Quantum Darwinism in practice

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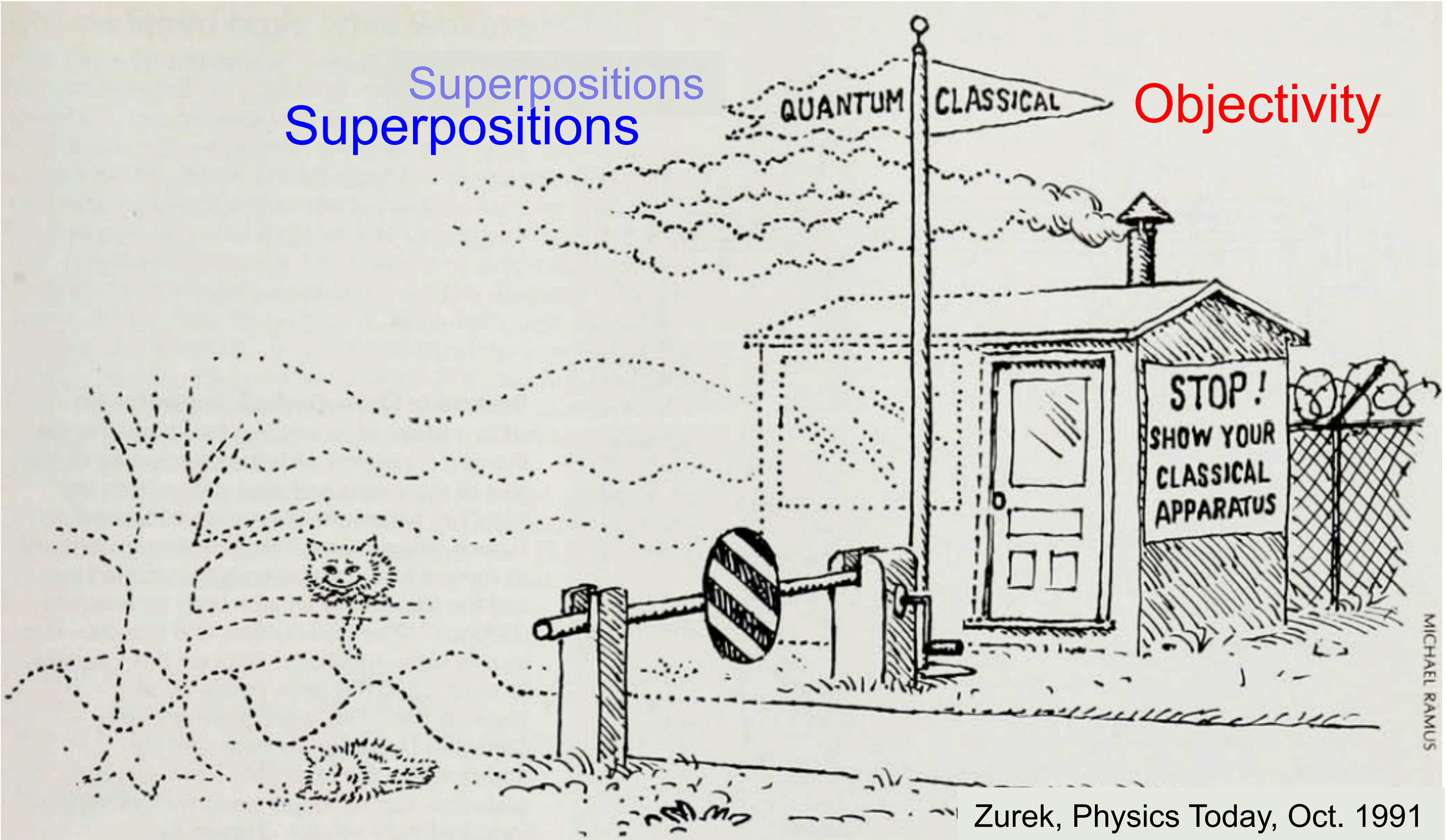
Quantum vs. Classical

How does the classical world arise from the quantum substrate?

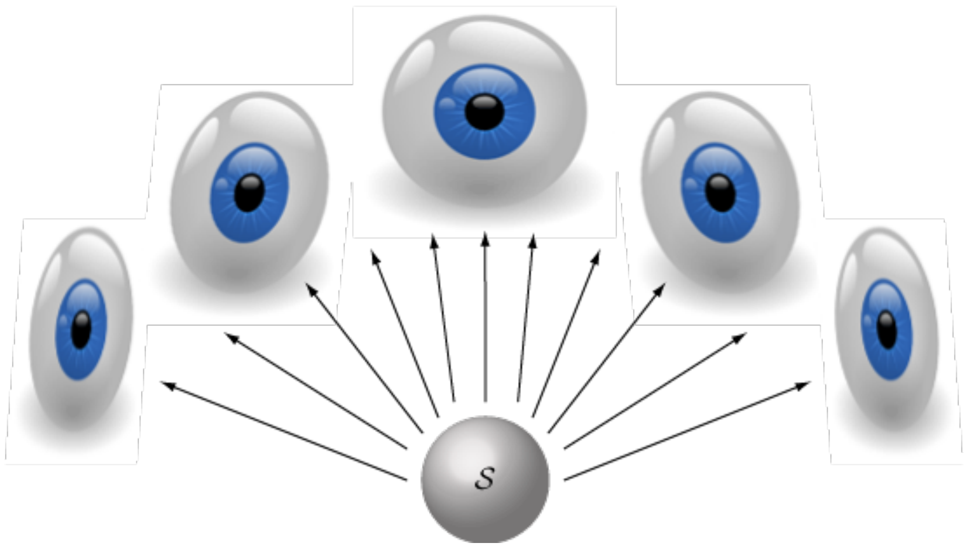


Zurek, Physics Today, Oct. 1991

Quantum Darwinism

Q-Darwinism recognizes the environment's role as a communication channel

- Many observers independently gather information about a system indirectly by intercepting some small fragment of the environment
- Redundant information in the environment gives rise to objectivity



Quantum Darwinism

Q-Darwinism recognizes the environment's role as a communication channel

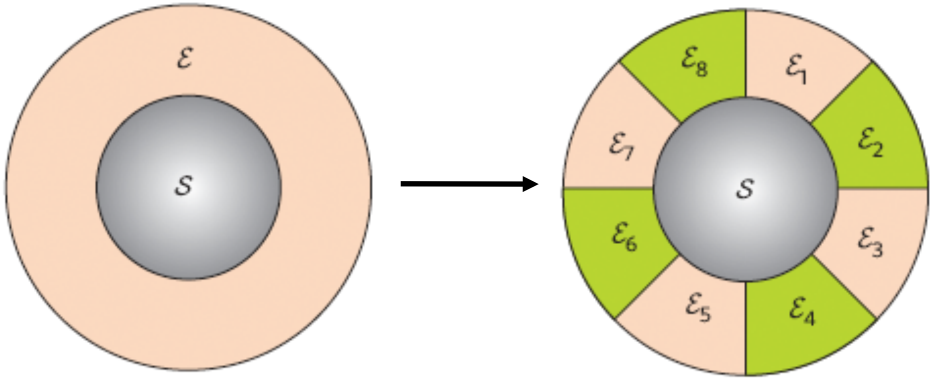
- Many observers independently gather information about a system indirectly by intercepting some small fragment of the environment
- Redundant information in the environment gives rise to objectivity

Why Darwinism?

- Classicality is the result of a selection of preferred states – the pointer states - and the redundant proliferation of information
- Fitness = ability of a state to survive and "procreate" (i.e., spawn the most information theoretic progeny)

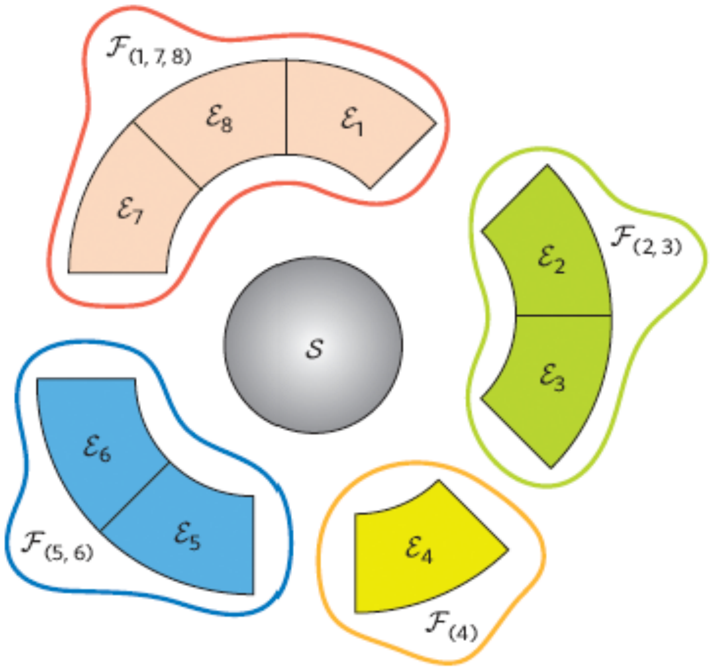
Decoherence vs. Q-Darwinism

Decoherence Paradigm



Redundancy Paradigm

What fragment size gives “complete” information about the system?



Under decoherence many fragments give complete information

Ollivier et al.,
PRL 93, 220401 (2004)
Blume-Kohout, Zurek,
PRA 73, 062310 (2006)

Correlated qubits

$$\left(|0\rangle_{\mathcal{S}} + |1\rangle_{\mathcal{S}}\right) |\overset{\mathcal{E}_1}{0}\overset{\mathcal{E}_2}{0}\rangle_{\mathcal{E}}/\sqrt{2}$$

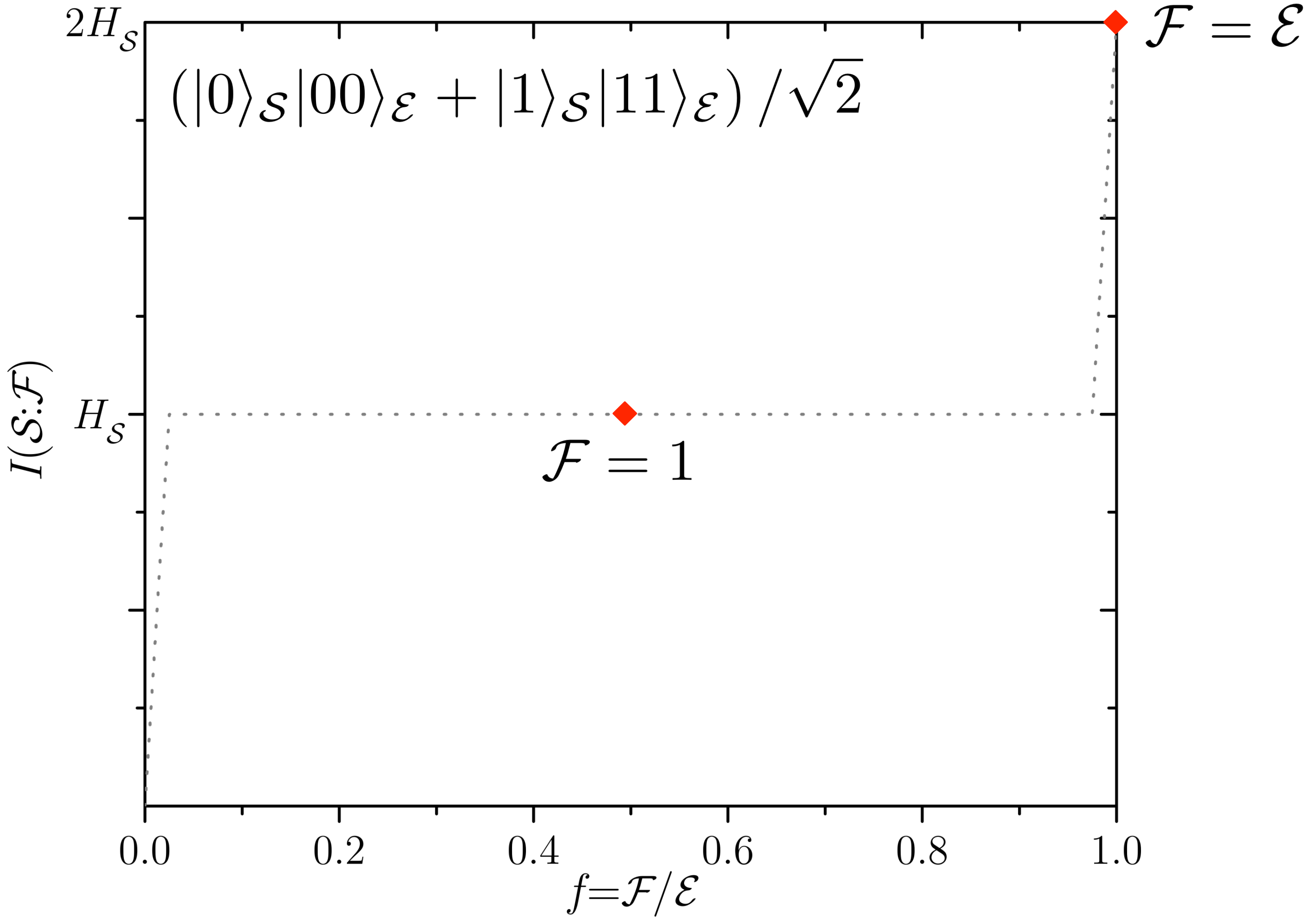
$$\xrightarrow[\text{CNOT}]{} \left(|0\rangle_{\mathcal{S}}|00\rangle_{\mathcal{E}} + |1\rangle_{\mathcal{S}}|11\rangle_{\mathcal{E}}\right)/\sqrt{2}$$

$$\xrightarrow[\mathrm{tr}_{\mathcal{E}/\mathcal{F}}]{\mathcal{F} = \mathcal{E}_1 \,\mathrm{or}\, \mathcal{E}_2} \rho_{\mathcal{S}\mathcal{F}} = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$$

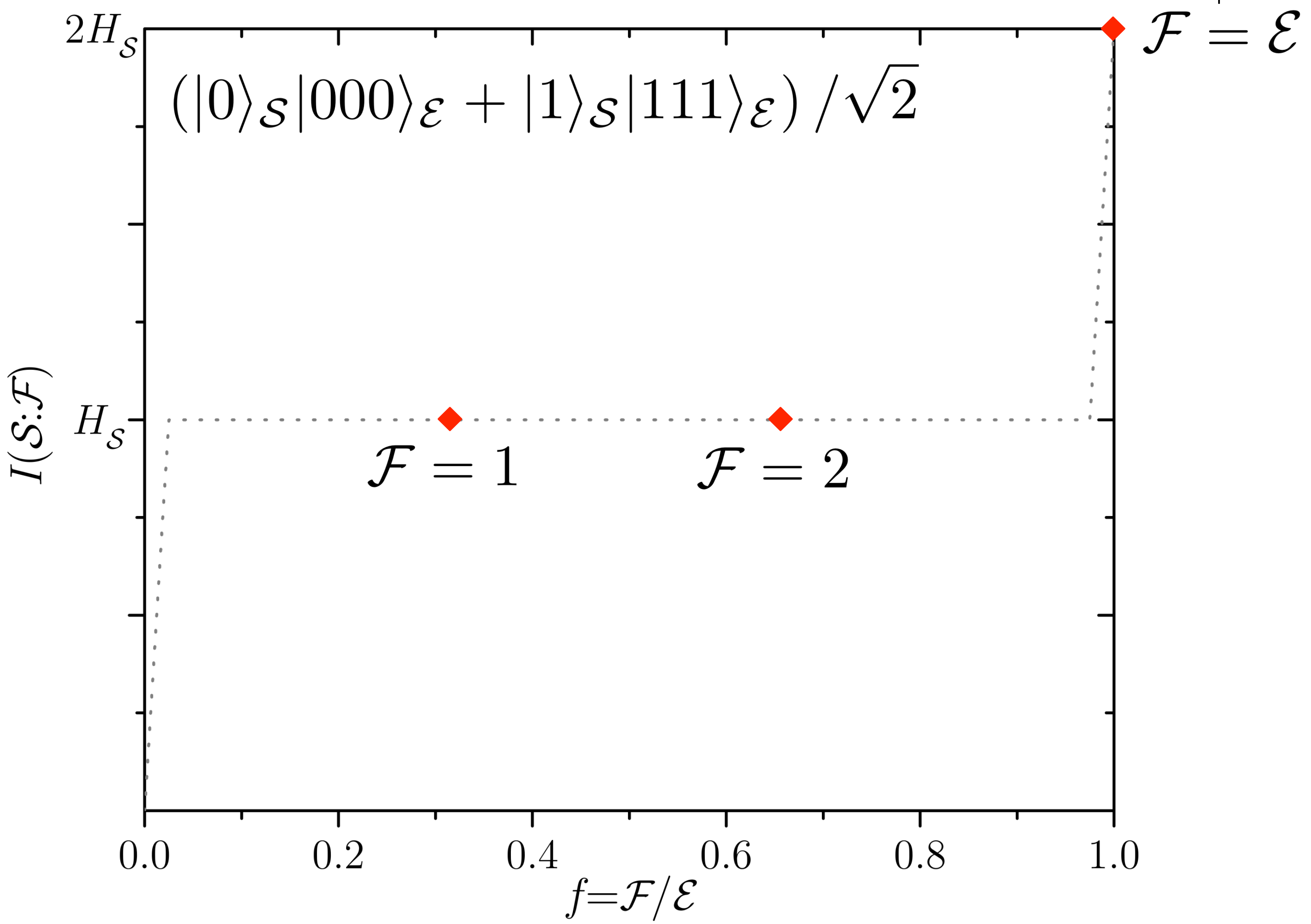
$\mathcal{S}$, $\mathcal{F}$ are classically correlated:

$$p_{0,0} = \frac{1}{2};\ p_{1,1} = \frac{1}{2}$$

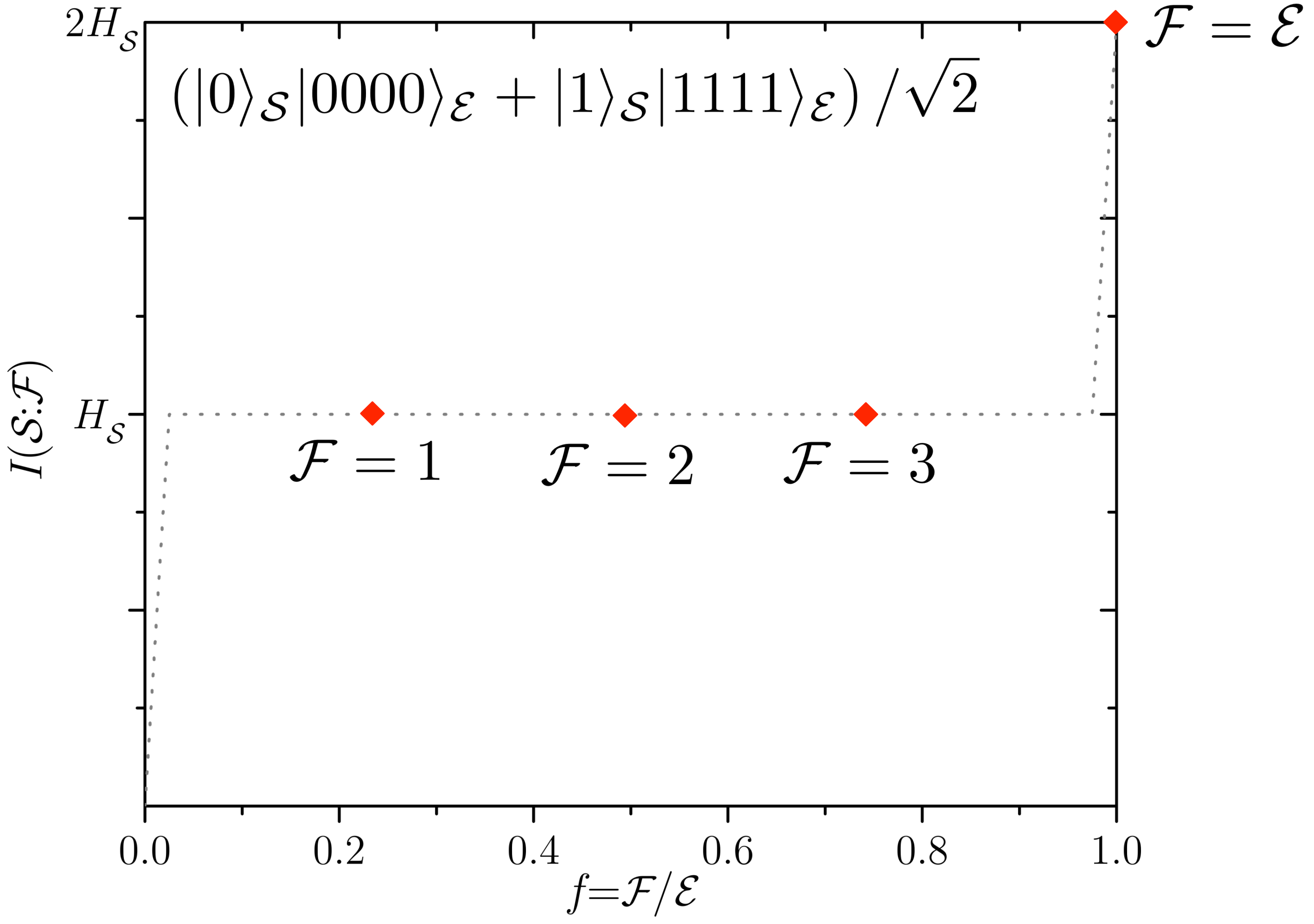
Information plot



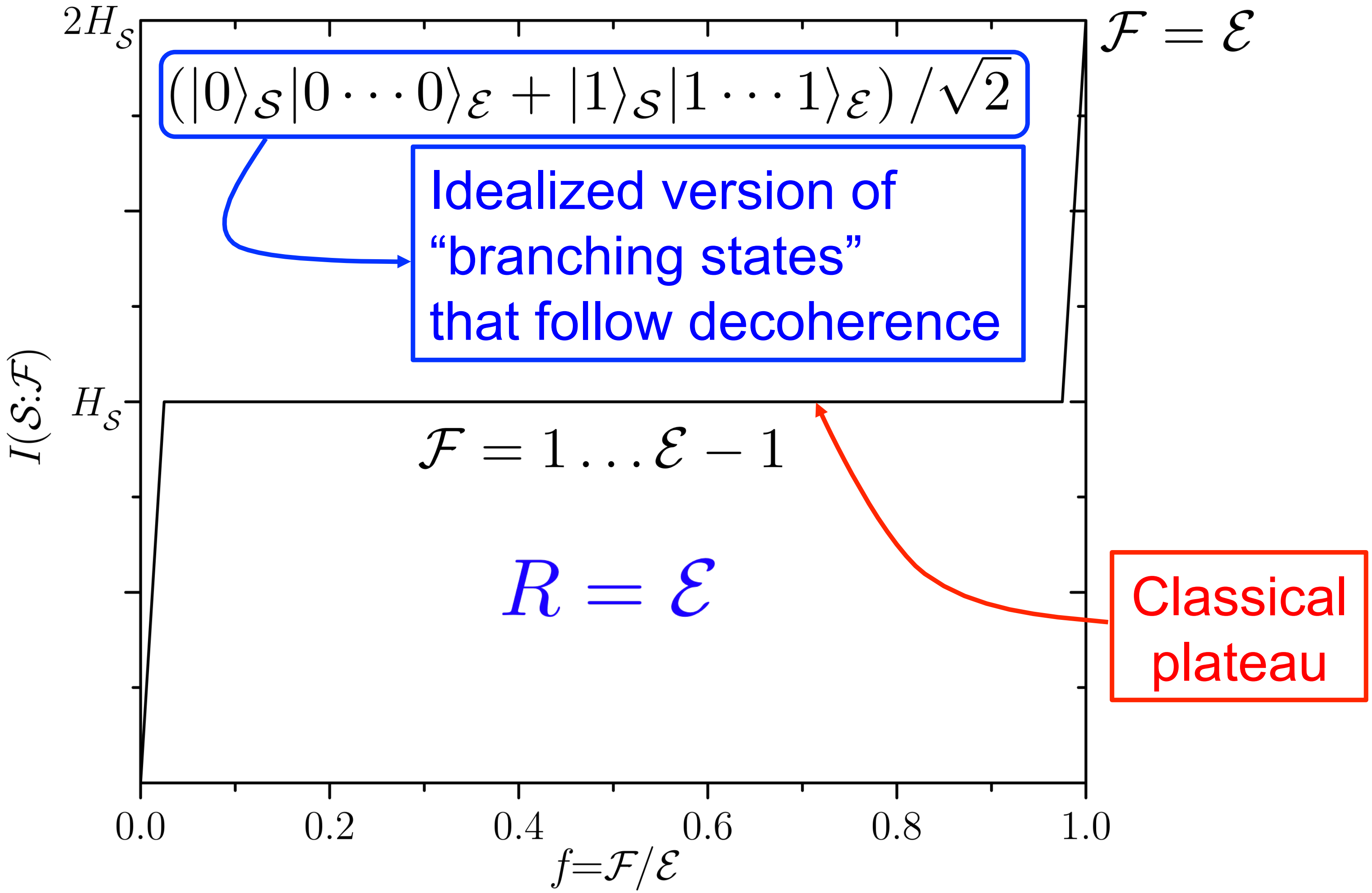
Information plot



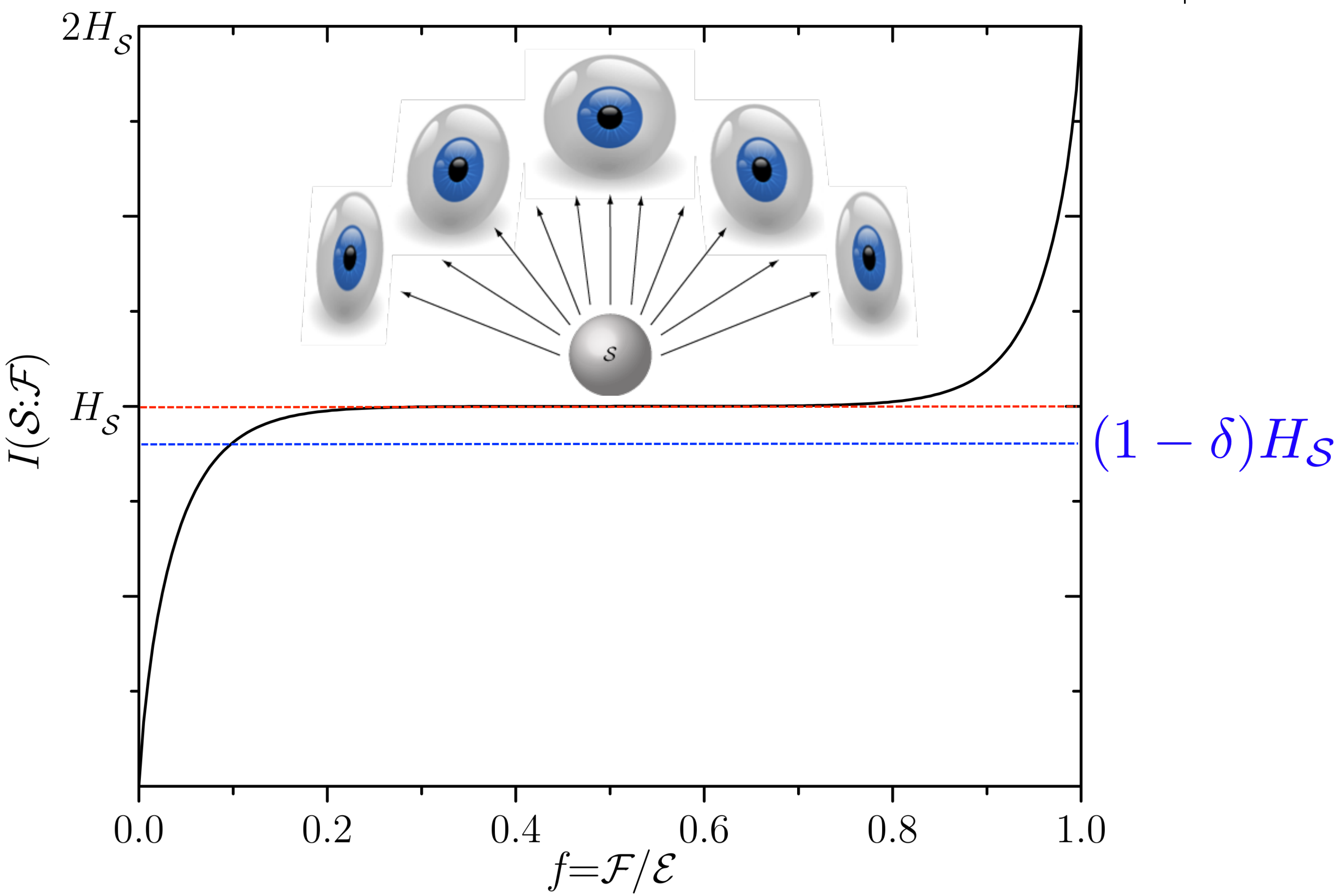
Information plot



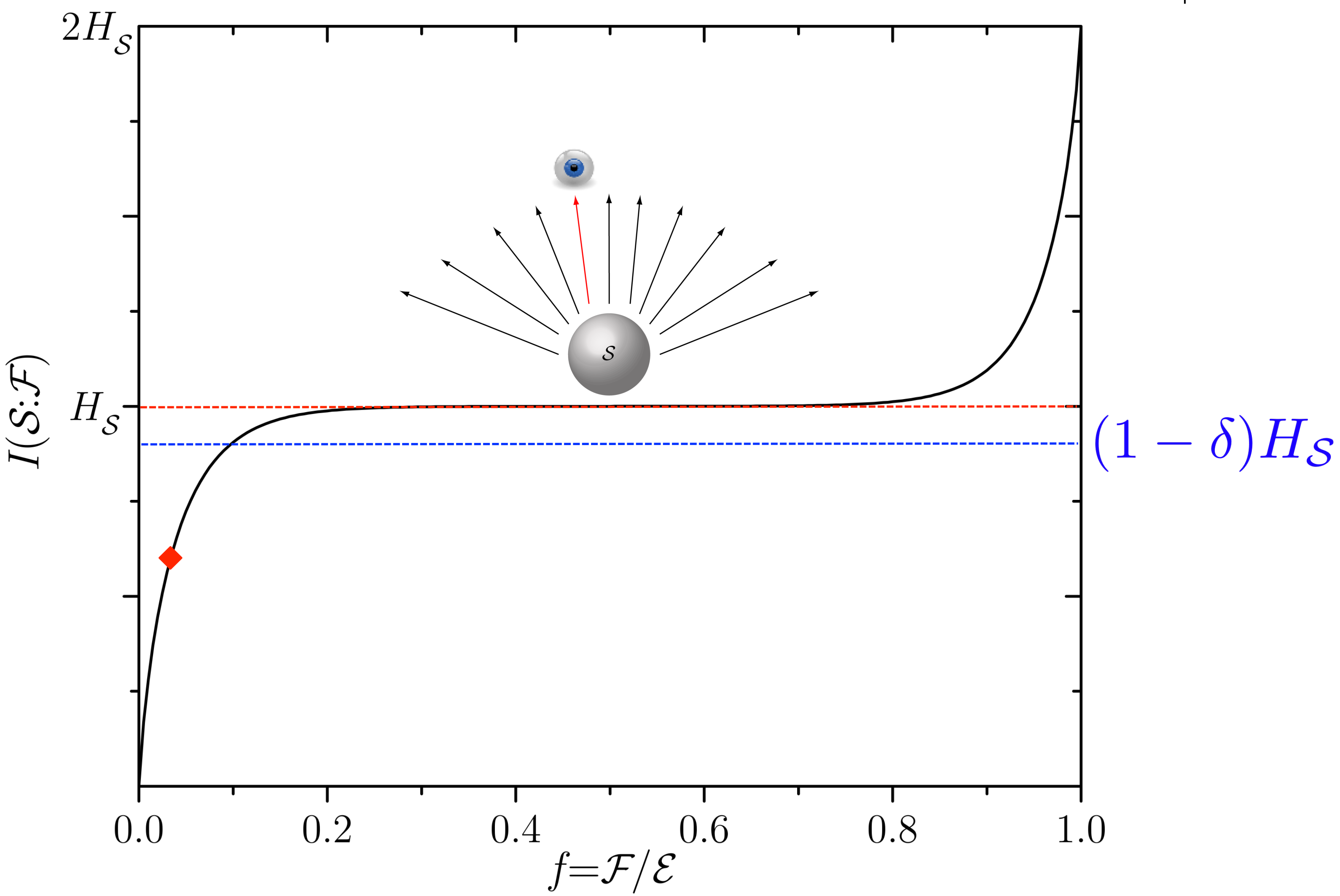
Information plot



Information plot

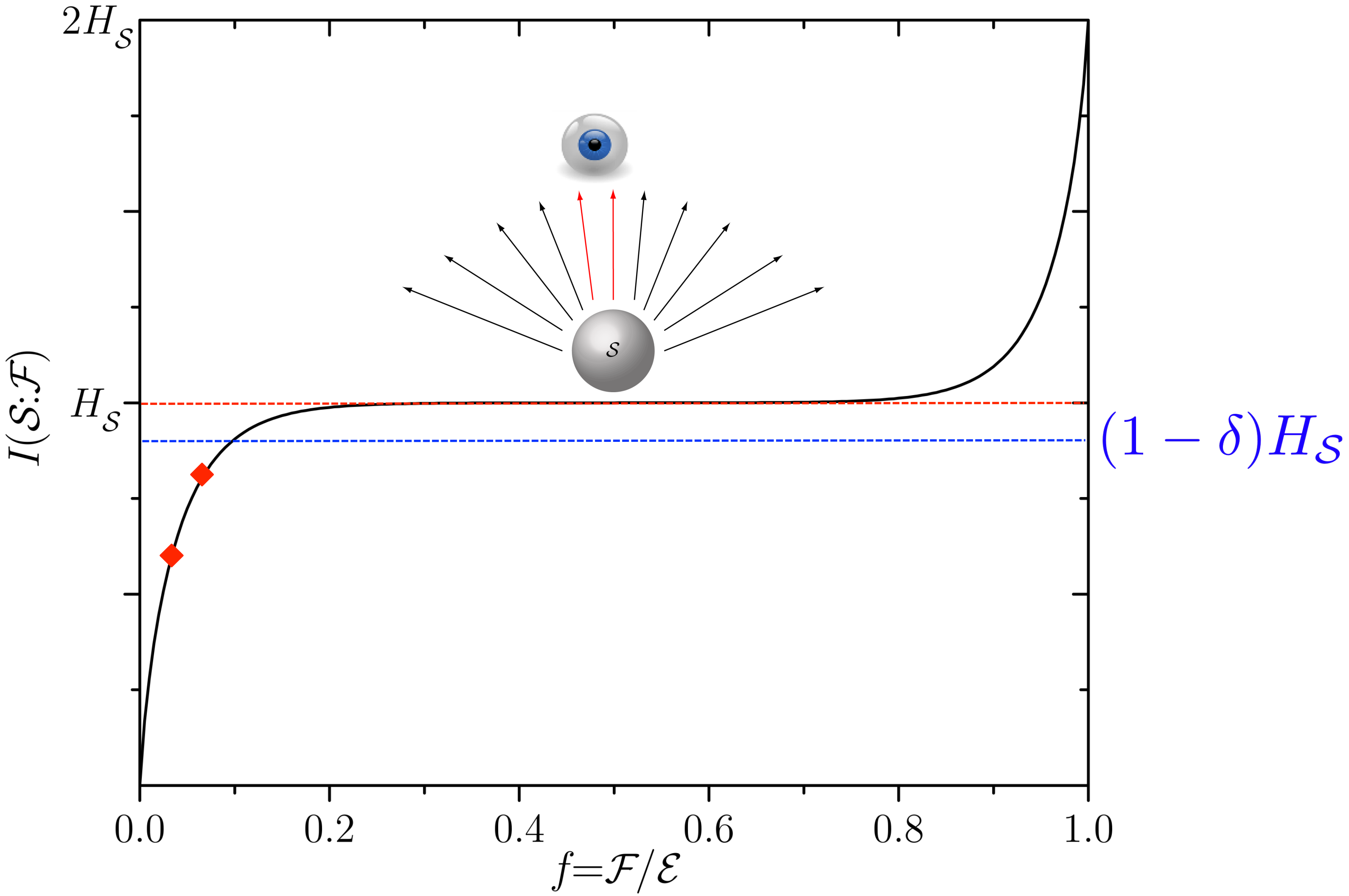


Redundancy



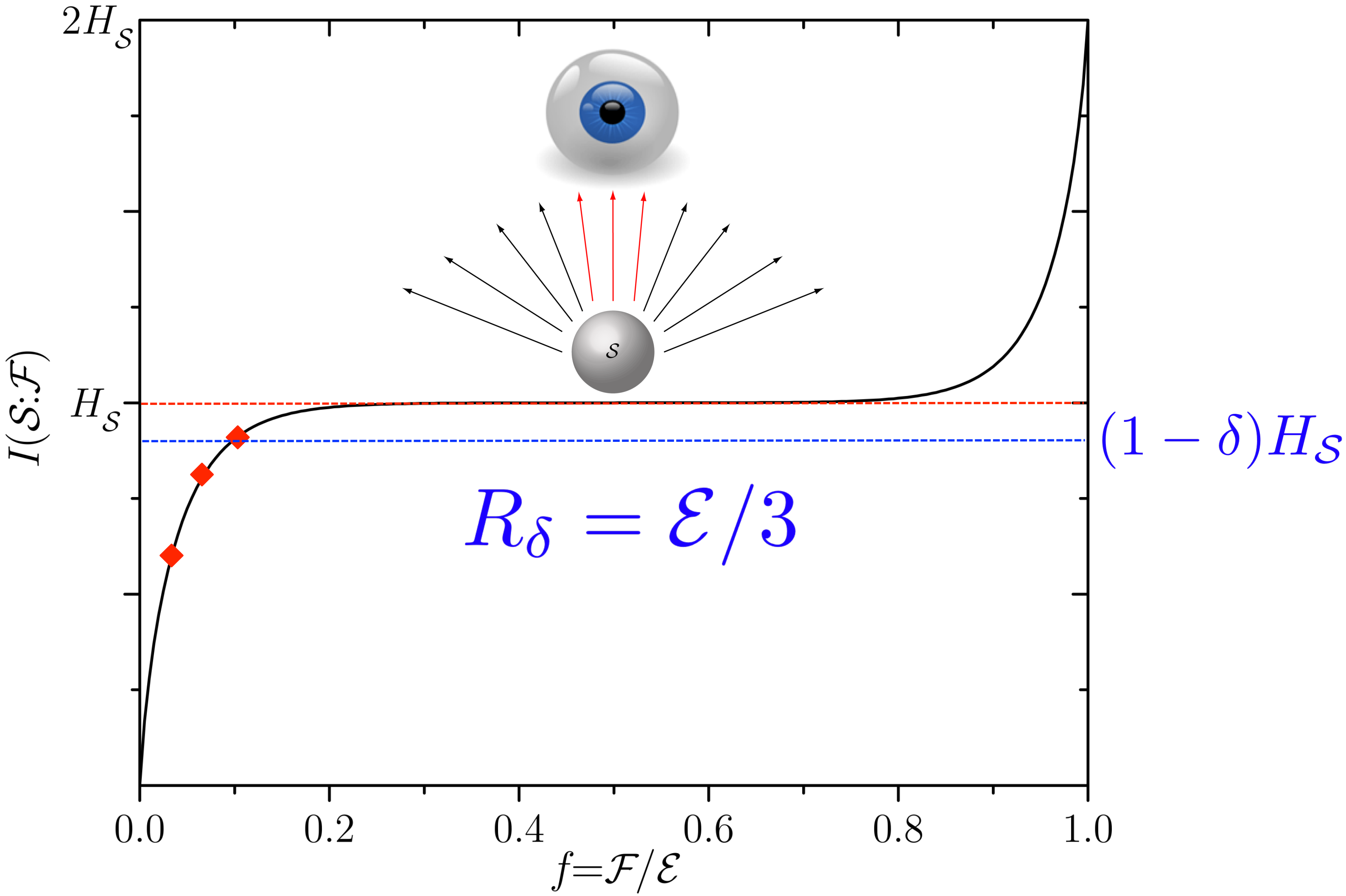
$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
$I(\mathcal{S}{:}\mathcal{F})$
$\mathcal{S}$
$(1-\delta)H_{\mathcal{S}}$
0.0
0.2
0.4
0.6
0.8
1.0
$f{=}\mathcal{F}/\mathcal{E}$

Redundancy



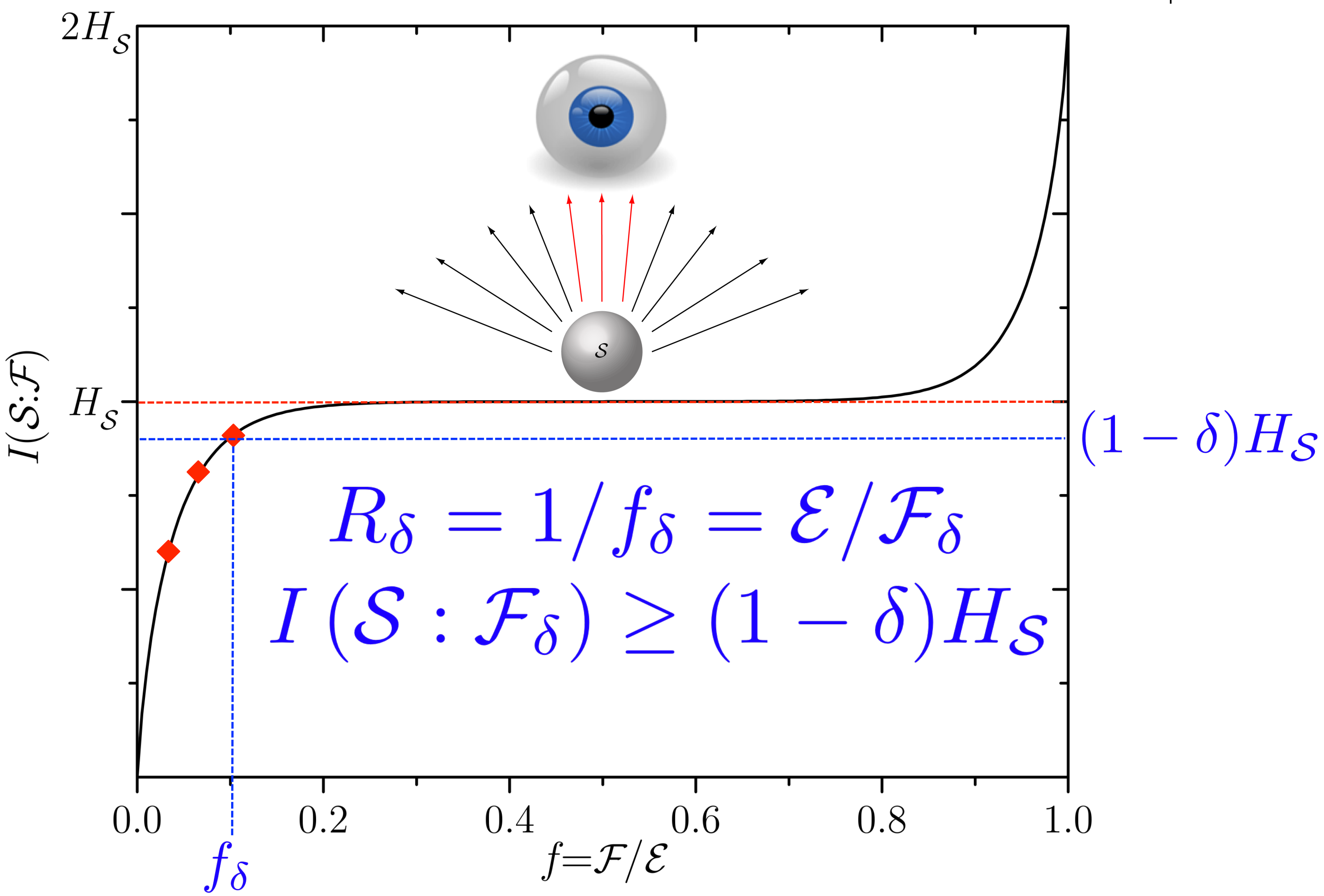
$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
$I(\mathcal{S}{:}\mathcal{F})$
$\mathcal{S}$
$(1-\delta)H_{\mathcal{S}}$
0.0
0.2
0.4
0.6
0.8
1.0
$f{=}\mathcal{F}/\mathcal{E}$

Redundancy



$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
$I(\mathcal{S}:\mathcal{F})$
$\mathcal{S}$
$(1-\delta)H_{\mathcal{S}}$
$R_\delta = \mathcal{E}/3$
0.0
0.2
0.4
0.6
0.8
1.0
$f = \mathcal{F}/\mathcal{E}$

Redundancy



$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
$I(\mathcal{S}{:}\mathcal{F})$
$\mathcal{S}$
$(1-\delta)H_{\mathcal{S}}$
$R_\delta = 1/f_\delta = \mathcal{E}/\mathcal{F}_\delta$
$I\left(\mathcal{S}:\mathcal{F}_\delta\right) \geq (1-\delta)H_{\mathcal{S}}$
0.0
0.2
0.4
0.6
0.8
1.0
f_δ
$f{=}\mathcal{F}/\mathcal{E}$

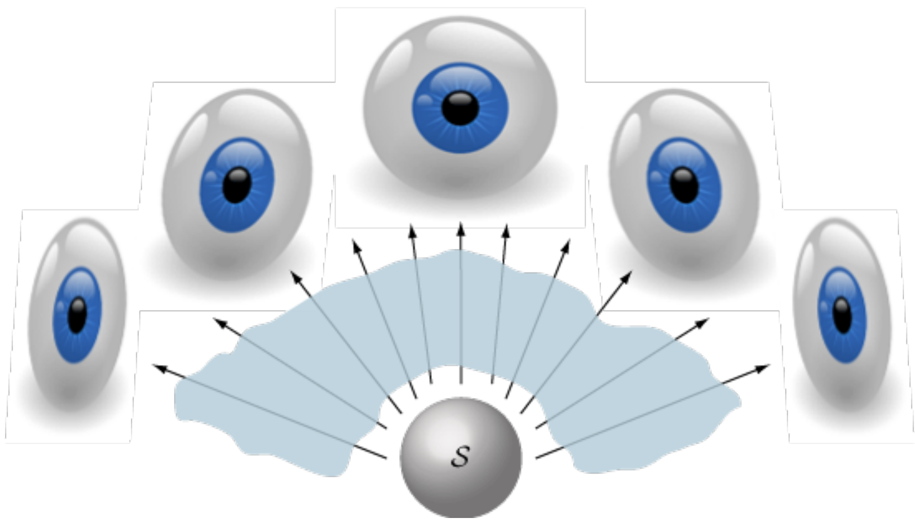
Quantum Darwinism

Environment as a communication channel

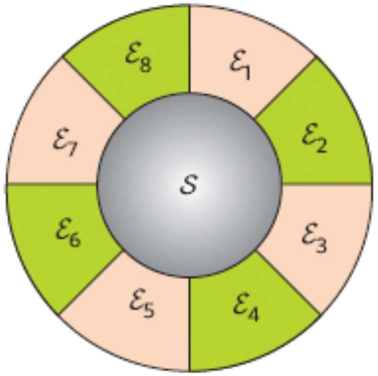
- Preferred states that survive and redundantly proliferate information into the environment
- Observers can independently determine the state of the system and reach consensus, aka, classicality

Q-Darwinism in practice

1. Example system in non-ideal environments
2. State distinguishability and Decohering interactions



1. Example: symmetric spin $\mathcal{E}$



$$\mathbf{H}_{\mathcal{S}\mathcal{E}} = \frac{1}{2}\sigma_{\mathcal{S}}^z \sum_{k=1}^{\mathcal{E}} \sigma_k^z$$

$$\rho(0) = \rho_{\mathcal{S}}(0) \otimes \rho_r^{\otimes \mathcal{E}}$$

Purely decohering Hamiltonian with independent environment components

Decoheres in pointer basis

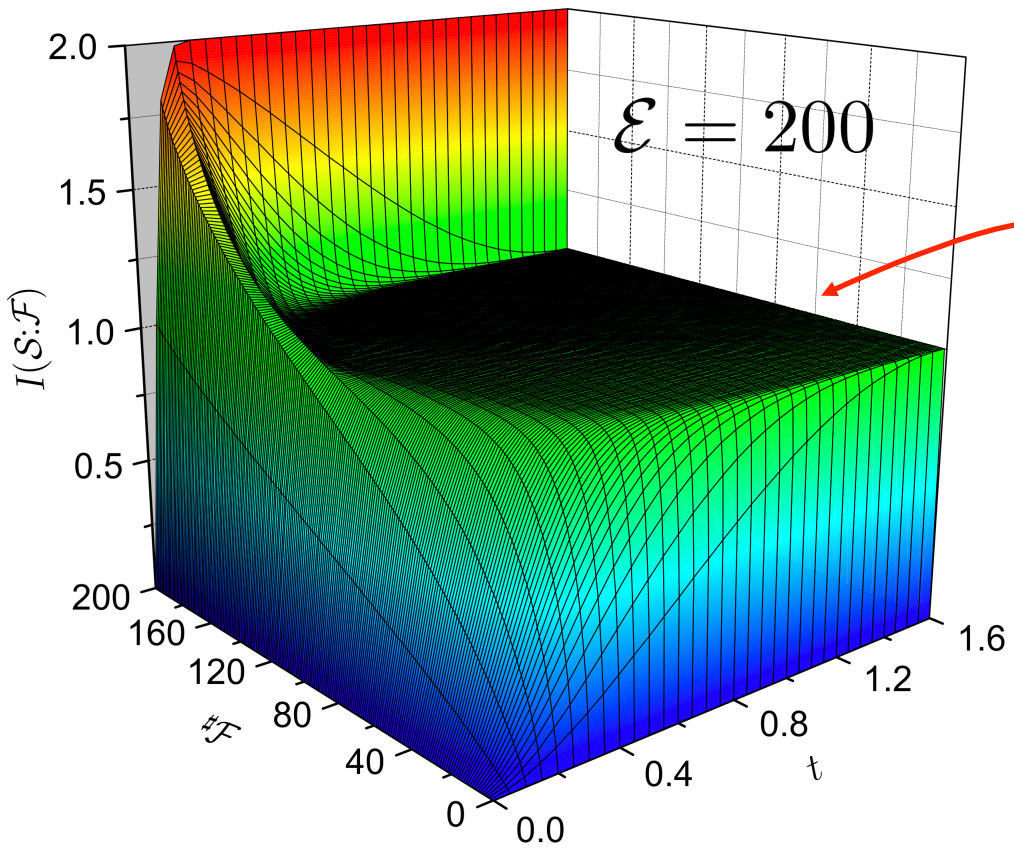
$$\rho_{\mathcal{S}}(t) = \begin{pmatrix} s_{00} & s_{01}\Lambda_{\mathcal{E}}(t) \\ s_{10}\Lambda_{\mathcal{E}}^{\star}(t) & s_{11} \end{pmatrix} \longrightarrow \rho_{\mathcal{S}} = \begin{pmatrix} s_{00} & 0 \\ 0 & s_{11} \end{pmatrix}$$

$$\Lambda_{\mathcal{E}}(t) = [\Lambda_k(t)]^{\mathcal{E}} = [\cos(t) - \imath\sigma \sin(t)]^{\mathcal{E}}$$

$$\sigma = r_{00} - r_{11} \quad \rho_r = \begin{pmatrix} r_{00} & r_{01} \\ r_{10} & r_{11} \end{pmatrix}$$

Mutual Information

Information flow into the environment



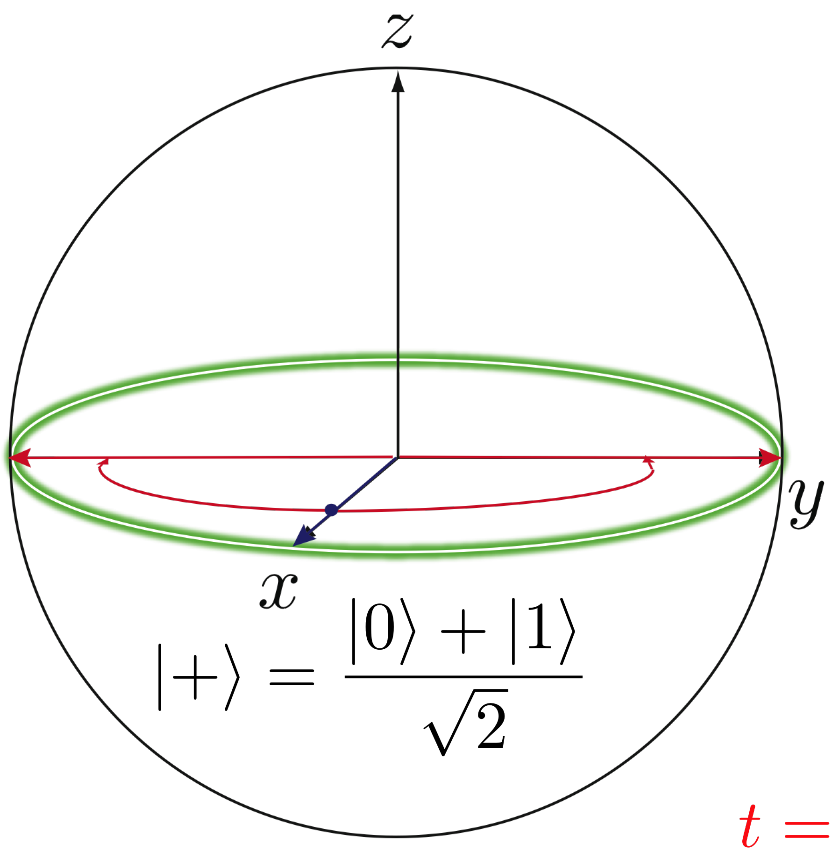
$$|\psi(0)\rangle = |+\rangle \otimes |+\rangle^{\otimes \mathcal{E}}$$

Classical plateau

$$t \to \frac{\pi}{2},\ R_\delta \to \mathcal{E}$$

Single environment qubit

$$|\psi(0)\rangle = |+\rangle|+\rangle^{\otimes\mathcal{E}} \rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}|0\rangle|\tilde{0}(t)\rangle^{\otimes\mathcal{E}} + \frac{1}{\sqrt{2}}|1\rangle|\tilde{1}(t)\rangle^{\otimes\mathcal{E}}$$



$$|\tilde{1}\rangle = \frac{|0\rangle + e^{-\imath t}|1\rangle}{\sqrt{2}}$$

$$|\tilde{0}\rangle = \frac{|0\rangle + e^{\imath t}|1\rangle}{\sqrt{2}}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$t = \frac{\pi}{2} \rightarrow |\langle\tilde{1}|\tilde{0}\rangle|^2 = 0$$

Non-ideal $\mathcal{E}$

- Another extreme:

$$\rho(0) = \rho_{\mathcal{S}}(0) \otimes \bar{\mathrm{I}}^{\otimes \mathcal{E}}$$

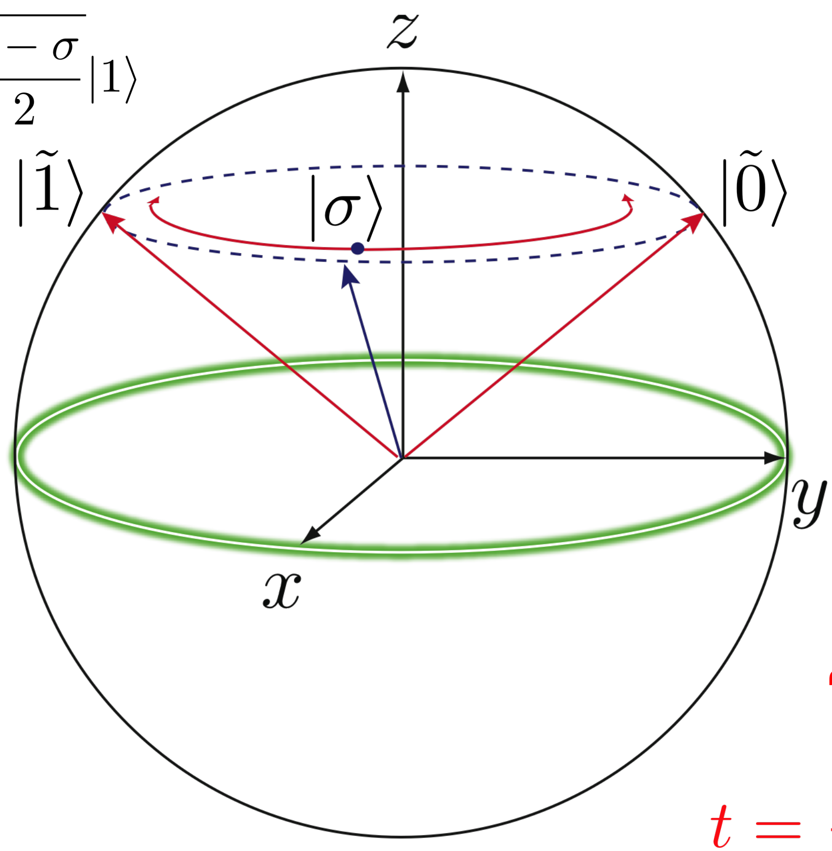
$$\longrightarrow I(\mathcal{S} : \mathcal{F}) = 0$$

- How does this picture change when starting with different environment states?

Single environment qubit

$$|\psi(0)\rangle = |+\rangle|\sigma\rangle^{\otimes\mathcal{E}} \rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}|0\rangle|\tilde{0}(t)\rangle^{\otimes\mathcal{E}} + \frac{1}{\sqrt{2}}|1\rangle|\tilde{1}(t)\rangle^{\otimes\mathcal{E}}$$

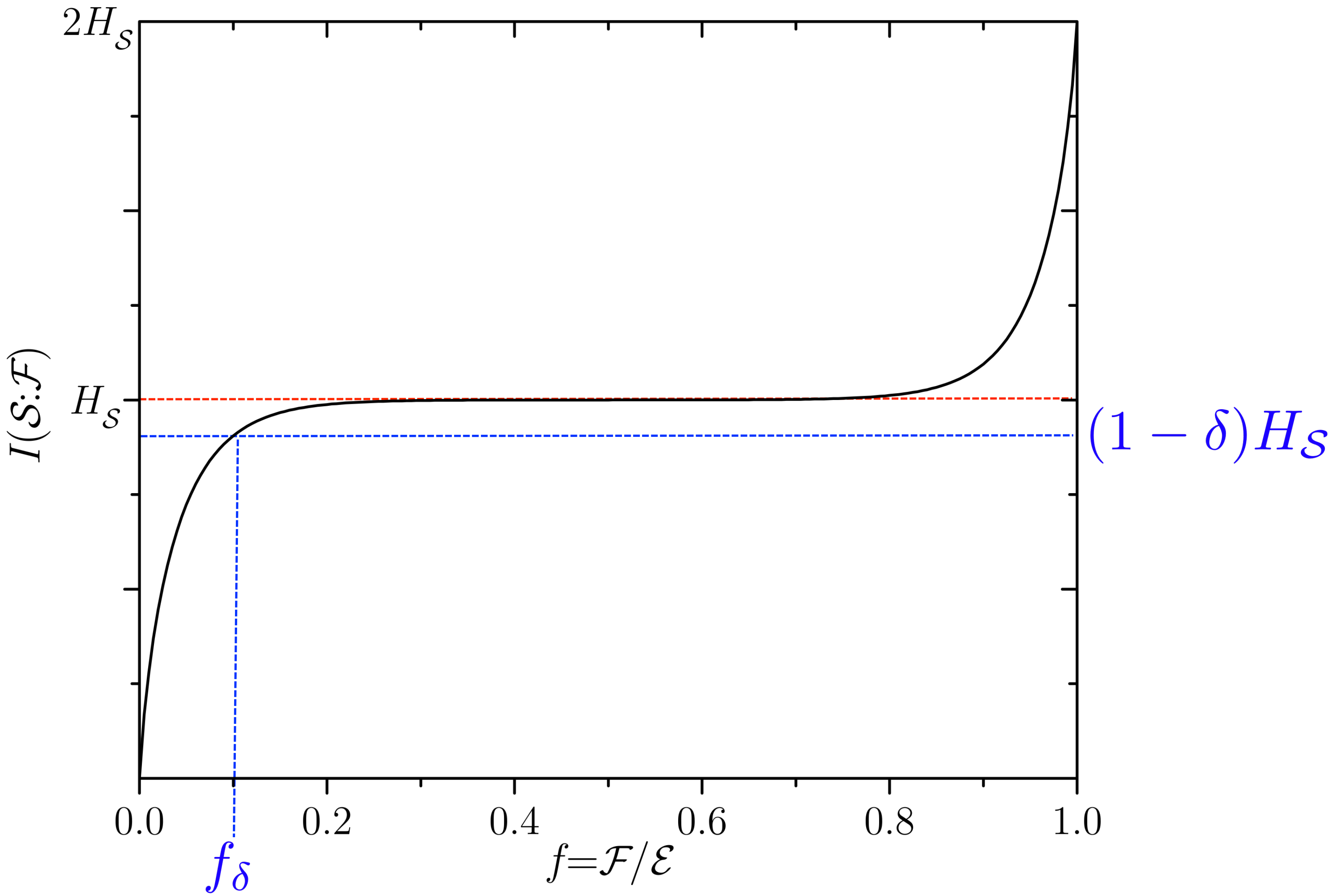
$$|\sigma\rangle = \sqrt{\frac{1+\sigma}{2}}|0\rangle + \sqrt{\frac{1-\sigma}{2}}|1\rangle$$



"misalignment"

$$t = \frac{\pi}{2} \rightarrow |\langle\tilde{1}|\tilde{0}\rangle|^2 = \sigma^2$$

Redundancy



$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
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$(1-\delta)H_{\mathcal{S}}$
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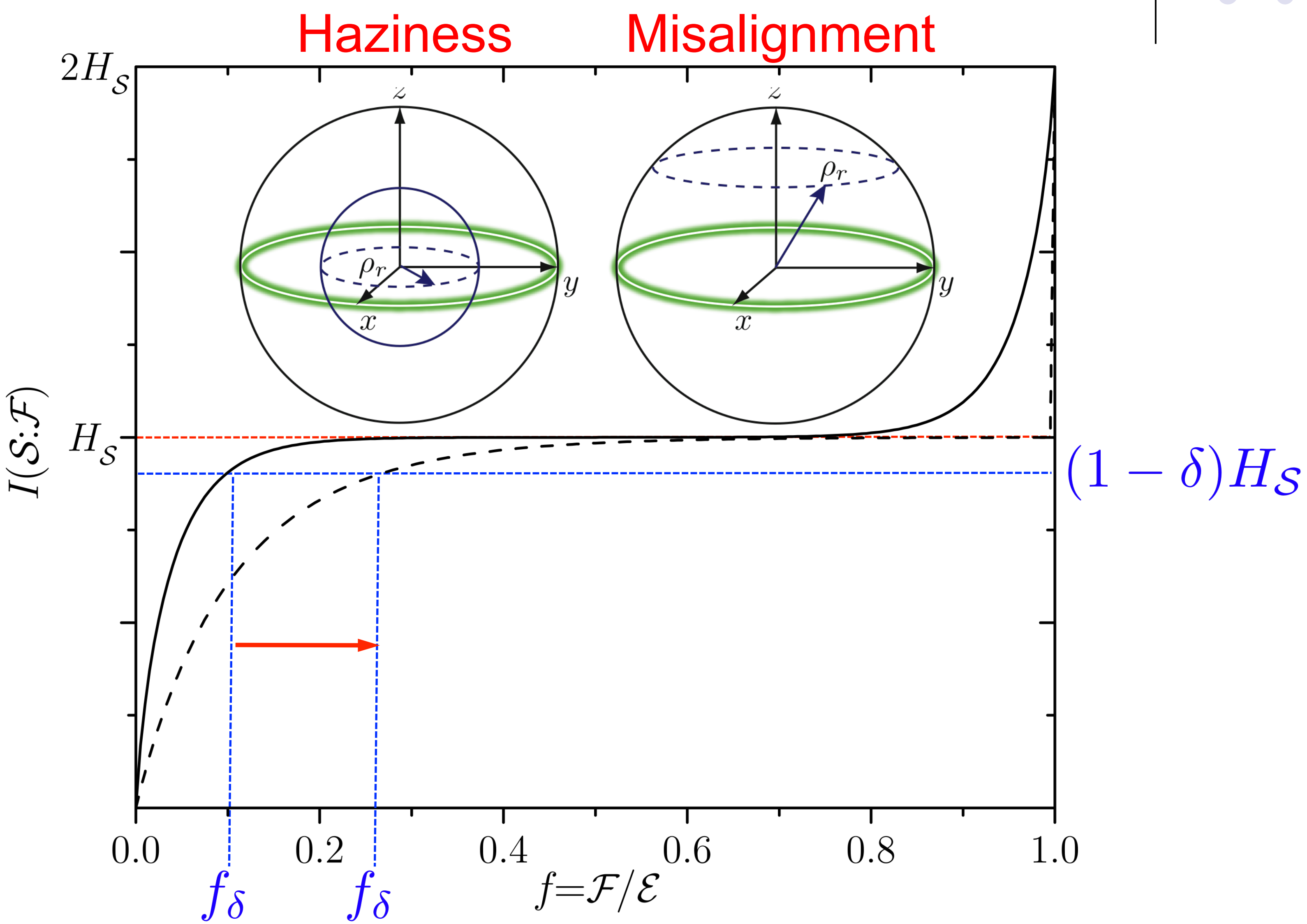
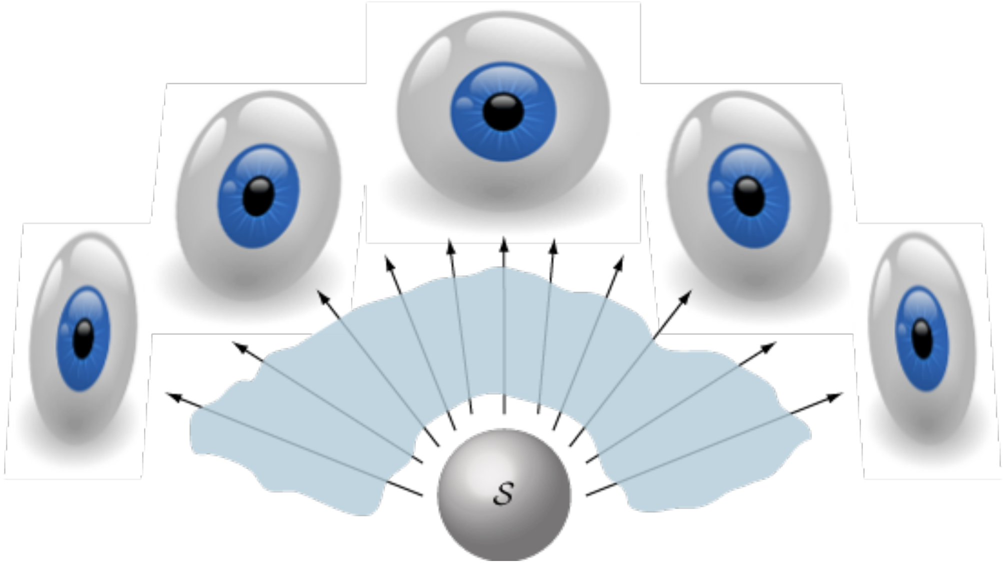
Redundancy



Haziness
Misalignment
$2H_{\mathcal{S}}$
$H_{\mathcal{S}}$
$I(\mathcal{S}\!:\!\mathcal{F})$
$(1-\delta)H_{\mathcal{S}}$
z
y
x
ρ_r
z
y
x
ρ_r
0.0
0.2
0.4
0.6
0.8
1.0
f_δ
f_δ
$f\!=\!\mathcal{F}/\mathcal{E}$

Hazy $\mathcal{E}$



Haziness $\quad h \equiv H\left(\rho_r\right)$

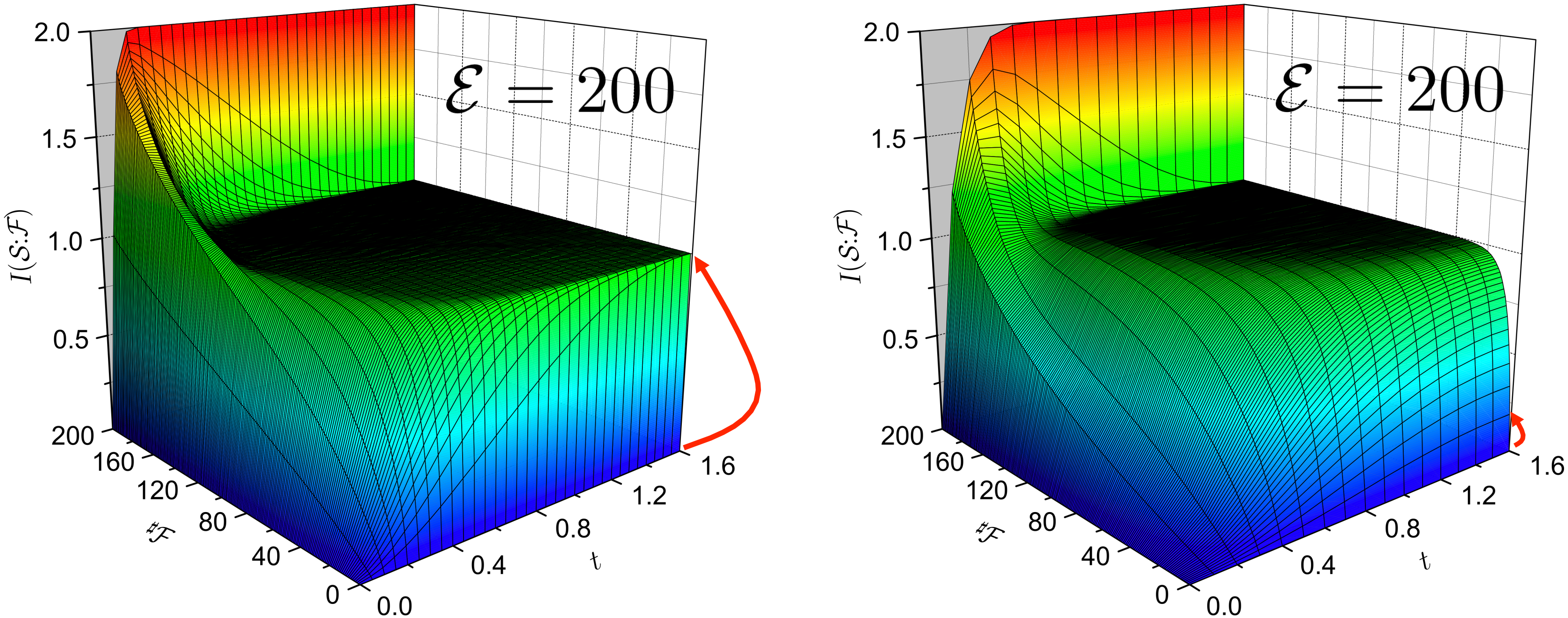
The initial entropy of an environment component

Extreme: $\rho\left(0\right) = \rho_{\mathcal{S}}\left(0\right) \otimes \bar{\mathrm{I}}^{\otimes \mathcal{E}} \longrightarrow I(\mathcal{S} : \mathcal{F}) = 0$

Hazy $\mathcal{E}$

$$\rho(0) = \rho_{\mathcal{S}}(0) \otimes \rho_r^{\otimes \mathcal{E}} \qquad h \equiv H(\rho_r)$$

Pure environment, *h*=0 Hazy environment, *h*=0.8



Classical plateau still forms even for quite hazy environments

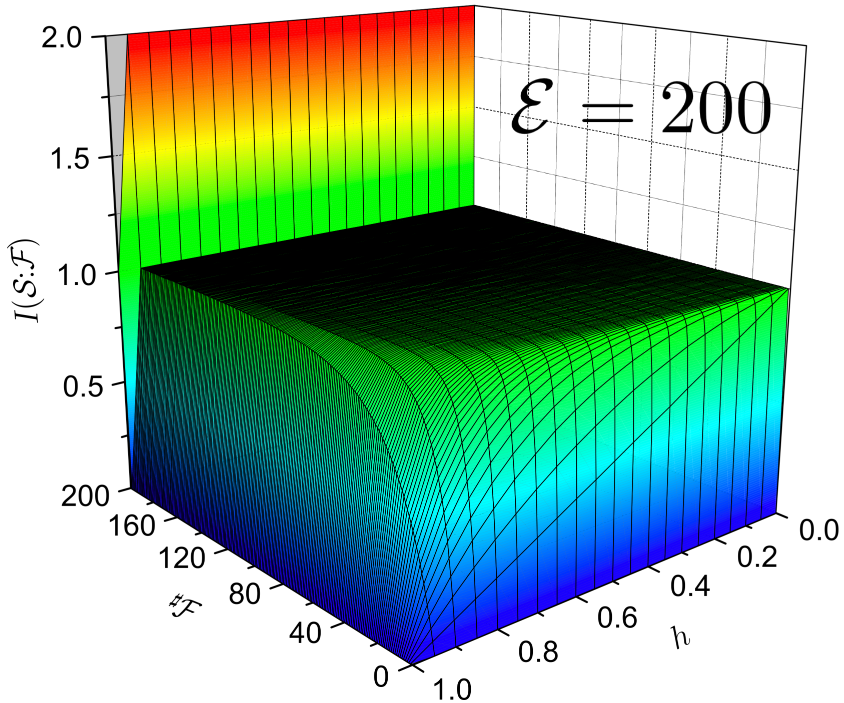
Initial rate of information gain is 1-*h*

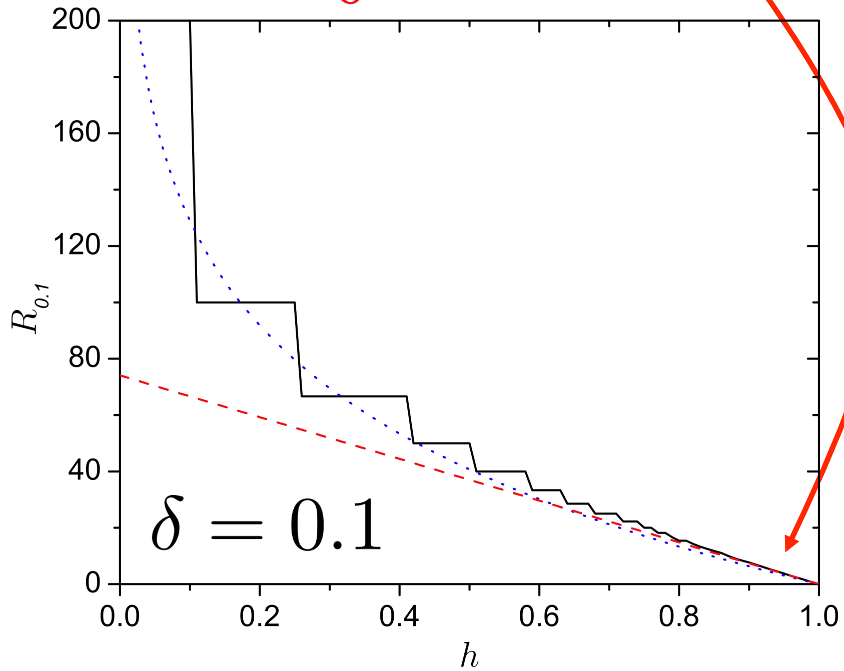
Hazy $\mathcal{E}$

$$t = \frac{\pi}{2}$$

Noisy channel

$$R_\delta \sim 1 - h$$





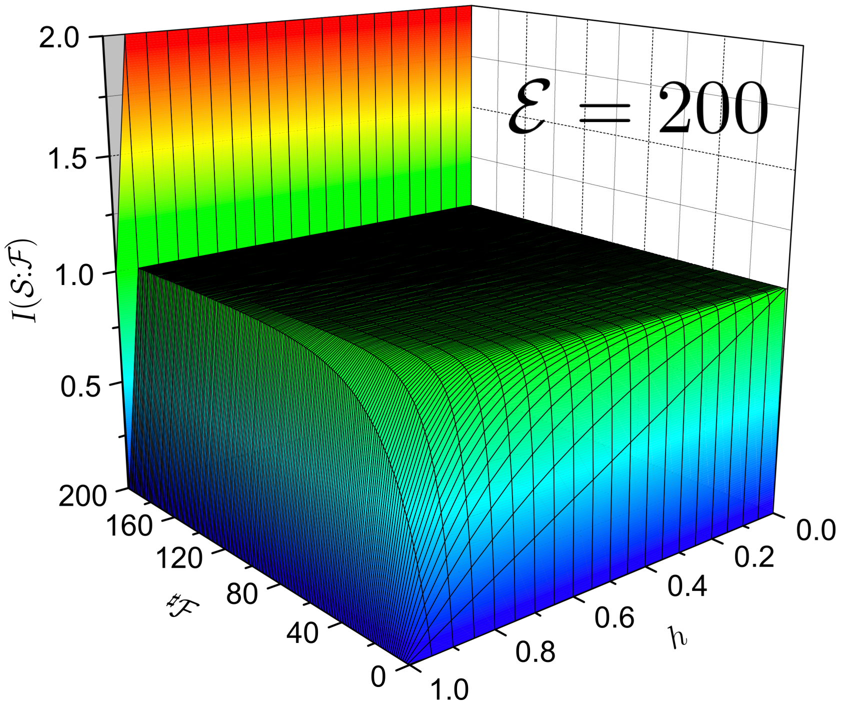
$$R_\delta = \frac{\mathcal{E}}{\mathcal{F}_\delta} \quad \mathcal{F}_\delta \rightarrow I(\mathcal{S}:\mathcal{F}) \geq (1-\delta) H_\mathcal{S}$$

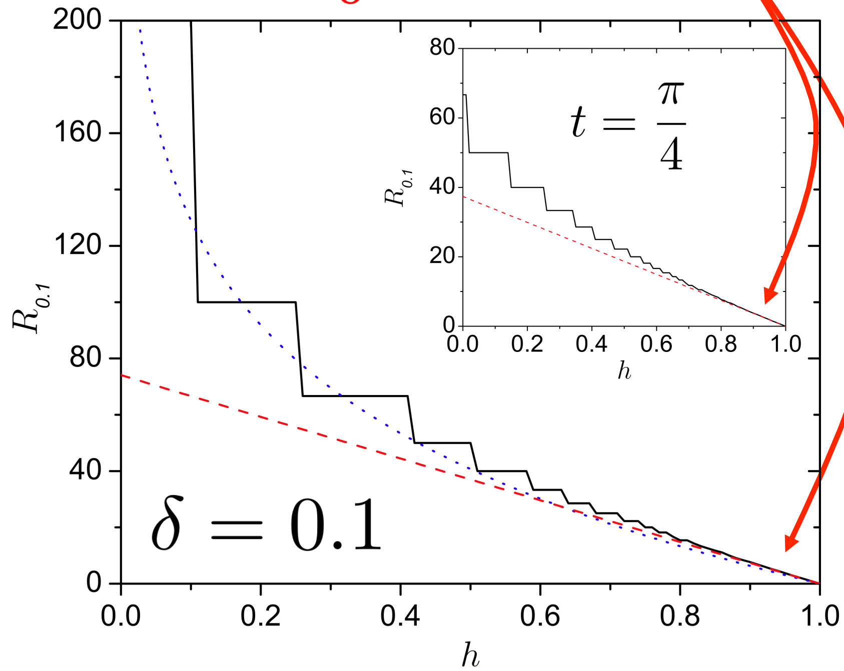
Hazy $\mathcal{E}$

$$t = \frac{\pi}{2}$$

Noisy channel

$$R_\delta \sim 1 - h$$



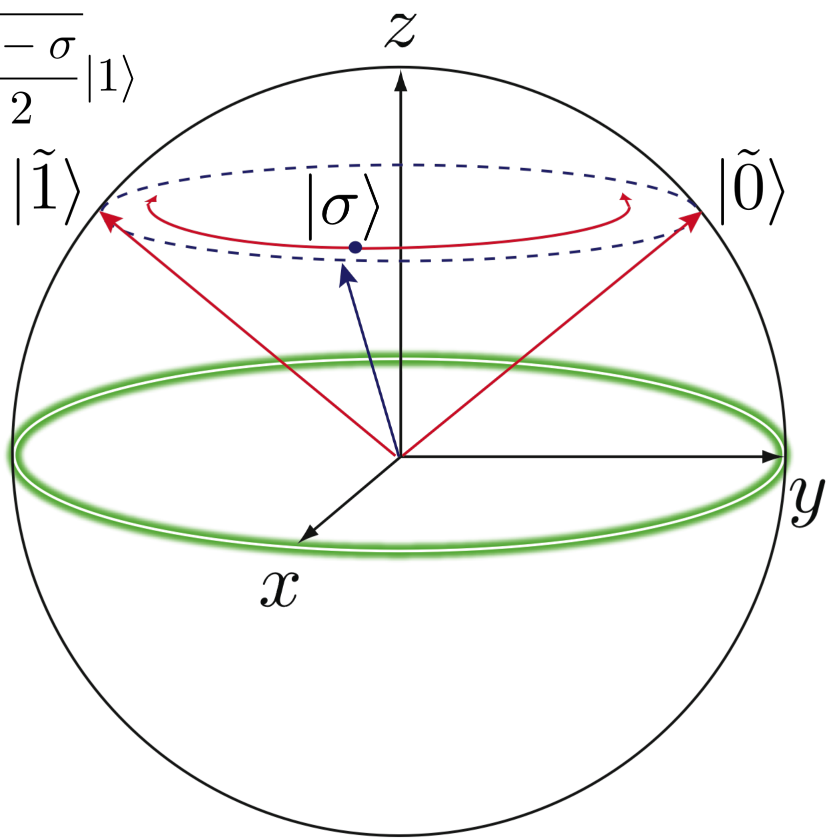


$$R_\delta = \frac{\mathcal{E}}{\mathcal{F}_\delta} \qquad \mathcal{F}_\delta \rightarrow I(\mathcal{S}:\mathcal{F}) \geq (1-\delta)H_\mathcal{S}$$

Misaligned, pure $\mathcal{E}$

$$|\sigma\rangle = \sqrt{\frac{1+\sigma}{2}}|0\rangle + \sqrt{\frac{1-\sigma}{2}}|1\rangle$$

$$|\psi(0)\rangle = |+\rangle|\sigma\rangle^{\mathcal{E}}$$

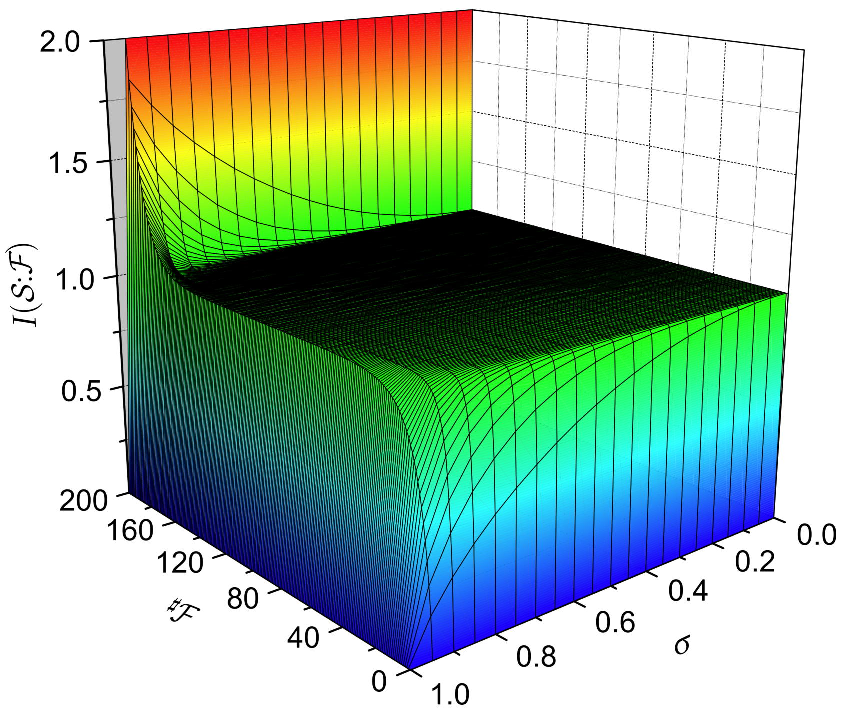


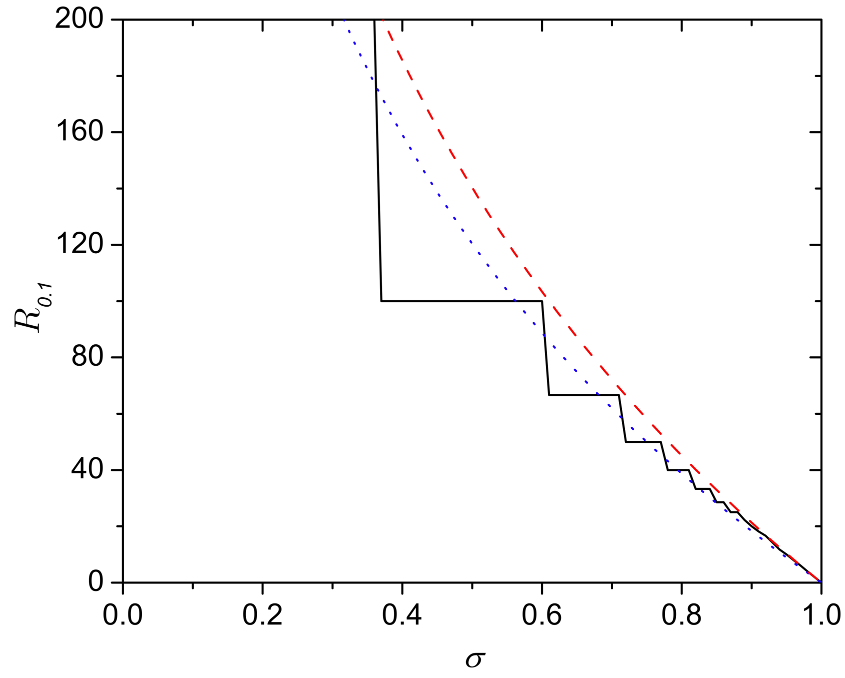
Misalignment: rotation out of x-y plane

Extreme: $|\sigma| = 1 \longrightarrow I(\mathcal{S} : \mathcal{F}) = 0$

Misaligned, pure $\mathcal{E}$

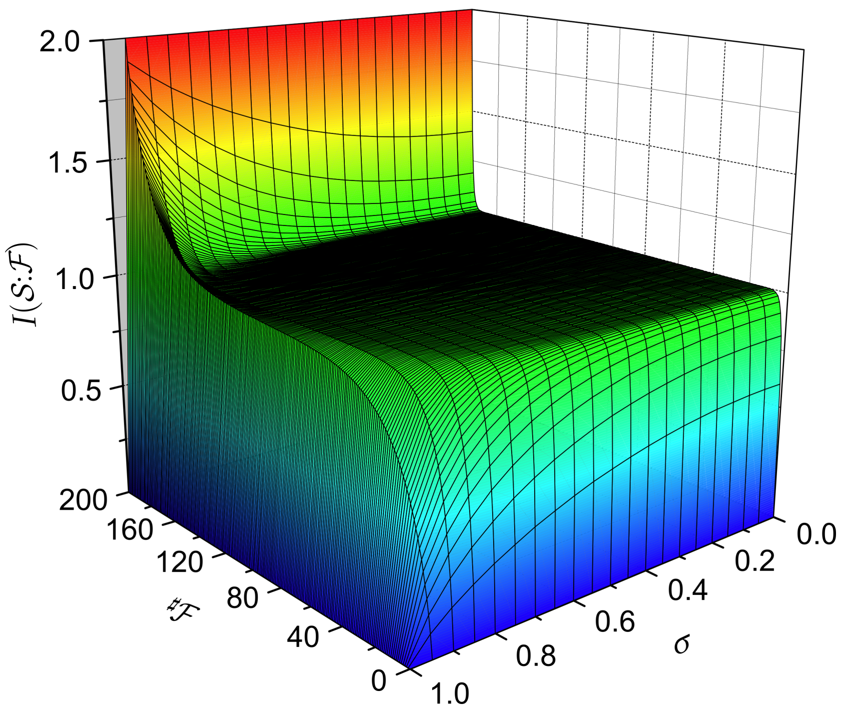
$$t = \frac{\pi}{2}$$

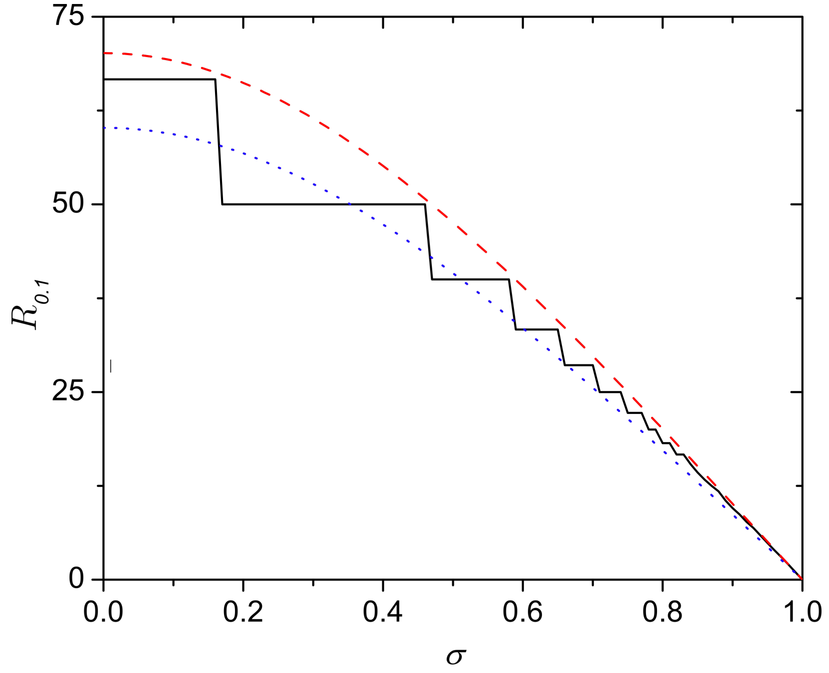




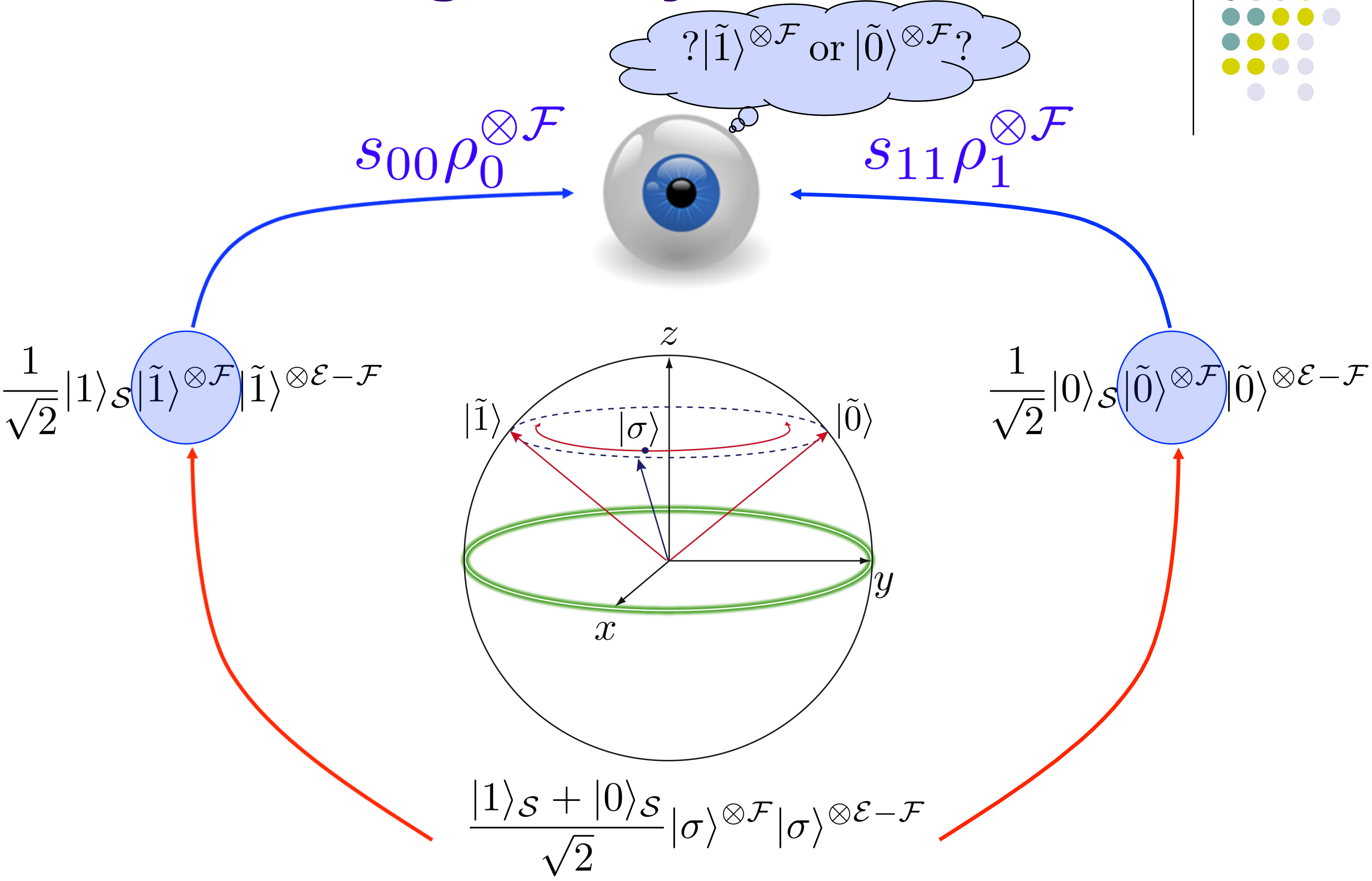
Misaligned, pure $\mathcal{E}$

$$t = \frac{\pi}{4}$$





2. Inferring the system's state



Proliferation of information under decohering interactions

$$I(\mathcal{S}:\mathcal{F}) = H_{\mathcal{S}}(t) + H_{\mathcal{F}}(t) - H_{\mathcal{S}\mathcal{F}}(t)$$

$$\underbrace{\mathbf{H}_{\mathcal{S}\mathcal{E}} = \sum_{k=1}^{\mathcal{E}} \Pi_{\mathcal{S}} \Upsilon_k \quad \rho(0) = \rho_{\mathcal{S}}(0) \otimes \left[\bigotimes_{k=1}^{\mathcal{E}} \rho_k(0) \right]}$$

A purely decohering Hamiltonian with independent environment components

Proliferation of information under decohering interactions

$$I(\mathcal{S}:\mathcal{F}) = H_{\mathcal{S}}(t) + H_{\mathcal{F}}(t) - H_{\mathcal{S}\mathcal{F}}(t)$$

$$I(\mathcal{S}:\mathcal{F}) = \boxed{\chi(\mathcal{S}\to\mathcal{F})} + \boxed{\boldsymbol{\delta}(\mathcal{S}:\mathcal{F})_{\{\Pi_{\mathcal{S}}\}}}$$

Quantum Discord

$$\chi(\mathcal{S}\to\mathcal{F}) = H\left(\sum_i s_{ii}\rho_i^{\otimes\mathcal{F}}\right) - \sum_i s_{ii} H\left(\rho_i^{\otimes\mathcal{F}}\right)$$

Holevo Bound: Maximum classical information transmittable by a quantum channel

Quantum Discord

$$\delta\left(\mathcal{S}:\mathcal{F}\right)_{\{\Pi_{\mathcal{S}}\}} = H_{\mathcal{S}d\mathcal{E}}(t) - \boxed{H_{\mathcal{S}d(\mathcal{E}/\mathcal{F})}(t)}$$

System decohered only by $\mathcal{E}/\mathcal{F}$

"Good decoherence" – when $\mathcal{E}$ and $\mathcal{E}/\mathcal{F}$ suffice to decohere $\mathcal{S}$

$$H_{\mathcal{S}d\mathcal{E}}(t) - H_{\mathcal{S}d(\mathcal{E}/\mathcal{F})}(t) \approx 0$$

Mutual information is only the classical information

$$I\left(\mathcal{S}:\mathcal{F}\right) \approx \chi\left(\mathcal{S}\to\mathcal{F}\right)$$

Classical Information

$$\chi(\mathcal{S}\to\mathcal{F}) = H\left(\overbrace{\boxed{\sum_i s_{ii}\rho_i^{\otimes\mathcal{F}}}}^{\rho_{\mathcal{F}}(t)}\right) - \sum_i s_{ii} H\left(\rho_i^{\otimes\mathcal{F}}\right)$$

$$\chi(\mathcal{S}\to\mathcal{F}) = H_{\mathcal{F}}(t) - H_{\mathcal{F}}(0)$$

Information transmitted is suppressed by initial entropy

$$\boxed{H(\mathcal{S}\leftrightarrow\mathcal{F})} \geq \chi(\mathcal{S}\to\mathcal{F}) \geq \boxed{I(\Phi_D[\mathcal{S}]:\Phi_M[\mathcal{F}])}$$

Exchange entropy　　　　Measure the fragment

Information transmitted is tightly bounded

Roga, Fannes, Życzkowski, PRL 105, 040505 (2010)

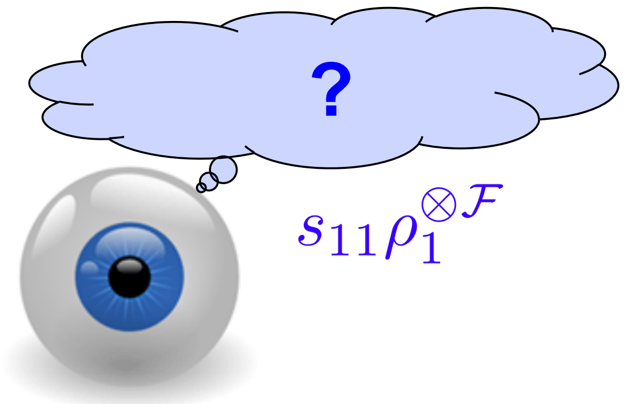
Lower bound

$$\chi\left(\mathcal{S} \to \mathcal{F}\right) \geq I\left(\Phi_D\left[\mathcal{S}\right] : \Phi_M\left[\mathcal{F}\right]\right)$$

$\Phi_D\left[\mathcal{S}\right]$ kills any remaining coherence in the system

$\Phi_M\left[\mathcal{F}\right]$ distinguishes the states of the fragment:

$s_{00}\rho_0^{\otimes\mathcal{F}}$



$s_{11}\rho_1^{\otimes\mathcal{F}}$

$$\rho_0^{\otimes\mathcal{F}} \Rightarrow \begin{cases} 0 & \sim 1 - P_e \\ 1 & \sim P_e \end{cases}$$

$$\rho_1^{\otimes\mathcal{F}} \Rightarrow \begin{cases} 0 & \sim P_e \\ 1 & \sim 1 - P_e \end{cases}$$

$$P_e = \frac{1}{2}\left(1 - \mathrm{tr}\left|s_{11}\rho_1^{\otimes\mathcal{F}} - s_{00}\rho_0^{\otimes\mathcal{F}}\right|\right)$$ à la Helstrom

$$\chi\left(\mathcal{S} \to \mathcal{F}\right) \geq I\left(\Phi_D\left[\mathcal{S}\right] : \Phi_M\left[\mathcal{F}\right]\right) \sim H_S - H(P_e)$$

Classical Information, $D_{\mathcal{S}} = 2$

$$H\left(\left(1-\sqrt{\left(s_{00}-s_{11}\right)^2+4P_e}\right)/2\right) \geq \chi\left(\mathcal{S}\to\mathcal{F}\right) \geq H_{\mathcal{S}} - H\left(P_e\right)$$

$$H_{\mathcal{S}} - \mathcal{O}\left(P_e^2\right) \geq \chi\left(\mathcal{S}\to\mathcal{F}\right) \geq H_{\mathcal{S}} - \mathcal{O}\left(P_e \log P_e\right)$$

$$\begin{aligned} P_e &= \frac{1}{2}\left(1-\mathrm{tr}\left|s_{11}\rho_1^{\otimes\mathcal{F}} - s_{00}\rho_0^{\otimes\mathcal{F}}\right|\right) \\ &\sim e^{-\mathcal{F}\xi_{QCB}} \end{aligned}$$

$$\xi_{QCB} = -\ln \min_{0\leq s\leq 1} \mathrm{tr}\rho_0^s\rho_1^{1-s}$$

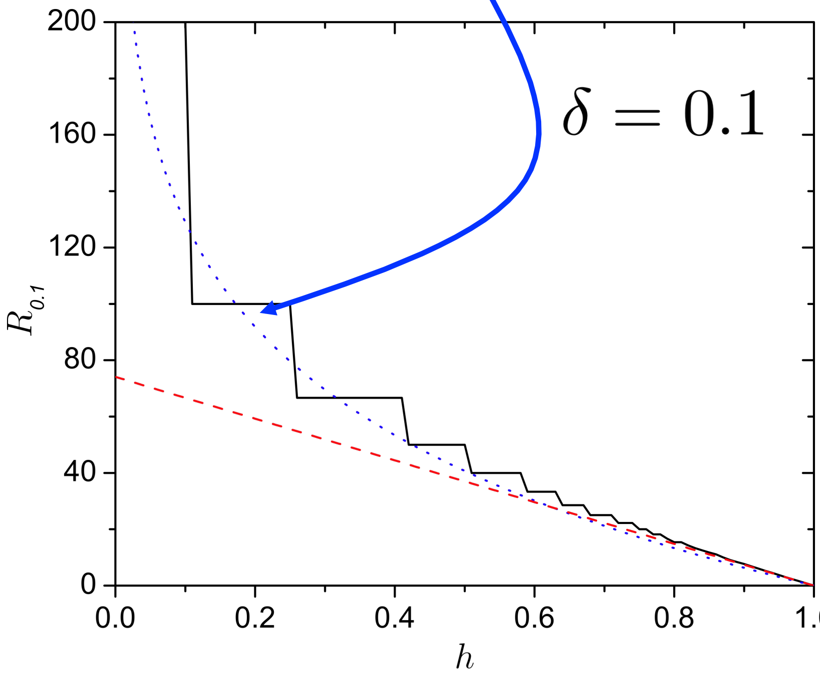
QCB: Audenaert et al., PRL 98, 160501 (2007)

$$\Rightarrow R_\delta = \mathcal{E}\frac{\xi_{QCB}}{\ln(1/\delta)}$$

"Chernoff information" gives the Redundant Information

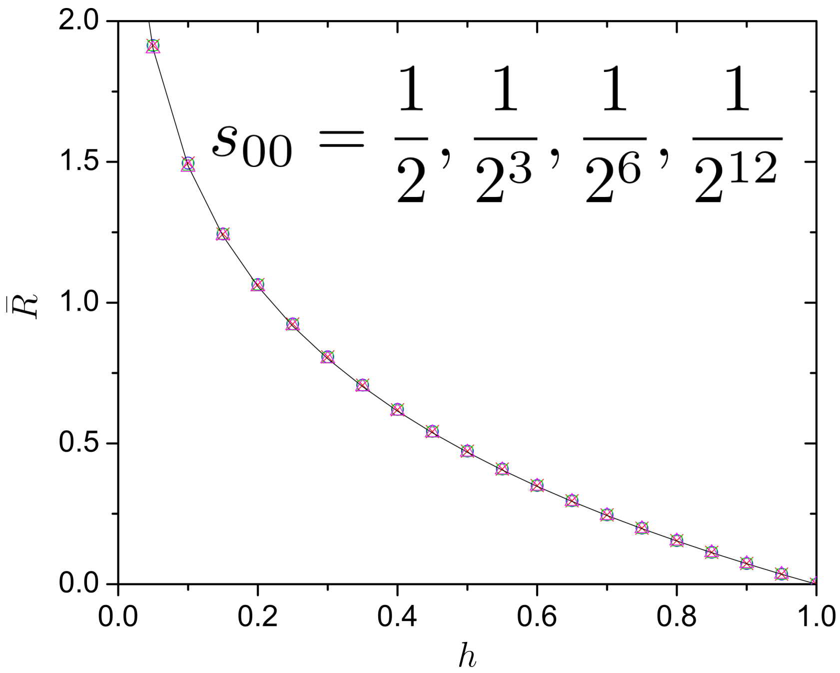
Redundancy, hazy $\mathcal{E}$, $t = \pi/2$

$$R_\delta = \mathcal{E}\frac{\ln 2\sqrt{\lambda_-\lambda_+}}{\ln(\delta)}$$



Normalized Redundancy

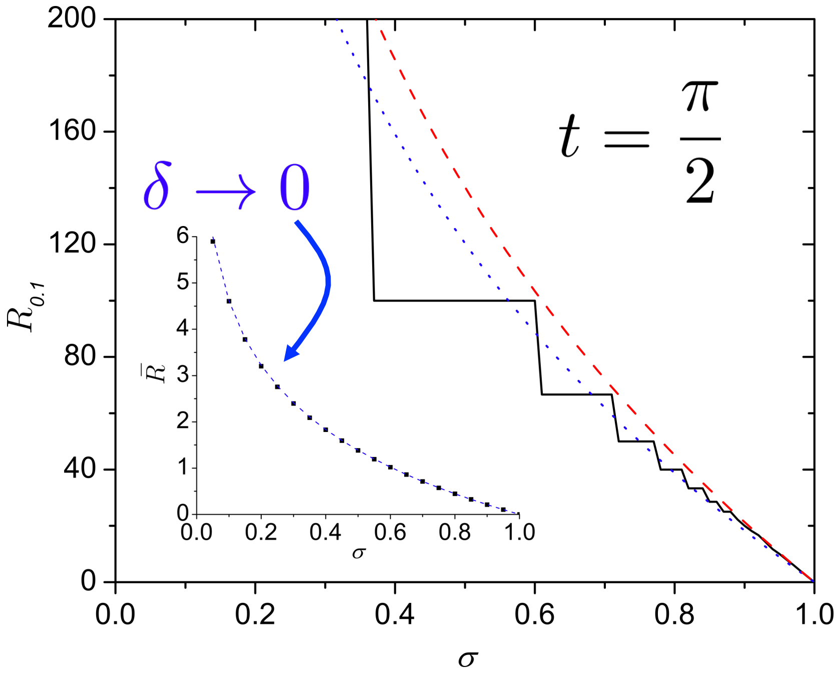
$$\bar{R} = \lim_{\delta\to 0}\frac{-R_\delta \ln\delta}{\mathcal{E}} = \xi_{QCB}$$

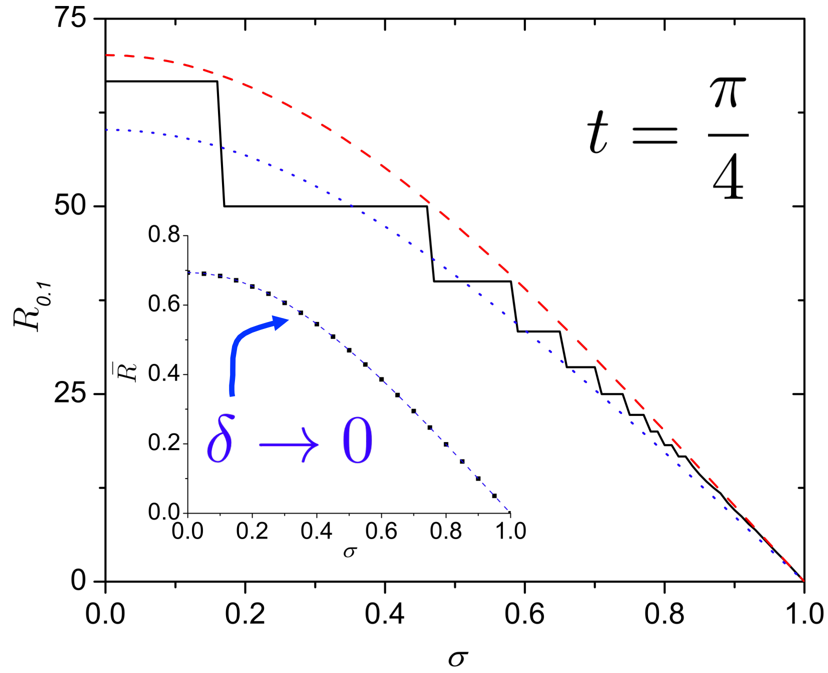


Redundancy collapses onto universal curve

Misaligned, pure $\mathcal{E}$

$$R_\delta \approx \frac{\mathcal{E} \ln |\Lambda_k(t)|^2}{\ln \delta}$$





$$\Lambda_k(t) = \cos(t) - \imath\sigma \sin(t)$$

Quantum Darwinism in Practice

- Preferred states – the pointer states - survive and spawn the most information theoretic progeny, which is communicated by $\mathcal{E}$
- Non-ideal environments: $I(\mathcal{S}:\mathcal{F}) \approx H_{\mathcal{F}}(t) - H_{\mathcal{F}}(0)$
  - Rate of information gain reduced by the initial entropy or misalignment, i.e., noisy channel: 1-*h*
  - Pointer states can still be exhaustively and redundantly determined from the environment
- Redundancy is given by the "Chernoff Information"

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MZ, HTQ, WHZ, PRL 103, 110402 (2009);
PRA 81, 062110 (2010);
See also http://mike.zwolak.org