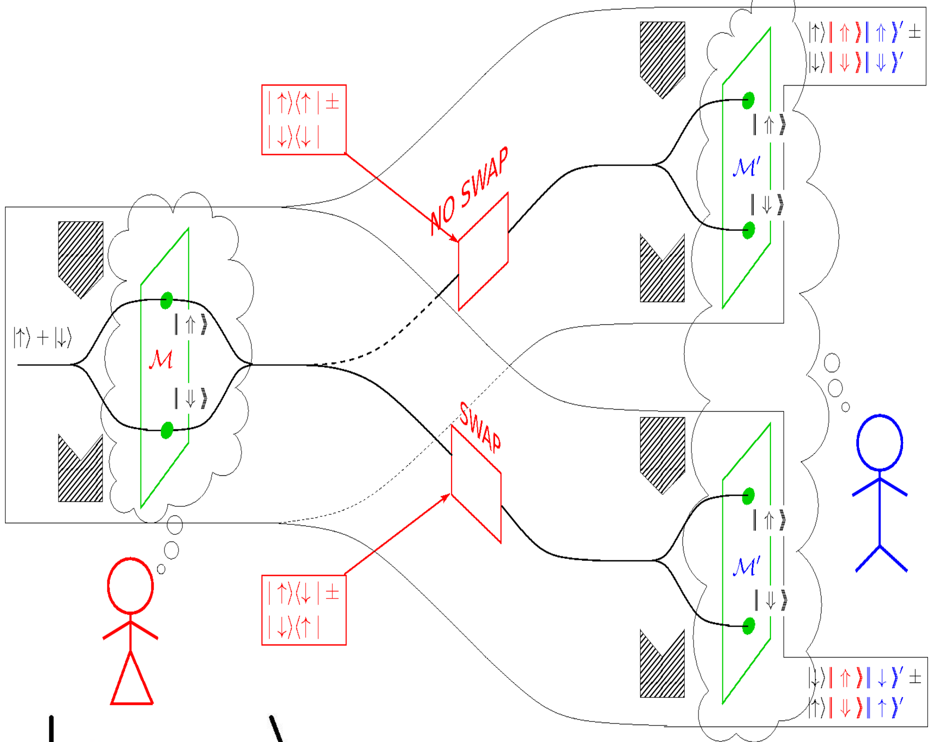
$$U_O \equiv |\uparrow\rangle\langle\uparrow| \otimes |\Uparrow\rangle\langle ?| + |\downarrow\rangle\langle\downarrow| \otimes |\Downarrow\rangle\langle ?| + C$$

$|?\rangle$

$|\Uparrow\rangle = |\uparrow\rangle^N$

initial memory st.

redundancy (necessary for decoherence)



1. uniform state $\sum_\ell |a_\ell\rangle_{\mathcal{S}}$

2. measure: $\sum_\ell |a_\ell\rangle_{\mathcal{S}} |a_\ell\rangle^N_{\mathcal{M}}$

3. swap the system: $|a_\ell\rangle_{\mathcal{S}} \rightarrow |a_{\ell\oplus k}\rangle_{\mathcal{S}}$

$\boxed{\text{“shift-and-multiply” instead of Pauli op. } U(k,j) = \sum_\ell e^{2\pi i\, j\ell/d} |a_{\ell\oplus k}\rangle_s \langle a_\ell|}$

4. measure again, on a memory $\mathcal{M}'$

$$\sum_\ell |a_{\ell\oplus k}\rangle_{\mathcal{S}} |a_\ell\rangle^N_{\mathcal{M}} |a_{\ell\oplus k}\rangle^N_{\mathcal{M}'}$$

first observer second observer

Summarizing

1. States postulate **NPQM**
2. Schroedinger equation
3. Tensor product structure
4. Locality

$$\sum_{\ell} \omega^{j\ell} |a_{\ell \oplus k}\rangle_s \boxed{|a_\ell\rangle_m^N} \boxed{|a_{\ell\oplus k}\rangle_{m'}^N}$$

Summarizing

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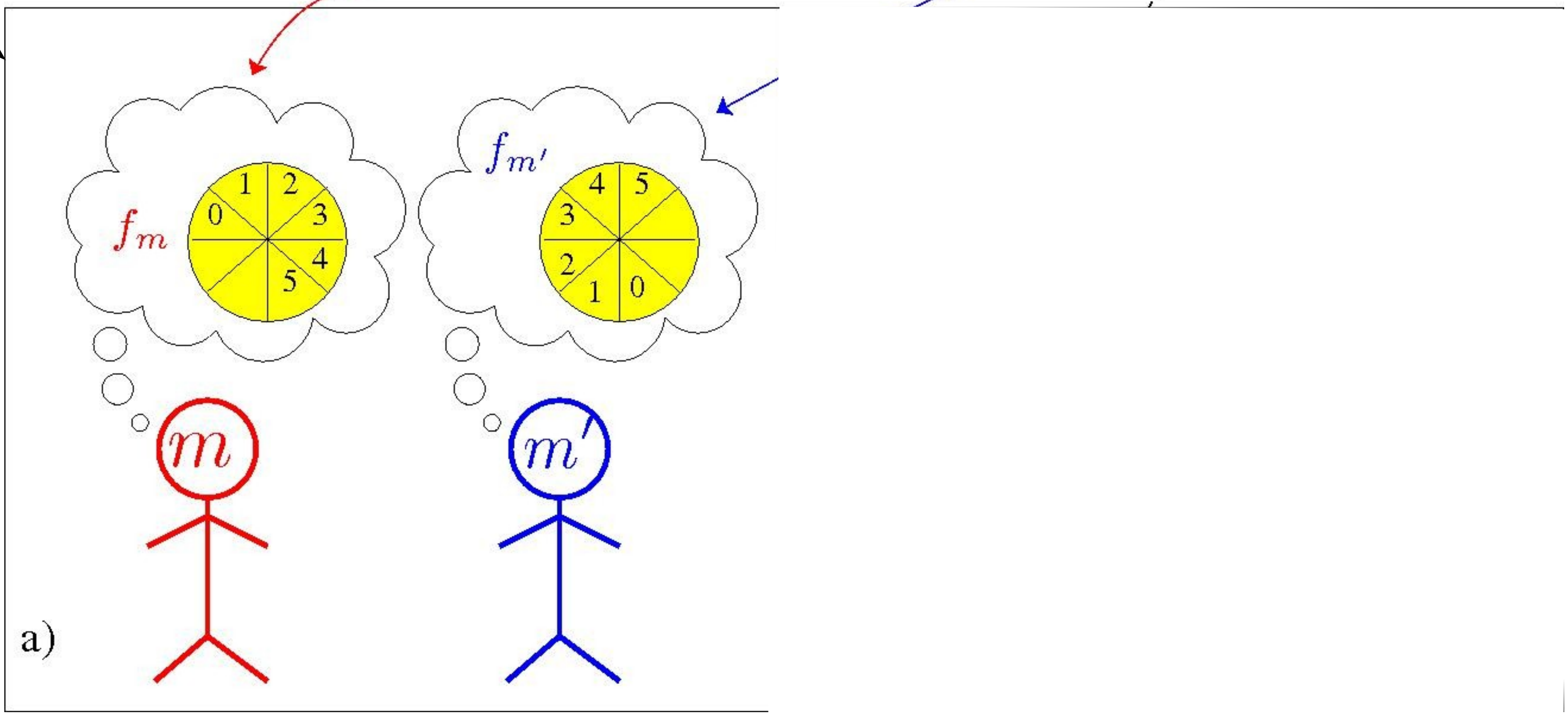
$$\sum_\ell \omega^{j\ell} |a_{\ell\oplus k}\rangle_s \boxed{|a_\ell\rangle_m^N} \boxed{|a_{\ell\oplus k}\rangle_{m'}^N}$$

envariance



Summarizing

1. States postulate **NPQM**
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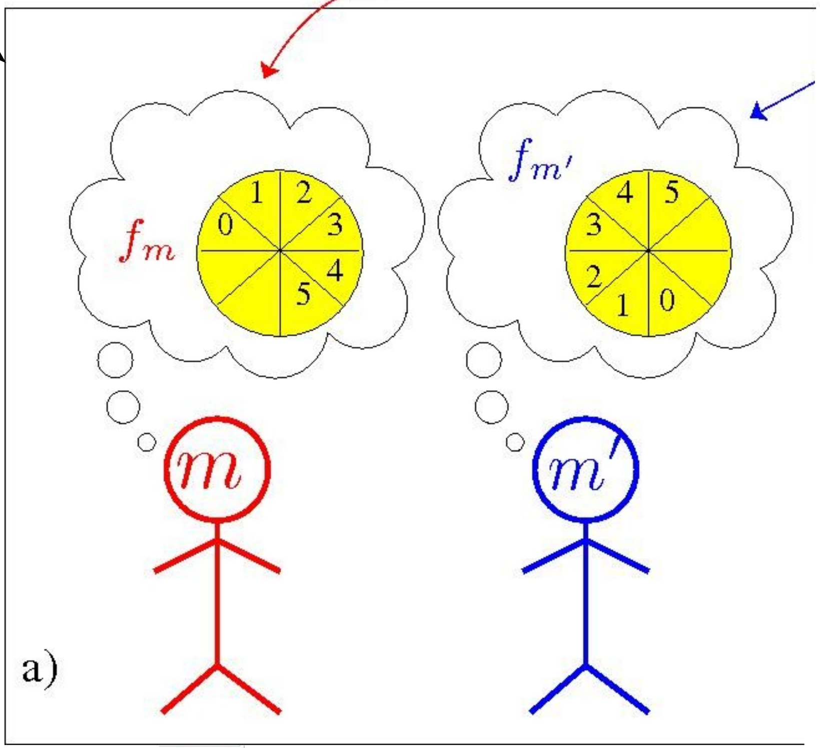
envariance



$$\Downarrow$$

$$f(|a_0\rangle, |a_1\rangle, \cdots) = f(|a_k\rangle, |a_{k\oplus 1}\rangle, \cdots) \quad \text{for all } k$$

Summarizing

1. States postulate **NPQM**
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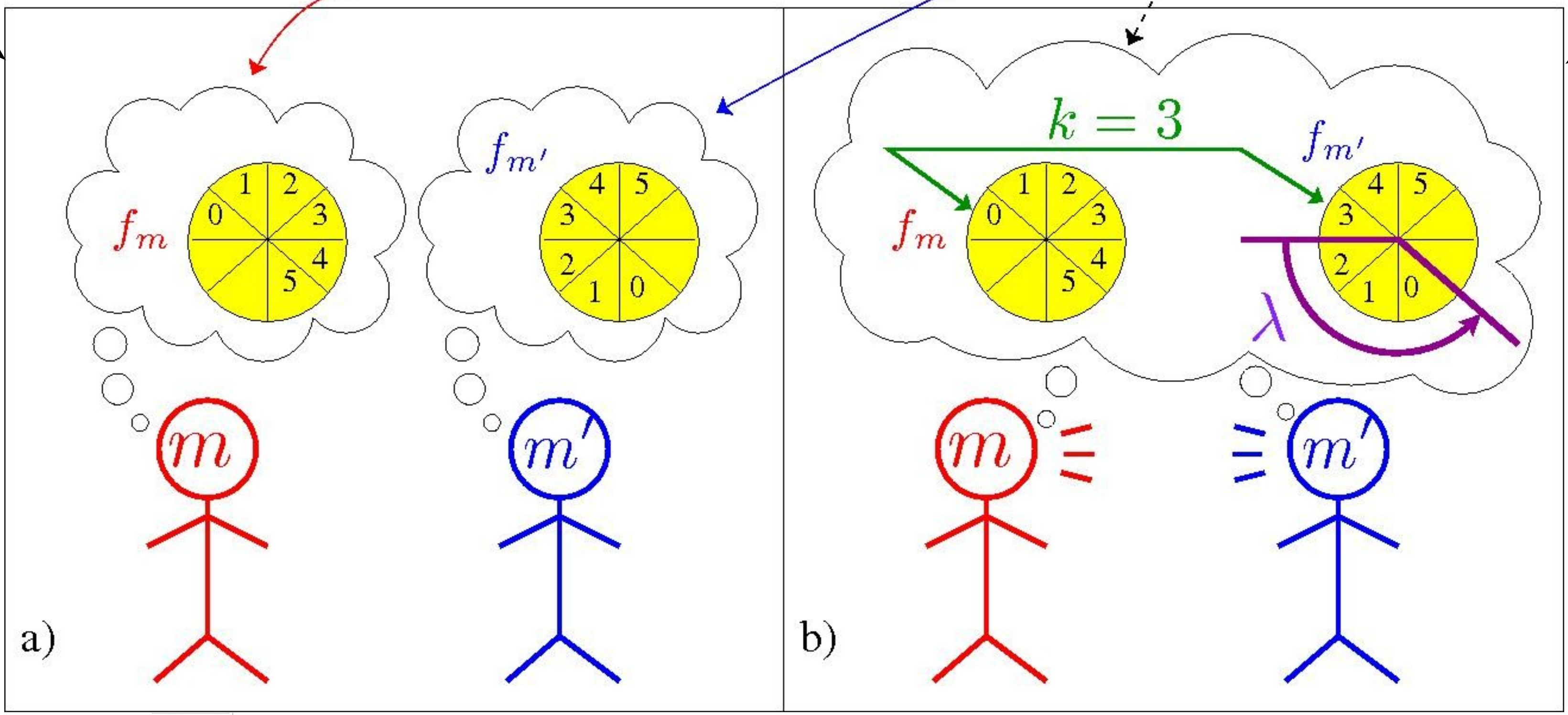
$$\sum_{\ell} \omega^{j\ell} |a_{\ell\oplus k}\rangle_s \boxed{\boxed{|a_\ell\rangle_m^N} \boxed{|a_{\ell\oplus k}\rangle_{m'}^N}}$$

envariance

correlations



$\Downarrow$

$$f(|a_0\rangle, |a_1\rangle, \cdots) = f(|a_k\rangle, |a_{k\oplus 1}\rangle, \cdots) \quad \text{for all } k$$

Summarizing

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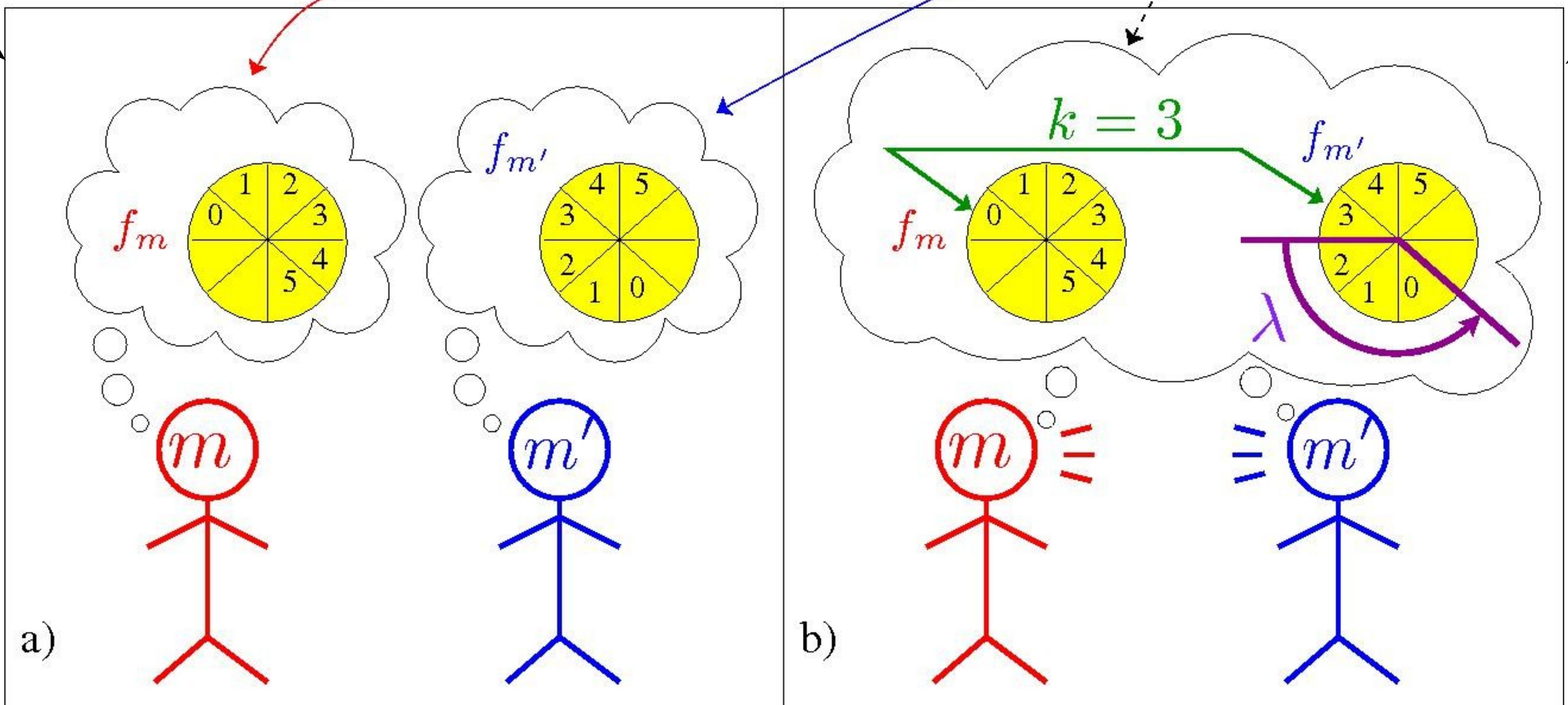
$$\sum_{\ell} \omega^{j\ell} |a_{\ell\oplus k}\rangle_s \boxed{|a_\ell\rangle_m^N} \boxed{|a_{\ell\oplus k}\rangle_{m'}^N}$$

envariance

correlations



$$f(|a_0\rangle, |a_1\rangle, \cdots) = f(|a_k\rangle, |a_{k\oplus 1}\rangle, \cdots) \quad \text{for all } k$$

$\exists\lambda$ that allows them to compare outcomes:

$\exists\ d$ values λ_ℓ such that $f_{\mathcal{M}}(\lambda_\ell) = |a_\ell\rangle$

Summarizing

1. States postulate **NPQM**
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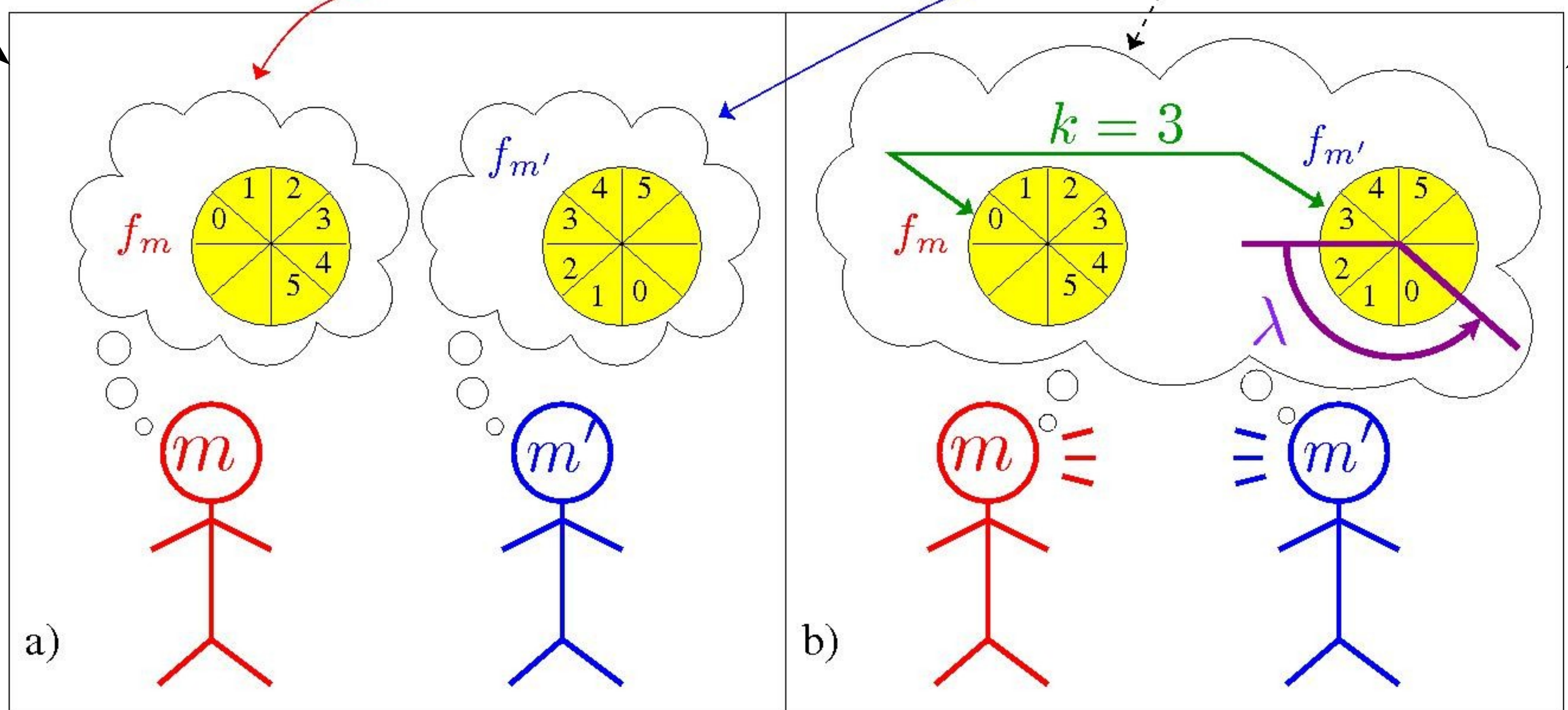
4. Locality

$$\sum_{\ell} \omega^{j\ell} |a_{\ell\oplus k}\rangle_s \, |a_\ell\rangle_m^N \, |a_{\ell\oplus k}\rangle_{m'}^N$$

envariance

correlations



$$f(|a_0\rangle, |a_1\rangle, \cdots) = f(|a_k\rangle, |a_{k\oplus 1}\rangle, \cdots) \quad \text{for all } k$$

symmetry

form of outcomes

$\exists \lambda$ that allows them to compare outcomes:

$\exists\ d$ values λ_ℓ such that $f_{\mathcal{M}}(\lambda_\ell) = |a_\ell\rangle$

Summarizing

1. States postulate **NPQM**
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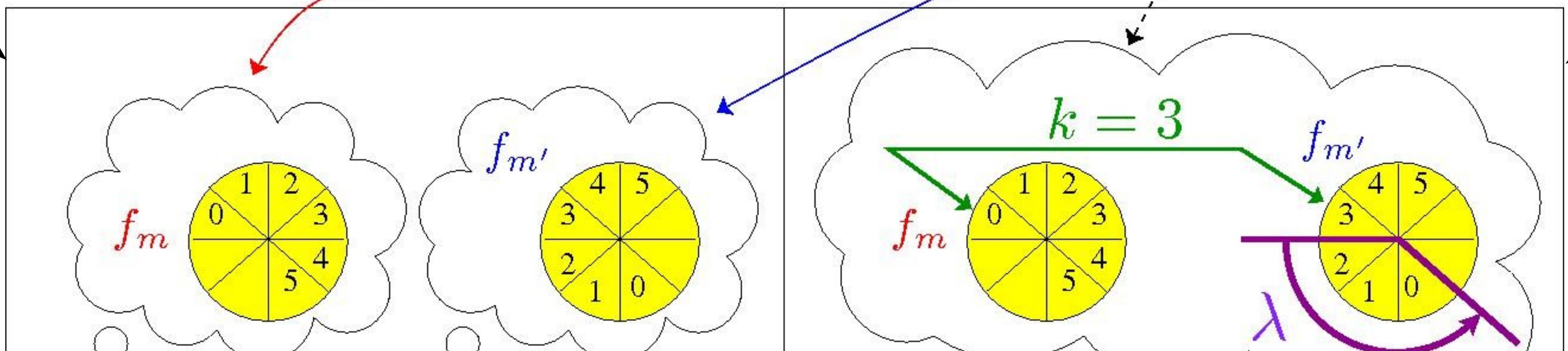
$$\sum_\ell \omega^{j\ell} |a_{\ell\oplus k}\rangle_s \boxed{|a_\ell\rangle^N_m} \boxed{|a_{\ell\oplus k}\rangle^N_{m'}}$$

envariance

correlations



d-dimensional extension of

$$\exists\ 2 \text{ values } \lambda_\ell \text{ such that } f_{\mathcal{M}}(\lambda_0) = |\uparrow\rangle,\ f_{\mathcal{M}}(\lambda_1) = |\downarrow\rangle$$

form of outcomes

$\exists \lambda$ that allows them to compare outcomes:

$$\exists\ d \text{ values } \lambda_\ell \text{ such that } f_{\mathcal{M}}(\lambda_\ell) = |a_\ell\rangle$$

Bertlemann's socks inequality

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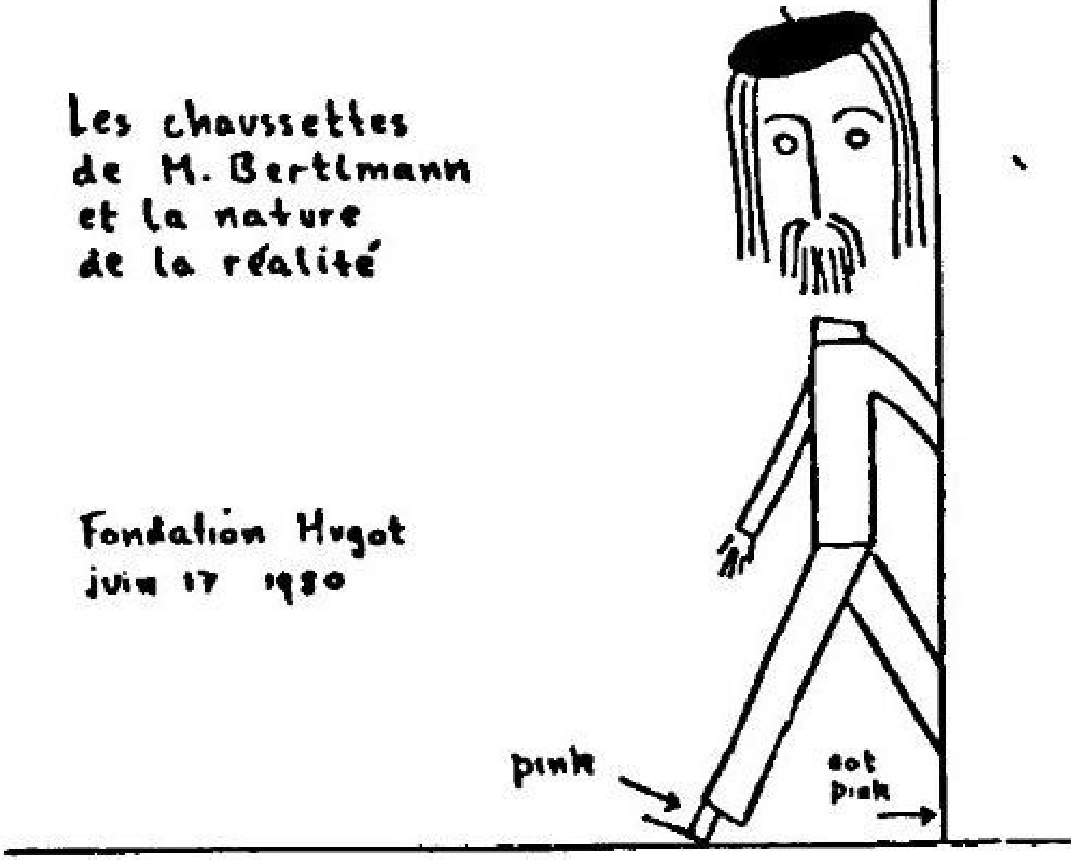

Bertlemann's socks inequality

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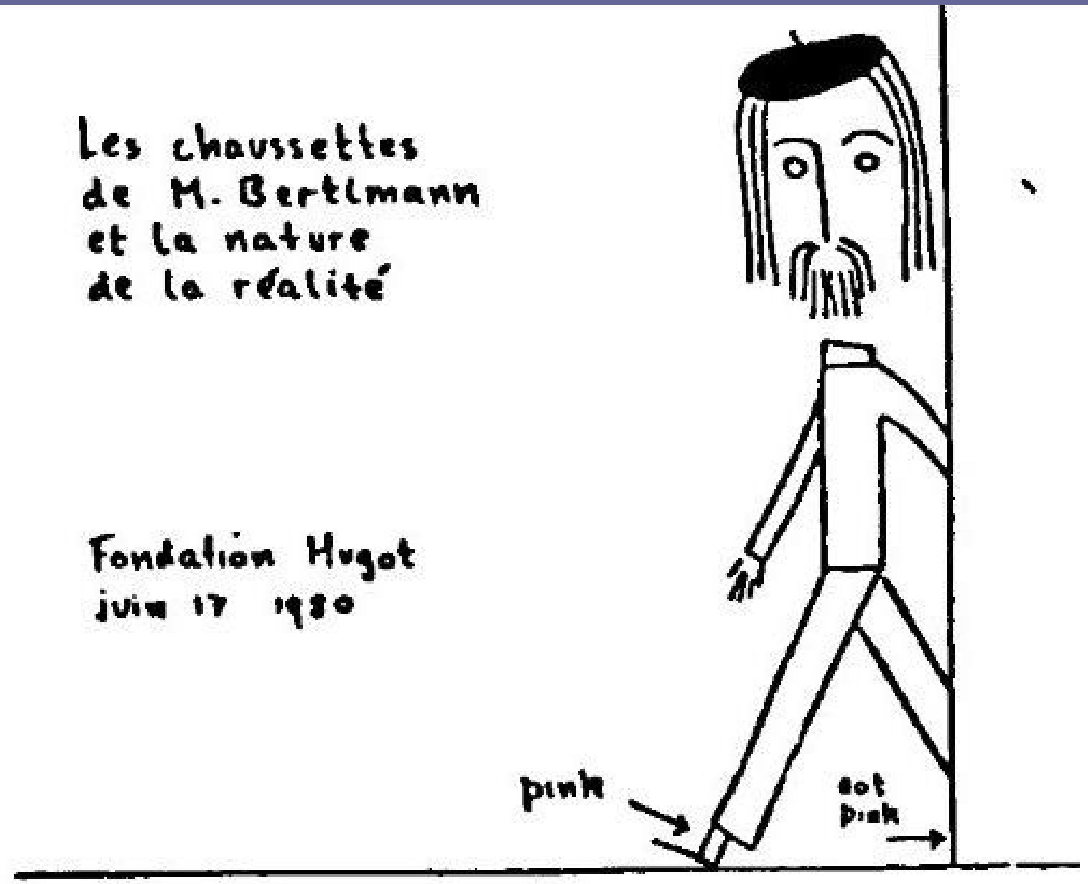

no Born rule, only use the properties of the measurement outcome $f_{\mathcal{M}}(\lambda)$ derived from symmetry considerations, namely

$$\boxed{\exists\ d \text{ values } \lambda_\ell \text{ such that } f_{\mathcal{M}}(\lambda_\ell) = |a_\ell\rangle}$$

Bertlemann's socks inequality

usual B.i. setup: two qubits + three observables

$$A = \{|0\rangle, |1\rangle\}$$
$$B = \{|b_0\rangle \equiv |0\rangle + \sqrt{3}|1\rangle, |b_1\rangle \equiv \sqrt{3}|0\rangle - |1\rangle\}$$
$$C = \{|c_0\rangle \equiv |0\rangle - \sqrt{3}|1\rangle, |c_1\rangle \equiv \sqrt{3}|0\rangle + |1\rangle\}$$

Bertlemann's socks inequality

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$$C = \{|c_0\rangle \equiv |0\rangle - \sqrt{3}|1\rangle, |c_1\rangle \equiv \sqrt{3}|0\rangle + |1\rangle\}$$

state of the qubits:

$$|b_0\rangle|b_0\rangle + |b_1\rangle|b_1\rangle$$

Bertlemann's socks inequality

1. States postulate **NPQM**
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4. Locality

measure *A* on the first and *B* on the second...

$$|b_0 b_0\rangle + |b_1 b_1\rangle = |0\rangle(|b_0\rangle + \sqrt{3}|b_1\rangle) + |1\rangle(\sqrt{3}|b_0\rangle - |b_1\rangle)$$

Bertlemann's socks inequality

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$$= |0\rangle(|b_0\rangle|a\rangle_g + |b_1\rangle(|b\rangle_g + |c\rangle_g + |d\rangle_g)) + \ldots$$

Bertlemann's socks inequality

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$$= |0\rangle(|b_0\rangle|a\rangle_g + |b_1\rangle(|b\rangle_g + |c\rangle_g + |d\rangle_g)) + \ldots$$

$$\Downarrow$$

$$f^{(A,B)}(\lambda) = \begin{cases} |0\rangle|b_0\rangle|a\rangle_g \text{ for } \lambda_0 & |1\rangle|b_0\rangle|e\rangle_g \text{ for } \lambda_4 \\ |0\rangle|b_1\rangle|b\rangle_g \quad \text{''} \quad \lambda_1 & |1\rangle|b_0\rangle|f\rangle_g \quad \text{''} \quad \lambda_5 \\ |0\rangle|b_1\rangle|c\rangle_g \quad \text{''} \quad \lambda_2 & |1\rangle|b_0\rangle|g\rangle_g \quad \text{''} \quad \lambda_6 \\ |0\rangle|b_1\rangle|d\rangle_g \quad \text{''} \quad \lambda_3 & |1\rangle|b_1\rangle|h\rangle_g \quad \text{''} \quad \lambda_7. \end{cases}$$

Bertlemann's socks inequality

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measure *A* on the first and *B* on the second...

$$|b_0b_0\rangle + |b_1b_1\rangle = |0\rangle(|b_0\rangle + \sqrt{3}|b_1\rangle) + |1\rangle(\sqrt{3}|b_0\rangle - |b_1\rangle)$$

$$= |0\rangle(|b_0\rangle|a\rangle_g + |b_1\rangle(|b\rangle_g + |c\rangle_g + |d\rangle_g)) + \dots$$

$$\Downarrow$$

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and analogously for *B* and *C*....

Bertlemann's socks inequality

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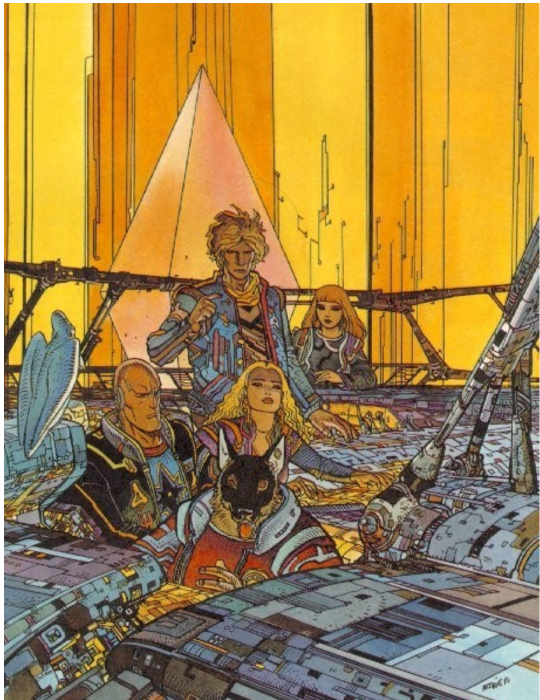
$$f^{(A,C)}(\gamma) = \begin{cases} |0\rangle|c_0\rangle|a\rangle_g \text{ for } \gamma_0 & |1\rangle|c_0\rangle|e\rangle_g \text{ for } \gamma_4 \\ |0\rangle|c_1\rangle|b\rangle_g \quad \text{"} \quad \gamma_1 & |1\rangle|c_0\rangle|f\rangle_g \quad \text{"} \quad \gamma_5 \\ |0\rangle|c_1\rangle|c\rangle_g \quad \text{"} \quad \gamma_2 & |1\rangle|c_0\rangle|g\rangle_g \quad \text{"} \quad \gamma_6 \\ |0\rangle|c_1\rangle|d\rangle_g \quad \text{"} \quad \gamma_3 & |1\rangle|c_1\rangle|h\rangle_g \quad \text{"} \quad \gamma_7, \end{cases}$$

$$f^{(B,C)}(\theta) = \begin{cases} |b_0\rangle|c_0\rangle|a\rangle_g \text{ for } \theta_0 & |b_1\rangle|c_0\rangle|e\rangle_g \text{ for } \theta_4 \\ |b_0\rangle|c_1\rangle|b\rangle_g \quad \text{"} \quad \theta_1 & |b_1\rangle|c_0\rangle|f\rangle_g \quad \text{"} \quad \theta_5 \\ |b_0\rangle|c_1\rangle|c\rangle_g \quad \text{"} \quad \theta_2 & |b_1\rangle|c_0\rangle|g\rangle_g \quad \text{"} \quad \theta_6 \\ |b_0\rangle|c_1\rangle|d\rangle_g \quad \text{"} \quad \theta_3 & |b_1\rangle|c_1\rangle|h\rangle_g \quad \text{"} \quad \theta_7. \end{cases}$$

$$f^{(B,B)}(\beta) = \begin{cases} |b_0\rangle|b_0\rangle \ \beta_0 \\ |b_1\rangle|b_1\rangle \ \beta_1. \end{cases}$$



Bertlemann's socks inequality

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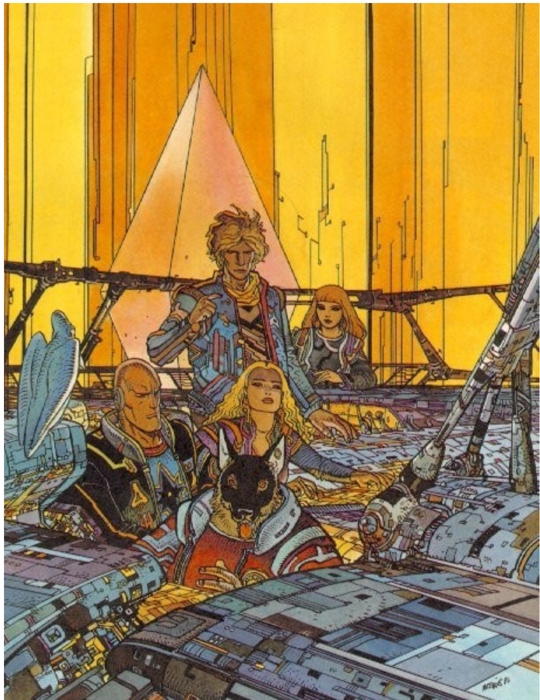
$$f^{(A,C)}(\gamma) = \begin{cases} |0\rangle|c_0\rangle|a\rangle_g \text{ for } \gamma_0 & |1\rangle|c_0\rangle|e\rangle_g \text{ for } \gamma_4 \\ |0\rangle|c_1\rangle|b\rangle_g \;\;\text{''}\;\; \gamma_1 & |1\rangle|c_0\rangle|f\rangle_g \;\;\text{''}\;\; \gamma_5 \\ |0\rangle|c_1\rangle|c\rangle_g \;\;\text{''}\;\; \gamma_2 & |1\rangle|c_0\rangle|g\rangle_g \;\;\text{''}\;\; \gamma_6 \\ |0\rangle|c_1\rangle|d\rangle_g \;\;\text{''}\;\; \gamma_3 & |1\rangle|c_1\rangle|h\rangle_g \;\;\text{''}\;\; \gamma_7, \end{cases}$$

$$f^{(B,C)}(\theta) = \begin{cases} |b_0\rangle|c_0\rangle|a\rangle_g \text{ for } \theta_0 & |b_1\rangle|c_0\rangle|e\rangle_g \text{ for } \theta_4 \\ |b_0\rangle|c_1\rangle|b\rangle_g \;\;\text{''}\;\; \theta_1 & |b_1\rangle|c_0\rangle|f\rangle_g \;\;\text{''}\;\; \theta_5 \\ |b_0\rangle|c_1\rangle|c\rangle_g \;\;\text{''}\;\; \theta_2 & |b_1\rangle|c_0\rangle|g\rangle_g \;\;\text{''}\;\; \theta_6 \\ |b_0\rangle|c_1\rangle|d\rangle_g \;\;\text{''}\;\; \theta_3 & |b_1\rangle|c_1\rangle|h\rangle_g \;\;\text{''}\;\; \theta_7. \end{cases}$$

$$f^{(B,B)}(\beta) = \begin{cases} |b_0\rangle|b_0\rangle \;\beta_0 \\ |b_1\rangle|b_1\rangle \;\beta_1. \end{cases}$$

cannot be preassigned locally!



Bertlemann's socks inequality

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.. if you don't believe it, we can prove it with an inequality (satisfied by every pre-determined $f_{\mathcal{M}}$ and violated by ours)!

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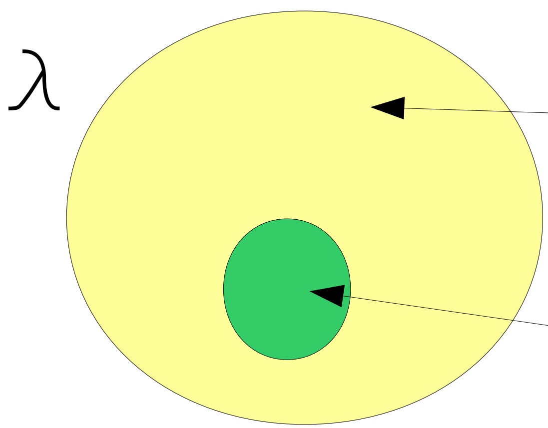

Fraction of the parameter space λ where A=B

Fraction of the parameter space λ where $A \neq B$

Bertlemann's socks inequality

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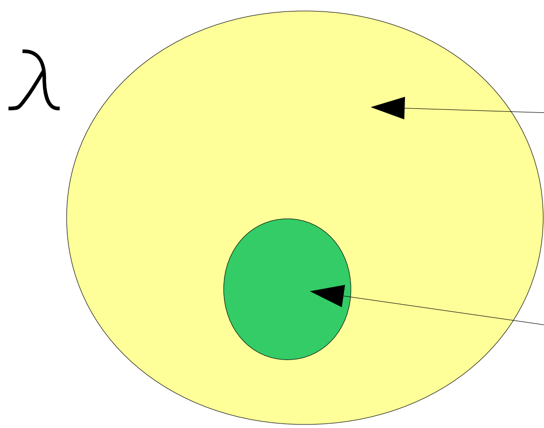

Fraction of the parameter space λ where *A*=*B*

Fraction of the parameter space λ where *A*≠*B*

$$\color{red}{1} = Fr(A_1 = B_2) + Fr(A_1 \neq B_2) =$$

Bertlemann's socks inequality

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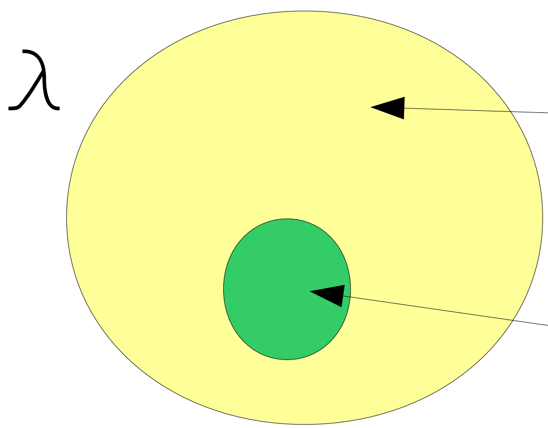


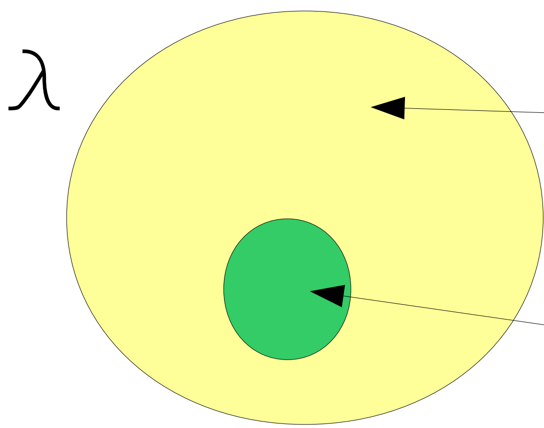
Fraction of the parameter space λ where A=B

Fraction of the parameter space λ where $A\neq B$

$$1 = Fr(A_1 = B_2) + Fr(A_1 \neq B_2) =$$
$$Fr(A_1 = B_2) + Fr(A_1 = C_2 \neq B_2) + Fr(A_1 \neq B_2 = C_2)$$

Bertlemann's socks inequality

.. if you don't believe it, we can prove it with an inequality (satisfied by every pre-determined $f_{\mathcal{M}}$ and violated by ours)!



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$$1 = Fr(A_1 = B_2) + Fr(A_1 \neq B_2) =$$

$$Fr(A_1 = B_2) + Fr(A_1 = C_2 \neq B_2) + Fr(A_1 \neq B_2 = C_2)$$

$$\leqslant Fr(A_1 = B_2) + Fr(A_1 = C_2) + Fr(B_1 = B_2 = C_2)$$

Bertlemann's socks inequality

NPQM

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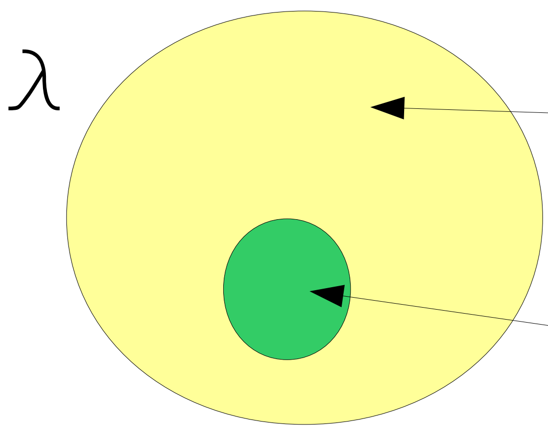

Fraction of the parameter space λ where A=B

Fraction of the parameter space λ where $A \neq B$

$$1 = Fr(A_1 = B_2) + Fr(A_1 \neq B_2) =$$
$$Fr(A_1 = B_2) + Fr(A_1 = C_2 \neq B_2) + Fr(A_1 \neq B_2 = C_2)$$
$$\leqslant Fr(A_1 = B_2) + Fr(A_1 = C_2) + Fr(B_1 = B_2 = C_2)$$

....what happens in our case?

Bertlemann's socks inequality

1. States postulate **NPQM**
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3. Tensor product structure
4. Locality

$$f^{(A,B)}(\lambda) = \begin{cases} \boxed{|0\rangle|b_0\rangle|a\rangle_g \text{ for } \lambda_0} & |1\rangle|b_0\rangle|e\rangle_g \text{ for } \lambda_4 \\ |0\rangle|b_1\rangle|b\rangle_g \quad '' \quad \lambda_1 & |1\rangle|b_0\rangle|f\rangle_g \quad '' \quad \lambda_5 \\ |0\rangle|b_1\rangle|c\rangle_g \quad '' \quad \lambda_2 & |1\rangle|b_0\rangle|g\rangle_g \quad '' \quad \lambda_6 \\ |0\rangle|b_1\rangle|d\rangle_g \quad '' \quad \lambda_3 & \boxed{|1\rangle|b_1\rangle|h\rangle_g \quad '' \quad \lambda_7} . \end{cases}$$

¼ of the cases

$$f^{(A,C)}(\gamma) = \begin{cases} \boxed{|0\rangle|c_0\rangle|a\rangle_g \text{ for } \gamma_0} & |1\rangle|c_0\rangle|e\rangle_g \text{ for } \gamma_4 \\ |0\rangle|c_1\rangle|b\rangle_g \quad '' \quad \gamma_1 & |1\rangle|c_0\rangle|f\rangle_g \quad '' \quad \gamma_5 \\ |0\rangle|c_1\rangle|c\rangle_g \quad '' \quad \gamma_2 & |1\rangle|c_0\rangle|g\rangle_g \quad '' \quad \gamma_6 \\ |0\rangle|c_1\rangle|d\rangle_g \quad '' \quad \gamma_3 & \boxed{|1\rangle|c_1\rangle|h\rangle_g \quad '' \quad \gamma_7} , \end{cases}$$

¼ of the cases

$$f^{(B,C)}(\theta) = \begin{cases} \boxed{|b_0\rangle|c_0\rangle|a\rangle_g \text{ for } \theta_0} & |b_1\rangle|c_0\rangle|e\rangle_g \text{ for } \theta_4 \\ |b_0\rangle|c_1\rangle|b\rangle_g \quad '' \quad \theta_1 & |b_1\rangle|c_0\rangle|f\rangle_g \quad '' \quad \theta_5 \\ |b_0\rangle|c_1\rangle|c\rangle_g \quad '' \quad \theta_2 & |b_1\rangle|c_0\rangle|g\rangle_g \quad '' \quad \theta_6 \\ |b_0\rangle|c_1\rangle|d\rangle_g \quad '' \quad \theta_3 & \boxed{|b_1\rangle|c_1\rangle|h\rangle_g \quad '' \quad \theta_7} . \end{cases}$$

¼ of the cases

$$Fr(A_1 = B_2) + Fr(A_1 = C_2) + Fr(B_1 = B_2 = C_2) = \frac{3}{4} \leqslant 1$$

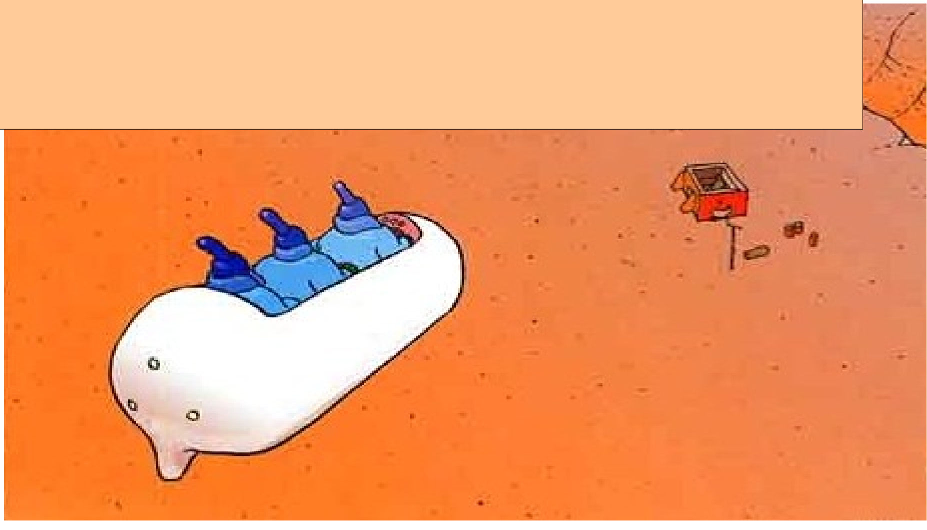
Bell inequality violation!

Summary

1. we used only the symmetry properties of $f_{\mathcal{M}}(\lambda)$

   to conclude $\exists\ d \text{ values } \lambda_\ell \text{ such that } f_{\mathcal{M}}(\lambda_\ell) = |a_\ell\rangle$

(the measurement outcome must explicitly depend on the eigenstates of the observable)



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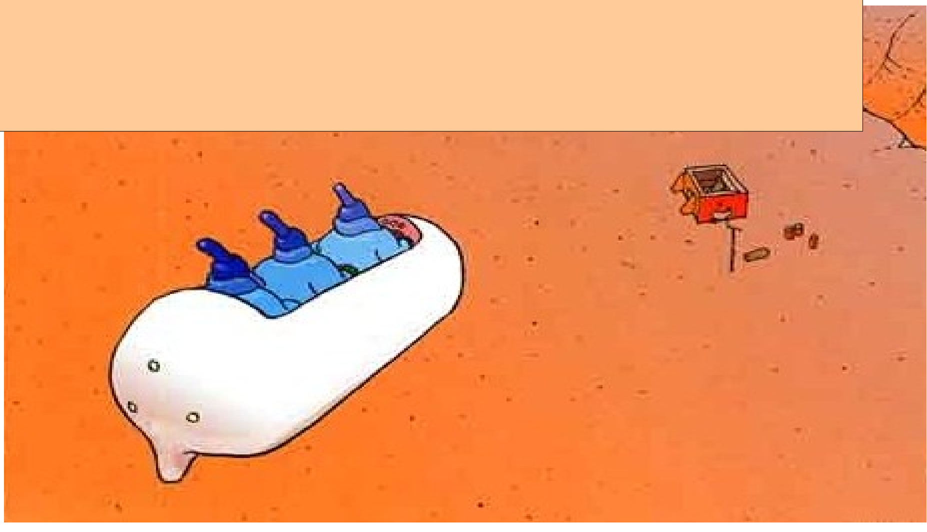

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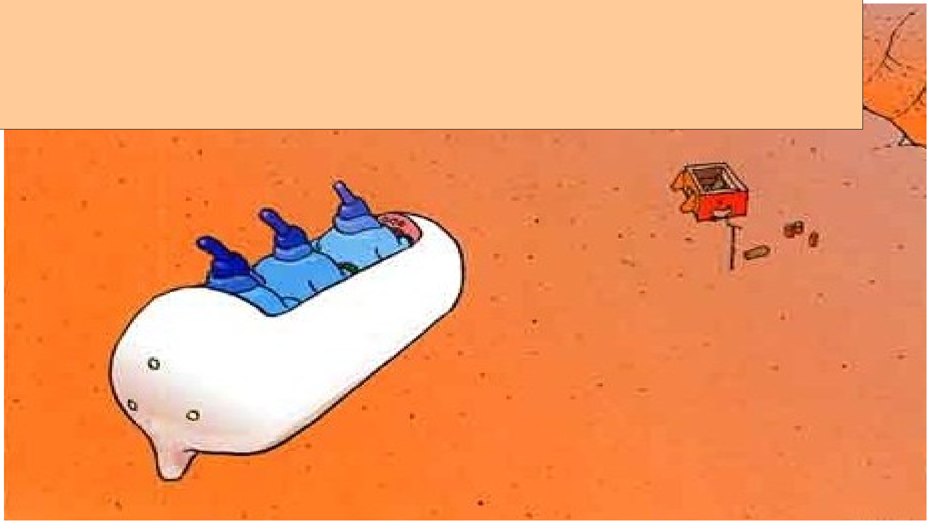

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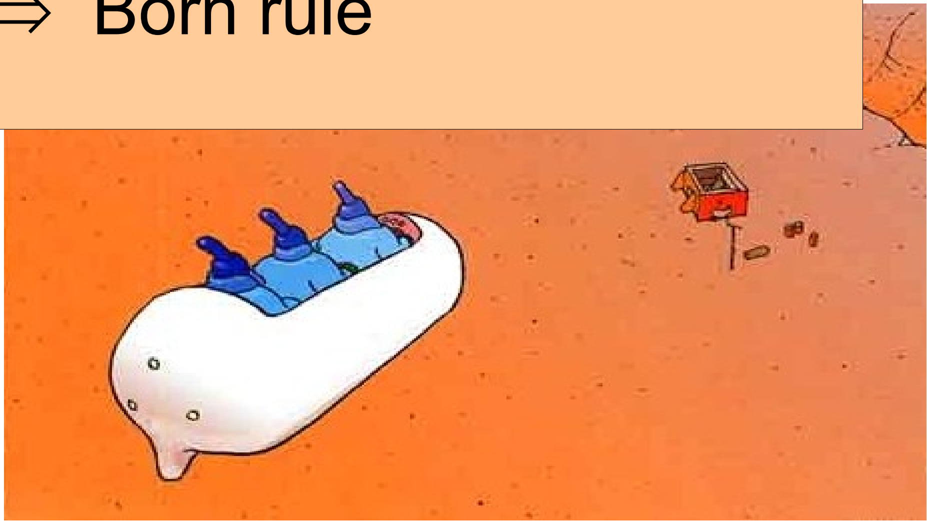
4. symmetries of $f_{\mathcal{M}}(\lambda)$ $\Longrightarrow$ Born rule



Take home message

The non-probabilistic
part of quantum mechanics (+ locality)
is sufficient to derive all of it!

NPQM

1. States postulate
2. Schroedinger equation
3. Tensor product structure
4. Locality

$\Rightarrow$ **QM**

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