CLASSICAL AND QUANTUM ASPECTS OF TOMOGRAPHY

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In this course I will consider some aspects of tomographic maps. In the first lecture I will introduce the Radon transform, which is the key mathematical tool for reconstructing the tomographic map of both the Wigner quasidistribution of a quantum state and the probability distribution on the phase space of a classical particle.

The original transform was introduced by Radon, who proved that a differentiable function on the 3-dimensional Euclidean space can be determined explicitly by means of its integral over the planes. I will prove the original Radon inversion formula and its generalization to n-dimensional spaces.

In the second lecture I will consider a broader framework and look at the more general problem of expressing a function on a manifold in terms of its integrals over certain submanifolds. This has become an important topic in integral geometry with many applications ranging from partial differential equations, group representations, and X-ray technology.

The focus will be on invariant (or equivariant) transformations under some symmetry groups from the space of functions on one geometrical space to the space of functions on another geometrical space.

In the last lecture I will show some possible generalizations of the above picture to the quantum case. A straightforward generalization derives from the phase-space description of quantum mechanics through Wigner quasidistribution functions. We will see how this map can be considered as a specific tomographic version of the star-product quantization.

REFERENCES