Stabilization of Gas Flow by Compressors

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We consider a system that consists of two gas pipes of finite length that are coupled by a compressor. An essential effect in the flow of gas through the pipes is the pressure loss caused by the friction at the pipe walls. To model this effect, it is essential that a source term appears in the isothermal Euler equations for space dimension one, that we use for the description of the gas flow in the pipes.

The flow through the compressor is described by a system of two equations (which are not differential equations): The first equation states that the flow rate for the gas that enters the compressor is the same as for the gas that leaves the compressor. The second equation gives the increase in pressure as a function of the compressor control. If the compressor is switched on, it causes a pressure jump for the flow.

In order to have desirable system states, we want to use controls that generate classical solutions starting from classical initial states. We consider subsonic flow. The existence of classical solutions on finite time-intervals (semi-global solutions) is shown in the paper: *Existence of classical solutions and feedback stabilization for the flow in gas networks*, ESAIM: COCV (2009) using a result by Zhiquiang Wang (Chin. Ann. Math. 2006).

To stabilize the system locally around a stationary state we first analyze these time-independent states for our system: For stationary states, the flow rate is constant and the pressure is strictly decreasing along the space interval. In particular, if the flow rate is greater than zero, the stationary state is **not** constant and the classical solution exists only on a finite space interval. At the end of the interval, the state becomes critical and the space derivative of the pressure tends to minus infinity.

To stabilize the system, we control the compressor in such a way that the flow rate at the compressor is constant with the value of the flow rate of our stationary state. By extending ideas of Coron, Bastin and d'Andréa-Novel in this way we obtain a LAYPUNOV-function with exponential weights that decays exponentially on our time interval. This yields exponential decay in L^2 for the difference to the desired stationary state. Open problems are:

1. The extension of the result to the infinite time-interval $[0, \infty)$.
2. The extension of the result to large pipeline networks.
3. To find conditions that guarantee the exponential decay in the sense of continuously differentiable states.