On the origin of probability in quantum mechanics

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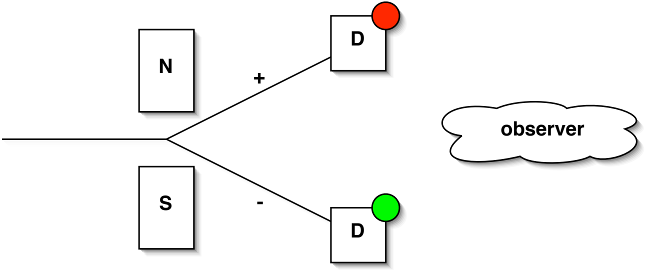
Benasque, September 2010

Outline

1. “No Collapse” quantum mechanics
2. Does the Born rule (probabilities) emerge?
3. Possible resolutions

R. Buniy, S. Hsu and A. Zee, Phys. Lett. B640, 219 (2006)

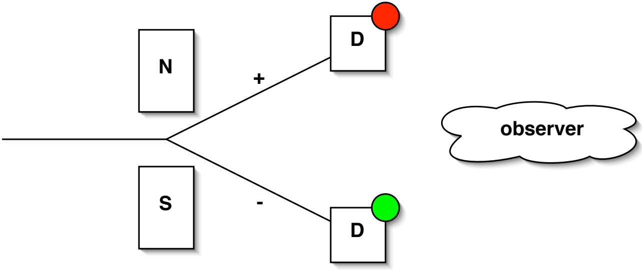
Do we need wavefunction collapse?



Stern-Gerlach device (magnet, detector, red/green lights) plus observer.

$$|+\rangle \longrightarrow |up\rangle \otimes |\color{red}{red}\rangle \otimes |observed\ \color{red}{red}\rangle$$
$$|-\rangle \longrightarrow |down\rangle \otimes |\color{green}{green}\rangle \otimes |observed\ \color{green}{green}\rangle$$

Do we need wavefunction collapse?



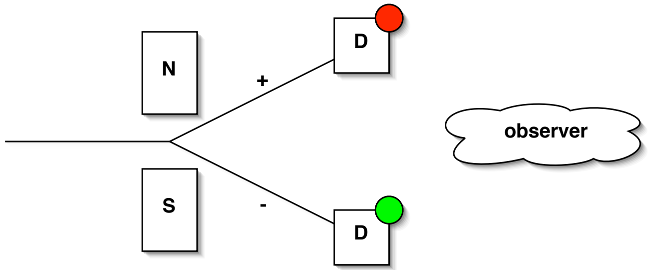
Suppose initial state is a superposition: $|\psi\rangle = c_{+}|+\rangle + c_{-}|-\rangle$.

Schrodinger evolution (linear) leads to:

$$c_{+}|up\rangle \otimes |red\rangle \otimes |observed\ red\rangle \; + \; c_{-}|down\rangle \otimes |green\rangle \otimes |observed\ green\rangle \; .$$

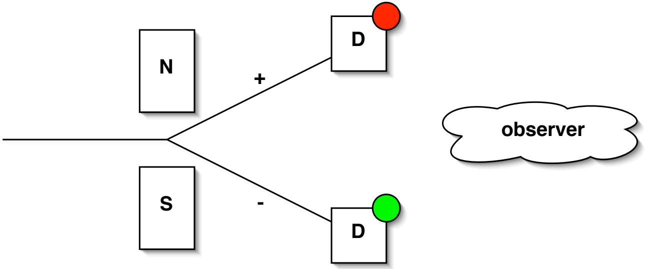
Or does the wavefunction “collapse”?

Do we need wavefunction collapse?

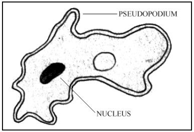


Does it matter if *observer* = *nothing* ? (Or a few atoms? Many atoms?)

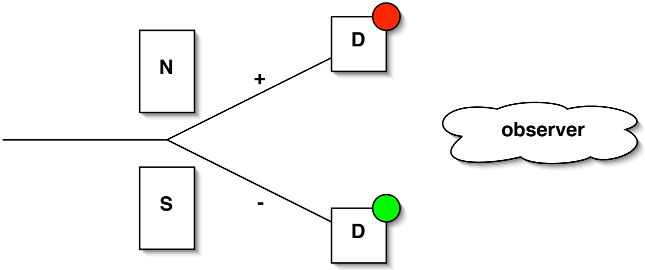
Do we need wavefunction collapse?



Does it matter if *observer* = *amoeba* ?

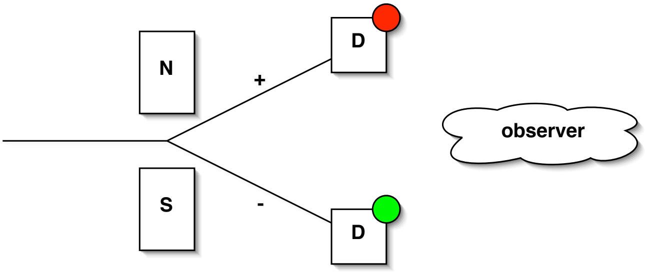


Do we need wavefunction collapse?



Does it matter if *observer* = *cat* ?
(Schrodinger's cat?)

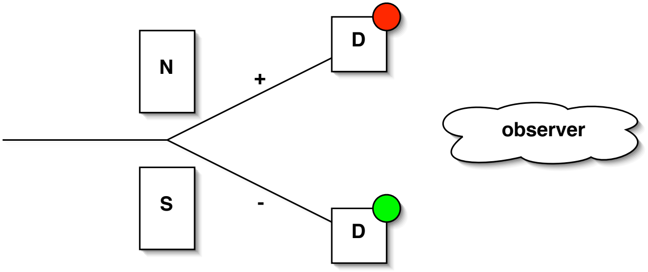
Do we need wavefunction collapse?



Does it matter if *observer* = *Gork the robot* ?

(S. Coleman, *Quantum Mechanics, In Your Face!*)

Do we need wavefunction collapse?



Does it matter if *observer* = *a scientist* ?

Time evolution in QM

Quantum mechanics, as conventionally formulated, has two types of time evolution:

U: Unitary, deterministic: $\psi(t) = e^{-iHt}\psi(0)$

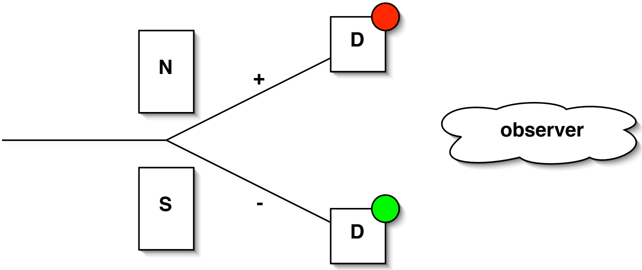
C: Copenhagen, Collapse: discontinuous, probabilistic

(von Neumann projection: $|\psi\rangle \longrightarrow |a\rangle$, for outcome a.)

Is **C** really necessary? Perhaps **C** is an *apparent* phenomena which emerges from unitary evolution **U**!

H. Everett, 1958

Many Worlds Interpretation



= "No collapse"

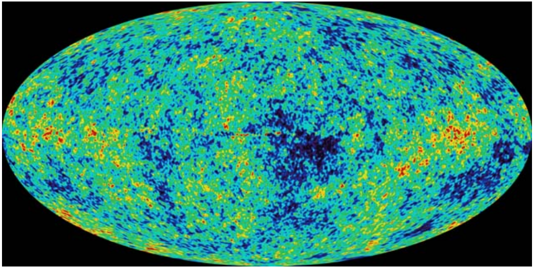
Macroscopic beings will *perceive* a collapse due to decoherence of distinct branches.

DeWitt, Hartle, Gell-Mann, Feynman, Hawking, Coleman, Weinberg, Guth, Deutsch, Zeh, Zurek, ...

Quantum cosmology

The Universe is a closed system and (presumably) obeys quantum mechanics.

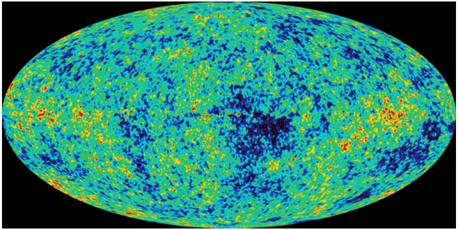
Inflationary cosmology now well-supported by precision CMB observations.



Quantum cosmology

Origin of structure in the universe (galaxies, planets, people) due to *quantum* fluctuations of inflaton field. Spectrum of fluctuations is measured and agrees with predictions from inflation.

There were no ″observers″ in the early universe to ″measure″ the local energy density. How did the specific pattern of density fluctuations emerge?



Many Worlds Interpretation

It is *plausible* (but of course unproven) that purely unitary evolution of a pure state in a closed system can reproduce, *for semi-classical creatures inside the system*, all of the phenomenology of the Copenhagen interpretation.

Dynamical mechanisms: Decoherence, Quantum Darwinism (objectivity), Pointer States, ...

The most challenging aspect is Born's rule and probabilities!

Note: Born's rule fundamental to decoherence, Second Law, emergence of semi-classical reality, etc. Circularity!

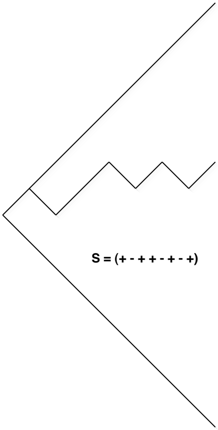
Einstein's Mistakes, S. Weinberg, Physics Today 2005

... Bohr's version of quantum mechanics was deeply flawed, but not for the reason Einstein thought. The Copenhagen interpretation describes what happens when an observer makes a measurement, but the observer and the act of measurement are themselves treated classically. This is surely wrong: Physicists and their apparatus must be governed by the same quantum mechanical rules that govern everything else in the universe. But these rules are expressed in terms of a wavefunction (or, more precisely, a state vector) that evolves in a perfectly deterministic way. So where do the probabilistic rules of the Copenhagen interpretation come from?

Einstein's Mistakes, S. Weinberg, Physics Today 2005

... Considerable progress has been made in recent years toward the resolution of the problem, which I cannot go into here. It is enough to say that neither Bohr nor Einstein had focused on the real problem with quantum mechanics. The Copenhagen rules clearly work, so they have to be accepted. But this leaves the task of explaining them by applying the deterministic equation for the evolution of the wavefunction, the Schrodinger equation, to observers and their apparatus. The difficulty is not that quantum mechanics is probabilistic—that is something we apparently just have to live with. The real difficulty is that it is also deterministic, or more precisely, that it combines a probabilistic interpretation with deterministic dynamics.

Measurements on N spins



Assume Born's rule works for observer.

Does it work for the experiment?

Ensemble state, N identical spin states:
$\Psi = \otimes_{i=1}^{N} \psi_i = \psi_1 \otimes \psi_2 \cdots \otimes \psi_N.$

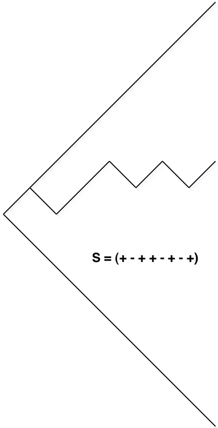
$|\psi_i\rangle = c_+|+\rangle + c_-|-\rangle$

Each of 2^N branches or histories S appears in Ψ:

$\Psi = \sum_{\{s_1,\ldots s_N\}} c_{s_1 \ldots s_N} |s_1, \ldots s_N\rangle$

$S = (s_1, s_2, \ldots s_N)$, $s_i = \pm$.

Born rule



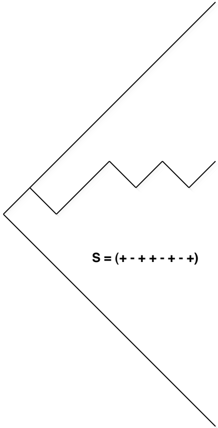
How does Born rule ($p_\pm = |c_\pm|^2$) arise in No Collapse interpretation?

Each observer perceives a particular history S, but all branches are possible. Structure of tree completely independent of $c_\pm$!

Multiplicity of branches S dominated by $n_+ \approx n_- = N/2$.

On most branches S, physicists would not have deduced the Born rule (unless $c_+ = c_- = .5$)!

Born rule



Spacetime perspective: initial wavefunction of the universe Ψ plus Schrodinger evolution leads to realization of all paths or branches.

A derivation of the Born rule must answer the question: Why do we happen to live on a particular branch where the Born rule is seen to hold?

Born rule

$S = (+ + - + + - + - - + \cdots)$

Let $n = n_+$, $f = f_+ = n/N$ and $p = p_+ = |c_+|^2$.

Compute the norm squared of branches S with given statistical properties, such as number of + outcomes, n. (We can use our experience with the Born rule as a shortcut.)

$$\sum_{n + \text{outcomes}} |c_{s_1,\ldots,s_N}|^2 \;=\; P(n) \;=\; \binom{N}{n} p^n (1-p)^{N-n}$$

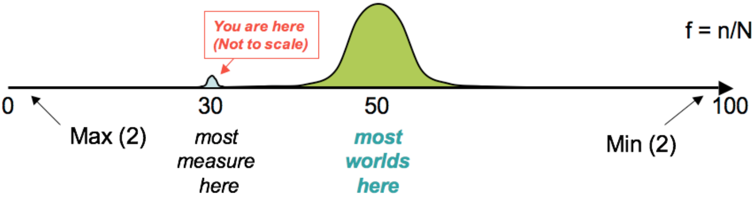
1) combinatorial factor $\binom{N}{n}$ peaked at $n = N/2$ or $f = 1/2$.

2) individual probability factor $p^n(1-p)^{N-n}$ peaked at $n = 0, N$ or $f = 0, 1$.

Competition between (1) and (2): $P(n)$ peaked at $n = pN$ or $f = p$.

Born rule

Example: $p = p_+ = .3$



Key observation: Born rule probability is just the norm squared for a set of outcomes. In conventional QM, probability = measure. Branches which are rare according to Born rule must have small norm.

Norm of Ψ dominated by $|s_1, \ldots, s_N\rangle$ with $f \approx p$ (fraction of + values = .3).

Branches with $f \neq p$ are called maverick worlds (Everett).

Born rule

For $N \to \infty$, states with $f \neq p$ (maverick worlds) have *measure zero*. Only states with fraction of + values given by Born rule have non-zero norm.

Everett asserts: zero norm states are unphysical. IF we remove them, we have derived the Born rule within the No Collapse interpretation.

(Everett, DeWitt, Hartle)

Our horizon is finite

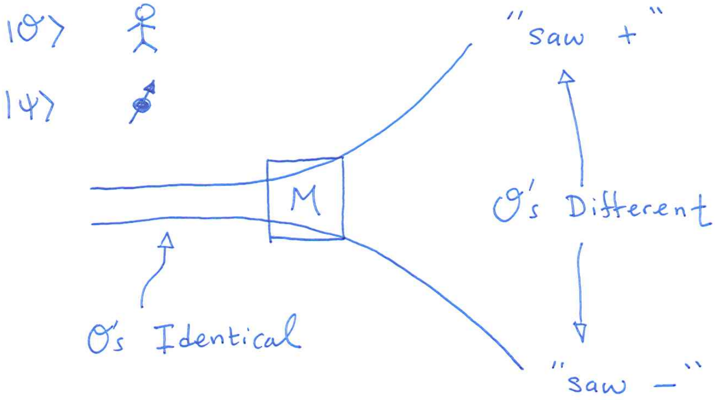
Can $N \to \infty$?

Our causal horizon is finite. Decoherence times are fast, but not zero. Hence N is finite.

For N finite, does the argument still work?

Maverick worlds have small, but non-zero, norm. We have no justification for removing components of Ψ with small but non-zero norm. In fact, in the absence of wavefunction collapse, the norm of a subcomponent $|s_1, \ldots, s_N\rangle$ plays no role in quantum mechanics!

A question of identity. MW "superdeterministic"

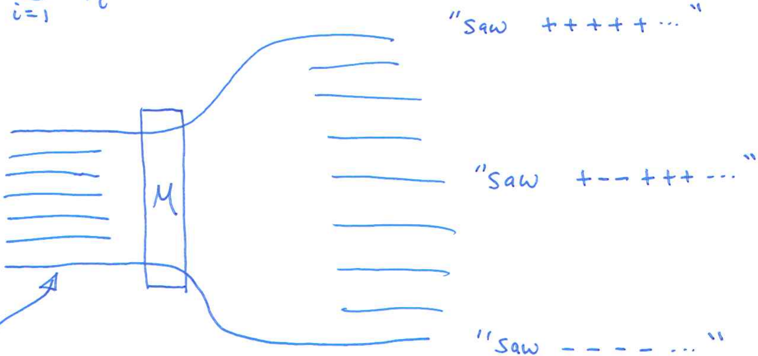


Before: $(|+\rangle + |-\rangle)|O\rangle$

After: $|+\rangle|O_+\rangle + |-\rangle|O_-\rangle$

A question of identity. MW "superdeterministic"

$$\Psi = \bigotimes_{i=1}^{N} \psi_i$$



Need Prob. Measure over $|s_1 s_2 \cdots s_N\rangle$ outcomes

Resolutions?

1. Anthropism (but we could certainly tolerate more violation of Born's rule than we see)

2. Foundations of probability theory (e.g., decision theory, envariance). But these are "normative" and could be *empirically* wrong. (Certainly don't work for people living on Maverick branches!)

"Therefore the true logic for this world is the calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind." –James Clerk Maxwell, 1850

Discrete quantum state space

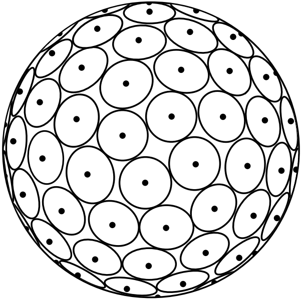
Quantum mechanics + gravity = minimum length.

No operational meaning to distance less than the Planck length: l_P (Calmet, Graesser and Hsu, PRL 2004.)

This suggests a discreteness to quantum state space, or Hilbert space, in abuse of terminology. (Buniy, Hsu and Zee, PLB 2005.)

A measurement capable of distinguishing arbitrarily similar states might require so much energy it is prevented by gravitational collapse.

Discrete quantum state space



Consider rotation of an object by small angle ϵ. If ϵ sufficiently small, no component is displaced by more than a Planck length l_P. The corresponding spin eigenstates are indistinguishable.

We are motivated to consider a *minimum norm* in Hilbert space.

$$|\psi - \psi'| < \epsilon \quad \rightarrow \quad |\Psi - \Psi'| < \sqrt{N}\epsilon.$$

Born rule from discrete state space

Suppose we are allowed to drop components of Ψ below some threshold in norm $\sqrt{N}\epsilon \ll 1$.

The collective norm squared of all maverick states $|\delta, N\rangle$ with frequency deviation $|\delta| = |f - p|$ greater than δ_0 is

$$\sum_{|\delta|>\delta_0} \langle \delta, N|\delta, N\rangle \approx 2N \int_{p+\delta_0}^{\infty} df \; P(fN) \, . \tag{1}$$

where $P(fN) \approx [2\pi N p(1-p)]^{-1/2} \exp\left[-\frac{N(f-p)^2}{2p(1-p)}\right]$.

Requiring that this collective norm squared is less than $N\epsilon^2$ yields

$$\delta_0 > N^{-1/2}\left[2p(1-p)|\ln(N\epsilon^2)|\right]^{1/2} . \tag{2}$$

Born rule from discrete state space

If, for finite N, an experimenter could measure all N outcomes which define his branch of the wavefunction, he might find a deviation from the predicted Born frequency $f = p$ as large as

$$|\ln (N\epsilon^2)|^{1/2}$$

standard deviations (i.e., measuring the deviation in units of $N^{-1/2}$).

Born rule from discrete state space

An experimenter is unlikely to be able to measure more than a small fraction of the outcomes that determine his branch.

A particular branch of the wavefunction is specified by the sequence of outcomes $S = (s_1, s_2, \ldots, s_N)$.

N is the *total number* of decoherent outcomes on a branch, so it is typically enormous – at least Avogadro's number if the system contains macroscopic objects such as an experimenter.

Born rule from discrete state space

The experimental outcomes available to test Born's rule will be some much smaller number $N_* \ll N$ corresponding to some subset of the s_i directly related to the experiment.

Any deviation from the Born rule of order $N^{-1/2}$ will be well within the experimental statistical error of order $N_*^{-1/2}$.

The Born rule will be observed to hold in all the branches which remain after truncation due to discreteness.

Some numbers

Let $\epsilon \sim 10^{-100}$ – a tiny discreteness scale!

Assume $N \sim 10^{160} \sim H^{-4}$ in Fermis.

Then $N\epsilon^2 \sim 10^{-40}$ and

$$|\ln (N\epsilon^2)|^{1/2} \sim 10$$

so the predicted deviation from Born rule is 10 standard deviations *for the entire universe*.

Unless experimenters can measure more than $N/(10)^2$ of all N branchings, their statistical accuracy will not be enough to exclude this deviation.

Summary

Original (Everett) derivation of Born rule in No Collapse interpretations is flawed: only applies when $N = \infty$, and in a contrived way. *But our universe is finite*.

Other proposals? Envariance?

Very small discreteness of quantum state space is enough to restore the result – if $\exists$ minimum norm in Hilbert space, detectable Maverick worlds can be excluded.