Quantum Process Tomography: An efficient approach

JUAN PABLO PAZ

Departamento de Fisica, FCEyN

QUFIBA: Quantum Foundations and Information @ Buenos Aires

Universidad de Buenos Aires, Argentina

http://www.qufiba.df.uba.ar

Benasque

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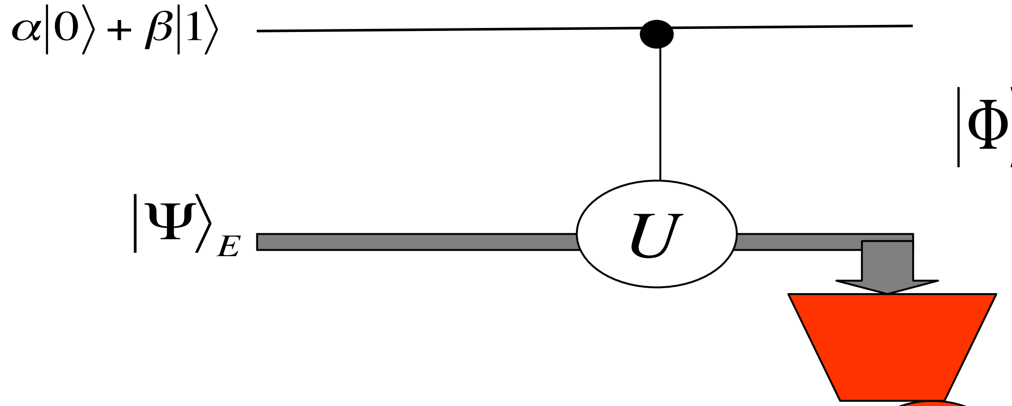
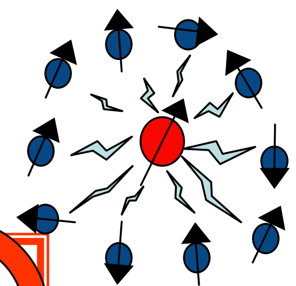
DECOHERENCE: FRIEND AND FOE

DECOHERENCE IS THE ENEMY FOR QUANTUM INFORMATION PROCESSING



$$|\Phi\rangle_{S,E} = \alpha|0\rangle_s \otimes |\Psi\rangle_E + \beta|1\rangle_s \otimes U|\Psi\rangle_E$$



$$\rho_A = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1|\cancel{\langle\Psi|U^+|\Psi\rangle} + \beta\alpha^*|1\rangle\langle 0|\cancel{\langle\Psi|U|\Psi\rangle}$$

QUANTUM BIT + DECOHERING ENVIRONMENT = CLASSICAL BIT

HOW TO FIGHT AGAINST DECOHERENCE?
USE QUANTUM ERROR CORRECTION, FAULT TOLERANCE

TO FIGHT AGAINST DECOHERENCE WE NEED TO CHARACTERIZE IT (NOISE CHARACTERIZATION). HOW?

This talk

QUANTUM PROCESS TOMOGRAPHY (QPT):

- What is QPT? Why is it necessary? Why is it hard? Standard QPT. Real experiments.
- Other methods. Ancillary assisted, etc. Shortcomings…
- A new method: Selective and Efficient Quantum Process Tomography (SEQPT).
- First experiment @ Buenos Aires

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Selective and Efficient Estimation of Parameters for Quantum Process Tomography

Ariel Bendersky,[1] Fernando Pastawski,[2] and Juan Pablo Paz[1]
o de Física, FCEyN UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina
e Matemática Astronomía y Física, UNC, Ciudad Universitaria, 5000 Córdoba, Argentina
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PHYSICAL REVIEW LETTERS

e and Efficient Quantum Process Tomography with Single Photons

stian Tomás Schmiegelow,[1] Miguel Antonio Larotonda,[2] and Juan Pablo Paz[1]
Física and IFIBA, FCEyN, UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina
LAP, CITEDEF, J.B. de La Salle 4397, 1603 Villa Martelli, Buenos Aires, Argentina
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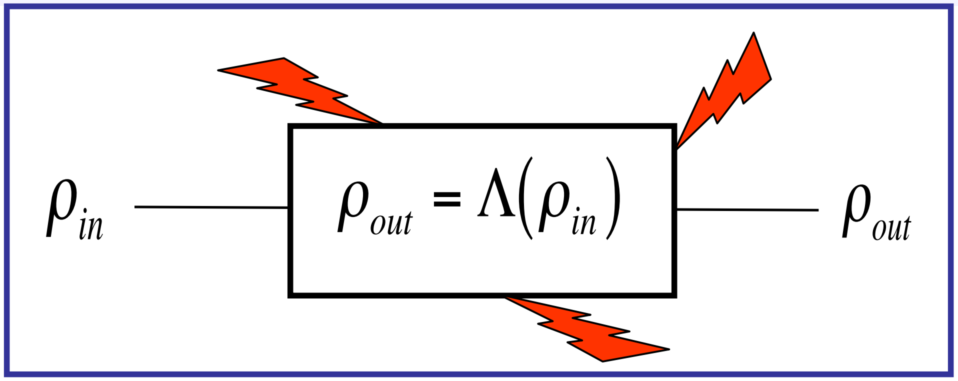
WHAT IS QUANTUM PROCESS TOMOGRAPHY



A QUANTUM PROCESS IS A LINEAR MAP (PRESERVING HERMITICITY, TRACE AND POSITIVITY)

ANY LINEAR MAP IS DEFINED BY ITS 'CHI-MATRIX'

$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{nm} \chi_{mn} E_n \, \rho_{in} \, E_m^+$$

$$P_0 = I, \quad P_1 = X, \quad P_2 = Y, \quad P_3 = Z$$

$$E_m = P_{m_1} \otimes P_{m_2} \otimes \ldots\ldots\ldots \otimes P_{m_n}$$

$$E_0 = I_1 \otimes I_2 \otimes \ldots\ldots\ldots \otimes I_n$$

$$Tr(E_m E_n) = D\delta_{mn}$$

- MAP IS HERMITIAN $\quad \chi_{mn} = \chi_{nm}^*$
- MAP PRESERVES TRACE $\quad \sum_{mn} \chi_{mn} E_m^+ E_n = I$

MAP $\Lambda(\rho)$ IS COMPLETELY POSITIVE (CP) $\Leftrightarrow$ MATRIX χ_{mn} IS POSITIVE

$$\chi_{mn} = U_{ma} D_a U_{an}^+ \Rightarrow \Lambda(\rho) = \sum_a D_a A_a \rho A_a^+; \quad A_a = \sum_m U_{ma} E_m$$

$$|\chi_{mn}|^2 \leq \chi_{mm} \, \chi_{nn}$$

KRAUS FORM OF THE MAP (ONLY FOR CP MAPS)

WHY IS QUANTUM PROCESS TOMOGRAPHY HARD

$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{nm} \chi_{mn} E_n \, \rho_{in} \, E_m^{+}, \quad \sum_{mn} \chi_{mn} E_m^{+} E_n = I$$

•1) THERE ARE EXPONENTIALLY MANY COEFFICIENTS χ_{mn}

(i.e. There are $D^2 \times D^2$ of them where $D = 2^n$)

• 2) TO FIND OUT ANY ONE OF THEM WE NEED EXPONENTIAL RESOURCES

STANDARD QUANTUM PROCESS TOMOGRAPHY (SQPT)

Chapter 10, Nielsen & Chuang's book

$$P_{ik} = Tr(\rho_k \Lambda(\rho_i))$$

• EXPERIMENTALLY DETERMINE "TRANSITION PROBABILITIES"

$$P_{ik} = \sum_{nm} \chi_{mn} Tr(\rho_k E_n \, \rho_i E_m^{+})$$

• FIND χ_{mn} INVERTING (HUGE) LINEAR SYSTEM.

"STANDARD QUANTUM PROCESS TOMOGRAPHY"
(NIELSEN & CHUANG, CHAPTER 10)

QUANTUM PROCESS TOMOGRAPHY IS HARD!

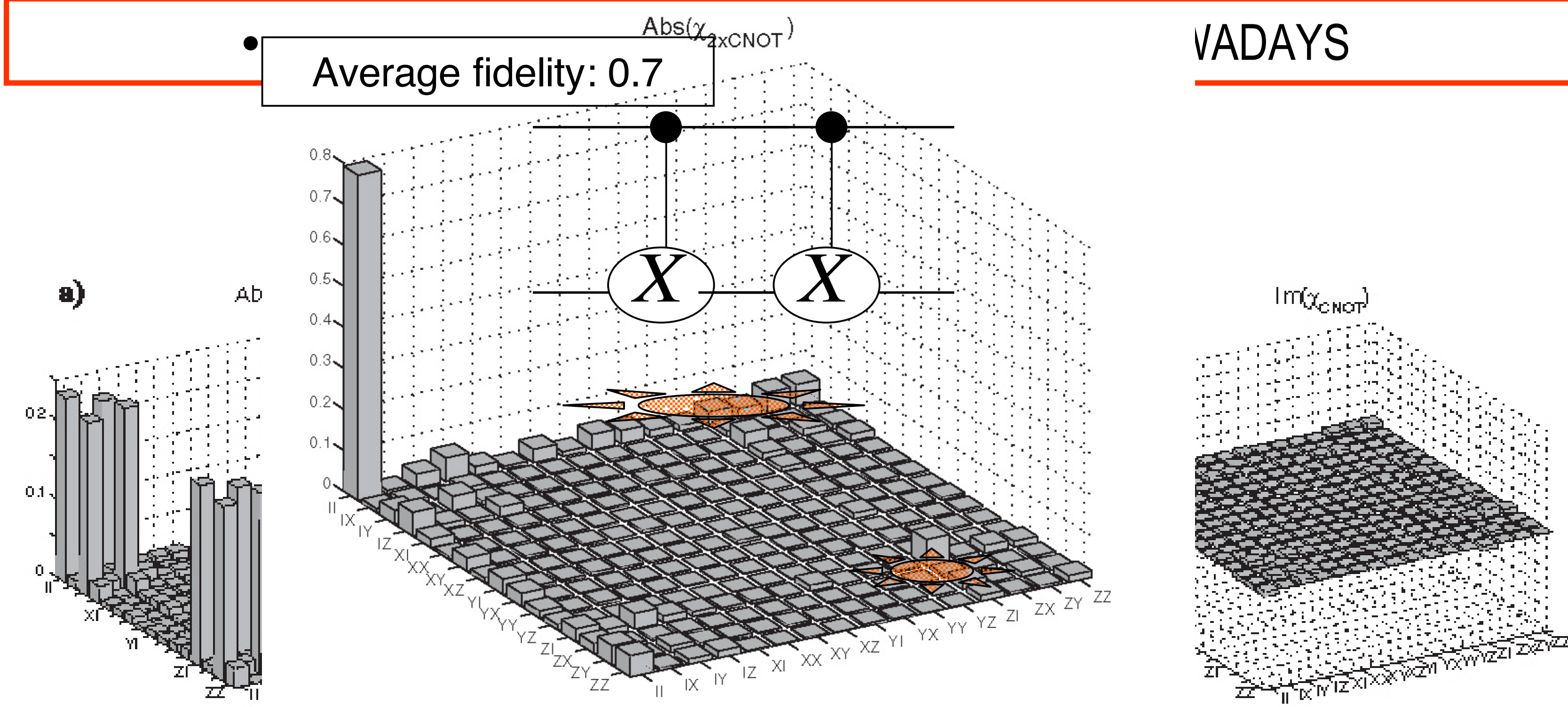
QUANTUM RESOURCES (qbits, operations, state preparations, measurements, etc):

- prepare D^2 initial states and detect D^2 final states
- repeat experiments (exponentially) many times to geta fixed precision in χ_{mn}

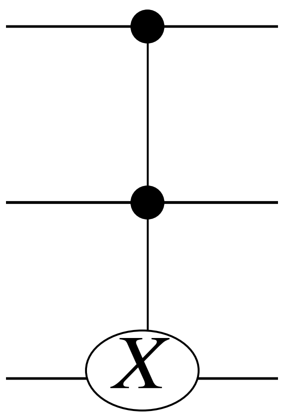
CLASSICAL RESOURCES (data processing in classical computers):

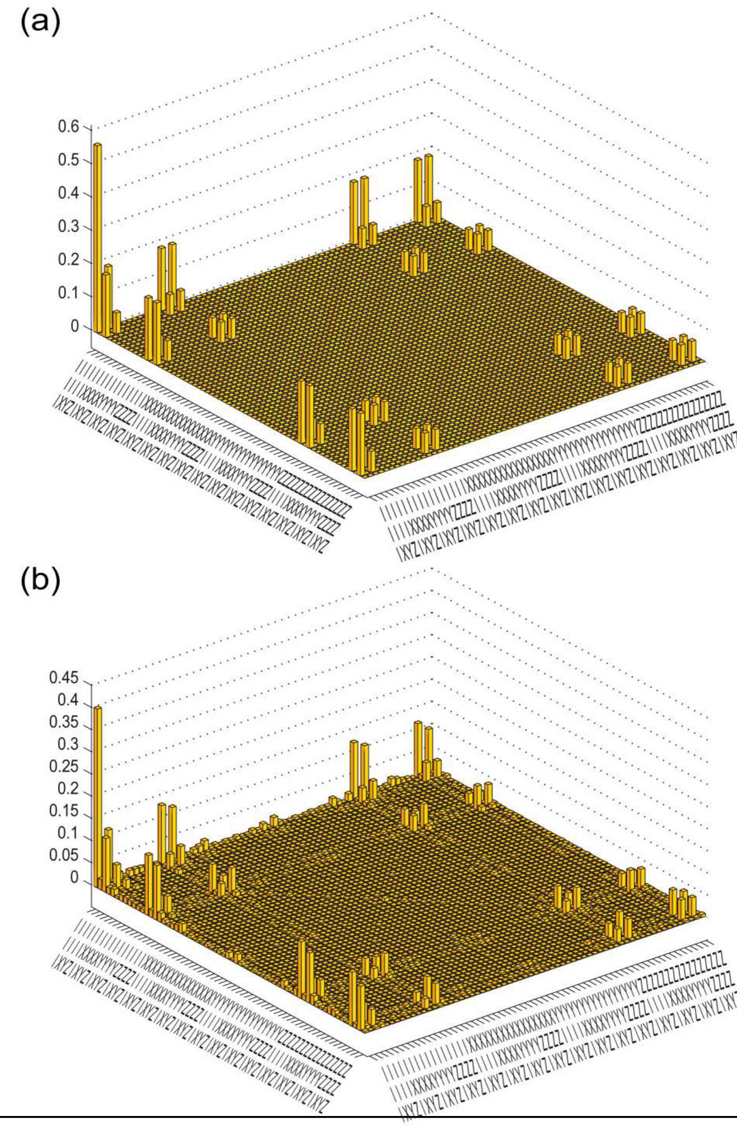
- invert an exponentially large (linear) system

• VADAYS



Realization of the quantum Tofoli gate with trapped ions” T Monz, K. Kim, W. Hänsel, M. Riebe, A. S. Villar, P. Schindler, M. Chwalla, M. Hennrich, and R. Blatt, <u>Physical Review Letters 102, 040501 (2009)</u>





64x64 matrix. Obtained after inverting a 4096x4096 linear system formed with all the probabilities measured after perfirnubg 4096 experiments (prepare each of 64 independent states and measure each of 64 independent transition probabilities.

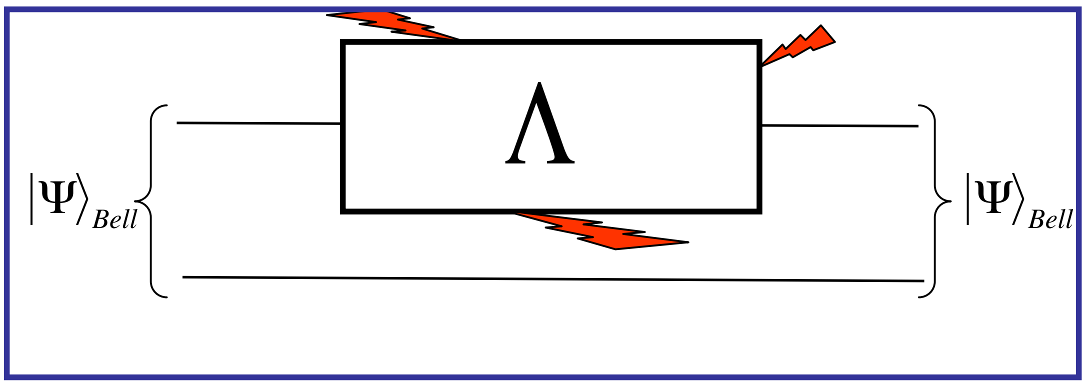
Average fidelity: 0.67
Measured Chi-matrix shows the same “fingerprint” of the ideal one (Tofoli)

QPT IS HARD… (continuation)

• STANDARD QUANTUM PROCESS (NIELSEN & CHUANG) IS EXPONENTIALLY HARD EVEN TO ACHIEVE PARTIAL CHARACTERIZATION!!

ARE THERE OTHER METHODS? DCQP (‘DIRECT CARACTERIZATION OF A QUANTUM PROCESS). D. Lidar and M. Mohseni, Phys. Rev. A 77, 032322 (2008)

• DIAGONAL MATRIX ELEMENTS χ_{nn} ARE SURVIVAL PROBABILITIES OF SYSTEM PLUS ANCILLA (A VERY EXPENSIVE RESOURCE!)



$$|\Psi\rangle_{Bell} = \frac{1}{\sqrt{D}} \sum_{j=1}^{D} |j\rangle \otimes |j\rangle$$

$$\chi_{00} = {}_{Bell}\langle\Psi|\Lambda \otimes I\left(|\Psi\rangle_{Bell}\langle\Psi|_{Bell}\right)|\Psi\rangle_{Bell}$$

• BUT OFF DIAGONAL ELEMENTS ARE STILL EXPONENTIALLY HARD TO CALCULATE WITHIN THIS METHOD. UNTIL RECENTLY THERE WAS NO EFFICIENT METHOD TO ESTIMATE ALL CHI-MATRIX COEFFICIENTS

SELECTIVE AND EFFICIENT Q.P.T.

• AN ALTERNATIVE METHOD FOR QUANTUM PROCESS TOMOGRAPHY

• 1) SELECT A COEFFICIENT χ_{mn} (OR A SET OF THEM)

• 2) DIRECTLY MEASURE THEM WITHOUT DOING FULL QUANTUM PROCESS TOMOGRAPHY

QUANTUM RESOURCES Poly(Log(D))

CLASSICAL RESOURCES Poly(Log(D))

METHOD BASED ON INTERESTING PROPERTY OF CHI-MATRIX

• ALL MATRIX ELEMENTS χ_{mn} ARE AVERAGE FIDELITIES OF QUANTUM CHANNELS.

$$F_{mn}(\Lambda) = \int d|\Psi\rangle \langle\Psi| \Lambda\left(E_m|\Psi\rangle\langle\Psi|E_n\right)|\Psi\rangle$$

$$F_{mn}(\Lambda) = \frac{1}{(D+1)}\left(D\chi_{mn} + \delta_{mn}\right)$$

$\boxed{Estimate \quad F_{mn}(\Lambda)}$ $\Longrightarrow$ $\boxed{Estimate \quad \chi_{mn}}$

MORE PROPERTIES OF CHI-MATRIX

- ALL MATRIX ELEMENTS χ_{mn} ARE AVERAGE FIDELITIES OF QUANTUM CHANNELS.

$$F_{mn}(\Lambda) = \int d|\Psi\rangle \, \langle\Psi| \, \Lambda\left(E_m|\Psi\rangle\langle\Psi|E_n\right) |\Psi\rangle$$

$$F_{mn}(\Lambda) = \frac{1}{(D+1)}\left(D\chi_{mn} + \delta_{mn}\right)$$

- A simple consequence of the following identity

$$\int d|\Psi\rangle \, \langle\Psi| \, A \, |\Psi\rangle \, \langle\Psi| \, B|\Psi\rangle = \frac{1}{D(D+1)}\left(Tr(AB) + Tr(A)Tr(B)\right)$$

$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{nm} \chi_{mn} E_n \, \rho_{in} \, E_m{}^{+}, \quad \sum_{mn} \chi_{mn} E_m{}^{+} E_n = I$$

MAP $\Lambda(\rho)$ IS POSITIVE (P) $\Leftrightarrow$ MATRIX F_{mn} IS POSITIVE

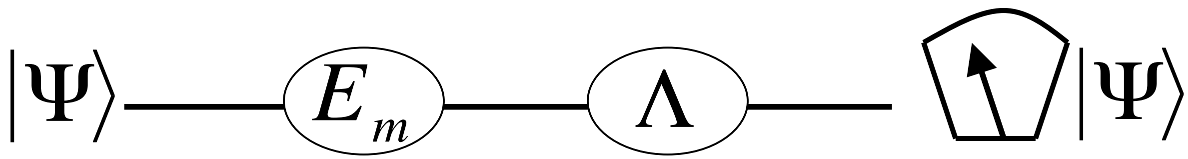
$$\left|F_{mn}\right|^2 \le F_{mm} \, F_{nn} \Leftrightarrow \left|\chi_{mn}\right|^2 \le \chi_{mm}\chi_{nn} + \frac{1}{D}\left(\chi_{nn} + \chi_{mm}\right) + \frac{1}{D^2}$$

MAP $\Lambda(\rho)$ IS COMPLETELY POSITIVE (CP) $\Leftrightarrow$ MATRIX χ_{mn} IS POSITIVE

$$\left|\chi_{mn}\right|^2 \le \chi_{mm} \, \chi_{nn}$$

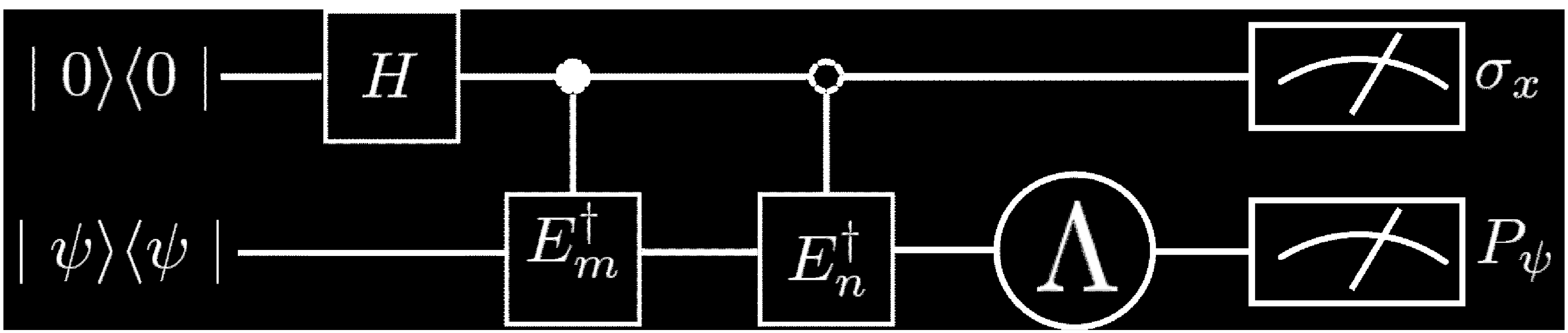
SELECTIVE AND EFFICIENT Q.P.T.

TO MEASURE DIAGONAL COEFFICIENTS: FOLLOW THIS PROCEDURE:



$$\frac{1}{(D+1)}\left(D\chi_{mm}+\delta_{mm}\right)=\int d|\Psi\rangle\,\langle\Psi|\,\Lambda\left(E_m|\Psi\rangle\langle\Psi|E_m\right)|\Psi\rangle$$

A DIFFERENT STRATEGY IS NEEDED FOR OFF-DIAGONAL COEFFICIENTS



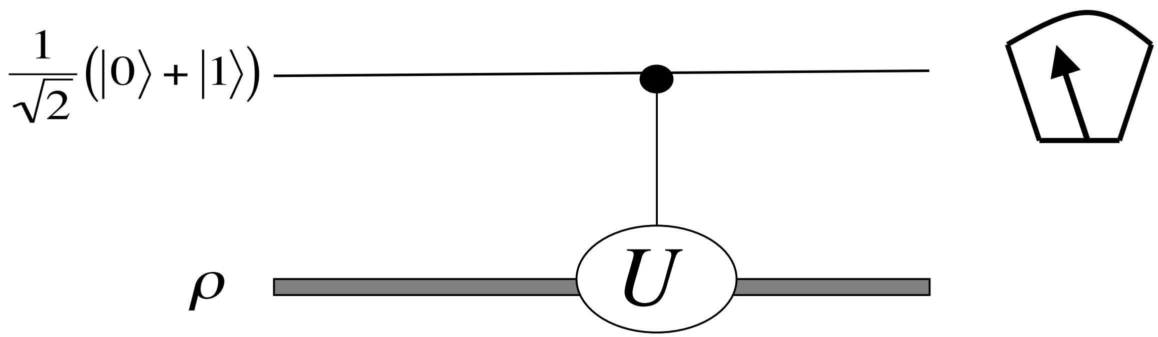
EXTRA RESOURCE: A CLEAN QUBIT: ITS POLARIZATION (CONDITIONED ON STATE SURVIVAL) REVEALS χ_{mn}

Method inspired in a known quantum algorithm

AN ALGORITHM TO MEASURE THE EXPECTATION VALUE OF A UNITARY OPERATOR U USING AN EXTRA (ANCILLARY) QUBIT

“THE SCATTERING ALGORITHM”

THE BASIC INGREDIENT FOR DQC1 MODEL OF QUANTUM COMPUTATION



$$\langle \sigma_x \rangle = \mathrm{Re}(Tr(\rho U))$$

$$\langle \sigma_y \rangle = \mathrm{Im}(Tr(\rho U))$$

Polarization of ancillary qubit reveals the expectation value of U!

INTEGRATING IN HILBERT SPACE USING 2-DESIGNS

- HOW TO PERFORM THE INTEGRAL OVER ALL STATES???

- USE 2-DESIGNS!
- A SET OF STATES (S) IS A 2-DESIGN IF AND ONLY IF

$$\int d|\Psi\rangle \langle\Psi| A |\Psi\rangle \langle\Psi| B |\Psi\rangle = \frac{1}{\#(S)} \sum_{|\Phi_J\rangle \in S} \langle\Phi_j| A |\Phi_j\rangle \langle\Phi_j| B |\Phi_j\rangle$$

- 2-designs are powerful tools!! USEFUL RESULTS

a) 2-DESIGNS EXIST!

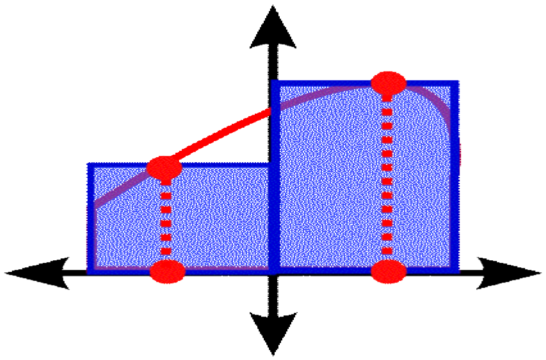
b) THEY HAVE AT LEAST D^2 STATES

c) STATES OF (D+1) MUTUALLY UNBIASED BASIS FORM A 2-DESIGN

d) EFFICIENT ALGORITHMS TO GENERATE 2-DESIGNS EXIST

INTERLUDE ON 2-DESIGNS

• IS THE EXISTENCE OF 2-DESIGN A SURPRISE?

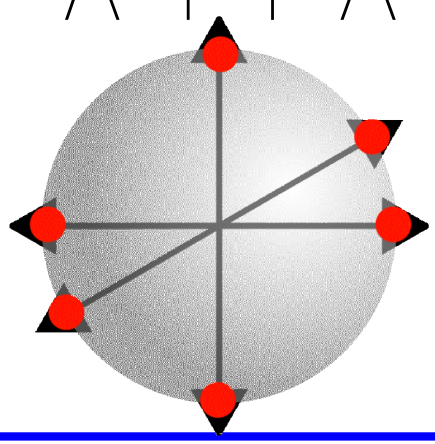


$$\int_0^1 dx\ f(x) = \int_0^1 dx\left(a + bx + cx^2\right) = \frac{1}{2}\left(f(x_1) + f(x_2)\right)$$

$$x_{\frac{1}{2}} = \frac{1}{2} \pm \sqrt{\frac{1}{12}} \Rightarrow S = \{x_1, x_2\} \rightarrow 2 - design$$

• 2-DESIGNS FOR SPIN 1/2: ENABLE TO COMPUTE AVERAGES OF PRODUCTS OF TWO EXPECTATION VALUES (integrals of functions that depend upon TWO bras and TWO kets)

$$\int d|\Psi\rangle \langle\Psi| A |\Psi\rangle \langle\Psi| B |\Psi\rangle = \frac{1}{6}\langle\uparrow_x|A|\uparrow_x\rangle\langle\uparrow_x|B|\uparrow_x\rangle + \frac{1}{6}\langle\downarrow_x|A|\downarrow_x\rangle\langle\downarrow_x|B|\downarrow_x\rangle$$



$$+ \frac{1}{6}\langle\uparrow_y|A|\uparrow_y\rangle\langle\uparrow_y|B|\uparrow_y\rangle + \frac{1}{6}\langle\downarrow_y|A|\downarrow_y\rangle\langle\downarrow_y|B|\downarrow_y\rangle$$

$$+ \frac{1}{6}\langle\uparrow_z|A|\uparrow_z\rangle\langle\uparrow_z|B|\uparrow_z\rangle + \frac{1}{6}\langle\downarrow_z|A|\downarrow_z\rangle\langle\downarrow_z|B|\downarrow_z\rangle$$

IS THE METHOD REALLY EFFICIENT?

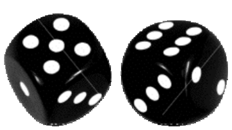
2-DESIGNS HELP US TO GO FROM INTEGRALS TO SUMS

$$F(\Lambda) = \frac{1}{(D+1)}\left(D\chi_{00}+1\right) = \frac{1}{\#(S)} \sum_{|\Phi_j\rangle \in S} \langle \Phi_j | \Lambda\left(|\Phi_j\rangle\langle\Phi_j|\right) |\Phi_j\rangle$$

• BUT: THERE ARE AN EXPONENTIALLY LARGE NUMBER OF ELEMENTS IN S

$$|\Psi\rangle \text{ — } E_m \text{ — } \Lambda \text{ — } |\Psi\rangle$$

• SOLUTION!: USE RANDOMNESS… SAMPLE RANDOMLY OVER S: AFTER M REPETITIONS YOU ESTIMATE THE AVERAGE WITH AN ERROR $\approx 1/\sqrt{M}$



$$|\Phi_j\rangle \rightarrow E_m \rightarrow \Lambda \rightarrow$$

$$|\Phi_j\rangle ?: \begin{cases} NO \Rightarrow x = 0 \\ YES \Rightarrow x = 1 \end{cases}$$

$$F(\Lambda_m) = \frac{1}{M}\sum_{i=1}^{M} x_i + O\left(\frac{1}{\sqrt{M}}\right) = \frac{1}{D}\left(D\chi_{mm}+1\right)$$

WHAT DOES S.E.Q.P.T. ACCOMPLISHES?

FIRST EFFICIENT METHOD TO DETERMINE ANY ELEMENT OF CHI MATRIX OF A QUANTUM PROCESS

Poly(Log(D)) QUANTUM GATES REQUIRED

Poly(Log(D)) CLASSICAL POST-PROCESSING REQUIRED

A SINGLE CLEAN QUBIT REQUIRED (WITH INTERACTIONS WITH SYSTEM)…

FURTHER EXTENSIONS:

FIND OUT LARGE CHI-ELEMENTS (A. Bendersky, F. Pastawski and J.P. Paz, Phys. Rev. A 80, 032116 (2009)

AVOID THE USE OF AN ANCILLARY CLEAN QUBIT (Unpublished)

PHYSICAL REVIEW A 80, 032116 (2009)

Selective and efficient quantum process tomography

Ariel Bendersky,[1] Fernando Pastawski,[2] and Juan Pablo Paz[1]
[1]*Departamento de Física, FCEyN, UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*
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GENERALIZATION: SEQPT without ancillary qubits

$$F_{mn}(\Lambda) = \frac{1}{(D+1)}(D\chi_{mn}+1) = \frac{1}{D(D+1)}\sum_{|\Phi_j\rangle\in S}\langle\Phi_j|\Lambda\left(E_m|\Phi_j\rangle\langle\Phi_j|E_n\right)|\Phi_j\rangle$$

ESTIMATE AUXILIARY FIDELITIES DEFINED AS

$$\tilde{F}^{(\pm)}{}_{mn}(\Lambda) = \frac{1}{D(D+1)}\sum_{|\Phi_j\rangle\in S}\langle\Phi_j|\Lambda\left((E_n \pm E_m)|\Phi_j\rangle\langle\Phi_j|(E_n \pm E_m)\right)|\Phi_j\rangle$$

$$F_{mn} = \tilde{F}^{(+)}{}_{mn} + \tilde{F}^{(-)}{}_{mn} - F_{nn} - F_{mm}$$

USE 2-DESIGN DEFINED BY THE (D+1) MUBs ASSOCIATED WITH THE OPERATOR BASIS E_m

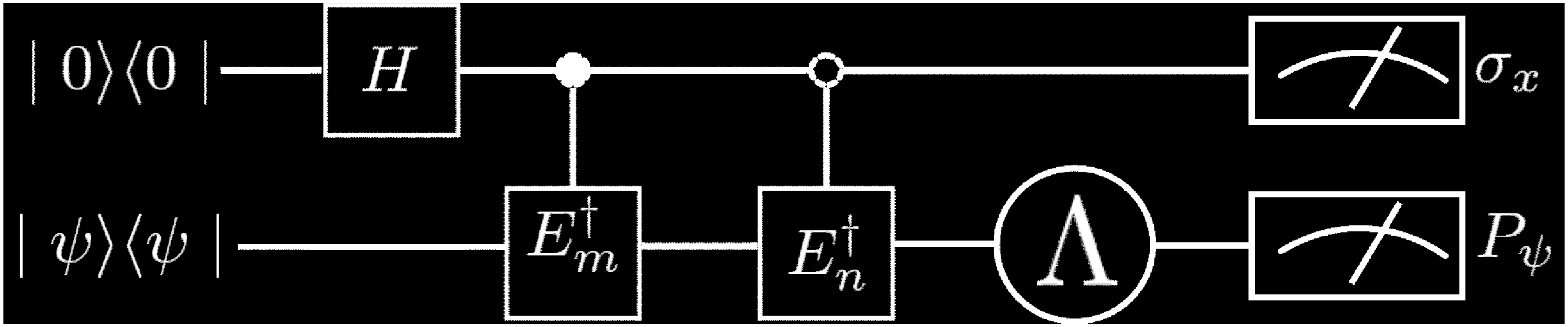
$$|\Phi_j\rangle \to |\Phi^{(b)}{}_k\rangle;\ b = 1,\dots,D;\quad k = 1,\dots,D+1$$

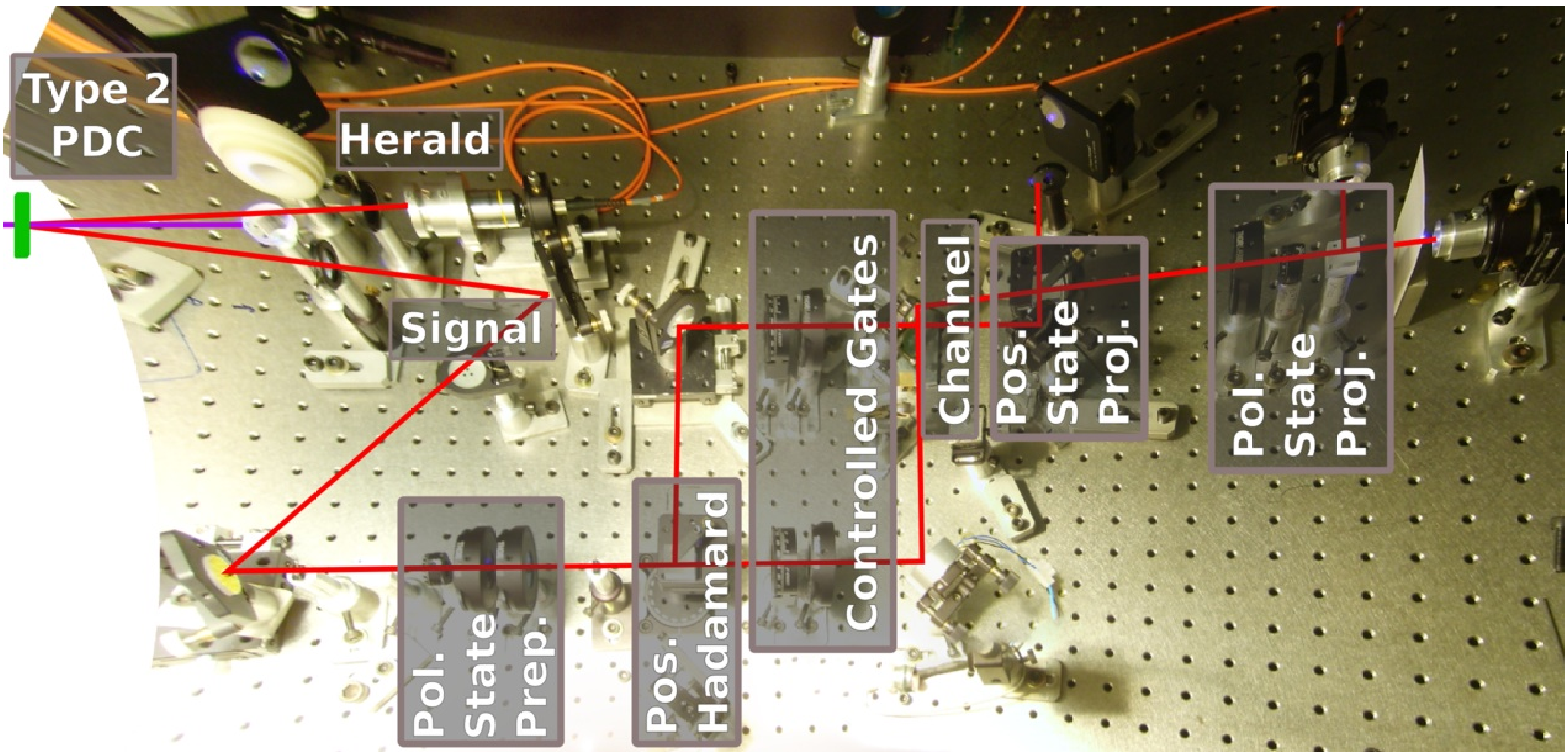
$$E_m|\Phi^{(b)}{}_k\rangle \approx |\Phi^{(b)}{}_{k'}\rangle$$

$$(E_m \pm E_n)|\Phi^{(b)}{}_k\rangle \approx \left(|\Phi^{(b)}{}_{k'}\rangle \pm |\Phi^{(b)}{}_{k''}\rangle\right)$$

EFFICIENT PROCCEDURE FOR PREPARING SUCH STATES EXIST!

PHOTONIC IMPLEMENTATION: FIRST EXPERIMENT IN OUR LAB IN BUENOS AIRES! (Schmiegelow et al,2010)





PHOTONIC IMPLEMENTATION: FIRST EXPERIMENT IN OUR LAB IN BUENOS AIRES! (Schmiegelow et al,2009)

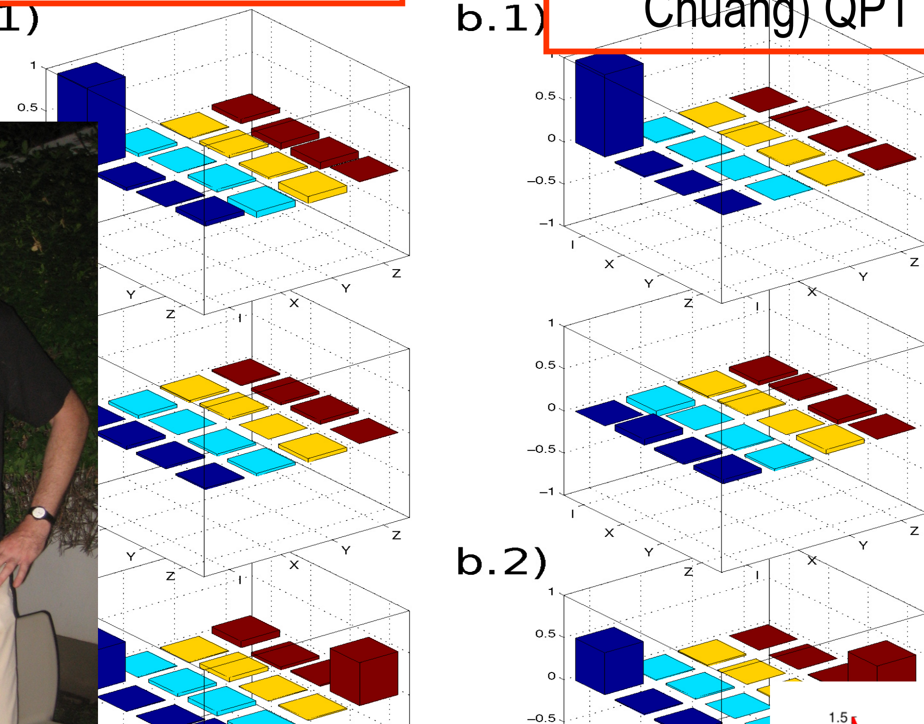
SEQPT

Standard (Nielsen & Chuang) QPT

a.1)

b.1)

b.2)



Average Fidelity requires estimating 16 transition probabilities

PHYSICAL REVIEW LETTERS

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[2]CEILAP, CITEDEF, J.B. de La Salle 4397, 1603 Villa Martelli, Buenos Aires, Argentina

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Next experiment: SEQPT on a 2-qubit channel (2010)

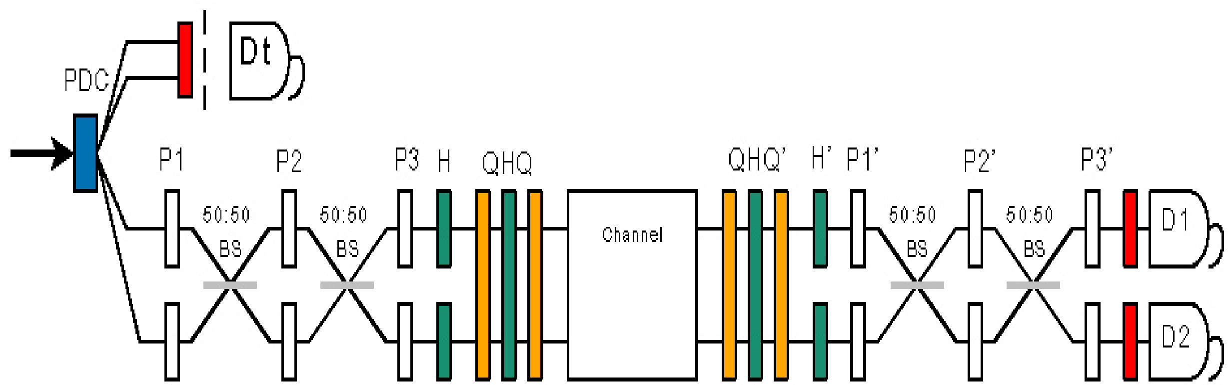
Base	$P1, P2, P3$	CNOT	QHQ
XX	$0, -\frac{\pi}{2}, 0$	✗	$\frac{\pi}{4}, \frac{\pi}{8}, 0$
YY	$\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2}$	✗	$0, \frac{\pi}{8}, 0$
ZZ	0,0,0	✗	0,0,0
belle	$0, -\frac{\pi}{2}, \pi$	✓	$\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$
beau	$0, -\frac{\pi}{2}, \pi$	✓	$-\frac{\pi}{4}, -\frac{\pi}{4}, 0$

TABLE II: Configurations for state preparation

Base	$P1, P2, P3$	CNOT	QHQ
XX	$0, -\frac{\pi}{2}, 0$	✗	$0, \frac{\pi}{8}, \frac{\pi}{4}$
YY	$\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}$	✗	$\frac{\pi}{2}, \frac{\pi}{8}, \frac{\pi}{8}$
ZZ	0,0,0	✗	0,0,0
belle	$\pi, \frac{\pi}{2}, 0$	✓	$-\frac{\pi}{4}, -\frac{\pi}{4}, 0$
beau	$\pi, \frac{\pi}{2}, 0$	✓	$\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$

TABLE III: Configurations for state readout

- Two qubits in one photon
- Five MUB are prepared by different settings of QWP, HQP, PS



The experiment with the 2-qubit implementation of SEQPT (without ancillary system) is almost complete.

SUMMARY

QUANTUM PROCESS TOMOGRAPHY (QPT):

- FULL QPT IS ALWAYS HARD. STANDARD METHODS FOR PARTIAL QPT ARE ALSO EXPONENTIALLY HARD
- THERE IS AN ALTERNATIVE METHOD FOR EFFICIENT AND SELECTIVE PARTIAL QUANTUM PROCESS TOMOGRAPHY
- IT INVOLVES ESTIMATION OF ‘SURVIVAL PROBABILITIES’ OF A SET OF STATES FORMING A 2-DESIGN (VERY USEFUL RESOURCE!)
- THE METHOD HAS BEEN EXPERIMENTALLY IMPLEMENTED (MORE IS COMING)

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Selective and Efficient Estimation of Parameters for Quantum Process Tomography

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