Networks of Balance Laws

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Let a graph be given where on each edge, dynamics described by a system of balance laws is given. If such a networked system of balance laws is considered, the question arises: How are the pdes on the edges coupled at the nodes? Often the coupling is performed by an algebraic system of equations. However, possibly this class of models is not optimal for the applications. Therefore the following direction of research is important:
Examine systems, where the coupling is given by a system of ordinary differential equations in terms of the boundary data!

Another problem concerns *transcritical flow*, where one of the eigenvalues of the system matrix for the solution attains both negative and positive values. This situation occurs for example in sewage systems where heavy rainfall may cause supercritical flow in a system that usually has a subcritical state. The exact controllability of transcritical systems has been studied by Coron, Glass, Gugat and Wang for graphs consisting of a single edge. To complete the picture, studies of the following type are important:
Does the exact controllability also hold for networked systems?

Also the stabilizability of transcritical flow is important:
Examine the boundary feedback stabilization of transcritical systems!
Possibly this requires to use feedback in a suitable weak sense. In particular it should be clarified how to handle the distribution of the places where the feedback is acting, because this distribution depends on time.

The last question concerns the class of graphs where exact boundary controllability is possible:
Is the exact boundary controllability and stabilizability of networked conservation laws possible for graphs that contain circles?
An excellent overview of the corresponding results for networks of strings is given in the book by Dager and Zuazua*: Wave propagation, observation and control in 1-d flexible multistructures*, Springer (2006).