Decoherence: quantum vs. classical

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1. Remark on “system + environment”

2. “classical” vs. “quantum” decoherence

3. examples:
   • Cavity QED (quantum)
   • Molecular vibrations (quantum)
   • Ion trap (classical)
   • Ion trap quantum computer (classical)

4. Is “pure decoherence” always “classical”?  
   • no: we construct 2-qubit example

5. Conclusions
Outline

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„total state perspective“ in open system dynamics:

1.) very good! (see also this talk ..)

2.) question: „system + environment = Alice + Bob“?

Continuous measurement, quantum trajectories:

measurement s on the environment exist such that the reduced state is unaffected by the measurement:

Crucial: environment large, short bath correlation time.
Example: standard model of open system dynamics

Environment of harmonic oscillators:

\[ H_{\text{tot}} = H_{\text{sys}} + \sum_{\lambda} g_{\lambda} ( L b_{\lambda}^+ + L^+ b_{\lambda} ) + \sum_{\lambda} \omega_{\lambda} b_{\lambda}^+ b_{\lambda} \]

Bath correlation function (here at zero temperature):

\[ \alpha(t-s) = \langle B(t) B^+(s) \rangle = \sum_i |g_i|^2 e^{-i\omega_i t} = \int d\omega J(\omega)e^{-i\omega t} \]

\[ J(\omega) : \text{Spectral density} \]

If spectral density flat \( \iff \) Markovian dynamics.

Reduced dynamics governed by \textit{Lindblad} master equation:

\[ \dot{\rho} = -\frac{i}{\hbar} [H_{\text{sys}}, \rho] + \frac{\gamma}{2} (L\rho, L^+) + [L, \rho L^+] \]
Total state: solve Schrödinger’s equation (T=0):

Model: \( H_{\text{tot}} = H_{\text{sys}} + \sum_{\lambda} g_{\lambda} (Lb_{\lambda}^+ + L^+b_{\lambda}) + \sum_{\lambda} \omega_{\lambda} b_{\lambda}^+ b_{\lambda} \)

Expand total state in a fixed (Bargmann) coherent state basis for the environmental degrees of freedom:

\[
|\Psi_t\rangle = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle \otimes |z\rangle
\]

System state \( |\psi_t(z^*)\rangle = \langle z|\Psi_t\rangle \) corresponds to a certain fixed configuration \( z = (z_1, z_2, z_3, ..., z_\lambda, ...) \) of the environment.

Note: \( z_{\lambda} = \frac{1}{\sqrt{2}} (q_{\lambda} + ip_{\lambda}) \).

Find:

\[
\dot{\psi}_t = -\frac{i}{\hbar} H_{\text{sys}} \psi_t + Lz_t \psi_t - L^+ \int_0^t ds \alpha(t-s) \frac{\delta \psi_t}{\delta z_s}
\]

[Note: \( L.\) Diosi, WTS, PLA 235, 569 (1997)]

with

\[
z_t = -i \sum_{\lambda} g_{\lambda} z_{\lambda}^* e^{i\omega_{\lambda}t}
\]

Note:

\[
\frac{\delta \psi_t}{\delta z_s} \bigg|_{s=t} = L\psi_t
\]
Total state: solve Schrödinger’s equation (T=0), Markov:

Model: \[ H_{\text{tot}} = H_{\text{sys}} + \sum_{\lambda} g_{\lambda} (Lb^\lambda_++L^+b^\lambda_-) + \sum_{\lambda} \omega_{\lambda} b^\lambda_+ b^\lambda_- \]

Expand total state in a fixed (Bargmann) coherent state basis for the environmental degrees of freedom:

\[ |\Psi_t\rangle = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle \otimes |z\rangle \]

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Note: \[ z_\lambda = \frac{i}{\sqrt{2}} (q_\lambda + ip_\lambda) \].

Find:

\[ \dot{\psi}_t = -\frac{i}{\hbar} H_{\text{sys}} \psi_t + Lz_t \psi_t - \frac{1}{2} L^+ L \psi_t \]

with \[ z_t = -i \sum_{\lambda} g_{\lambda} z^\lambda_+ e^{i\omega_{\lambda} t} \]

[L. Diosi and WTS, PLA 235, 569 (1997)]
Solving the Schrödinger equation (T=0):

\[ H_{\text{tot}} = H_{\text{sys}} + \sum_{\lambda} g_{\lambda} (Lb_{\lambda}^+ + L^+b_{\lambda}) + \sum_{\lambda} \omega_{\lambda} b_{\lambda}^+ b_{\lambda} \]

\[ |\Psi_t\rangle = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle \otimes |z\rangle \]

Find closed evolution equation for \[ |\psi_t(z^*)\rangle \]

• „quantum trajectories“: [L. Diosi and WTS, PLA 235, 569 (1997)]

\[ \rho(t) = \text{Tr}_{\text{env}} \left[ |\Psi(t)\rangle\langle\Psi(t)| \right] = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle\langle\psi_t(z^*)| \]

• better: let \(|z\rangle\) evolve with natural dynamics [WTS, L. Diosi, N. Gisin, PRL 82, 1801 (1999)]

• solution exists for harmonic oscillator and \(L=q\), for arbitrary alpha

[WTS and T. Yu, PRA 69, 052115 (2004)]
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   • no: we construct 2-qubit example

5. Conclusions
Decoherence caused by classical, fluctuating fields (Hamiltonians) („random external field“ (REF)-channel, random unitary channel):

\[ \rho(t) = \int d\mu(\omega) U_{\omega}(t) \rho(0) U_{\omega}^+(t) \]  

„classical“

Decoherence caused by genuine interaction with a „quantum environment“ (entanglement):

\[ H_{\text{tot}} = H_{\text{sys}} \otimes 1 + H_{\text{int}} + 1 \otimes H_{\text{env}} \]

\[ \rho(t) = \text{Tr}_{\text{env}} \left[ U_{\text{tot}}(t)(\rho(0) \otimes \rho_{\text{env}}(0))U_{\text{tot}}^+(t) \right] \]

„quantum“
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Paris decoherence experiment:

M. Brune et. al., PRL 77, 4887 (1996)

\[ \exp(-\gamma D^2 t) \]
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Femtosecond pump-probe spectroscopy

experiment: Stienkemeier group (Freiburg) PRA 80 042512 (2009) with \( \text{Rb}_2 \)

on He-nanodroplets!! (0.4 K)

Decay of revivals as an indicator for decoherence!

[M. Schlesinger and WTS, PRA 77, 012111 (2008)]
Femtosecond pump-probe spectroscopy: decay of revivals

\[ \exp(-\gamma D^2 t) \rightarrow \exp(-\gamma \int_0^t D(s)^2 ds) \]

\[ D(s)^2 = \Delta \chi(s)^2 + \Delta p(s)^2 \]

(simulations by Martin Schlesinger)  [see also M. Schlesinger and WTS, PRA 77, 012111 (2008)]
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Decoherence between coherent states:

\[ D = (q_1 - q_2) \]

\[ \exp(-\gamma D^2 t) \]

\[ H_{\text{tot}} = \frac{p^2}{2M} + V(q) + \sum_i g_i q_i + \sum_i \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) \]

(“entanglement with environment“ initial thermal bath state)

\[ \dot{\rho} = -i [H_{\text{sys}}, \rho] - \Gamma [q, [q, \rho]] \]

(unitary, stochastic dynamics)

\[ i \psi_t = (H_{\text{sys}} + \sqrt{\Gamma} q \xi(t)) \psi_t \]

\[ \langle \xi(t) \xi(s) \rangle = \delta(t - s) \]
\[ H_{\text{tot}} = \frac{p^2}{2M} + V(q) + q \sum_{i} g_i q_i + \sum_{i} \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) \]

Bath correlation function

\[ \alpha(t - s) = \langle \hat{F}(t)\hat{F}(s) \rangle = \int d\omega J(\omega) \left[ \coth\left(\frac{\hbar \omega}{2kT}\right) \cos(\omega t) - i \sin(\omega t) \right] \]

\[ i\dot{\psi}_t = (H_{\text{sys}} + \sqrt{D}q \xi(t))\psi_t \]

\[ \langle \xi(t)\xi(s) \rangle = \text{Re} \alpha(t - s) \]

(neglect damping = imaginary part)

[see also T. Grotz, L. Heaney, and WTS, PRA 74, 22102 (2006)]
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Innsbruck linear ion trap quantum computer

(Figs: (2007) Hartmut Häffner, Innsbruck)

\[
\omega_{x,y} \approx 1.5 - 5 \text{ MHz} \\
\omega_z \approx 0.7 - 2 \text{ MHz}
\]
Decoherence of a single qubit

Use general theoretical framework for (realistic) decoherence dynamics (channels)

Gaussian decoherence of single qubit („static disorder“)

\[ H = \frac{1}{2} (\omega + \Delta) \sigma_z \]

\[
\rho(0) = \begin{pmatrix} p & c \\ c^* & 1 - p \end{pmatrix}
\]

\[
\rho_\Delta(t) = \begin{pmatrix} p & ce^{-i(\omega+\Delta)t} \\ c^* e^{i(\omega+\Delta)t} & 1 - p \end{pmatrix}
\]

\[
\bar{\rho}(t) = \begin{pmatrix} p & ce^{-\sigma^2 t^2} e^{-i\omega t} \\ c^* e^{-\sigma^2 t^2} e^{i\omega t} & 1 - p \end{pmatrix}
\]

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1 Qubit: „quantum“ = „random unitary“

1-Qubit decoherence:

\[ \rho(t) = \begin{pmatrix} \rho_{00}(0) & c(t)\rho_{01}(0) \\ c^*(t)\rho_{10}(0) & \rho_{11}(0) \end{pmatrix} \]

\[ U_\Omega = e^{-i\frac{\Omega(t)}{2}\sigma_z} \]

\[ \bar{\rho}(t) = \begin{pmatrix} \rho_{00}(0) & \langle e^{-i\Omega} \rangle \rho_{01}(0) \\ \langle e^{i\Omega} \rangle \rho_{10}(0) & \rho_{11}(0) \end{pmatrix} \]

\[ \Omega(t) \] a stochastic variable (process)
In general: decoherence = unitary + stochastic?

May pure decoherence (phase damping) be described by stochastic, unitary evolution?

(no entanglement with „environment“ required)

Note: also relevant for quantum error correction: If quantum environment, total pure state, yet RU-channel: quantum information lost into the environment can be completely restored: „Quantum lost and found“ M. Gregoratti and R. F. Werner, J. Mod. Opt. 50, 915 (2003)
May pure decoherence (phase damping) be described by stochastic, unitary evolution?

(no entanglement with „environment“ required)

„for many practical purposes“: yes

for a single qubit: yes

for two qubits (and more): in general, no

\[ \rho(t) = \sum_{j \leq M} K_j(t) \rho(0) K_j^+(t) \] and \[ \langle n | \rho(t) | n \rangle = \text{const} \]

\[ \Rightarrow \rho_{ij}(t) = \langle \tilde{a}_i(t), \tilde{a}_j(t) \rangle \cdot \rho_{ij}(0) \]

with \( \tilde{a}_i = (a_{1i}, a_{2i}, \ldots, a_{di}) \) any set of normalized complex vectors

Note: \( tr \rho(t) = 1 \) and \( \text{id} \rightarrow \text{id} \) (unital)

Phase damping channels \( \iff \) Bistochastic (unital), diagonal q.o.
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Physics

Mathematics

Birkhoff: characterize extremal bistochastic maps of probability distributions

In general: decoherence = unitary+stochastic?

May pure decoherence (phase damping) be described by stochastic, unitary evolution?

(no entanglement with „environment“ required)

„for many practical purposes“: yes

for a single qubit: yes
for two qubits (and more): in general, no: Example??

„true quantum“ decoherence of two qubits

2 qubits coupled to 1 „environmental qubit“:

\[ H_{\text{tot}} = \kappa_A \sigma_z^{(A)} \sigma_z^{(R)} + \kappa_B \sigma_z^{(B)} \sigma_z^{(R)} + \Gamma \cdot \sigma \]

\[ = \sum_{j=1\ldots4} | j \rangle \langle j | \otimes H_j^{(R)} \]

Four environmental qubit-states \( | \Psi_j^{(R)} \rangle = \exp(-iH_j^{(R)}t) | \Psi_0 \rangle \)

with corresponding Bloch-vector \( \vec{r}_j \)

The 2-qubit dephasing channel \textbf{is RU}, iff the four Bloch vectors point to a plane.

see Julius‘ \textbf{poster}!

[and J. Helm, WTS, PRA 80, 042108 (2009), PRA 81, 042314 (2010)]
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\[ = \sum_{j=1..4} |j\rangle\langle j| \otimes H_j^{(R)} \]

Four environmental qubit-states

\[ |\Psi_j^{(R)}(t)\rangle = \exp(-iH_j^{(R)}t) |\Psi_0^{(R)}\rangle \]

with corresponding Bloch-vector \( \vec{r}_j \)

determine Volume V of tetrahedron: measure for being „non-RU“?

Compare with numerical determination of distance from the convex set of RU channels! (see Julius‘ poster)

[J. Helm, WTS, PRA 80, 042108 (2009), PRA 81, 042314 (2010)]
"true quantum" decoherence:

**open question** (for me):  

Given pure decoherence dynamics: 

\[ \rho_{ij}(t) = D_{ij}(t)\rho_{ij}(0) \]

(from process tomography, for instance)

Is there a "simple" criterion that tells us whether D is random unitary?
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(1) in practice (from a local point of view), pure decoherence is often of random unitary (RU) type:

*discussion of robust states etc. requires “total system point of view”*

(2) for a single qubit: Q=RU

(3) in general Q is larger: we construct a simple non-RU decoherence channel for two qubits

(4) Examples of quantum and classical decoherence: cavity QED, molecular dynamics, ion traps

(5) Open question: simple criterion to decide whether channel is RU (also relevant for “quantum lost and found”)
