Ribbons, rings and rough cavities: mesoscopic effects in transport through graphene structures

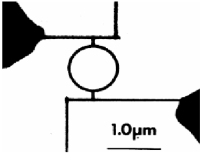
Klaus Richter

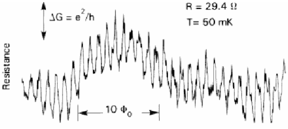
Universität Regensburg

Mesoscopic signatures

in phase coherent ...

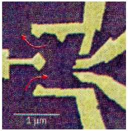
... diffusive systems

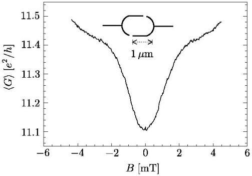




R.A. Webb *et al*, PRL 1985

... ballistic systems





Chang et al. (1994)

$\Rightarrow$ Aharonov-Bohm effect

$\Rightarrow$ (universal) conductance fluctuations

$\Rightarrow$ weak localization

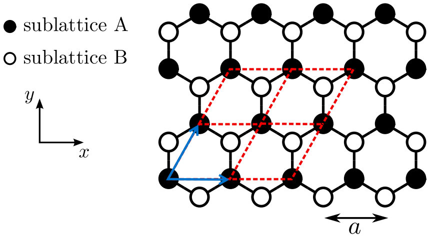
Outline:

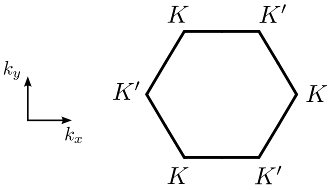
mesoscopic effects in graphene-based nanostructures:

1. transport formalism
2. Aharonov-Bohm effect in graphene rings
3. symmetry classes of graphene quantum dots:
   spectral statistics and weak localization
4. transport through zigzag nanoribbons:
   spin injection and spin conductance fluctuations

interplay of edge and interference effects

Tight-binding model for graphene





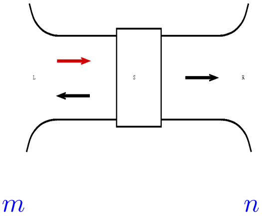
➜ tight-binding model

$$H = \underbrace{-\sum_{ij\text{ nn}} t\,|i\rangle\langle j|}_{\text{nearest-neighbor hopping}} - \underbrace{\sum_{ij\text{ nnn}} t'\,|i\rangle\langle j|}_{\text{next-nearest neighbor hopping}} + \underbrace{\sum_{i} U_i\,|i\rangle\langle i|}_{\text{on-site energy}}$$

$$U_i = \begin{cases} V_i + M_i & \text{for } i \text{ in sublattice A} \\ V_i - M_i & \text{for } i \text{ in sublattice B} \end{cases}$$

V: potential, M: staggered potential

Green function formalism for transport



Conductance:

$$G = (e^2/h)\,\mathcal{T} \qquad \text{with}$$

$$\mathcal{T} = \sum_{n=1}^{N'} \sum_{m=1}^{N} |t_{nm}|^2 = \mathrm{Tr}(\Gamma_l G^r \Gamma_{l'} G^a)$$

- retarded Green function: $G^r = (E - H_{scat} - \Sigma^r)^{-1}$
- self-energies: $\Sigma^r = \sum_{\text{leads}} \Sigma_l^r$
- coupling to lead l: $\Gamma_l = \mathrm{i}(\Sigma_l^r - \Sigma_l^a)$

- use **recursive Green function techniques** within Landauer- and Keldysh-approaches
- **matrix reordering strategies** (graph-theoretical approaches)

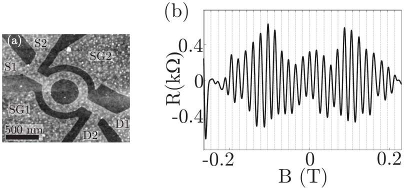
M. Wimmer and KR, arXiv:0806.2739

Aharonov-Bohm effect in graphene

Aharonov-Bohm effect: experiments

- transport in coherent, dirty regime

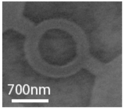
AB-effect in a side-gated ring

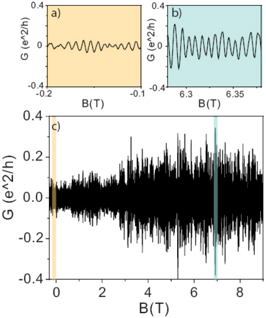


$\Rightarrow L_\phi = 1\mu$m at $T = 0.5$ K

F. Molitor et al., arXiv 0904.1364 (2009)

increased AB-oscillations at high B-field





S. Russo et al., Phys. Rev. B (2008)

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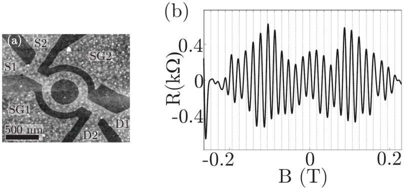

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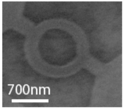

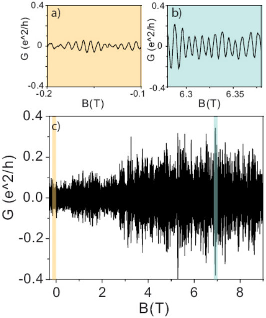

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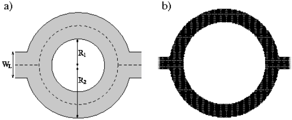
Theory?

- Rycerz, Acta Phys. Polonica (2009): valley polarization in few-mode regime
- Recher et al., Phys. Rev. B (2007): closed rings with effective mass boundary condition
- Luo et al. arXiv0907.3150 (2009): effective TRS breaking in armchair rings

Aharonov-Bohm effect: theory

magneto conductance
of large **ballistic** rings

geometry:



parameters:

radius: $R \simeq 55$ nm
ring width: $w_r \simeq 18$ nm
lead width: $w_l \simeq 14$ nm

$\rightarrow \quad \sim 10^5$ atoms

J. Wurm, M. Wimmer, H.U. Baranger, KR, Semicond. Sci. Techn. (2009)

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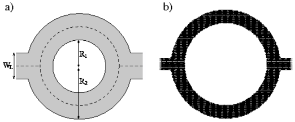

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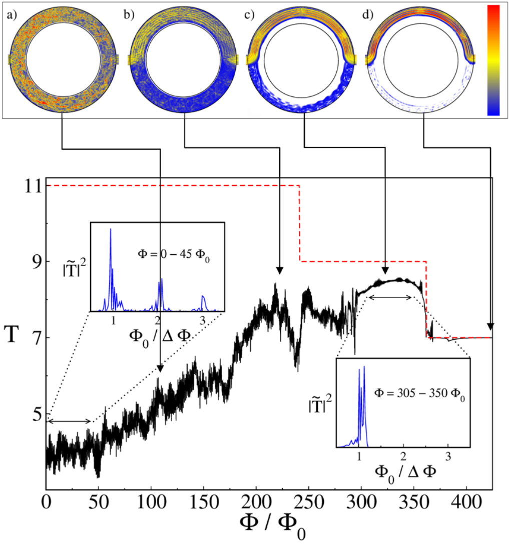


J. Wurm, M. Wimmer, H.U. Baranger, KR, Semicond. Sci. Techn. (2009)

AB effect: large-B signal

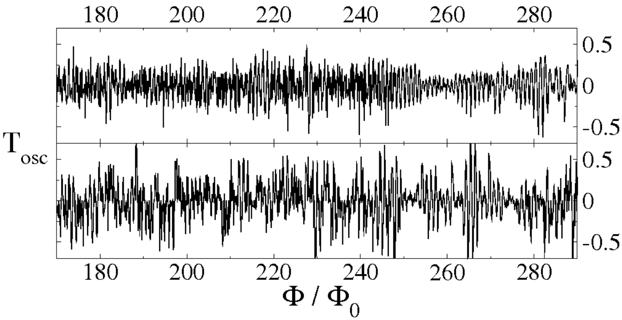
Russo-experiment: increase of AB oscillations at ~ 3Tesla: $w_r \geq 2r_{\rm cyc}$

numerics:

ring arms:

symmetric

asymmetric



⇒ no peculiar features in the numerical AB signal at $w_r \simeq 2r_{\rm cyc}$

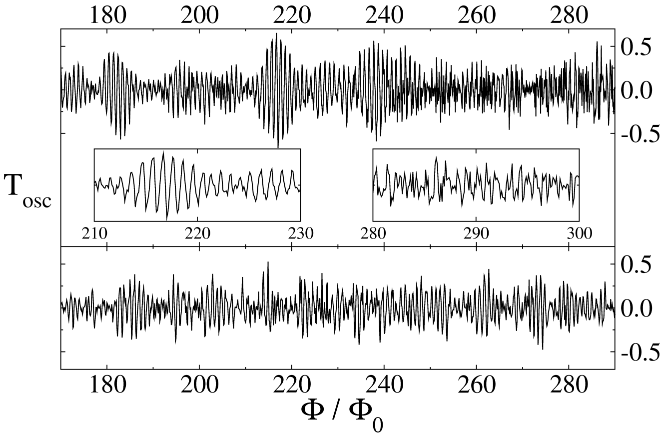
AB effect: disordered rings

Russo-experiment: strongly disordered regime

AB rings with

edge disorder

bulk disorder



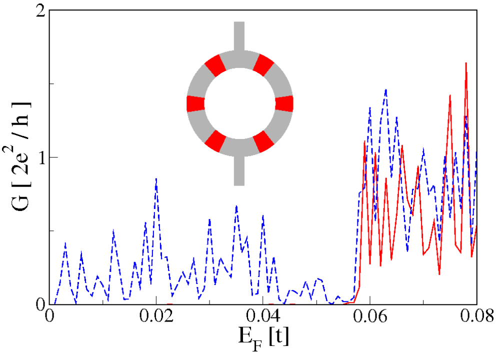
⇒ regimes of clean AB signal and aperiodic oscillations
(due to resonant tunneling at disordered edges)

AB rings: graphene-specific effects

- **Conductance suppression**

AB ring with:

- metallic and semiconducting armchair regions in different arms
- semiconducting armchair regions in both arms ⇒ conductance suppression



⇒ **effective barriers in bended graphene ribbons**

J. Wurm, M. Wimmer, I. Adagideli, KR, H.U. Baranger, arXiv (2009)

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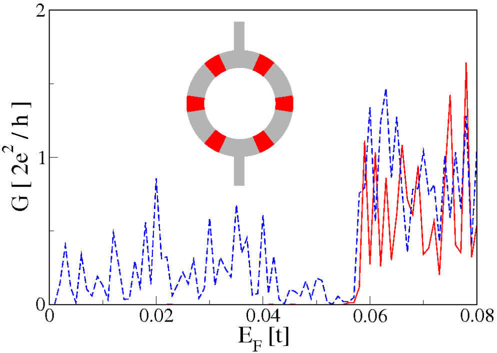


⇒ **effective barriers in bended graphene ribbons**

- **breaking the valley degeneracy** in AB rings with mass confinement

J. Wurm, M. Wimmer, I. Adagideli, KR, H.U. Baranger, arXiv (2009)

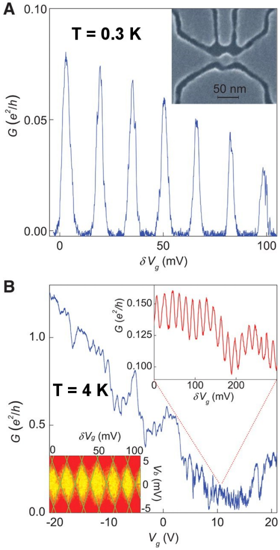
spectral statistics and weak localization in graphene quantum dots

Coulomb blockade experiments in graphene

- experiments in tunable graphene quantum dots show:
  - **Coulomb oscillations**
  - **Coulomb diamonds**
- "large" dots $\rightarrow$ equidistant peaks ($D \approx 250\ nm$)

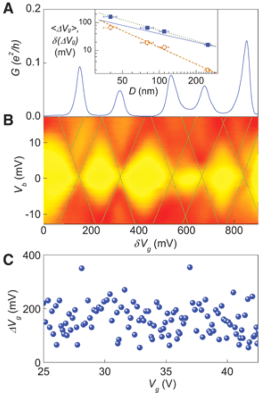
"Chaotic Dirac Billiard in Graphene Quantum Dots"
Ponomarenko *et. al*, *Science* **320**, 356 (2008)

see also related work:
Stampfer *et. al*, *APL* **92**, 012102 (2008),
Nano Lett. **8**, 2378 (2008)



Coulomb blockade experiments in graphene

- Small samples (size $D \lesssim 100\ nm$)
  $\rightarrow$ **size quantization** $\rightarrow$ non-periodic peaks



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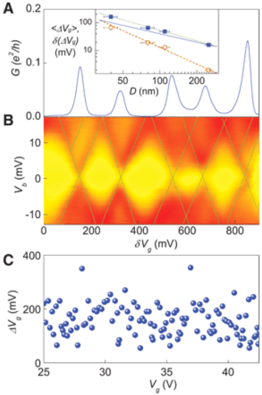
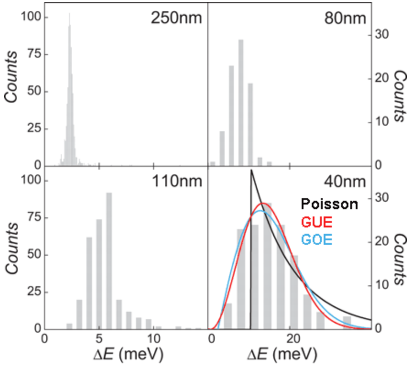




Ponomarenko et al.

- Wigner distribution of nearest neighbor peak spacings
  $\rightarrow$ level repulsion, signature of **quantum chaos** !
- Unitary statistics (GUE) $\rightarrow$ **TRS broken** at $B = 0$?

Symmetry Classes and Random Matrix Theory

- consider quantum system with chaotic classical dynamics
- conjecture: Random Matrix Theory (RMT) applicable
  $\rightarrow$ **universal predictions** for energy level (distributions)
  and transport (scattering) properties
- depending on time reversal symmetry (TRS) property, Hamiltonian H and scattering matrix S belong to different RMT ensembles:

	H	S
TRS	$GOE(GSE)$	$COE(CSE)$
no TRS	GUE	CUE

- orthogonal ensembles: e.g. $H^T = H$ (real symmetric)
  unitary ensemble: e.g. $H^\dagger = H$ (Hermitian)

Time reversal symmetries for graphene

Graphene hamiltonian with mass term

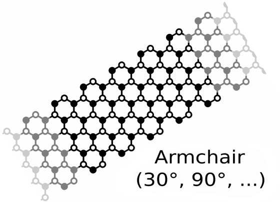
$$H_{\text{eff}} = v_F \pi_x \sigma_x \otimes \tau_z + v_F \pi_y \sigma_y \otimes \tau_0 + v_F^2 m(x,y) \sigma_z \otimes \tau_0$$

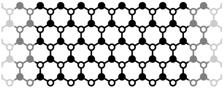
$$= v_F \begin{pmatrix} \vec{\sigma}\cdot\vec{\pi} & 0 \\ 0 & -\vec{\sigma}^*\cdot\vec{\pi} \end{pmatrix} + v_F^2 m(x,y)\sigma_z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- (special) time reversal symmetries :
  Suzuura, Ando, PRL (2002); McCann et al. PRL (2006); Ostrovsky et al. Eur. Phys. J. (2007)
  - conventional:
    $[\mathcal{T}, H_{\text{eff}}] = 0$
  - $\mathcal{T}^2 = 1$
  - special:
    $\mathcal{T}_{\text{sl}} = -i(\sigma_y \otimes \tau_0)\mathcal{C}$
    $\mathcal{T}_{\text{v}} = -i(\sigma_0 \otimes \tau_y)\mathcal{C}$
  - $\mathcal{T}^2_{\text{sl / v}} = -1$
- $m(x,y)$: breaks only $\mathcal{T}_{\text{sl}}$
- magnetic field B: breaks $\mathcal{T}, \mathcal{T}_{\text{sl}}, \mathcal{T}_{\text{v}}$

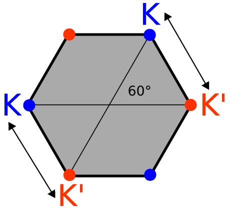
Graphene: boundary effects

intervalley scattering in graphene nanostructures





- **armchair edges**: contributions from both valleys are mixed



- **zigzag edges**: valleys form two disconnected subsystems

$\rightarrow$ intervalley scattering expected in graphene nanostructure

$\rightarrow$ mass confinement can suppress intervalley scattering

Predictions of Random Matrix Theory

- **strong intervalley scattering** (abrupt lattice termination):

	H
$B = 0$	GOE
$B \neq 0$	GUE

Predictions of Random Matrix Theory

- **strong intervalley scattering** (abrupt lattice termination):

	H
$B=0$	GOE
$B\neq 0$	GUE

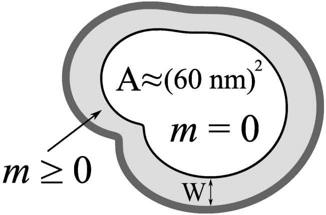
- **no intervalley scattering** (mass confinement):

	H
$B=0$	$\begin{pmatrix} GUE & 0 \\ 0 & GUE \end{pmatrix}$
$B\neq 0$	$\begin{pmatrix} GUE^{(1)} & 0 \\ 0 & GUE^{(2)} \end{pmatrix}$

GUE at $B=0$ expected (see Berry, Mondragon, Proc. R. Soc. London A (1987))

Spectral statistics of closed graphene dots

"Africa billiard":
prototype of a chaotic quantum system

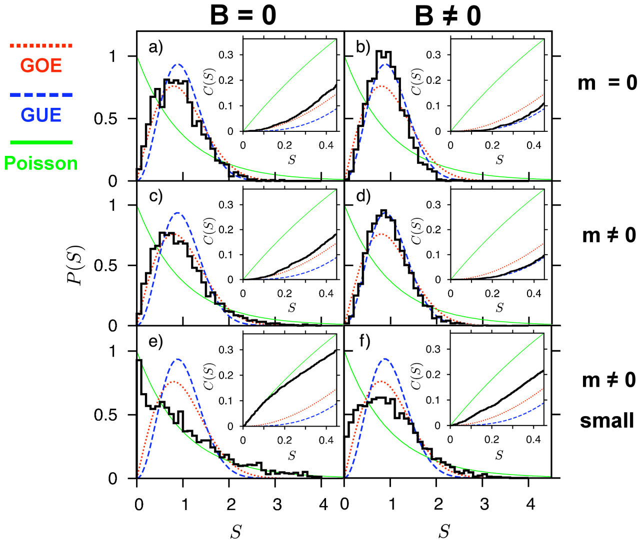


- quadratically increasing mass term

$$m(x, y) \sim (\delta(x, y) - W)^2$$

- calculate the density of states
- count number of adjacent levels with energy difference ΔE
- unfold the spectrum:
  get normalized distribution $P(S)$ with
  $S = \Delta E / \langle \Delta E \rangle$

Nearest-neighbor statistics



$\Rightarrow$ **no unitary** (GUE) statistics for $B = 0$!

Nearest-neighbor statistics

why **Poissonian statistics** for the small billiard with $m \neq 0$?

- **localized states** at zigzag type boundaries
- dominant for small enough systems

(see also De Raedt and Katsnelson (2008))

Nearest-neighbor statistics

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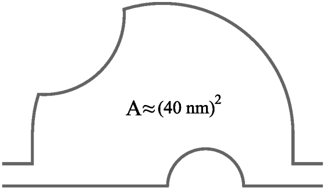
More importantly:

why **GOE statistics** for the larger billiard with $m \neq 0$?

- residual intervalley scattering
- relevant timescale for spectral statistics in closed systems: Heisenberg time $\tau_H = \frac{\hbar}{\langle \Delta E \rangle} \gg \tau_{KK'}$
- $\rightarrow$ intervalley scattering dominates: **no GUE expected**

Quantum transport in open graphene structures



- deformed half stadium
  (chaotic classical dynamics)
-

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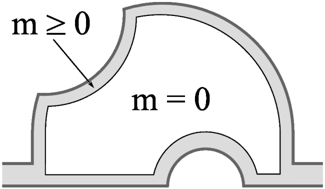
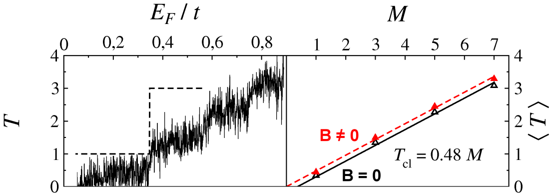


- deformed half stadium
  (chaotic classical dynamics)
- quadratically increasing mass term
  intervalley scattering suppressed !?

Weak localization in ballistic graphene structures

Calculation of the dimensionless conductance $\mathbf{T = \frac{h}{2e^2} G}$



- classical conductance $T_{\mathrm{cl}} \approx M/2$
- shift in $\langle T \rangle$ indicates *weak localization*

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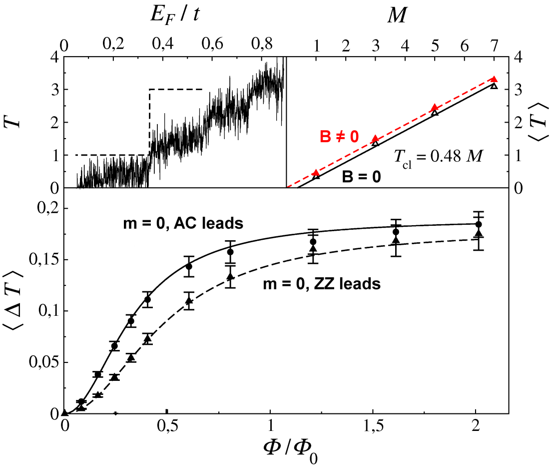

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- $m = 0$: intervalley scattering $\Rightarrow$ crossover: GOE $\longrightarrow$ GUE
- Lorentzian line shape (expected from semiclassics)

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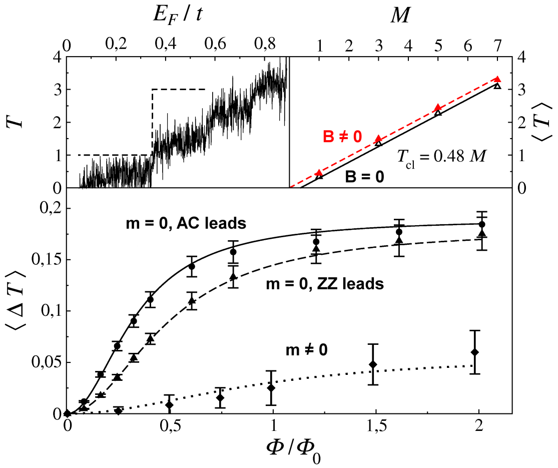

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- $m = 0$: intervalley scattering $\Rightarrow$ crossover: GOE $\longrightarrow$ GUE
- Lorentzian line shape (expected from semiclassics)
- $m \neq 0$: weak localization strongly suppressed for mass confinement !

symmetry classes: spectral vs. transport properties

for $B = 0$**:**

1. open and closed graphene systems with abrupt termination: GOE behavior

2. **transport** through open dots with smooth mass confinement: $\tau_{\rm esc} < \tau_{KK'} \longrightarrow$ GUE behavior for (WL and UCFs)

3. **spectral statistics** of closed dots with smooth mass confinement: $\tau_{\rm H} > \tau_{KK'} \longrightarrow$ GOE behavior

$\Rightarrow$ does not agree with experimental conclusions: GUE

J. Wurm, A. Rycerz, I. Adagideli, M. Wimmer, KR, H.U. Baranger, Phys. Rev. Lett. (2009)

Spin currents and mesoscopic spin conductance fluctuations in nanoribbons

Spins in graphene

- small intrinsic spin-orbit coupling

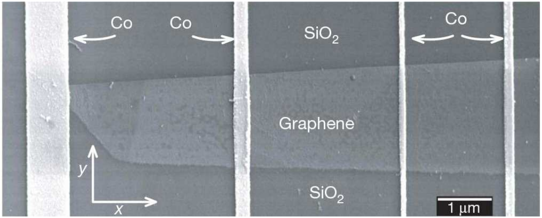
  $\Rightarrow$ long spin lifetimes expected

  $\Rightarrow$ graphene as prospective material for spin electronics

- successful spin injection from ferromagnetic contacts into graphene

  Hill et al., Trans. Magn. (2006); Oishi et al., Jpn. J. Appl. Phys. (2007);

  Tombros et al., Nature (2007):

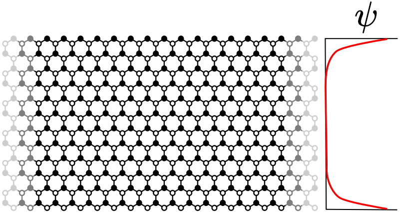


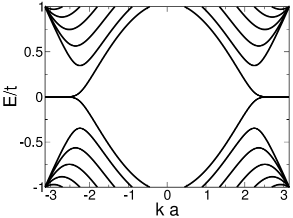
- how to generate spin currents in graphene without ferromagnets?

  $\Rightarrow$ **zigzag nanoribbons**

edge magnetism in zigzag nanoribbons

- Zigzag graphene nanoribbons:
  existence of a localized state at the edges





(see eg. Fujita, Wakabayashi, Nakada, Kusakabe (1996); Ezawa (2006); Peres, Guniea, Castro-Neto (2006); Brey and Fertig (2006); ...)

- experimental observation:
  (eg. Kobayashi et al. Phys. Rev B (2006); Niimi et al. Phys. Rev. B (2006); see also: Ritter and Lyding, Nature Mat. (2009))

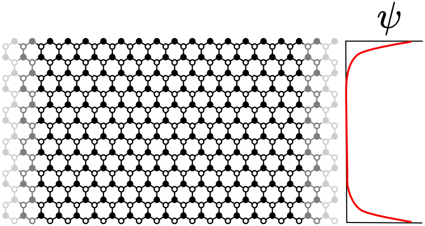
Edge magnetism in zigzag nanoribbons

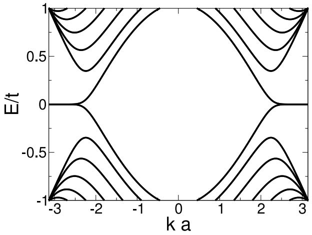
Zigzag graphene nanoribbons: flat-band ⇒ high density of states
⇒ magnetism of the edge state

Fujita *et al.*, JPSJ (1996) (mean field Hubbard model)

S. Okada and A. Oshiyama, Phys. Rev. Lett. (2001) (DFT)

Y.-W. Son *et al.*, Nature (2007) (DFT)





- edge magnetization within each sublattice A and B
- ground state: opposite magnetization of A and B edge states
- staggered magnetization

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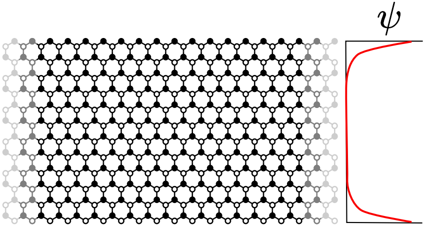
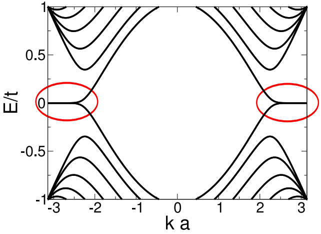




- edge magnetization within each sublattice A and B
- ground state: opposite magnetization of A and B edge states
- staggered magnetization

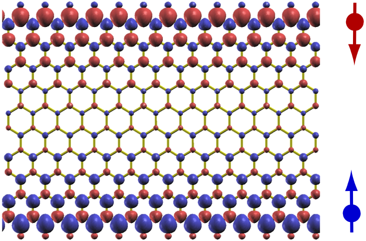
Edge magnetism in zigzag nanoribbons

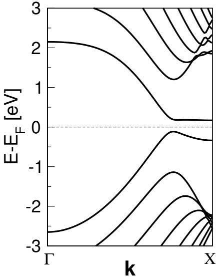
Zigzag graphene nanoribbons: flat-band $\Rightarrow$ high density of states
$\Rightarrow$ magnetism of the edge state

Fujita *et al.*, JPSJ (1996) (mean field Hubbard model)

S. Okada and A. Oshiyama, Phys. Rev. Lett. (2001) (DFT)

Y.-W. Son *et al.*, Nature (2007) (DFT)



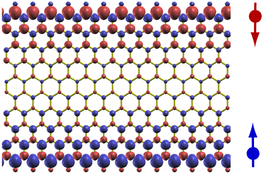


- edge magnetization within each sublattice A and B
- ground state: opposite magnetization of A and B edge states
- staggered magnetization

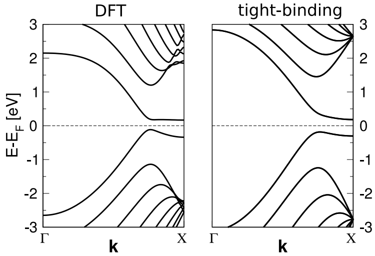
Modelling edge state magnetism

- staggered magnetization of edge state:

$$H_{\text{mag}} = \begin{cases} \mathbf{M} \cdot \mathbf{s} & \text{on sublattice A} \\ -\mathbf{M} \cdot \mathbf{s} & \text{on sublattice B} \end{cases}$$



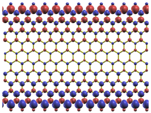
- fit magnetization to DFT calculations:



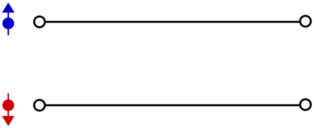
Spin transport at the edges

- magnetic edge states are strongly localized at the edges
- transport mediated through edge states
  nnn-coupling in tight-binding Hamiltonian essential !

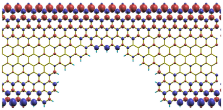
➜ two-wire mechanism of spin transport in graphene ribbon:



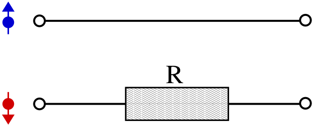
➜



➜ break up-down symmetry to achieve spin conductance !

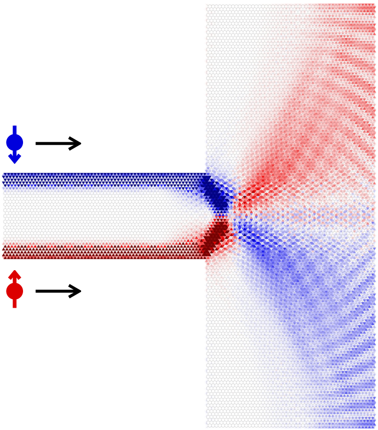


➜



(alternative proposal: Son *et al.* Nature (2007))

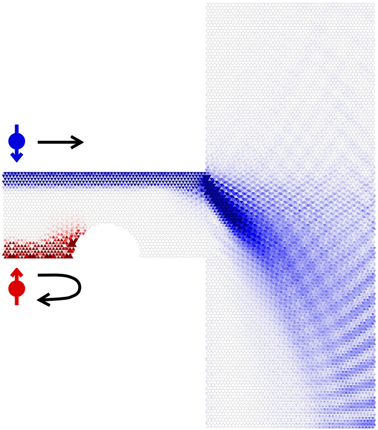
Spin injection in graphene



➜ spin Hall type effect

M. Wimmer, I. Adagideli, S. Berber, D. Tomanek, KR, Phys. Rev. Lett. (2009)

Spin injection in graphene

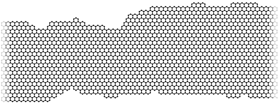


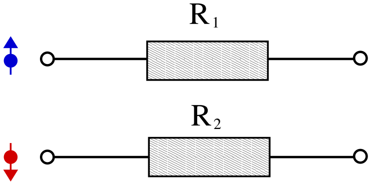
➜ net spin conductance !

M. Wimmer, I. Adagideli, S. Berber, D. Tomanek, KR, Phys. Rev. Lett. (2009)

Ribbons with rough edges

more realistic in today's experiments: rough edges:





Spin conductance: $G_s = G_\uparrow - G_\downarrow$

If $R_1 = R_2$, $\langle G_s \rangle = 0$.
However: **mesoscopic conductance fluctuations**!

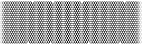
Spin conductance fluctuations:

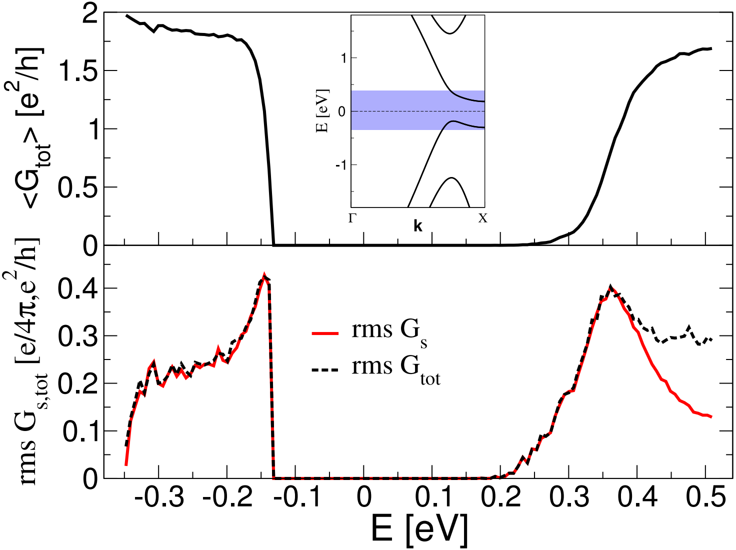
$$\mathrm{Var}\ G_s = \mathrm{Var}\ G_\uparrow + \mathrm{Var}\ G_\downarrow = \mathrm{Var}\ G_{\mathrm{tot}}$$

$$\mathrm{rms}\ G_s = \sqrt{\mathrm{Var}\ G_s} = \mathrm{rms}\ G_{\mathrm{tot}}$$

Spin conductance fluctuations

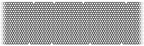
Disordered graphene nanoribbon: averaged properties

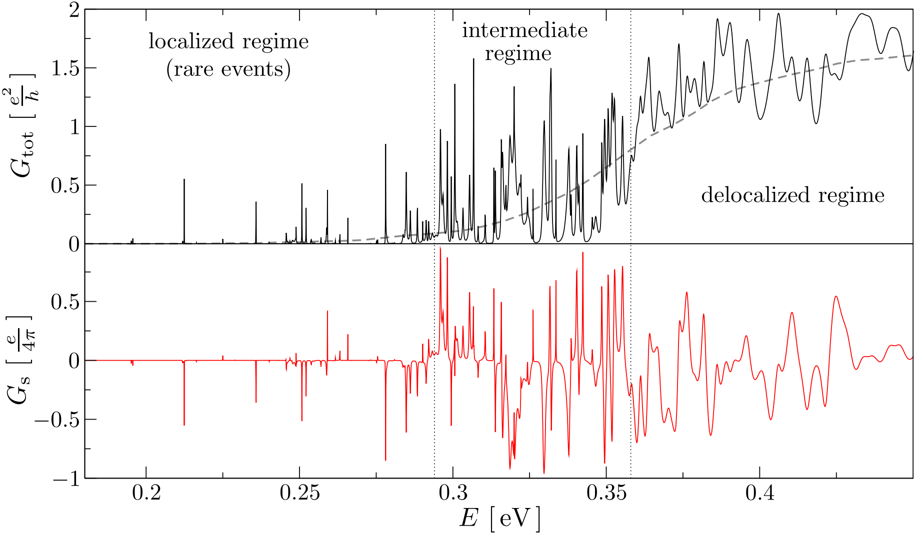




Spin conductance fluctuations

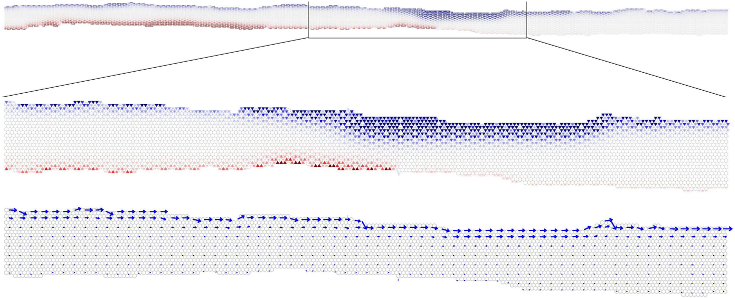
Disordered graphene nanoribbon: single ribbon





Spin conductance fluctuations

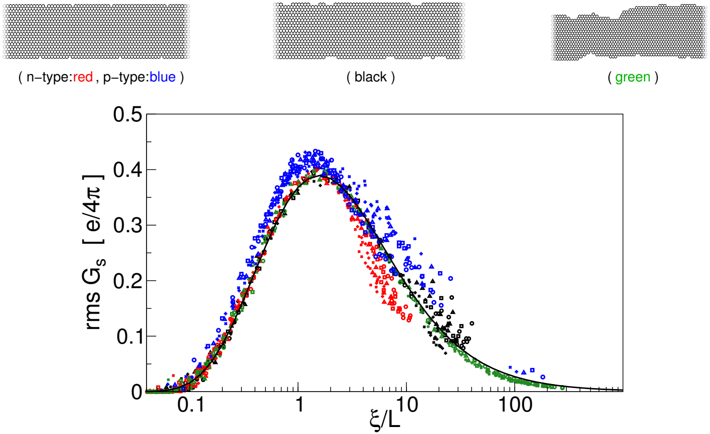
typical spin density in a disordered graphene nanoribbon:



corresponding spin current density

Universality of spin conductance fluctuations

different disorder models:



- fluctuations are universal (independent of type of disorder)
- correspond to transmission statistics through 1d disordered chain

  O. N. Dorokhov, JETP Lett. (1982); P. A. Mello *et al.*; Ann. Phys. (NY) (1988).

Thanks to ...

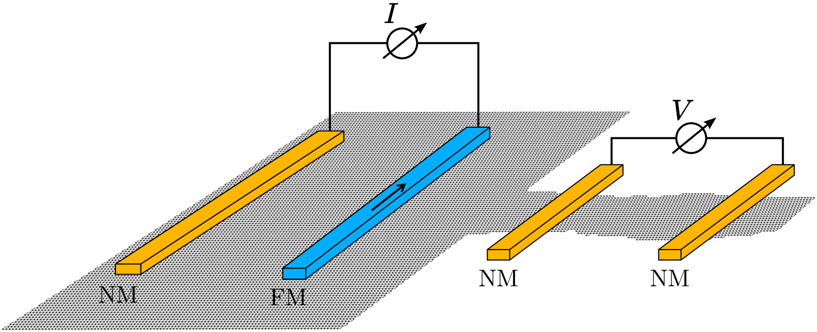
- Michael Wimmer (Leiden)
- Jürgen Wurm (Regensburg)
- Inanc Adagideli (Istanbul)
- Adam Rycerz (Regensburg / Krakow)
- Harold Baranger (Duke Univ.)
- Savas Berber (Istanbul)
- David Tomanek (East Lansing)

thanks to the German Science foundation (DFG)
and Alexander von Humboldt foundation

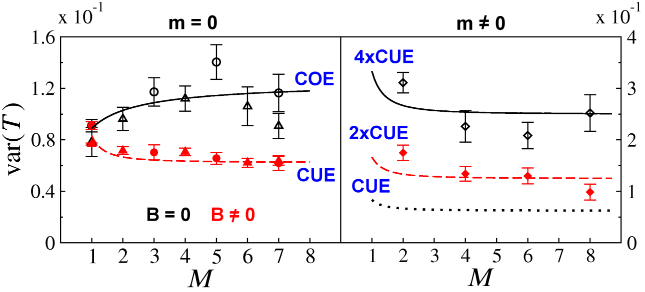
All-electrical detection of edge magnetism

Up to now: No direct experimental proof for edge magnetism

➜ Measuring the spin conductance G_s of a nanoribbon would yield conclusive evidence for edge magnetism.



Universal Conductance Fluctuations



- COE → CUE transition for systems with rough egdes (as in 2DEG billiards)
- 4 CUE → 2 CUE transition for systems with smooth mass boundary

- strong intervalley scattering (no mass, $\tau_{KK'} \ll \tau_{\text{esc}}$)

	S	WL	CF
$B = 0$	COE_{2M}	yes	var(COE)
$B \neq 0$	CUE_{2M}	no	var(CUE)

- strong intervalley scattering (no mass, $\tau_{KK'} \ll \tau_{\text{esc}}$)

	S	WL	CF
$B = 0$	COE_{2M}	yes	var(COE)
$B \neq 0$	CUE_{2M}	no	var(CUE)

- no intervalley scattering (mass confinement, $\tau_{KK'} \gg \tau_{\text{esc}}$)

	S	WL	CF
$B = 0$	$\begin{pmatrix} CUE_M & 0 \\ 0 & CUE_M \end{pmatrix}$	no	4var(CUE)
$B \neq 0$	$\begin{pmatrix} CUE_M^{(1)} & 0 \\ 0 & CUE_M^{(2)} \end{pmatrix}$	no	2var(CUE)