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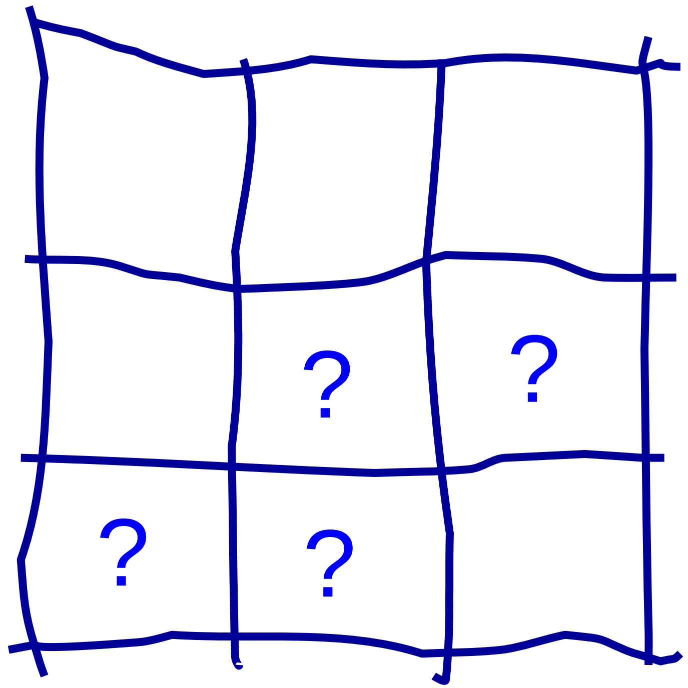
Decoherence: quantum vs. classical

Walter Strunz
Martin Schlesinger, Julius Helm → Poster

ITP, TU Dresden

Coherence and Decoherence , Benasque, September 2010



1 2 3 4 5 6 7 8 9

Outline

1. Remark on “system + environment”

2. “classical” vs. “quantum” decoherence

3. examples:
- Cavity QED (quantum)
- Molecular vibrations (quantum)(???)
- Ion trap (classical)
- Ion trap quantum computer (classical)

4. Is “pure decoherence” always “classical”?
- no: we construct 2-qubit example

5. Conclusions

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„total state perspective“ in open system dynamics:

1.) very good! (see also this talk ..)

2.) question: „system + environment = Alice + Bob“ ?

Continuous measurement , quantum trajectories:

measurement s on the environment exist such that the reduced state is unaffected by the measurement:

Crucial: environment large, short bath correlation time.

Example: standard model of open system dynamics

Environment of harmonic oscillators:

$$H_{\text{tot}} = H_{\text{sys}} + \sum_\lambda g_\lambda (L b_\lambda^+ + L^+ b_\lambda) + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda$$

Bath correlation function (here at zero temperature):

$$\alpha(t-s) = \langle B(t) B^+(s) \rangle = \sum_i | g_i |^2 e^{-i\omega_i t} = \int d\omega \, J(\omega) e^{-i\omega t}$$

$J(\omega)$ **: Spectral density**

If spectral density *flat* ⇨ **Markovian dynamics.**

Reduced dynamics governed by *Lindblad* master equation:

$$\dot{\rho} = -\tfrac{i}{\hbar}[H_{\text{sys}}, \rho] + \tfrac{\gamma}{2}\left([L\rho, L^+] + [L, \rho L^+]\right)$$

Total state: solve Schrödinger's equation (T=0):

Model: $H_{\text{tot}} = H_{\text{sys}} + \sum_\lambda g_\lambda (L b_\lambda^+ + L^+ b_\lambda) + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda$

Expand total state in a fixed (Bargmann) coherent state basis for the environmental degrees of freedom:

$$|\Psi_t\rangle = \int \frac{d^2 z}{\pi}\, e^{-|z|^2}\; |\psi_t(z^*)\rangle \otimes |z\rangle$$

System state $|\psi_t(z^*)\rangle = \langle z|\Psi_t\rangle$ **corresponds to a certain fixed configuration** $z = (z_1,\ z_2,\ z_3, \ldots, z_\lambda,\ \ldots)$ **of the environmet.**
Note: $z_\lambda = \frac{1}{\sqrt{2}}(q_\lambda + i p_\lambda)$.

Find: $$\dot{\psi}_t = -\tfrac{i}{\hbar} H_{\text{sys}} \psi_t + L z_t \psi_t - L^+ \int_0^t ds\; \alpha(t-s) \frac{\delta \psi_t}{\delta z_s}$$

[L. Diosi, WTS, PLA 235, 569 (1997)]

with $$z_t = -i \sum_\lambda g_\lambda z_\lambda^* e^{i\omega_\lambda t}$$

Note: $$\left.\frac{\delta \psi_t}{\delta z_s}\right|_{s=t} = L\psi_t$$

Total state: solve Schrödinger's equation (T=0), Markov:

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Model: $H_{\mathrm{tot}} = H_{\mathrm{sys}} + \sum_\lambda g_\lambda (L b_\lambda^+ + L^+ b_\lambda) + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda$

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Note: $z_\lambda = \frac{1}{\sqrt{2}}(q_\lambda + i p_\lambda)$.

Find:

$$\dot{\psi}_t = -\frac{i}{\hbar} H_{\mathrm{sys}} \psi_t + L z_t \psi_t - \frac{1}{2} L^+ L \psi_t$$

[L. Diosi and WTS, PLA 235, 569 (1997)]

with $z_t = -i \sum_\lambda g_\lambda z_\lambda^* e^{i\omega_\lambda t}$

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Solving the Schrödinger equation (T=0):

$$H_{\text{tot}} = H_{\text{sys}} + \sum_\lambda g_\lambda (L b_\lambda^+ + L^+ b_\lambda) + \sum_\lambda \omega_\lambda b_\lambda^+ b_\lambda$$

$$|\Psi_t\rangle = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle \otimes |z\rangle$$

Find closed evolution equation for $|\psi_t(z^*)\rangle$

- **„quantum trajectories“:** [L. Diosi and WTS, PLA 235, 569 (1997)]

$$\rho(t) = \text{Tr}_{\text{env}}\left[|\Psi(t)\rangle\langle\Psi(t)|\right] = \int \frac{d^2z}{\pi} e^{-|z|^2} |\psi_t(z^*)\rangle\langle\psi_t(z^*)|$$

- **better: let |z> evolve with natural dynamics** [WTS, L. Diosi, N. Gisin, PRL 82, 1801 (1999)]
- **solution exists for harmonic oscillator and L=q, for arbitrary alpha**

[WTS and T. Yu, PRA 69, 052115 (2004)]

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„classical“ vs. „quantum“ decoherence

Decoherence caused by classical, fluctuating fields (Hamiltonians) („random external field“ (REF)-channel, random unitary channel):

$$\rho(t) = \int d\mu(\omega) U_\omega(t) \rho(0) U_\omega^+(t)$$

„classical“

Decoherence caused by genuine interaction with a „quantum environment“ *(entanglement):*

$$H_{\text{tot}} = H_{\text{sys}} \otimes \mathbf{1} + H_{\text{int}} + \mathbf{1} \otimes H_{\text{env}}$$

$$\rho(t) = \mathrm{Tr}_{\text{env}}\left[U_{\text{tot}}(t) (\rho(0) \otimes \rho_{\text{env}}(0)) U_{\text{tot}}^+(t) \right]$$

„quantum“

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Paris decoherence experiment:

M. Brune et. al., PRL **77**, 4887 (1996)

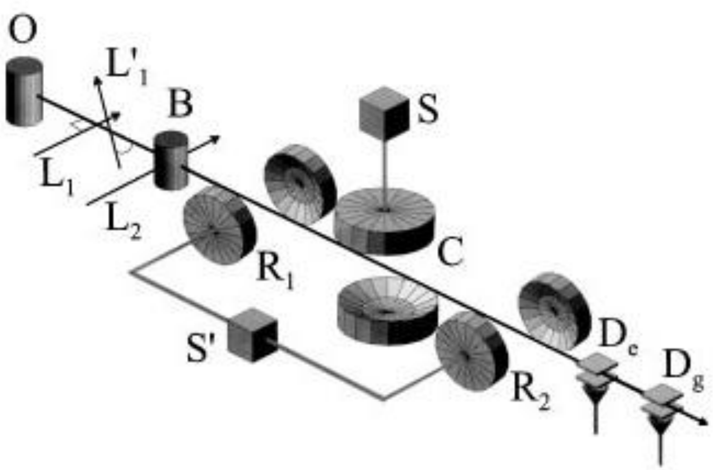
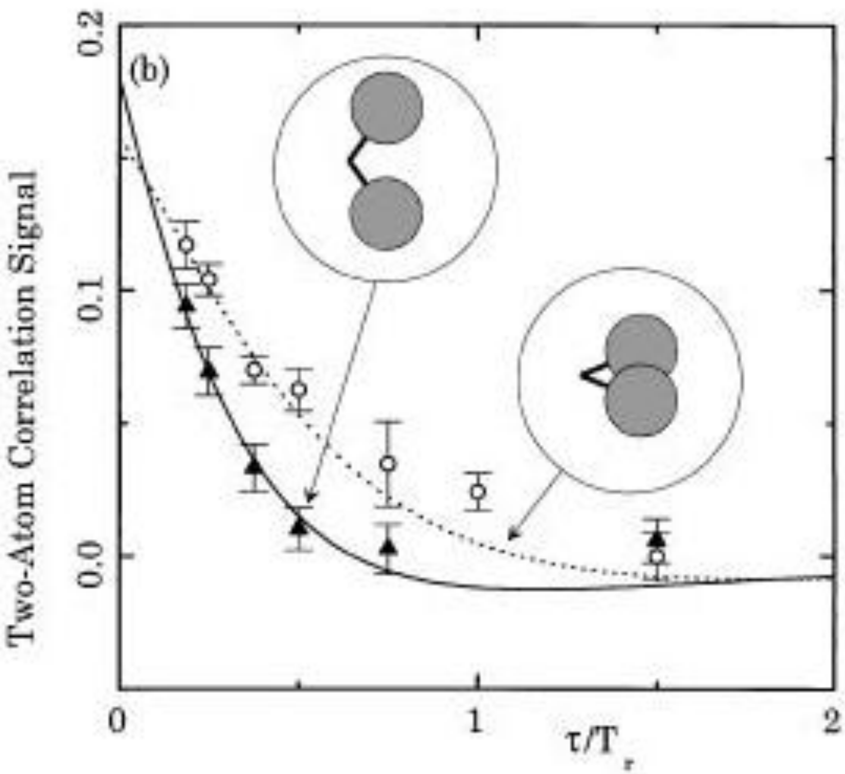


FIG. 2. Sketch of the experimental setup.



$$\exp(-\gamma D^2 t)$$

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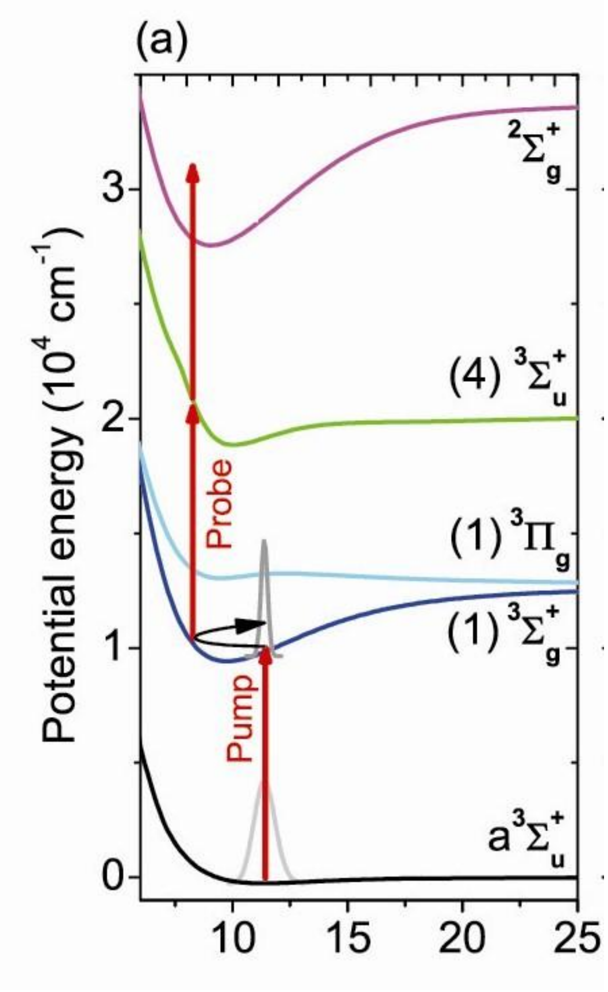
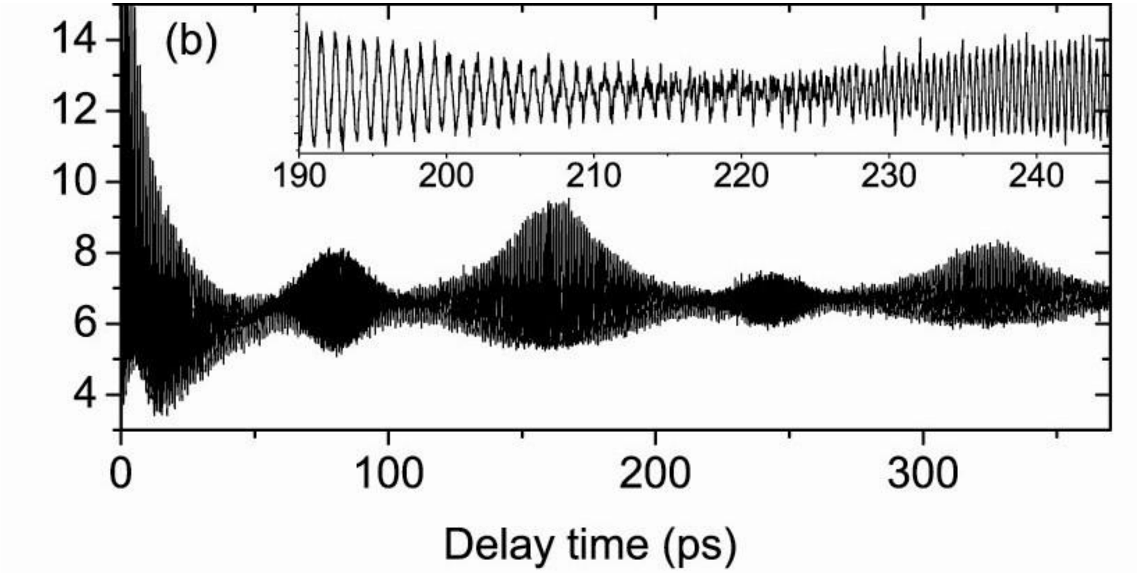
Femtosecond pump-probe spectroscopy



experiment: Stienkemeier group (Freiburg)
PRA 80 042512 (2009) with Rb_2

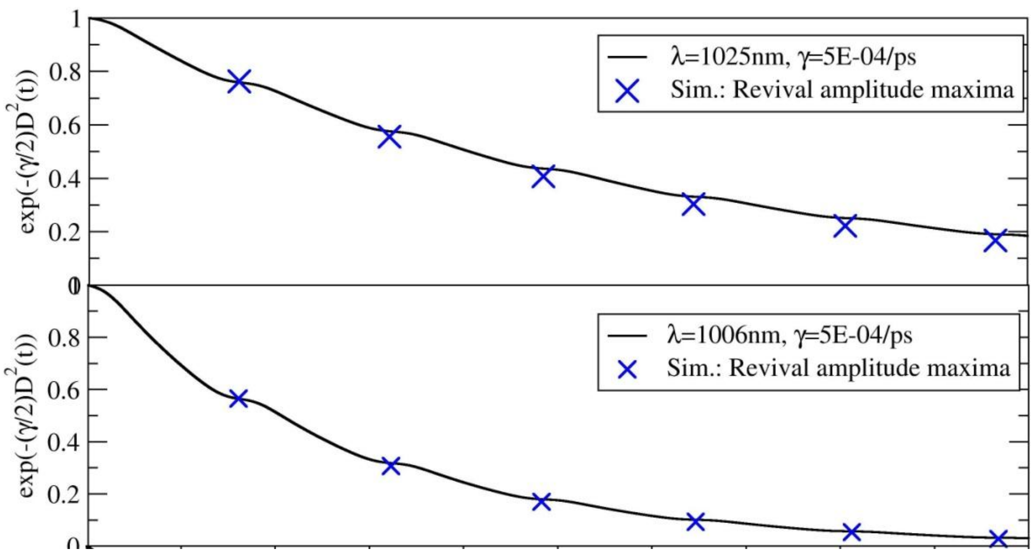
on He-nanodroplets!! (0.4 K)

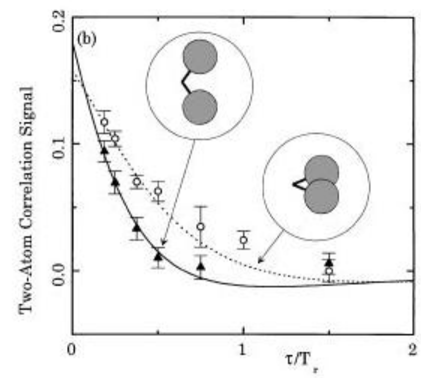


Decay of revivals as an indicator for decoherence!

[M. Schlesinger and WTS, PRA 77, 012111 (2008)]

Femtosecond pump-probe spectroscopy: decay of revivals





$$\exp(-\gamma D^2 t) \rightarrow \exp\left(-\gamma \int_0^t D(s)^2 ds\right)$$

$$D(s)^2 = \Delta x(s)^2 + \Delta p(s)^2$$

(simulations by Martin Schlesinger) [see also M. Schlesinger and WTS, PRA 77, 012111 (2008)]

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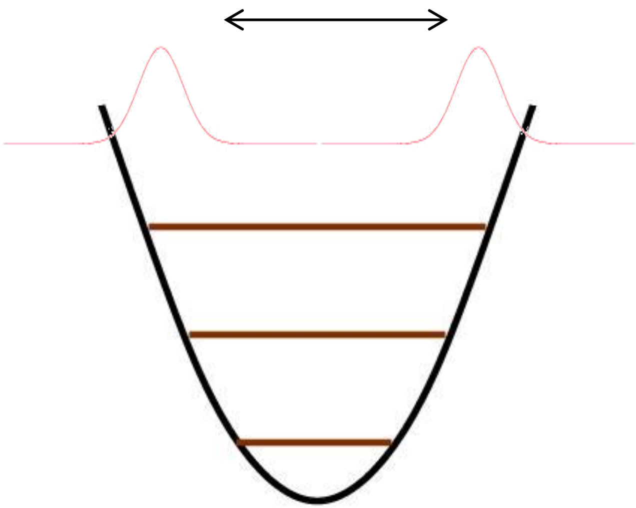
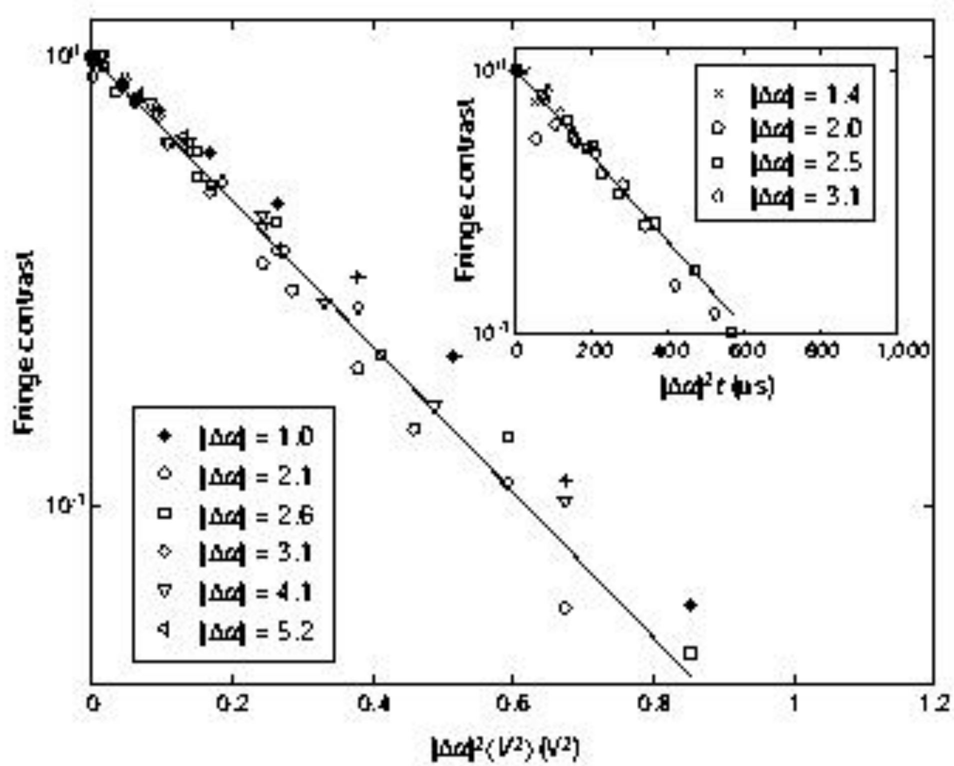
Boulder-experiment

Decoherence between *coherent states*:

$$\exp(-\gamma D^2 t)$$

$$D = (q_1 - q_2)$$





C. J. Myatt et. Al., Nature 403, 269 (2000).

„Caldeira-Leggett“ (without damping)

$$H_{\text{tot}} = p^2/2M + V(q) \;+\; q\sum_i g_i q_i \;+\; \sum_i (p_i^2/2m_i + \tfrac{1}{2} m_i \omega_i^2 q_i^2)$$

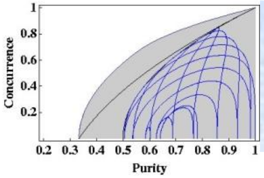
(„entanglement with environemt“
initial thermal bath state)

$$\dot{\rho} = -i[H_{\text{sys}}, \rho] - \Gamma[q,[q,\rho]]$$

(unitary, stochastic dynamics)

$$i\dot{\psi}_t = (H_{\text{sys}} + \sqrt{\Gamma} q \xi(t))\psi_t$$

$$\langle \xi(t)\xi(s) \rangle = \delta(t-s)$$



„Caldeira-Leggett“ without Markov

$$H_{\text{tot}} = p^2/2M + V(q) + q\sum_i g_i q_i + \sum_i (p_i^2/2m_i + \tfrac{1}{2} m_i \omega_i^2 q_i^2)$$

Bath correlation function

$$\alpha(t-s) = \langle \hat{F}(t)\hat{F}(s)\rangle = \int d\omega\, J(\omega)\left[\coth(\tfrac{\hbar\omega}{2kT})\cos(\omega t) - i\sin(\omega t)\right]$$

$$i\dot{\psi}_t = (H_{\text{sys}} + \sqrt{D}\, q\,\xi(t))\psi_t$$

$$\langle \xi(t)\xi(s)\rangle = \operatorname{Re}\alpha(t-s)$$

(neglect damping = imaginary part)

[see also T. Grotz, L. Heaney, and WTS, PRA 74, 22102 (2006)]

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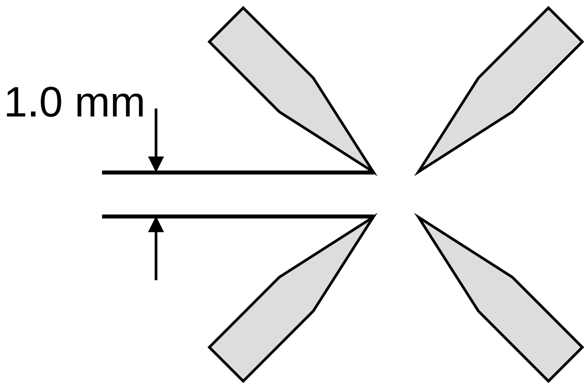
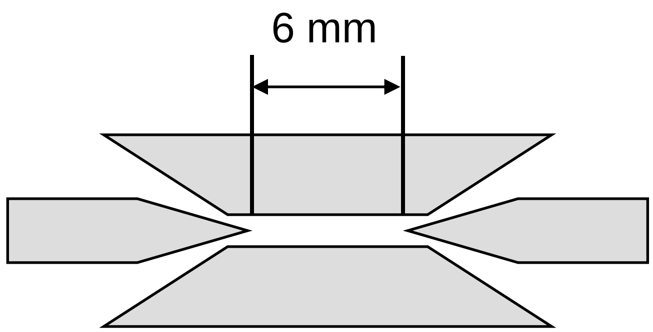
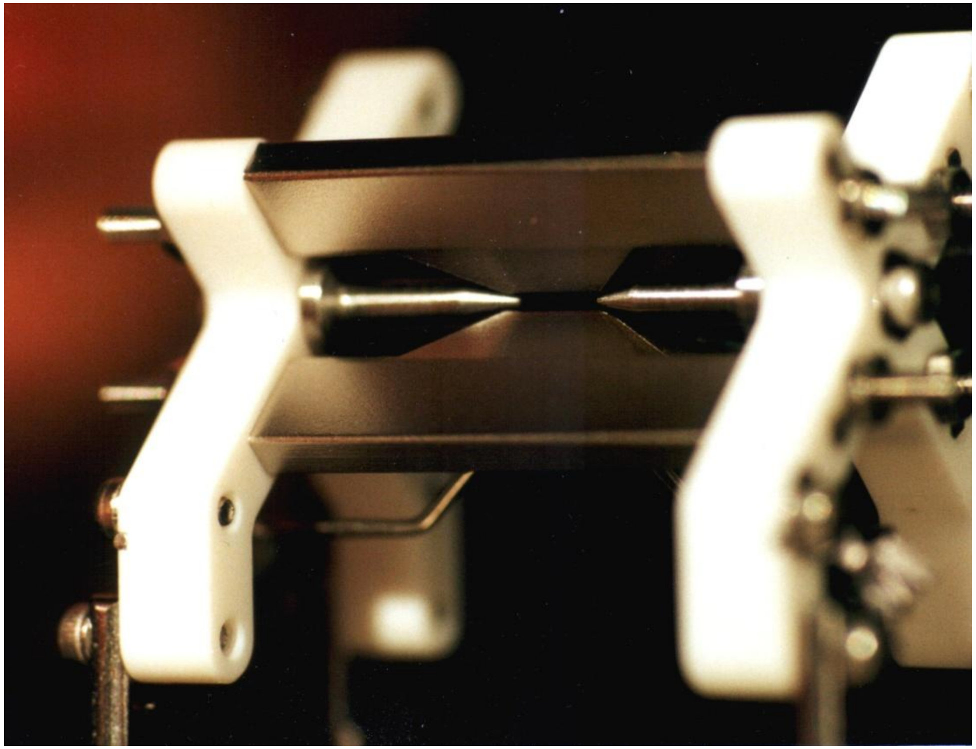
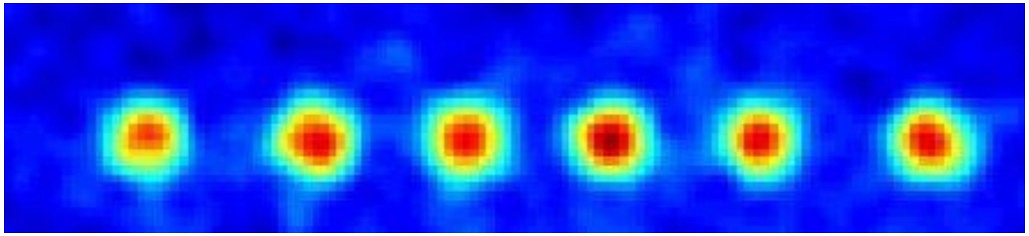
Innsbruck linear ion trap quantum computer

(Figs: (2007) Hartmut Häffner, Innsbruck)





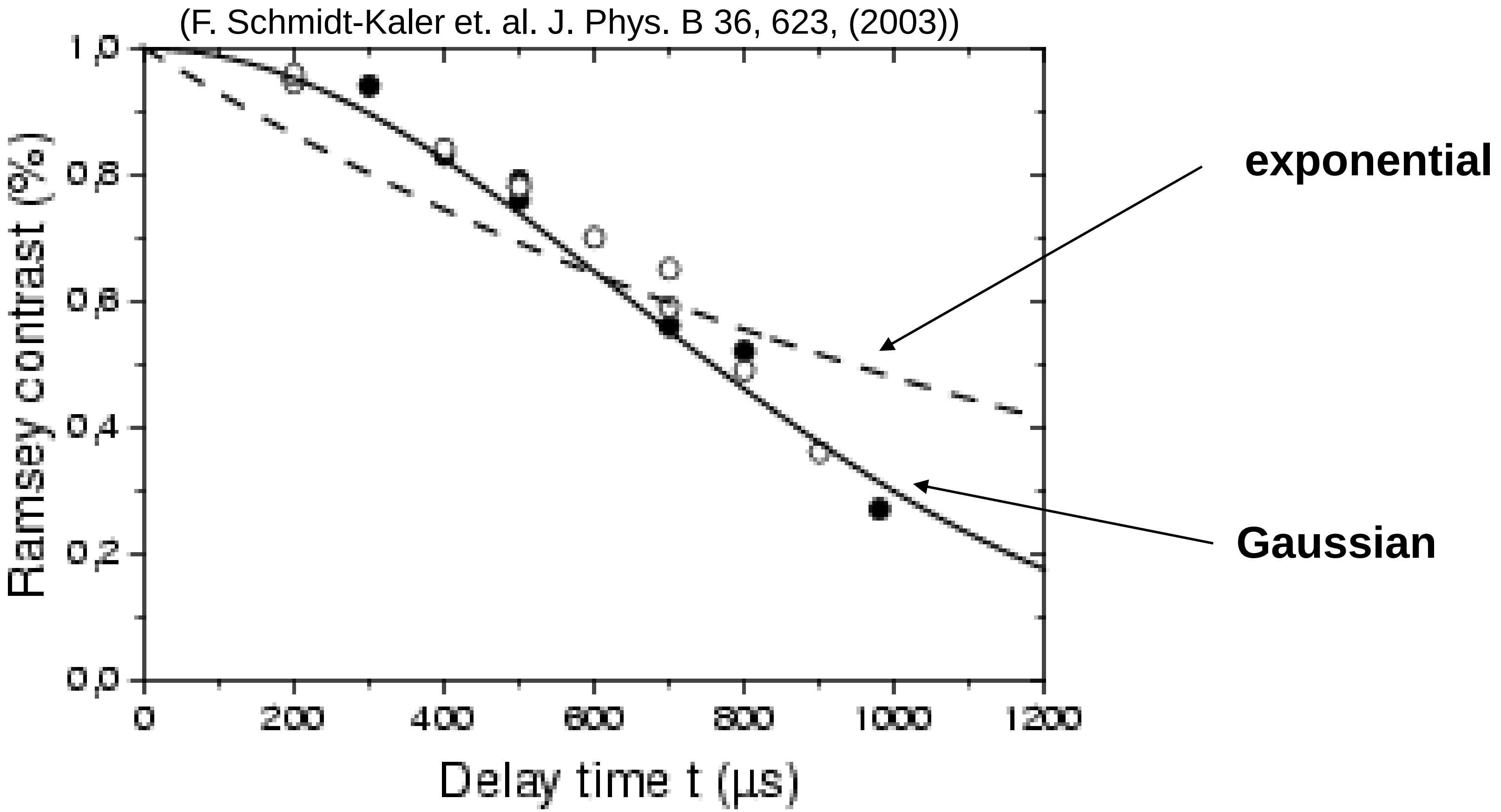




$$\omega_{x,y} \approx 1.5 - 5 \text{ MHz}$$

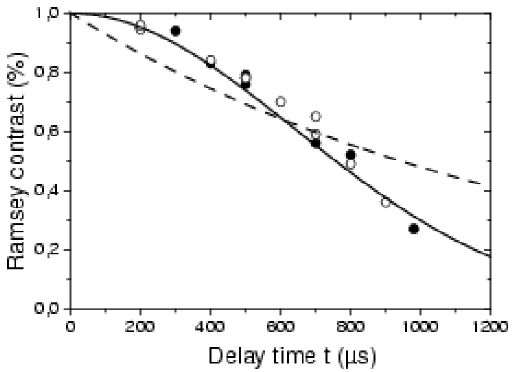
$$\omega_z \approx 0.7 - 2 \text{ MHz}$$

Decoherence of a single qubit

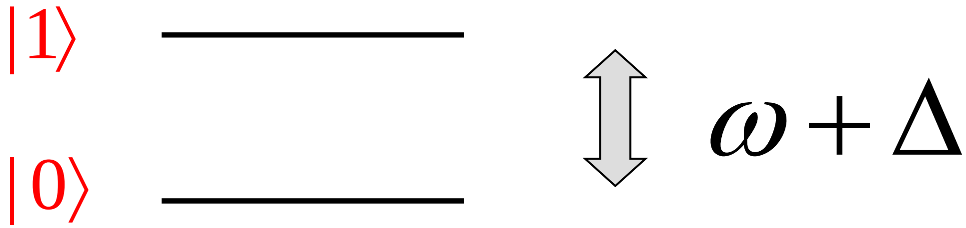


➡ Use general theoretical framework for (realistic) decoherence dynamics (channels)

Gaussian decoherence of single qubit („static disorder“)



(F. Schmidt-Kaler et. al. J. Phys. B 36, 623, (2003))



$$H = \frac{1}{2}(\omega+\Delta)\sigma_z$$

$$\rho(0) = \begin{pmatrix} p & c \\ c^* & 1-p \end{pmatrix}$$

$$\rho_\Delta(t) = \begin{pmatrix} p & ce^{-i(\omega+\Delta)t} \\ c^* e^{i(\omega+\Delta)t} & 1-p \end{pmatrix}$$

$$\bar{\rho}(t) = \begin{pmatrix} p & ce^{-\sigma^2 t^2} e^{-i\omega t} \\ c^* e^{-\sigma^2 t^2} e^{i\omega t} & 1-p \end{pmatrix}$$

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1 Qubit: „quantum“ = „random unitary“

1-Qubit decoherence:

$$\rho(t) = \begin{pmatrix} \rho_{00}(0) & c(t)\rho_{01}(0) \\ c^*(t)\rho_{10}(0) & \rho_{11}(0) \end{pmatrix}$$

$$U_\Omega = e^{-i\frac{\Omega(t)}{2}\sigma_z}$$

$\Omega(t)$ a stochastic variable (process)

$$\bar{\rho}(t) = \begin{pmatrix} \rho_{00}(0) & \langle e^{-i\Omega}\rangle \rho_{01}(0) \\ \langle e^{i\Omega}\rangle \rho_{10}(0) & \rho_{11}(0) \end{pmatrix}$$

In general: decoherence = unitary+stochastic?

May pure decoherence (phase damping) be described by
stochastic, unitary evolution ?

(no entanglement with „environment“ required)

Note: also relevant for quantum error correction:
If quantum environment, total pure state, yet RU-channel:
quantum information lost into the environment can be
completely restored: „Quantum lost and found“
M. Gregoratti and R. F. Werner, J. Mod. Opt. 50, 915 (2003)

In general: decoherence = unitary+stochastic?

May pure decoherence (phase damping) be described by
stochastic, unitary evolution ?

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„for many practical purposes“: yes

for a single qubit: yes

for two qubits (and more): in general, no

See: Landau and Streater, Linear Algebra Appl. 193: 107-127 (1993)

„pure decoherence“=„phase damping“ channels

$$\rho(t) = \sum_{j \le M} K_j(t)\rho(0)K_j^+(t)$$ and $$\langle n \,|\, \rho(t) \,|\, n\rangle = \text{const}$$

$$\Rightarrow \boxed{\rho_{\mathrm{ij}}(t) = \langle \vec{a}_{\mathrm{i}}(t), \vec{a}_{\mathrm{j}}(t)\rangle \cdot \rho_{\mathrm{ij}}(0)}$$

with $\vec{a}_{\mathrm{i}} = (a_{1\mathrm{i}}, a_{2\mathrm{i}}, \dots, a_{\mathrm{di}})$ any set of normalized complex vectors

Note: $tr\,\rho(t) = 1$ and $\mathrm{id} \rightarrow \mathrm{id}$ (unital)

Phase damping channels ⟺ *Bistochastic (unital), diagonal q.o.*

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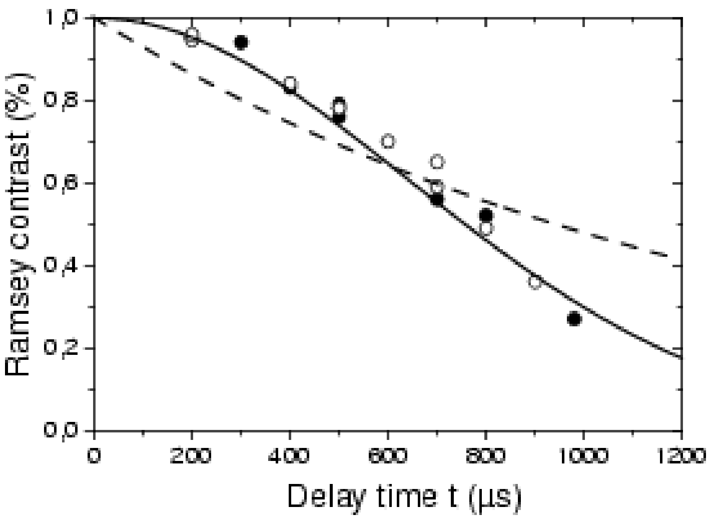
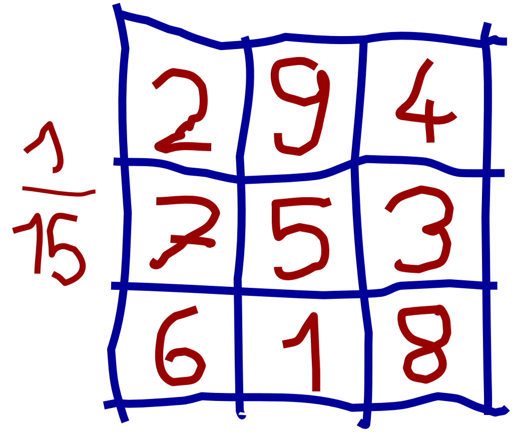
„Birkhoff‘s theorem“

Phase damping channels ⟺ *Bistochastic (unital), diagonal q.o.*



Physics



Mathematics

Birkhoff: characterize extremal *bistochastic* maps of probability distributions

Quantum case see: Landau and Streater, Linear Algebra Appl. 193: 107-127 (1993)

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for two qubits (and more): in general, no: Example??

See: Landau and Streater, Linear Algebra Appl. 193: 107-127 (1993)

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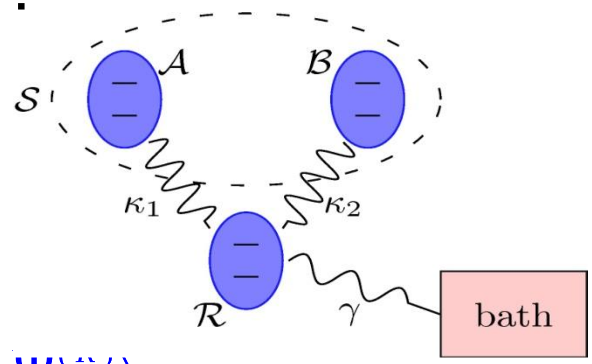
„true quantum“ decoherence of two qubits

2 qubits coupled to 1 „environmental qubit“:



$$H_{\text{tot}} = \kappa_A \sigma_z^{(A)} \sigma_z^{(R)} + \kappa_B \sigma_z^{(B)} \sigma_z^{(R)} + \vec{\Gamma} \cdot \vec{\sigma}^{(R)}$$

$$= \sum_{j=1..4} | j \rangle\langle j | \otimes H_j^{(R)}$$

Four environmental qubit-states $|\Psi_j^{(R)}\rangle = \exp(-iH_j^{(R)} t)|\Psi_0^{(R)}\rangle$

with corresponding Bloch-vector $\vec{r}_j$

The 2-qubit dephasing channel **is RU**, iff the four Bloch vectors point to a plane.

see Julius‘ **poster**!

[and J. Helm, WTS, PRA 80, 042108 (2009), PRA 81, 042314 (2010)]

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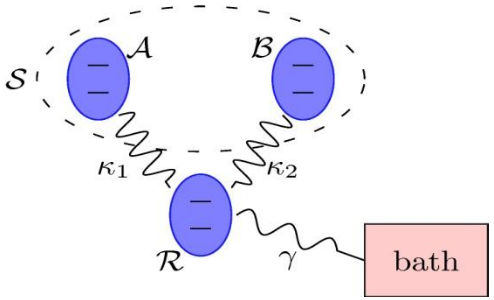

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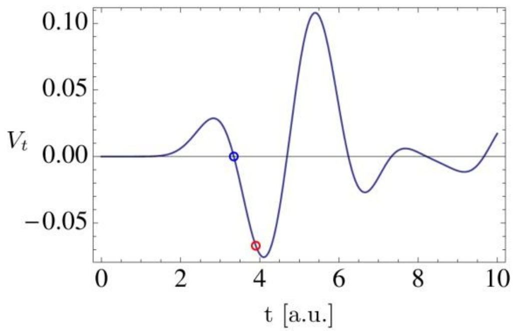
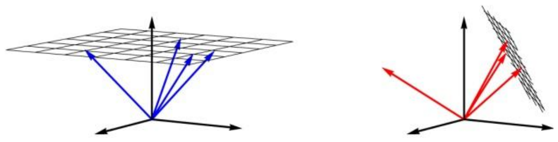


(a)

determine Volume V of tetrahedron:
measure for being „non-RU“?

Compare with numerical determination
of distance from the convex set of RU channels!
(see Julius‘ poster)


(b) (c)

[J. Helm, WTS, PRA 80, 042108 (2009), PRA 81, 042314 (2010)]

„true quantum“ decoherence:

open question (for me):

Given pure docoherence dynamics:

$$\rho_{ij}(t) = D_{ij}(t)\rho_{ij}(0)$$

(from process tomography, for instance)

Is there a „simple“ criterion that tells us whether D is random unitary?

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Conclusions

(1) in practice (from a local point of view), pure decoherence is often of random unitary (RU) type :

discussion of robust states etc. requires “total system point of view”

(2) for a single qubit: Q=RU

(3) in general Q is larger: we construct a simple non-RU decoherence channel for two qubits

(4) Examples of quantum and classical decoherence: cavity QED, molecular dynamics, ion traps

(5) Open question: simple criterion to decide whether channel is RU

(also relevant for “quantum lost and found”)