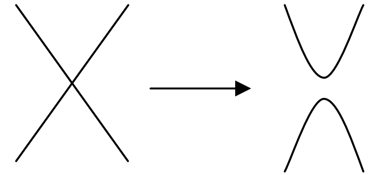
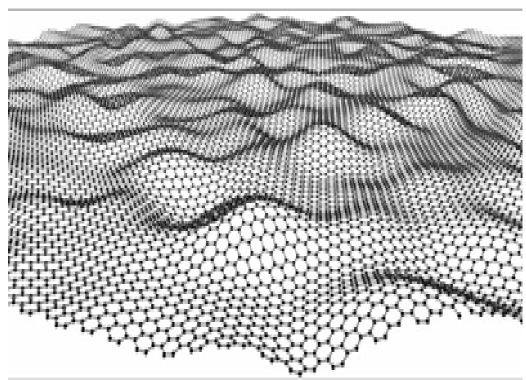
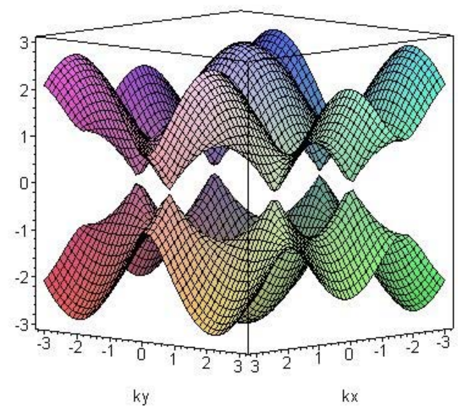
Transition from metallic to insulating behavior



Klaus Ziegler

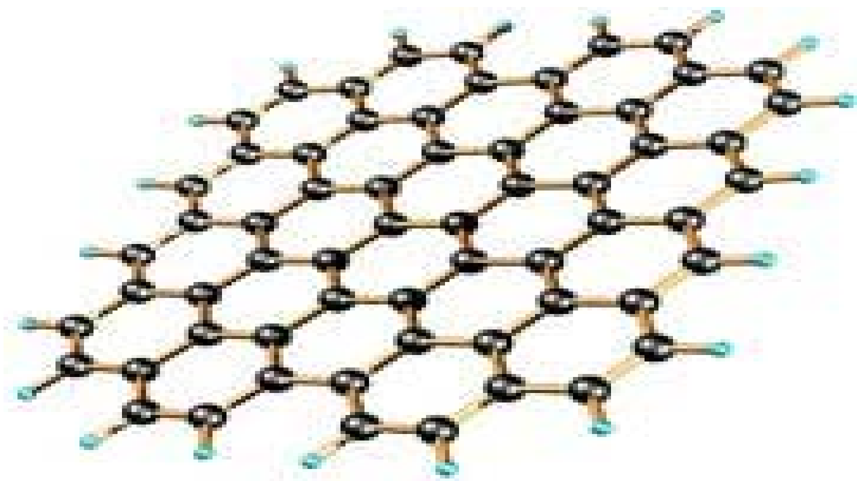


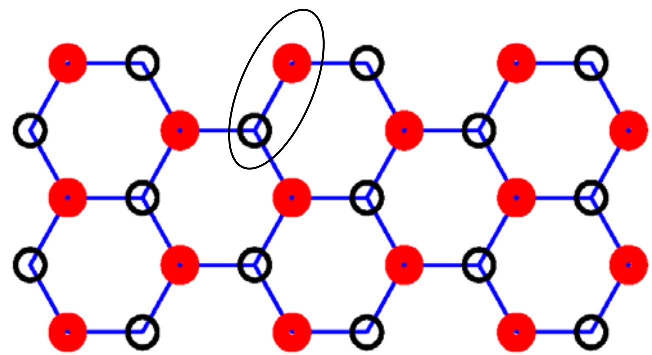


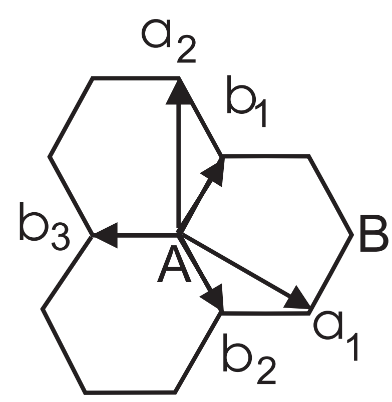
Workshop on Graphene, Benasque, July 2009

Structure of Graphene

Honeycomb lattice formed by carbon atoms





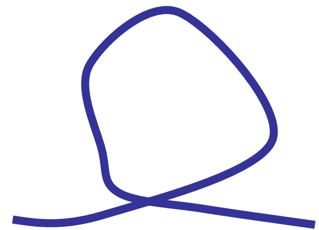


Semenoff, PRL 53, 2449 (1984)

Outline: gap opening and disorder

- monolayer vs. bilayer graphene
- some experimental facts
- diffusive regime: min. conductivity etc.
- gap opening: MI transition
- minimal conductivity for a random gap
- localized regime: hopping transport
- breaking valley symm.: Nernst conductivity
- Rabi oscillations between Landau levels
- local probing: local density of state correlations

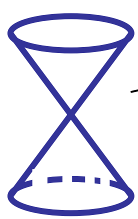
What is so special about the transport in graphene?

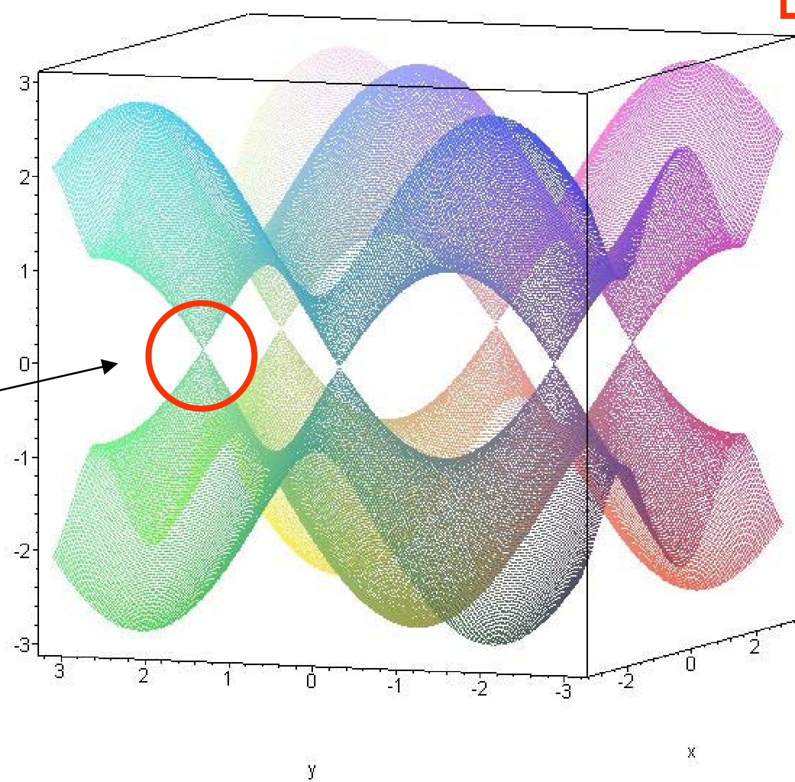


a) two-dimensional structure

b) Band structure: semimetal

$$\boxed{H = h_1\sigma_1 + h_2\sigma_2}$$

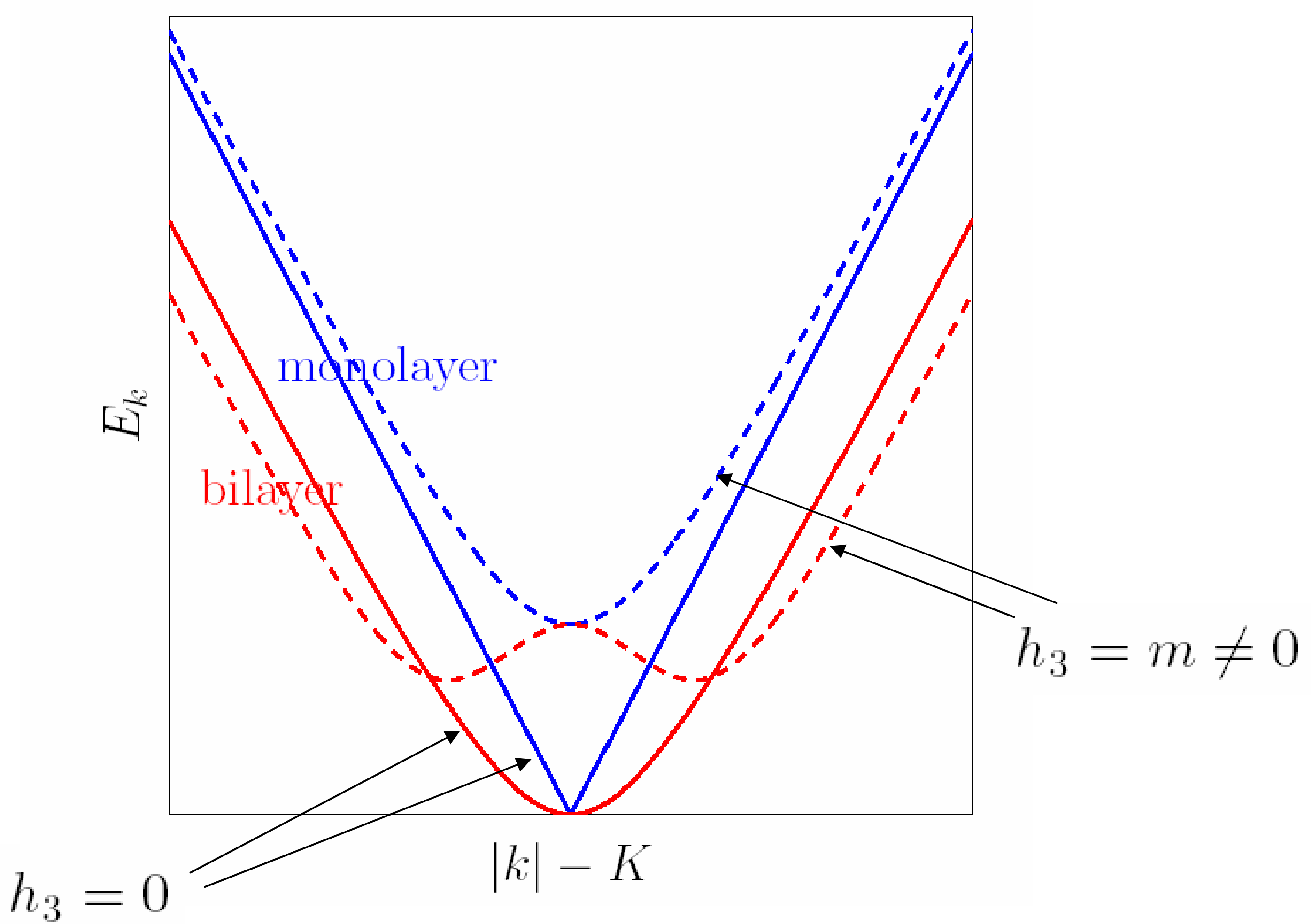




Dirac fermions

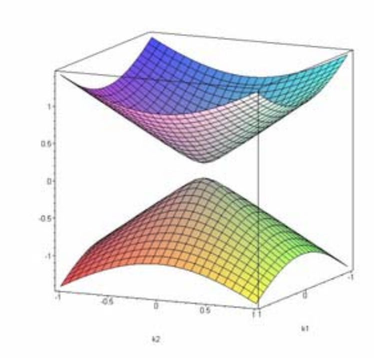
Low-energy spectrum

$$\boxed{H = h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3}$$

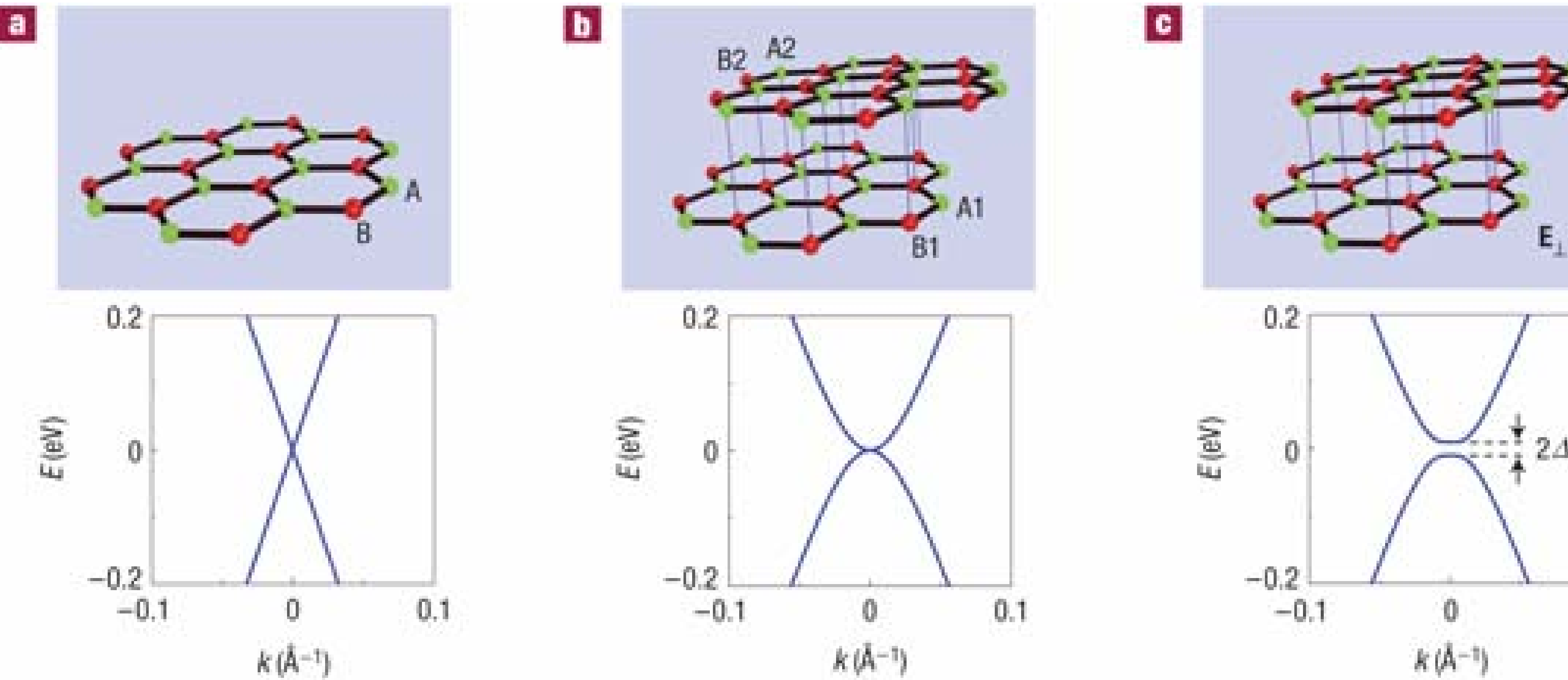


Gap opening: experimental facts

- gated graphene:
- continuous change between holes & electrons
- metallic behavior, high mobility
- Opening of a gap:
- ML: Hydrogenation
- BL: two gates
- magnetic field



monolayer & bilayer graphene:
low-energy dispersion

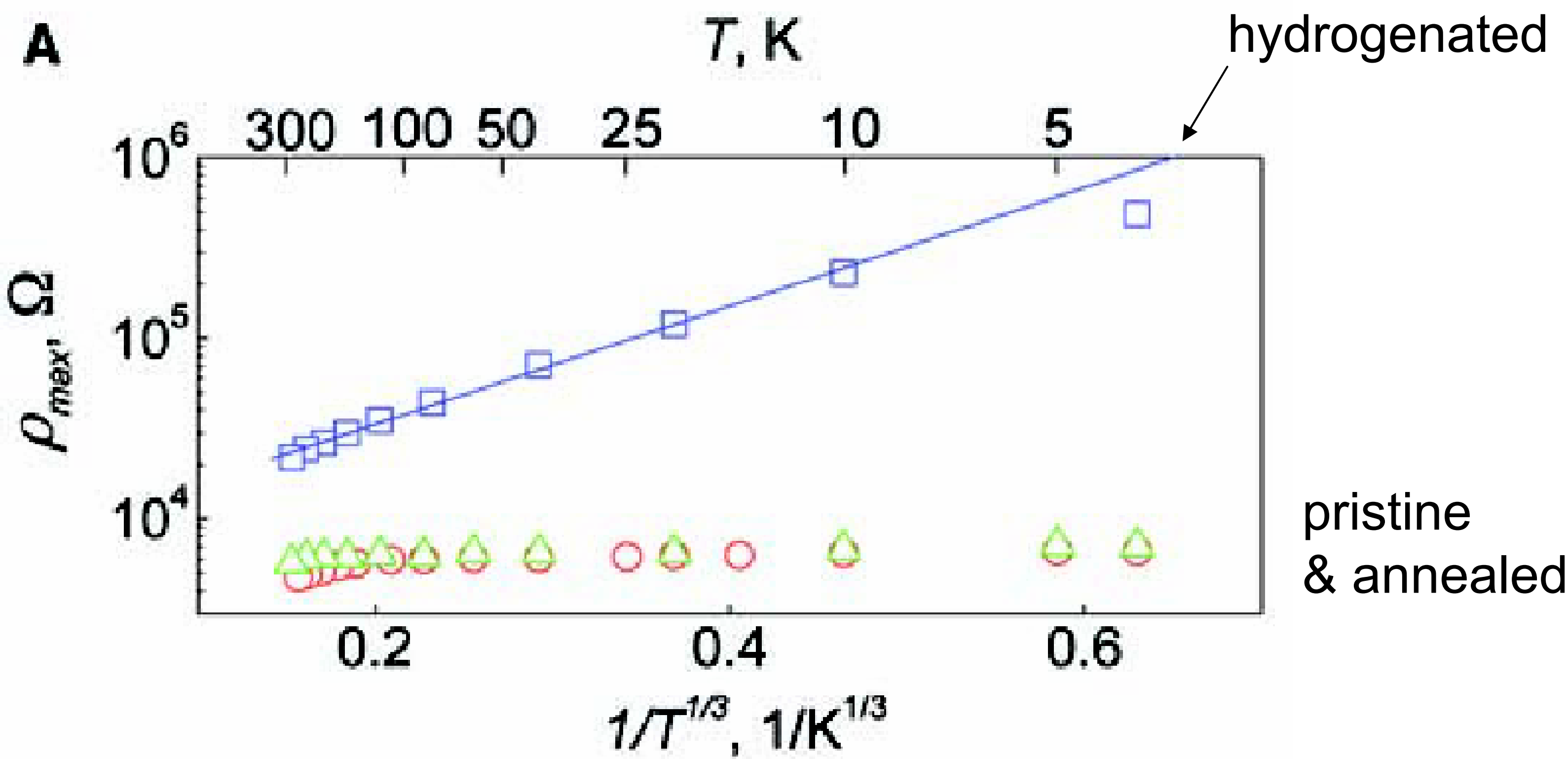


has been realized experimentally

Oostinga et al., Nature Materials 7, 151

graphene + hydrogen

variable-range hopping [Mott]: $\boxed{\sigma(T) \approx \sigma_0 e^{-(T_0/T)^{1/3}}}$

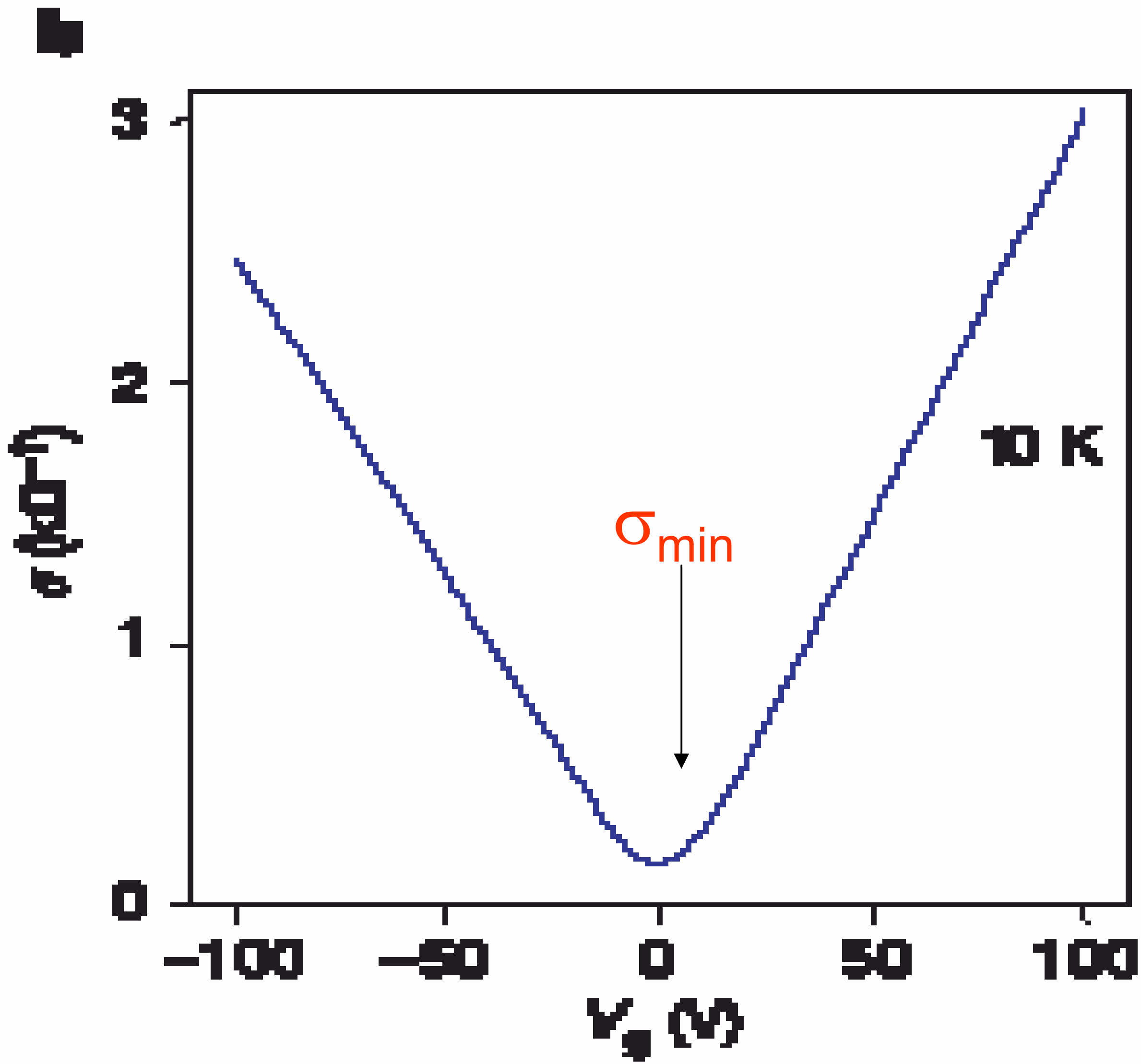


No gap but localized states!

Elias et al., Science 323, 610

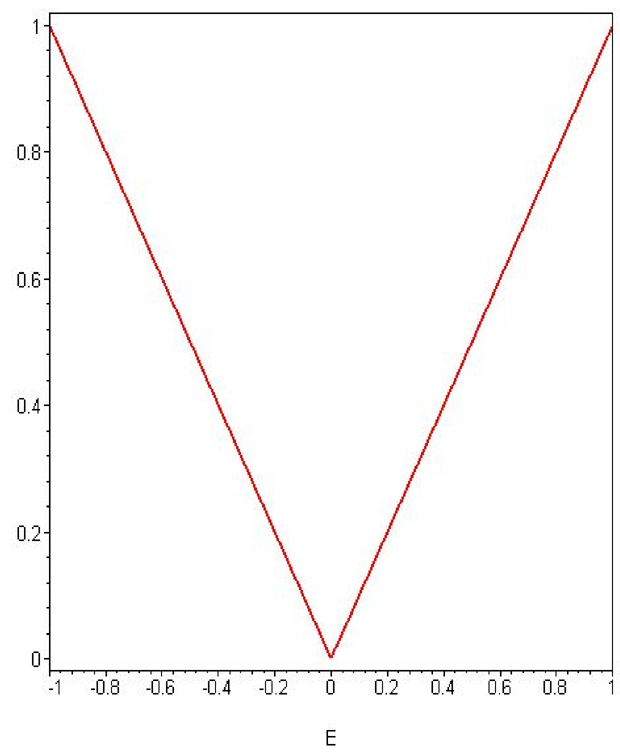
Conductivity in gated graphene

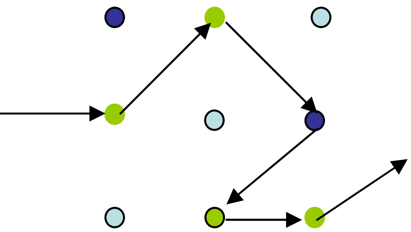
[Novoselov et al., Nature 438 (2005)]



$$\sigma(k_F) \propto \tau\rho$$

Density of States





Transport Theory

Scattering $\longrightarrow$ diffusive transport

Classical Boltzmann theory
for the conductivity

Density of states

Einstein relation: $$\sigma(k_F) \propto \tau\rho$$

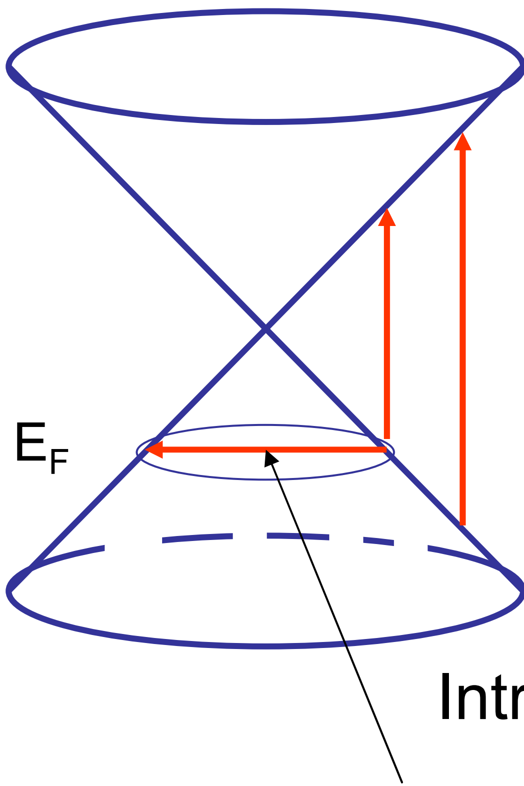
or: diffusion coefficient D

Drude theory: $$\sigma(\omega) = \frac{\sigma_0 \tau}{1 + i\omega\tau}$$

intra- & interband scattering

$\boxed{E_F < 0:}$

dynamical conductivity $\sigma(\omega)$



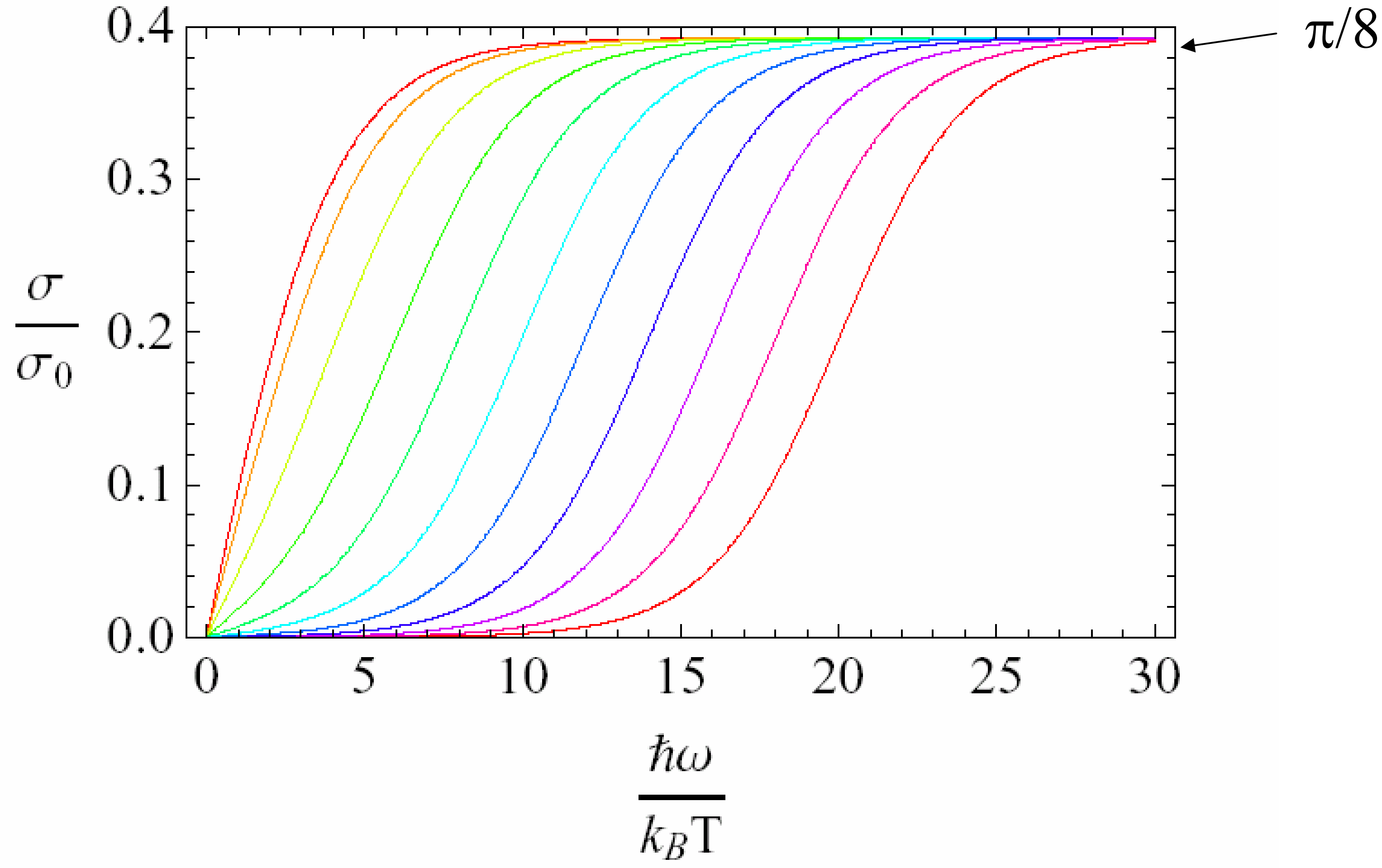
$-\omega/2$

Interband scattering: $\sigma = \pi/8$

Intraband scattering: $\sigma \sim \eta/\omega$

η: scattering rate

dynamical conductivity: Theory



Minimal conductivity:
intra- & interband scattering

$\boxed{E_F \sim 0:}$ Dirac point

inter- & intraband scattering are mixed up

$\boxed{\text{limiting processes } (T,\omega,\eta \longrightarrow 0) \text{ do not commute}}$

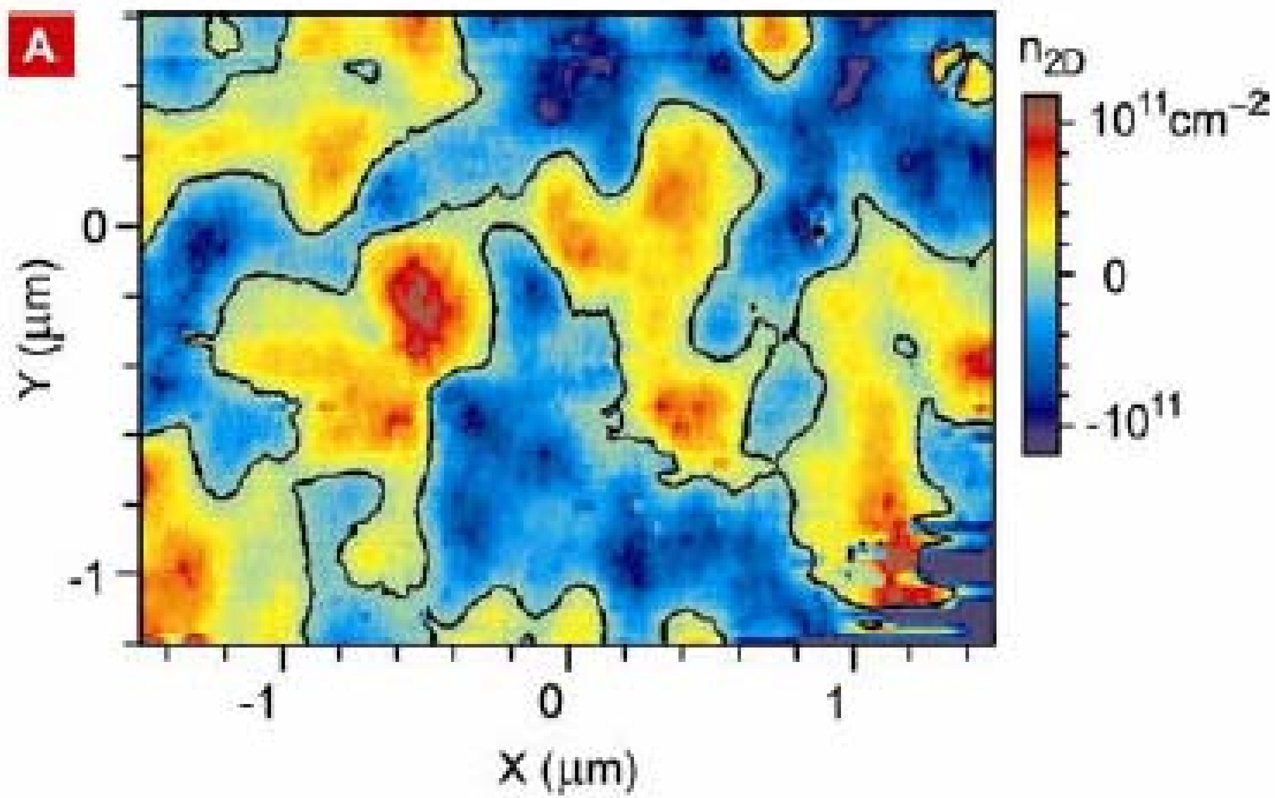
$$\sigma_1^{\min} = \frac{1}{\pi}\frac{e^2}{h}$$

Z, PRB75, 233407

$$\sigma_2^{\min} \approx \frac{\pi}{8}\frac{e^2}{h} \quad \text{for } \eta \approx 0 \qquad \sigma_3^{\min} \approx \frac{\pi}{4}\frac{e^2}{h} \quad \text{for } \eta \approx \omega$$

Disorder II

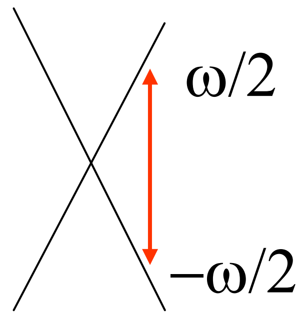
charge distribution @ T=0.3 K



Martin et al., Nature Physics 4 (2007)

Effect of disorder: transport I

Kubo formalism



Interband scattering:

$$\boxed{\sigma_0(\omega) = -\frac{e^2}{2h}\omega^2 \langle \Phi_{-\omega/2} | r_k^2 | \Phi_{\omega/2} \rangle}$$

$$= \frac{e^2}{2h}\omega^2 \sum_r r_k^2 Tr_2[\sigma_3 G_{r0}(\omega/2 + i\eta)\sigma_3 G_{0r}(\omega/2 + i\eta)]$$

Effect of disorder: transport II

Factorization:

$$\langle G^{+}_{rr',jj} G^{+}_{r'r,kk} \rangle \approx \langle G^{+}_{rr',jj} \rangle \langle G^{+}_{r'r,kk} \rangle$$

SCBA: $\langle G^{\pm} \rangle \approx (H_0 \pm i\,\eta)^{-1} \qquad \eta \sim e^{-\pi/g}$

(MLG)

Problem: exponential decay on scale $1/\eta$

Diffusion stops for scales L > $1/\eta$

Diffusion on large scales

large scale properties are determined by

symmetries

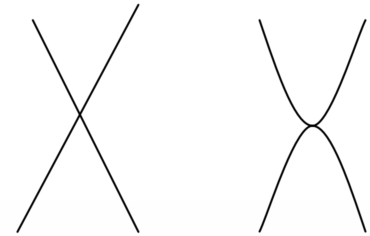
&

spontaneous symmetry breaking

classical examples in 2D:

-phase transition in the Ising model

-Kosterlitz-Thouless transition in the XY model



chiral symmetry

Hamiltonian (gapless low-energy quasiparticles of monolayers and multilayers)

$$\boxed{H = h_1\sigma_1 + h_2\sigma_2}$$

continuous symmetry transformation:

$$H \to e^{i\alpha\sigma_3} H e^{i\alpha\sigma_3} = H \quad (0 \le \alpha < 2\pi)$$

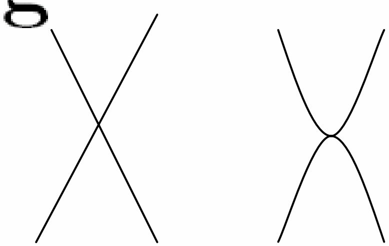
special case: $\alpha = \pi/2$

$$e^{i\alpha\sigma_3} = i\sigma_3$$

implies for eigenspinor $\mathbf{\Phi}_E$

$$\boxed{\sigma_3 \mathbf{\Phi}_E = \mathbf{\Phi}_{-E}}$$

Spontaneous symmetry breaking



order parameter for SCSB:

$$\lim_{\epsilon\longrightarrow 0}[(H+i\epsilon)^{-1}_{rr}-(H-i\epsilon)^{-1}_{rr}]\propto\rho_r(0)\propto\eta$$

diffusion propagator

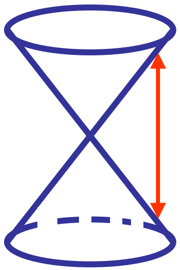
$$\boxed{\frac{1}{K(q)}\sim\frac{\eta^3/4}{i\omega+Dq^2}}$$

with the diffusion coefficient

$$D=\frac{1}{2\eta}\int_k\sum_j\left(\frac{\partial h_j}{\partial k_l}\frac{\partial h_j}{\partial k_l}-\frac{\partial^2 h_j}{\partial k_l^2}h_j\right)$$

PRB78, 125401

Transport and localization



localization:

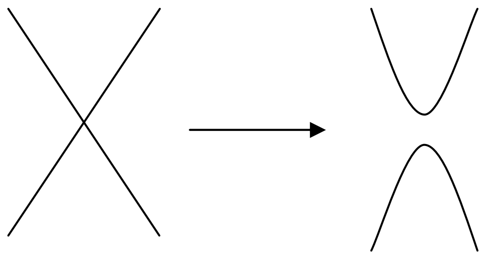
$$\langle \Phi_{\omega/2} | r_k^2 | \Phi_{-\omega/2} \rangle = -\frac{4}{\omega^2} \begin{cases} \eta^3 D/g^2 & \text{for } \eta \gg \omega \\ 1 & \text{for } \eta \ll \omega \end{cases}$$

Conductivity due to inter-band scattering:

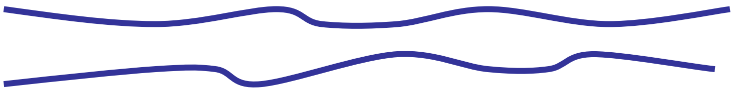
$$\sigma_0(\omega) = -\frac{e^2}{4\pi\hbar}\omega^2 \langle \Phi_{\omega/2} | r_k^2 | \Phi_{-\omega/2} \rangle \sim \frac{e^2}{\pi\hbar}\frac{\eta_0^2}{g^2} \int_k \sum_j \left(\frac{\partial h_j}{\partial k_l} \frac{\partial h_j}{\partial k_l} \right)$$

PRL100, 166801

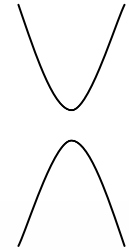
Breaking the sublattice symmetry:
Effect of gap opening on transport



- abrupt (first order) transition to insulator?
- Or smooth (second order) transition?
- Consequence of an inhomogeneous gap?



Opening of a gap by breaking the sublattice symmetry



broken chiral symmetry

Hamiltonian (gapful low-energy quasiparticles of monolayers and multilayers)

$$\boxed{H = h_1\sigma_1 + h_2\sigma_2 + m\sigma_3}$$

$m = V_A - V_B$

discrete symmetry transformations:

a) for $h_j^T = h_j$

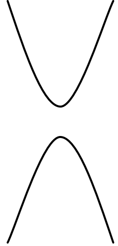
$$H \to -\sigma_2 H^T \sigma_2 = H$$

b) for $h_j^T = -h_j$

$$H \to -\sigma_1 H^T \sigma_1 = H$$

random gap: $\boxed{\langle m_r \rangle_m = \bar{m} \qquad \langle (m_r - \bar{m})(m_{r'} - \bar{m}) \rangle_m = g\,\delta_{r,r'}}$

Two-particle Green's function:



$$\hat{G}(i\epsilon) = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \begin{pmatrix} \sigma_0 & 0 \\ 0 & i\sigma_n \end{pmatrix} \times \begin{pmatrix} H + i\epsilon & 0 \\ 0 & H^T + i\epsilon \end{pmatrix}^{-1} \begin{pmatrix} \sigma_0 & 0 \\ 0 & i\sigma_n \end{pmatrix}.$$

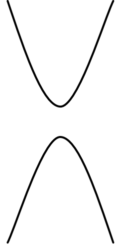
The extended Hamiltonian $\hat{H} = \mathrm{diag}(H, H^T)$ is invariant under a global "rotation"

$$\hat{H} \to e^{\hat{S}} \hat{H} e^{\hat{S}} = \hat{H}, \quad \hat{S} = \begin{pmatrix} 0 & \alpha\sigma_n \\ \alpha'\sigma_n & 0 \end{pmatrix} \qquad (18)$$

Symmetry is spontaneously broken: massless fermion mode

PRL80, 3113

Effective theory



scattering rate:

$$\boxed{\eta^2 = (m_c^2 - \bar{m}^2)\Theta(m_c^2 - \bar{m}^2)/4} \qquad m_c = \begin{cases} \dfrac{2\lambda}{\sqrt{e^{2\pi/g} - 1}} \sim 2\lambda e^{-\pi/g} & \text{(MLG)} \\ g/2 & \text{(BLG)} \end{cases}$$

disorder scaling function:

$$\langle\langle \Phi_{\omega/2} | r_k^2 | \Phi_{-\omega/2} \rangle\rangle_m = -\frac{\eta'^2}{(\omega/2)^2} \langle \Phi^0_{i\eta'} | r_k^2 | \Phi^0_{-i\eta'} \rangle$$

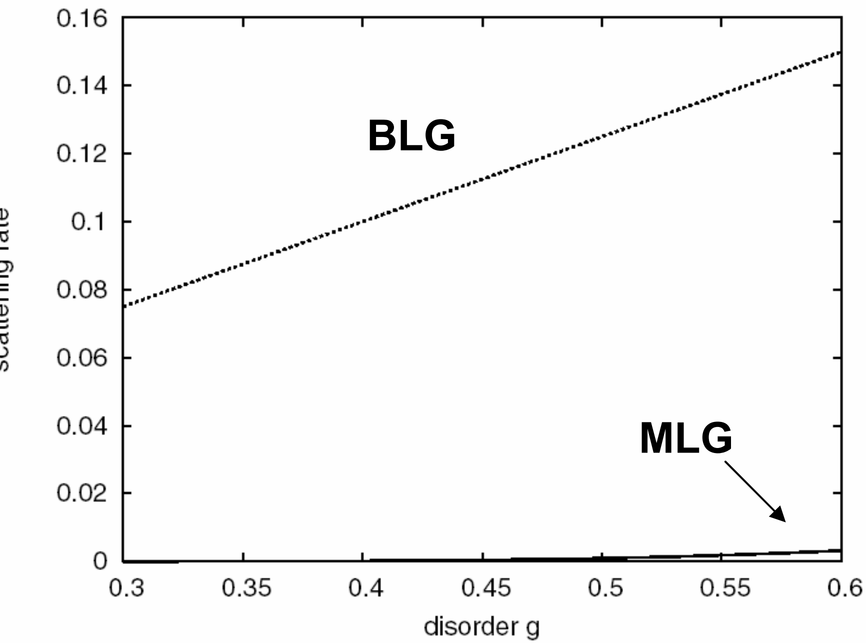
diffusion coefficient:

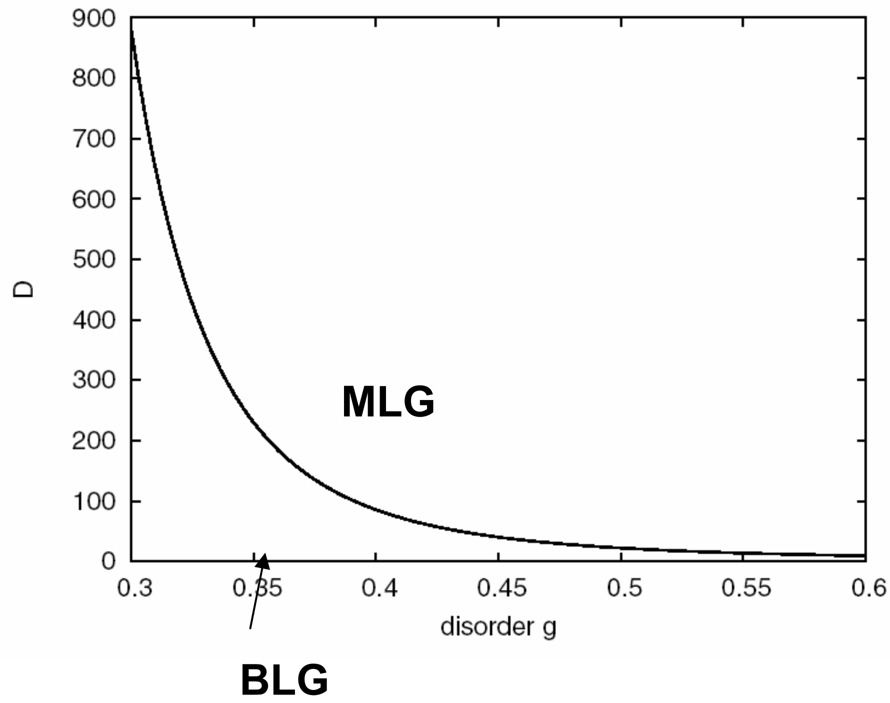
$$D = \frac{g\eta'}{2} \langle \Phi^0_{i\eta'} | r_k^2 | \Phi^0_{-i\eta'} \rangle$$

$$\boxed{\sigma_0(\omega) \sim \frac{4a\eta'^2}{\pi(4\eta'^2 + \bar{m}^2)} \Theta(m_c^2 - \bar{m}^2) \frac{e^2}{h}}$$

PRL102,126802; PRB79, 195424

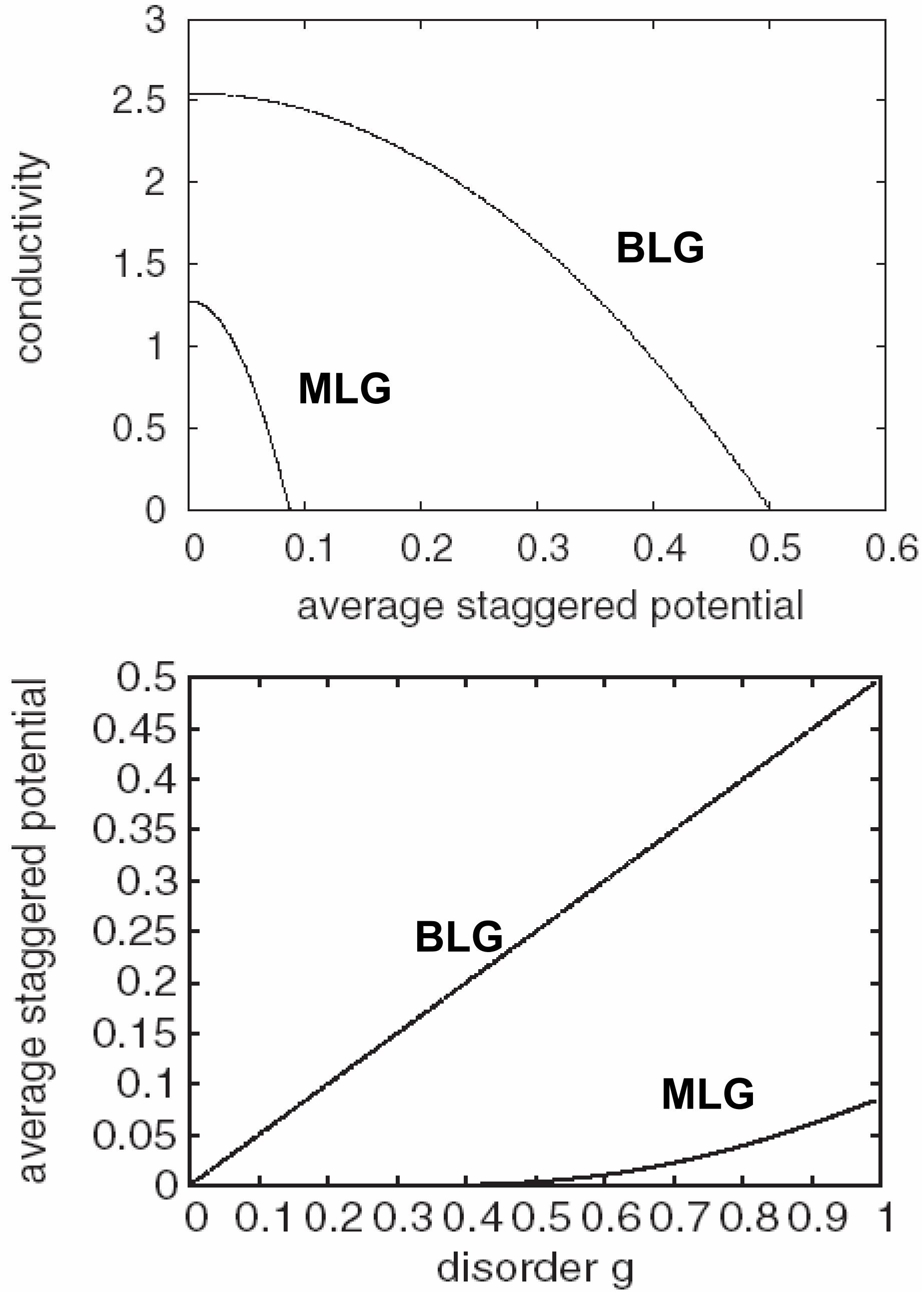
Scattering rate & diffusion coefficient





Z, PRB 79,195424

minimal conductivity



g=1

Z, PRL102, 12620

PHYSICAL REVIEW B, VOLUME 65, 012506

Thermal metal in network models of a disordered two-dimensional superconductor

J. T. Chalker,[1] N. Read,[2] V. Kagalovsky,[3,4] B. Horovitz,[4] Y. Avishai,[4] and A. W. W. Ludwig[5]

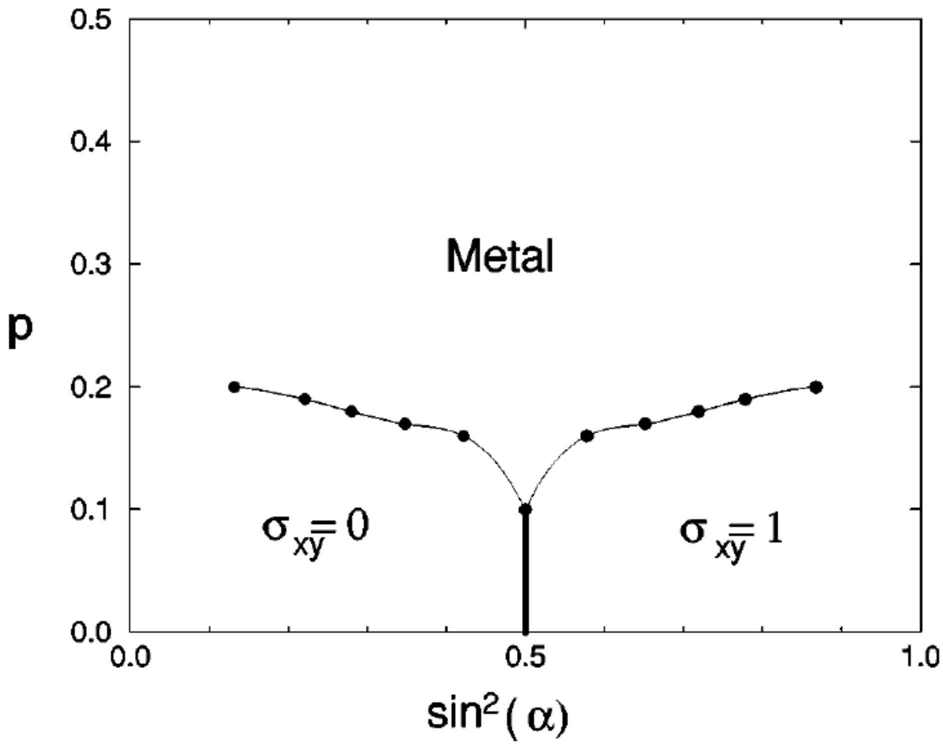
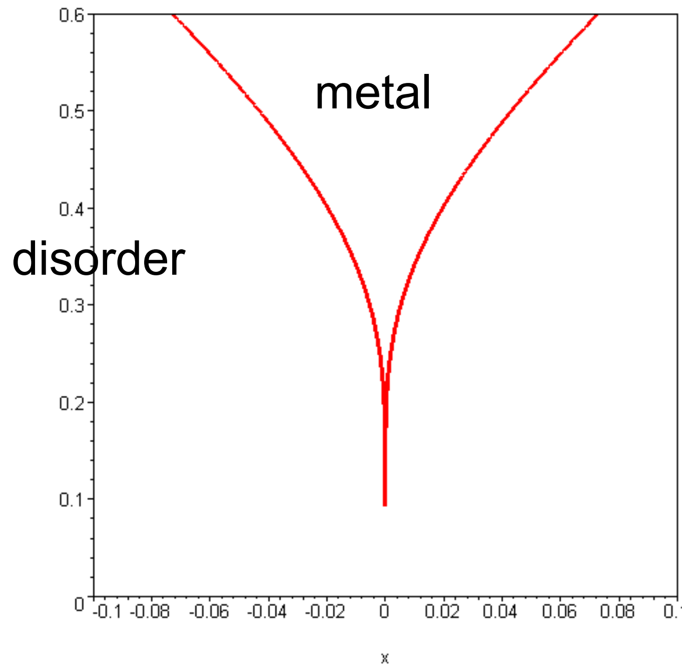


FIG. 1. The phase diagram of the CF model obtained from our numerical calculations.

phase diagram in the random-gap model

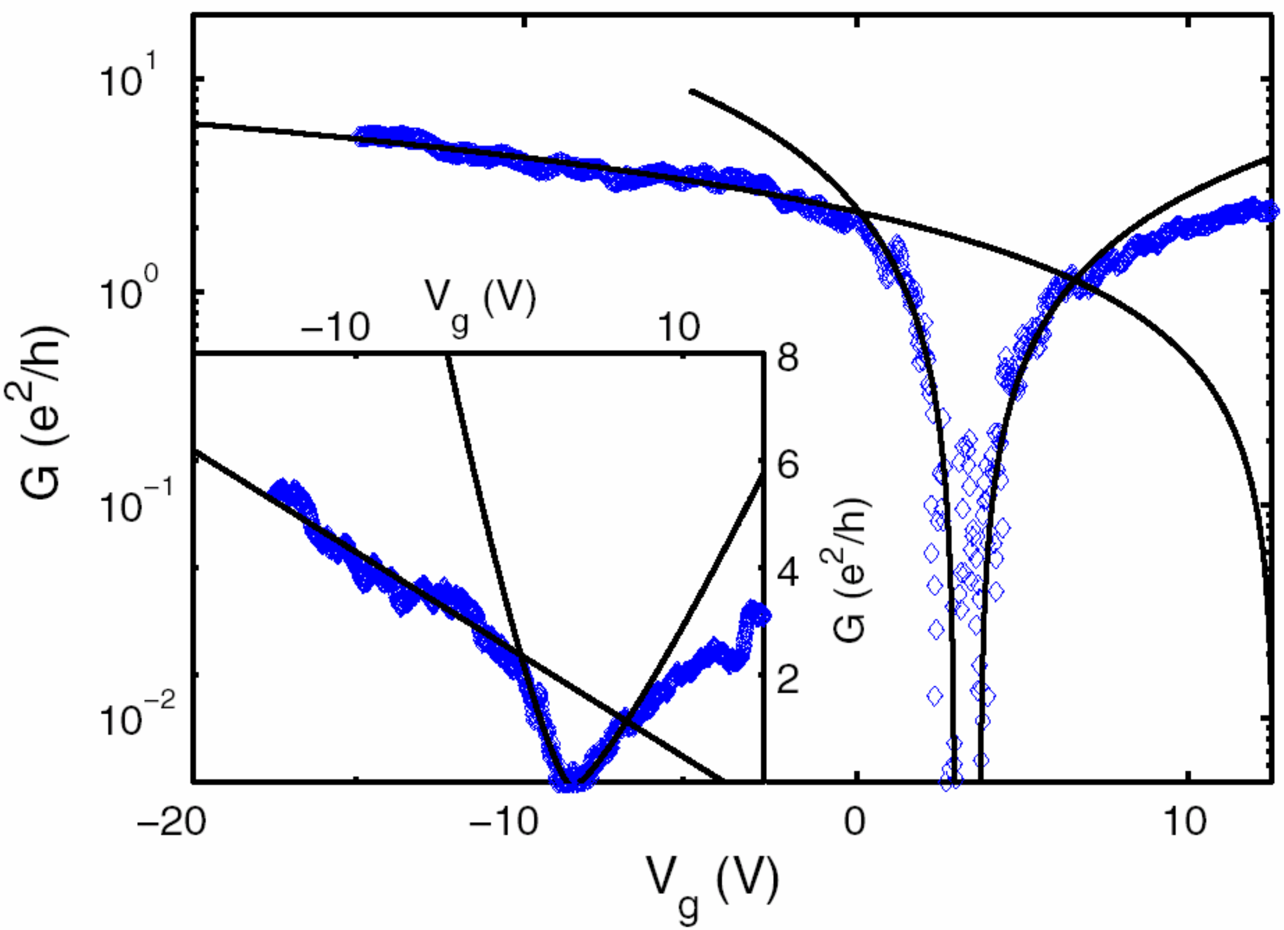


average gap parameter m

Conductivity transition due to gap opening

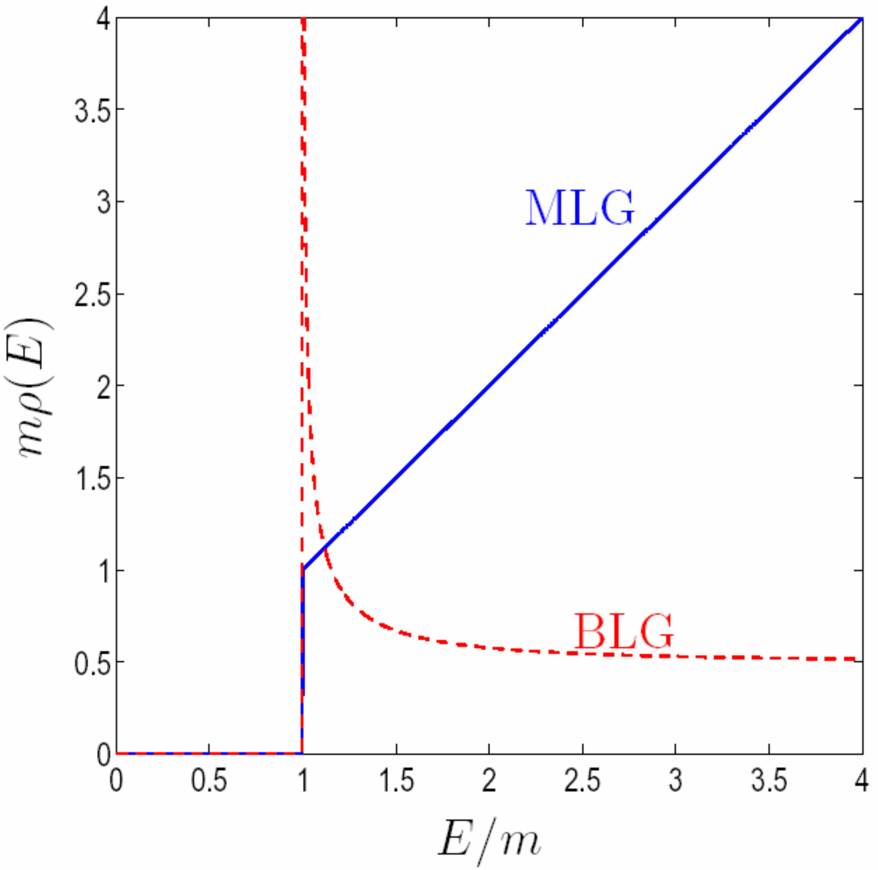
Adam et al., PRL101, 046404

fixed average gap



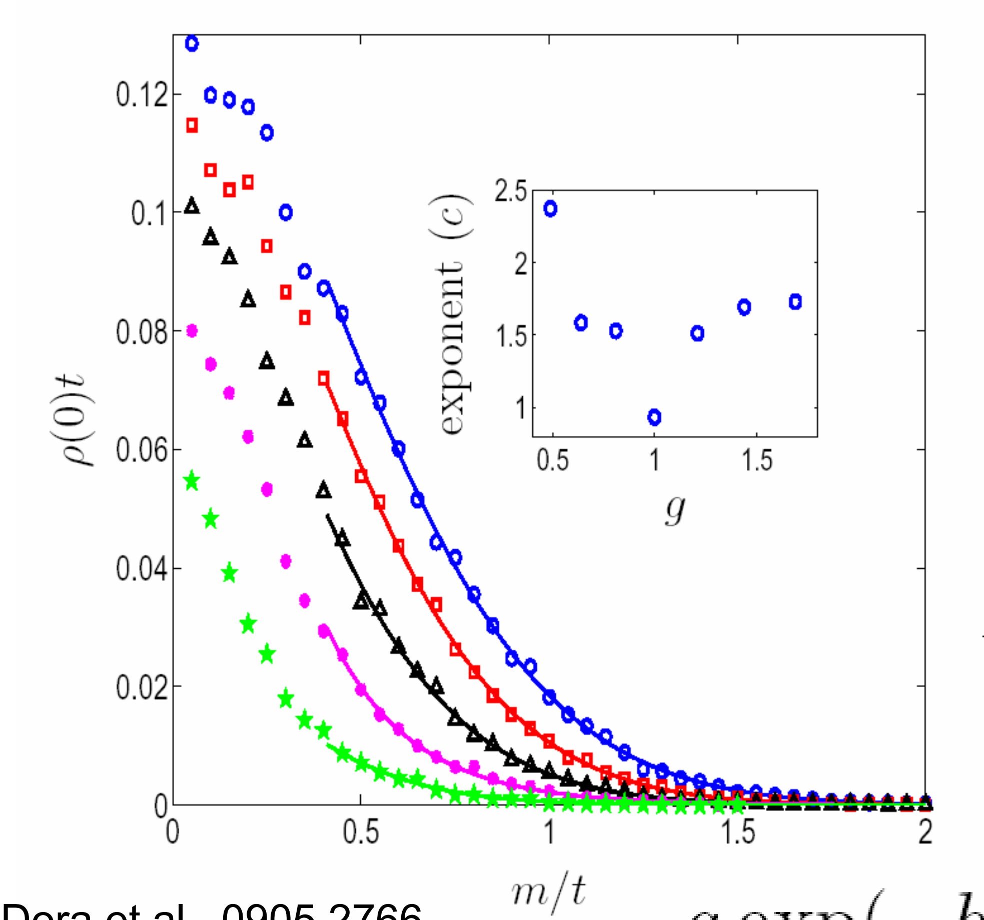
Behavior in the localized regime

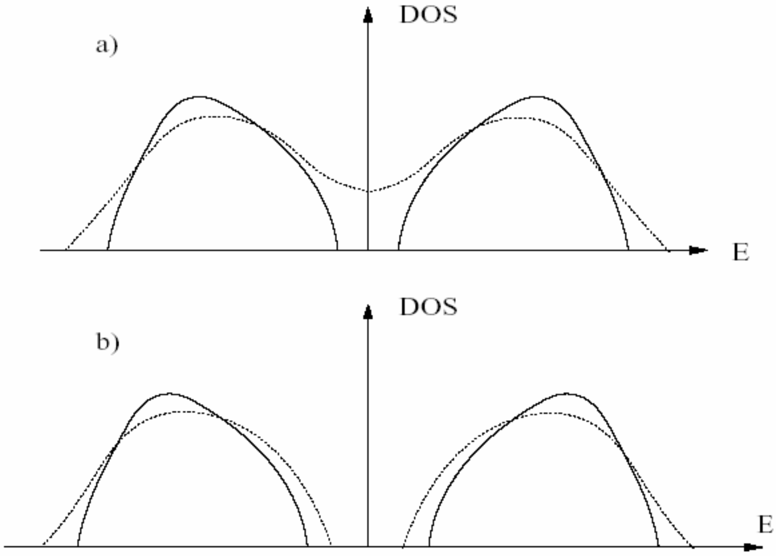
Density of states: uniform gap



Density of states: random gap model

DoS at the Dirac point: Lifshitz tails





Dora et al., 0905.2766

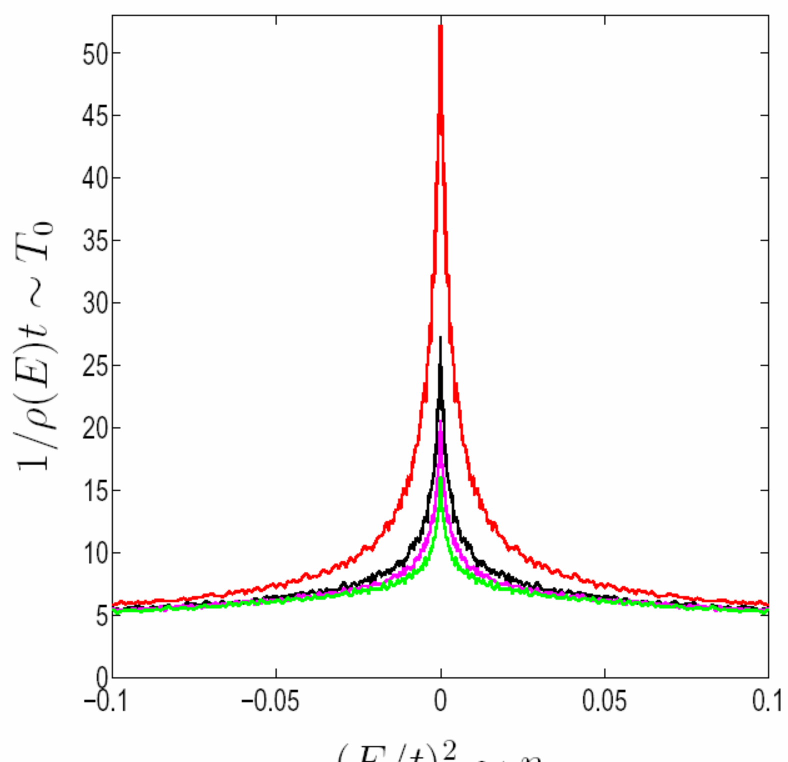
$$a\exp(-bm^c)$$

Hopping conductivity

variable-range hopping [Mott]:

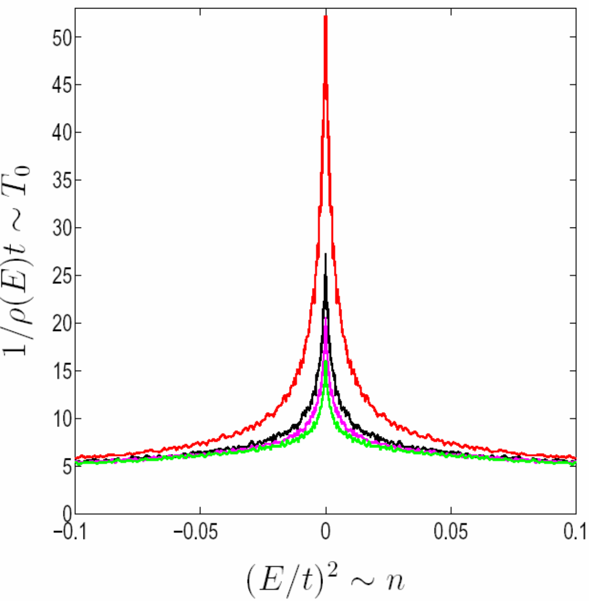
$$\sigma(T) \approx \sigma_0 e^{-(T_0/T)^{1/3}}$$

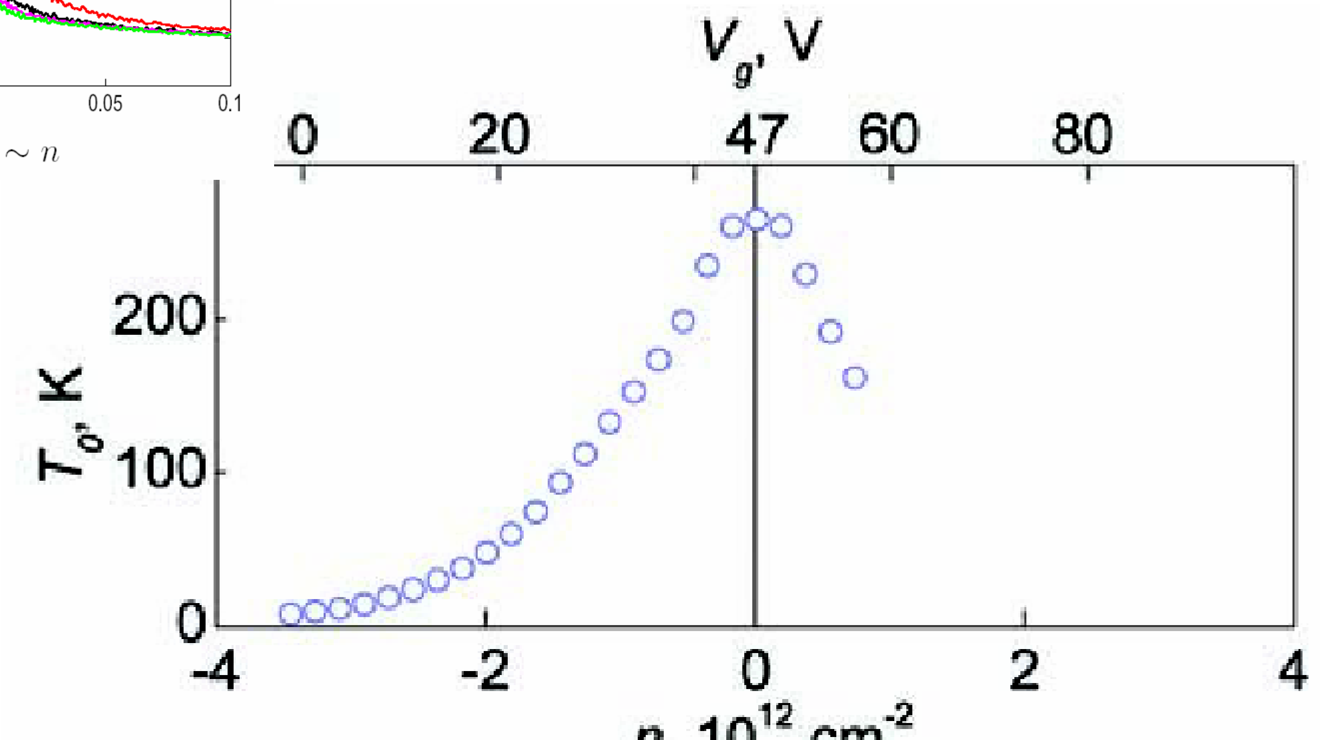
ora et al., 0905.2766



$$k_B T_0 \propto \frac{1}{\xi^2 \rho(E_F)}$$

Exponent T_0

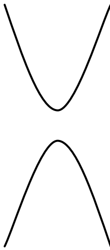




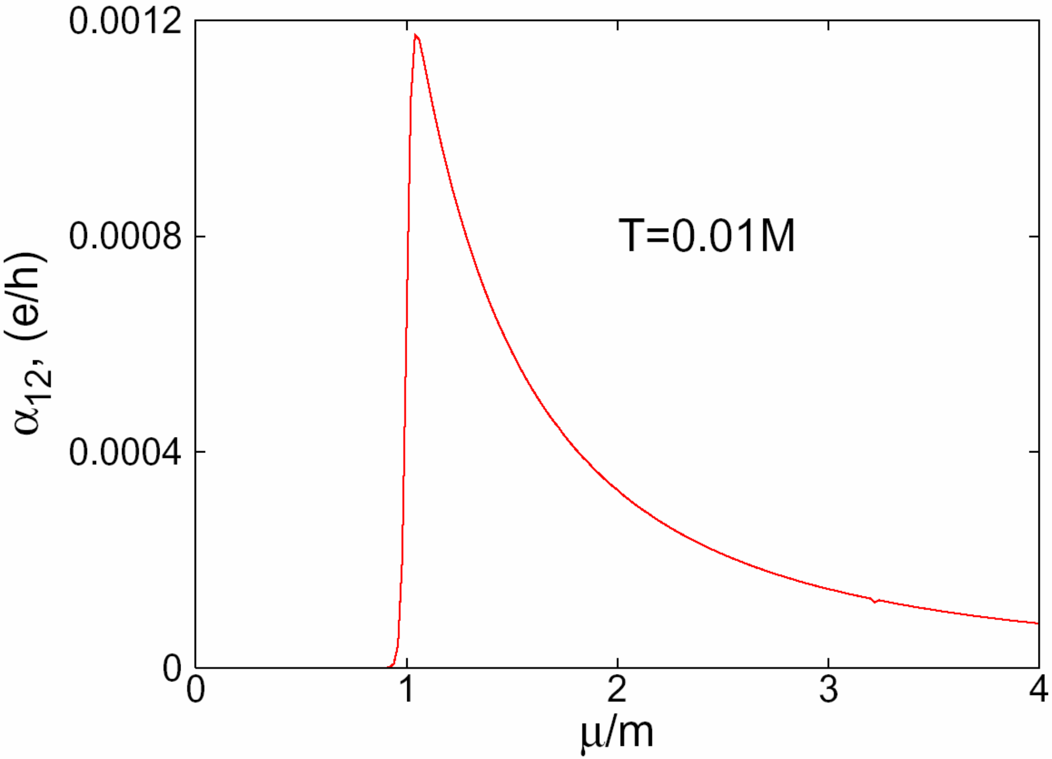
Thermal conductivity: anomalous Nernst effect

$$\boxed{J_x = \alpha_{xy}\,(-\partial_y T)}$$

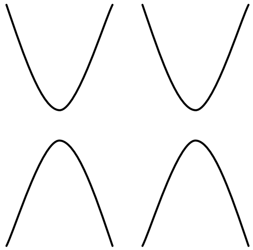
$$\boxed{H = h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3}$$



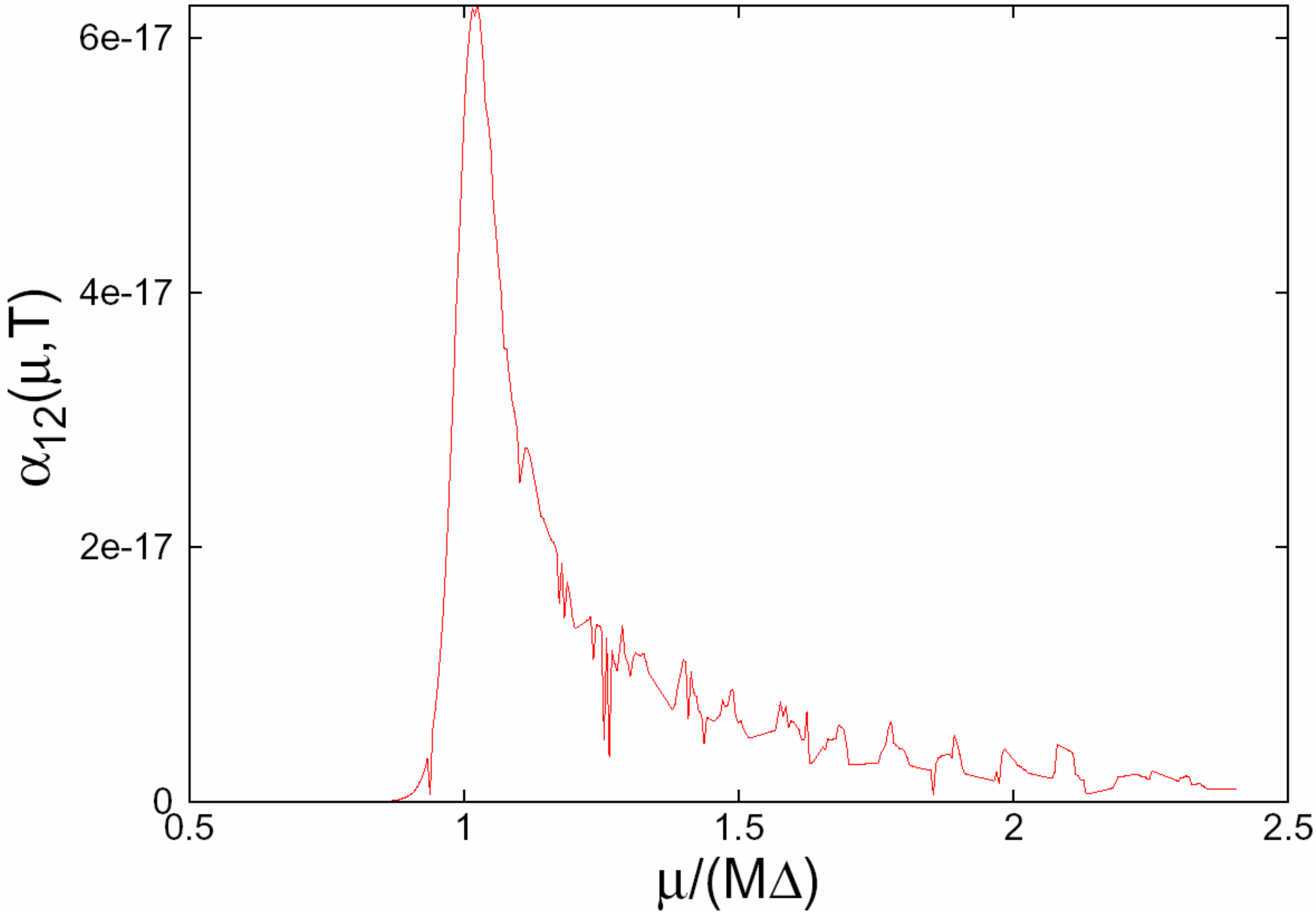
single massive Dirac cone:



Nernst coefficient II

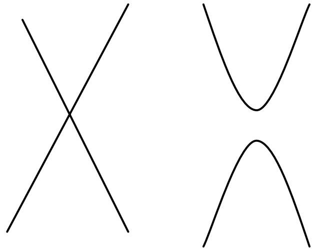


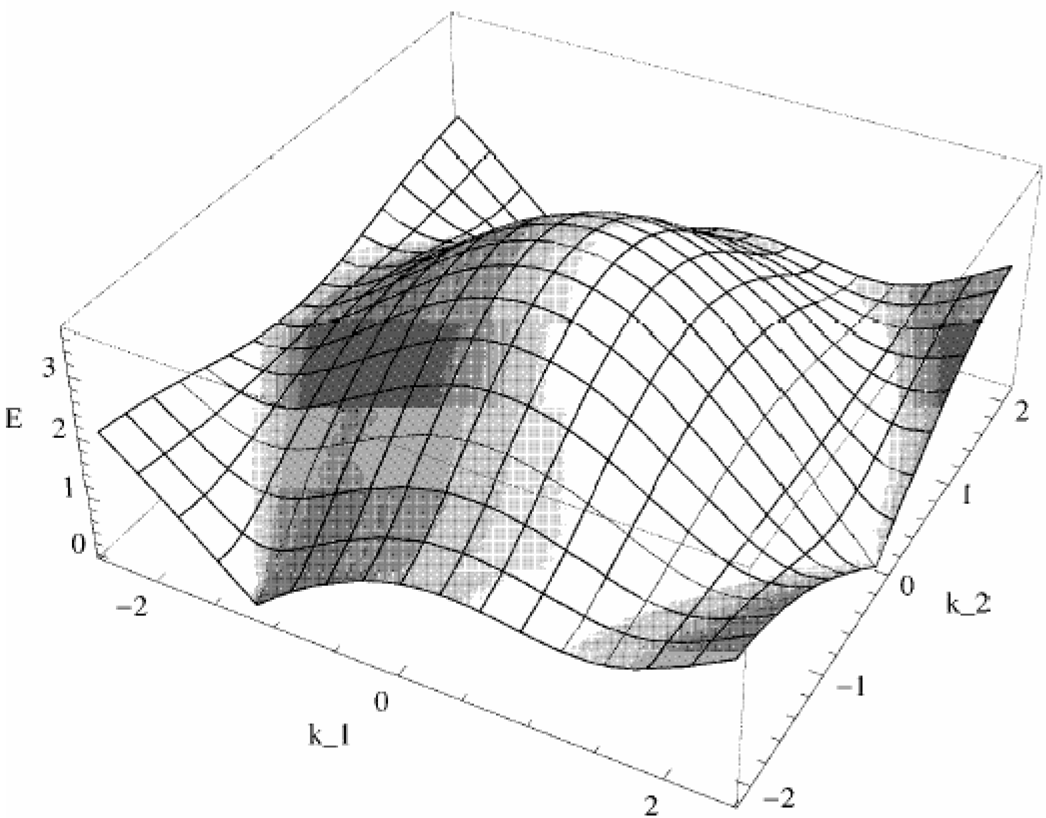
Honeycomb lattice with uniform gap



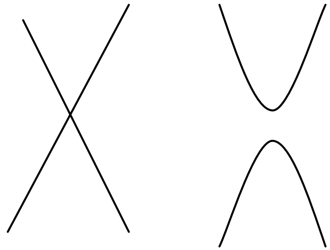
Breaking valley symmetry

$$H = h_1\sigma_1 + h_2\sigma_2 + h_3\sigma_3$$

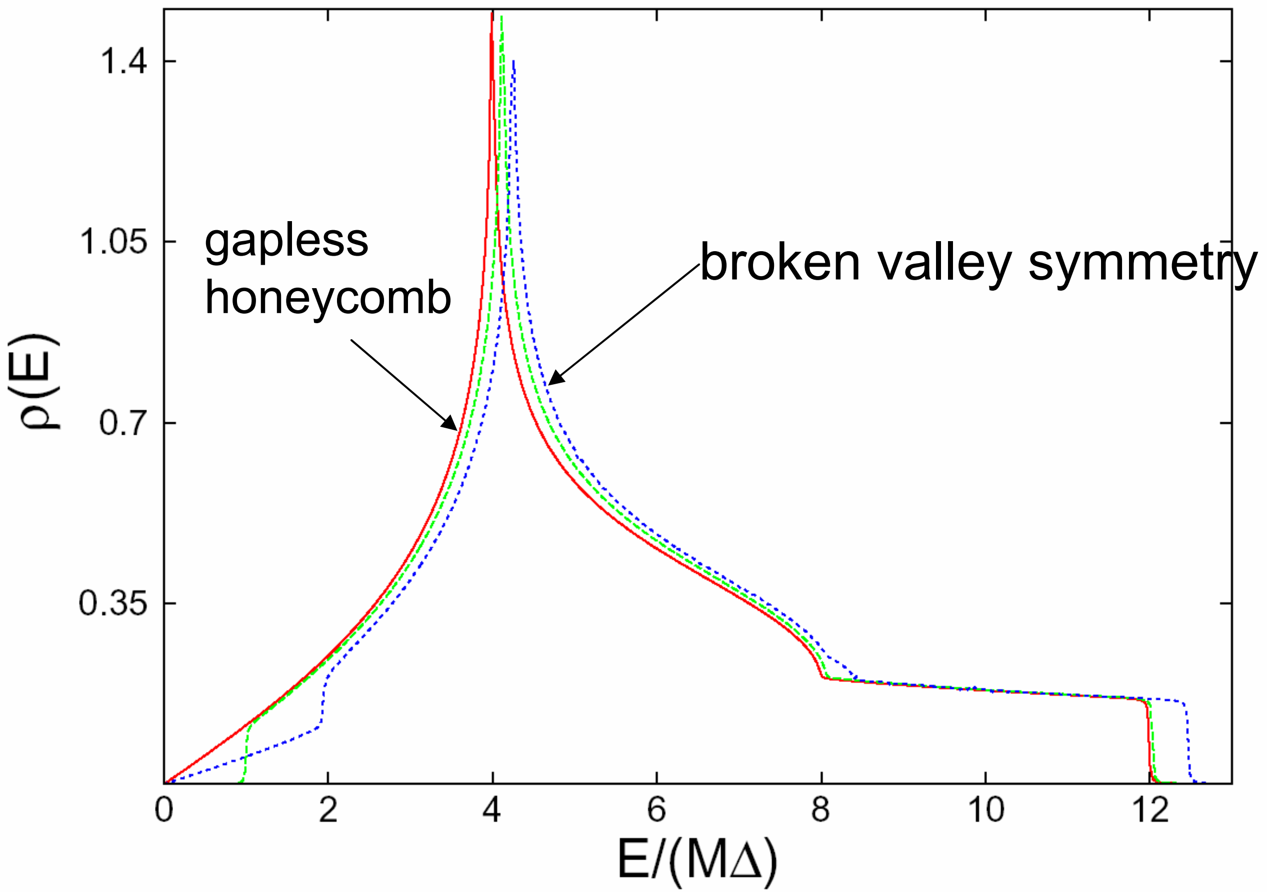




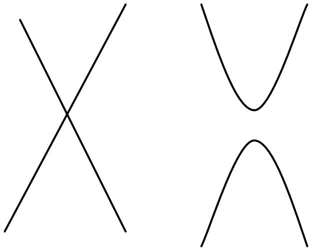
Densities of states



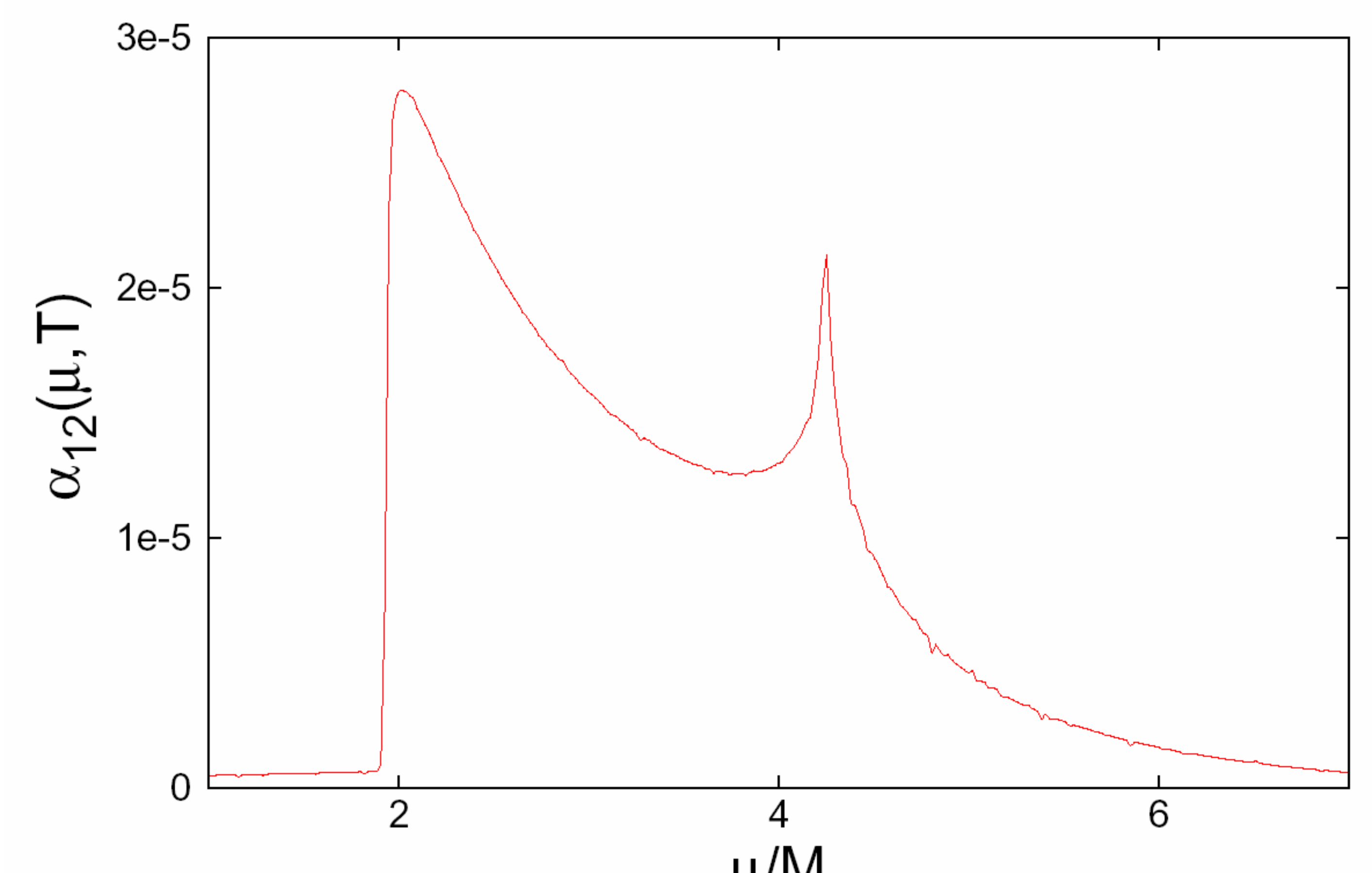
Honeycomb lattice with broken valley symmetry



Nernst coefficient III



Honeycomb lattice with broken valley symmetry



Conclusions

Interband scattering is relevant
Diffusion due to **massless modes**:
a) spontaneous chiral symmetry breaking (SCSB)
b) spontaneous super symmetry breaking (SSSB)
Constant optical conductivity

Fluctuating gap:
A) diffusion for small <m>
B) Metal-insulator transition for nonzero <m>
MIT is second order
gap formation: disorder creates localized states

Homogeneous magnetic field:
Rabi oscillations