

EXTRACTING OSCILLATION PARAMETERS FROM NEUTRINO DATA

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Outline of the course

Introduction

Lecture I: Solar Neutrinos (θ_{12} , Δm^2_{12})

Lecture II: Atmospheric Neutrinos (θ_{23} , Δm^2_{23})

Lecture III: Bounds on θ_{13} and δ

Lecture IV: Sterile Neutrinos

EXTRACTING Θ_{23}
AND ΔM^2_{23}
FROM
ATMOSPHERIC
NEUTRINO DATA

Outline

- The atmospheric neutrino fluxes
- The atmospheric neutrino problem
- Fits to the atmospheric neutrino data

Cosmic Rays

Cosmic Rays (CR) enter the solar system as an isotropic flux of particles. The bulk of this flux ($E_{CR} < 10^6$ GeV) comes from our galaxy. Exists also a SOLAR contribution to the CR flux due to occasional (high energy) flares of the SUN.

- The **ORIGIN** of CR is **NOT KNOW** precisely (supernova explosions, supernova shock waves)
- The **COMPOSITION** of CR is mainly H^+ (90%) and He^{++} (9%) but also heavier nuclei like **C** and **Fe** are present. The relative abundance is energy depend. The main part of the spectrum is in the range $0.1 - 10^3$ GeV/Nucleon and decrease like $E_{CR}^{-2.7}$ for higher energies
- **HADRONIC INTERACTIONS** between the CR and the nuclei of the atmosphere depends on the **NUMBER of NUCLEONS** (not of NUCLEI). So heavier components (C, Fe) have higher cross section and their contribution cannot be completely neglected.

ANISOTROPIES of CR flux are due to SOLAR WIND and GEOMAGNETIC FIELD

- For $E_{CR} < 10$ GeV the CR flux reaching the Earth is MODULATED by the SOLAR WIND. The stronger the solar wind the more difficult is for the less energetic CR to enter in the solar system. The high energy component of CR is unaffected by the solar wind.
- The terrestrial GEOMAGNETIC FIELD prevents primary CR with LOW RIGIDITY (momentum/charge) to enter in the atmosphere. This CUT-OFF is latitude dependent growing monotonically from 0 at the magnetic pole to a maximum at the equator.

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Only ratios of neutrino fluxes have reasonable errors

Atmospheric neutrinos are produced in three steps:

- (i) COSMIC RAYS HIT atmospheric NUCLEI and produce π^\pm, K^\pm (in the same way as a proton beam on a fixed target experiment):



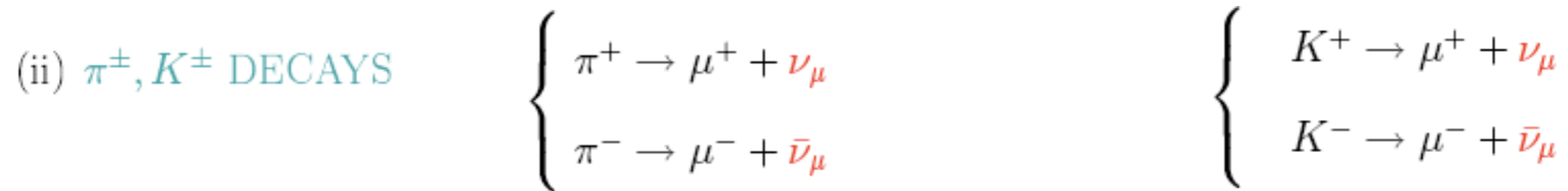
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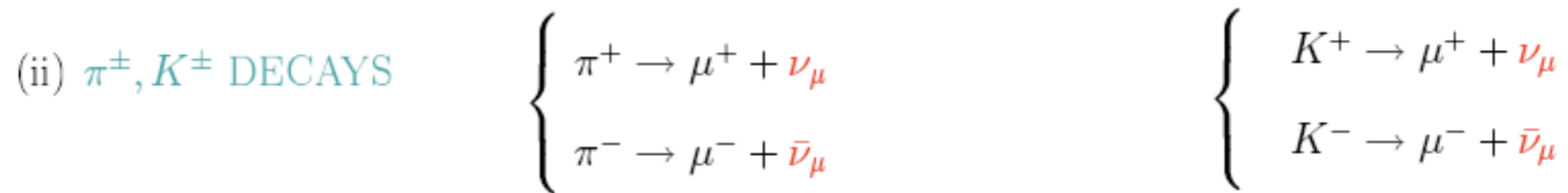


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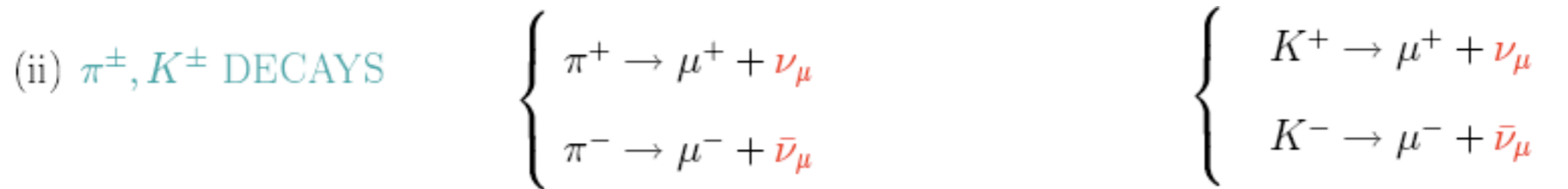


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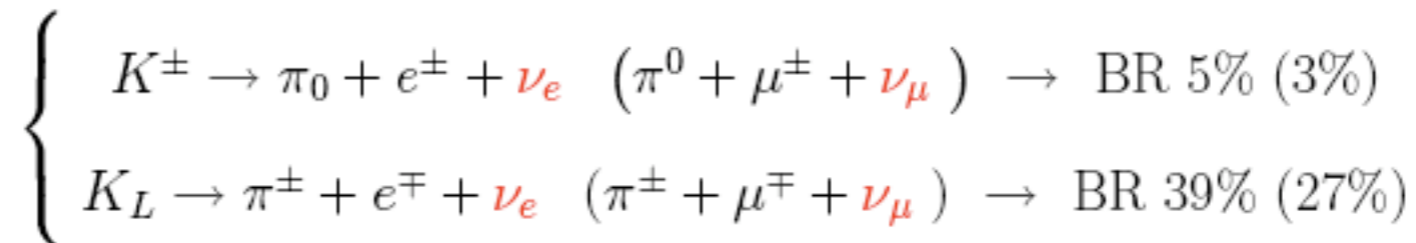
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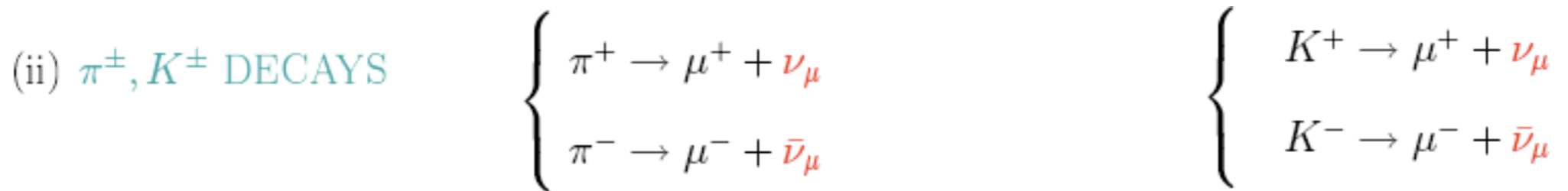


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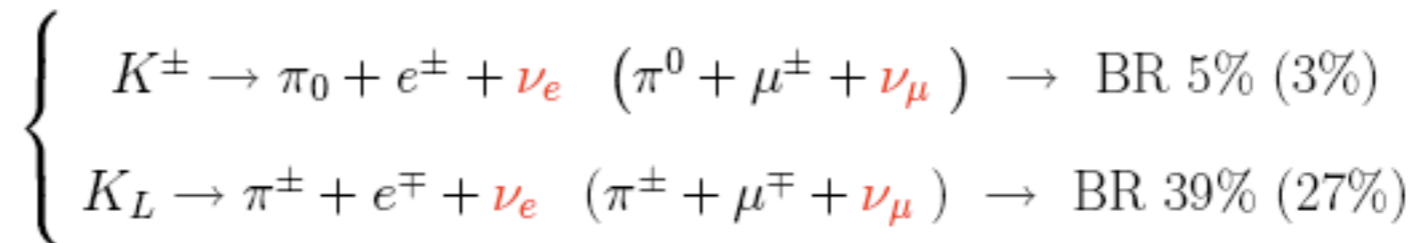
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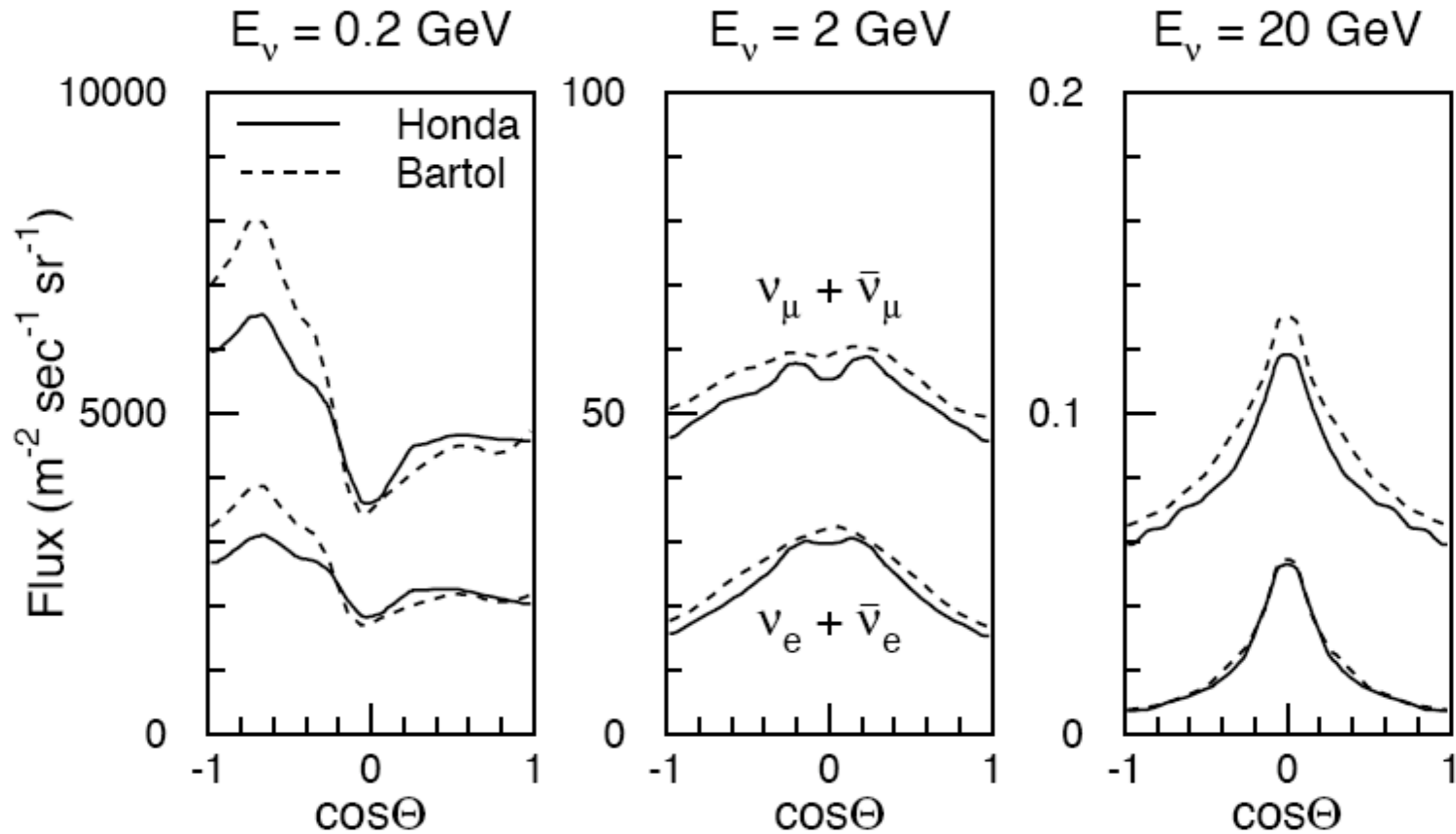


However,



The error on the ν_e/ν_μ fluxes is 5%

Two atmospheric fluxes computations



THE ATMOSPHERIC NEUTRINO PROBLEM

Atmospheric neutrino experiments

Atmospheric Neutrinos are observed in UNDERGROUND experiments (reduce cosmic rays muon background) in two different type of detectors:

- (1) **Water Cerenkov Detectors** (Kamiokande, IMB, SK): the target is a large volume of water surrounded by photo multipliers which detect the Cerenkov light produced by the charged leptons in $\nu_\ell + N \rightarrow \ell^\pm + X$:
 - e-like event \rightarrow DIFFUSE RING
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- (2) **Iron Calorimeter Detectors** (Frejus, Nusex, Soudan2, Macro): the target is composed by layers of (magnetized) IRON followed by layers of SCINTILLATORS which allow the reconstruction of the shower (e-like events) and/or the track (μ -like events) of the charged lepton produced in $\nu_\ell + N \rightarrow \ell^\pm + X$.

Typical neutrino energy: > 1 GeV

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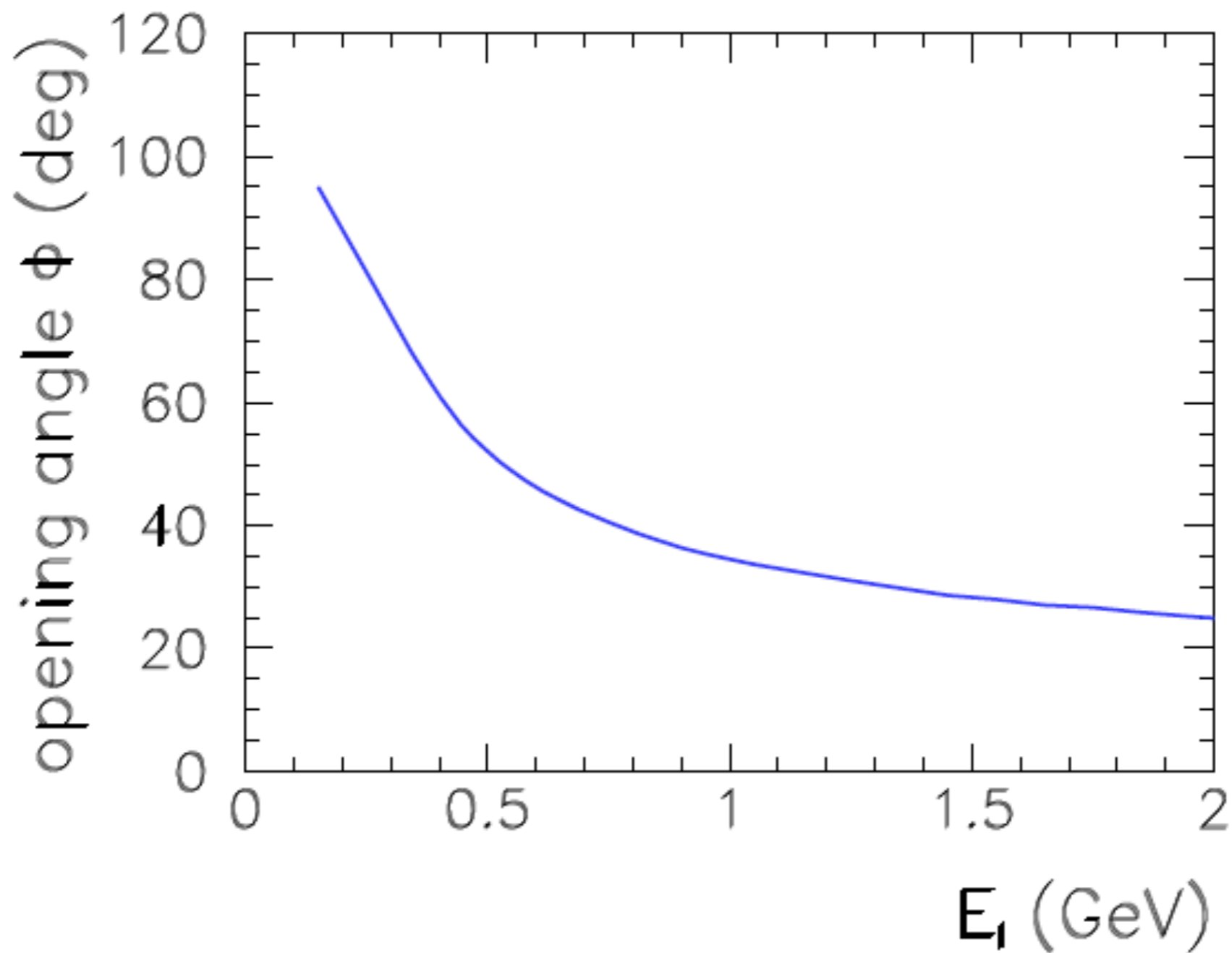
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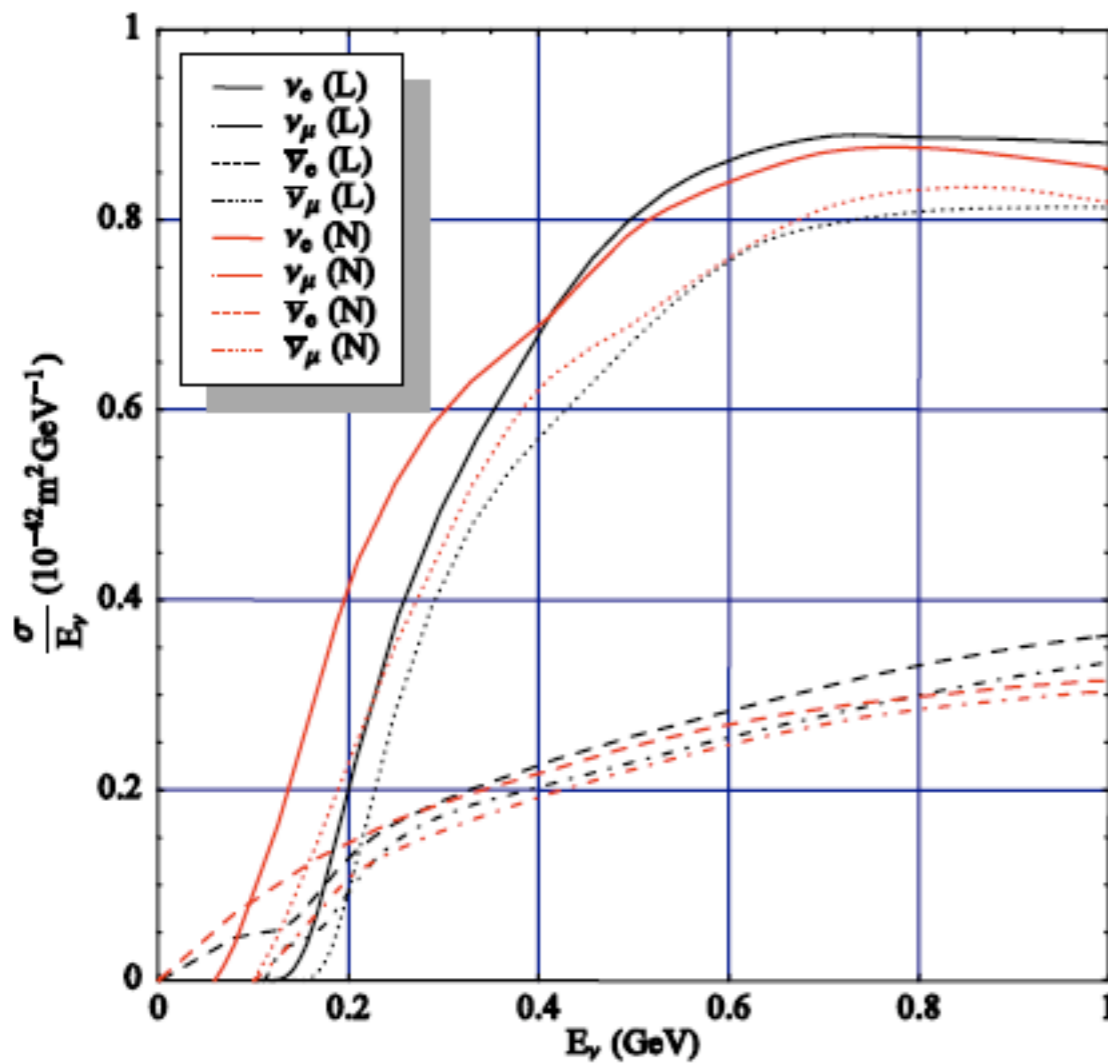
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- ★ Very good separation between μ -like and e-like events (SK separation efficiency $> 98\%$).

For $E > 1$ GeV, directional info is good

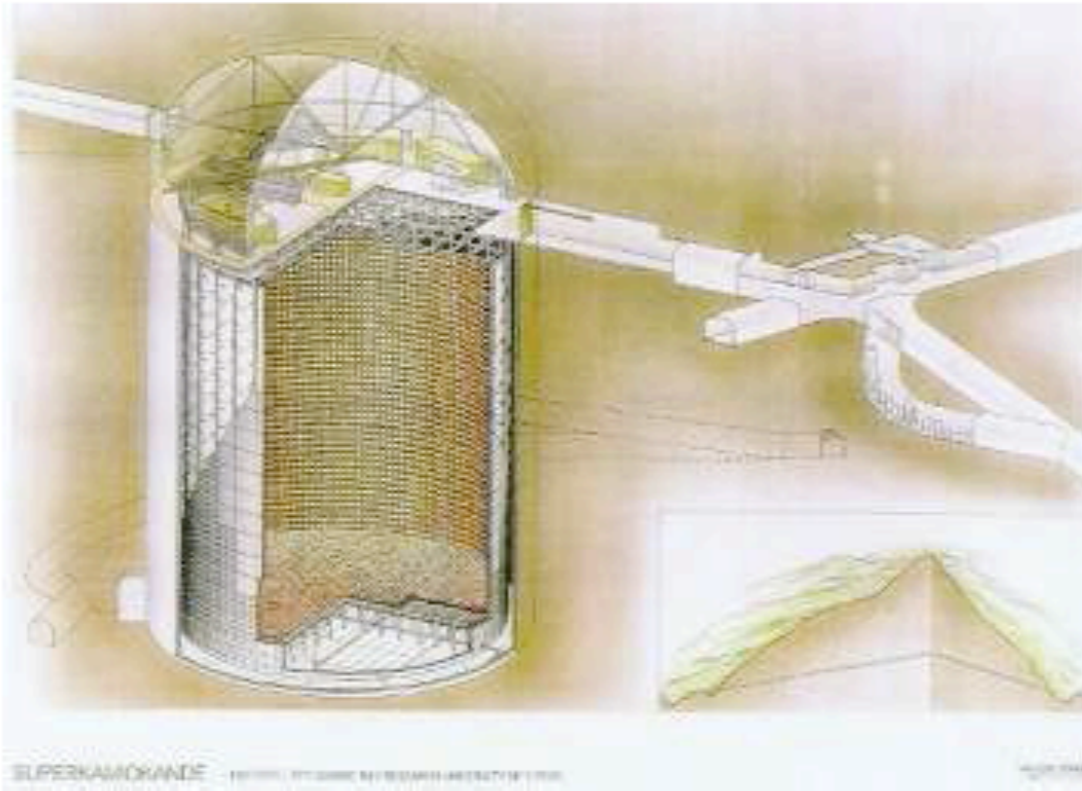


Comparison of νN cross-sections



- Different cross-sections can differ up to a factor of 2 below 0.5 GeV (at 0.2 GeV)
- Comparison of LIPARI (black) and **NUANCE (red)** cross-section
- The cross-sections will be measured by the experiments

Event separation at SuperKamiokaNDE



50,000 ton water Cherenkov detector
(22.5 kton fiducial volume)

1000m underground (2700 m.w.e.)

11,146 20-inch PMTs for inner detector

1,885 8-inch PMTs for outer detector

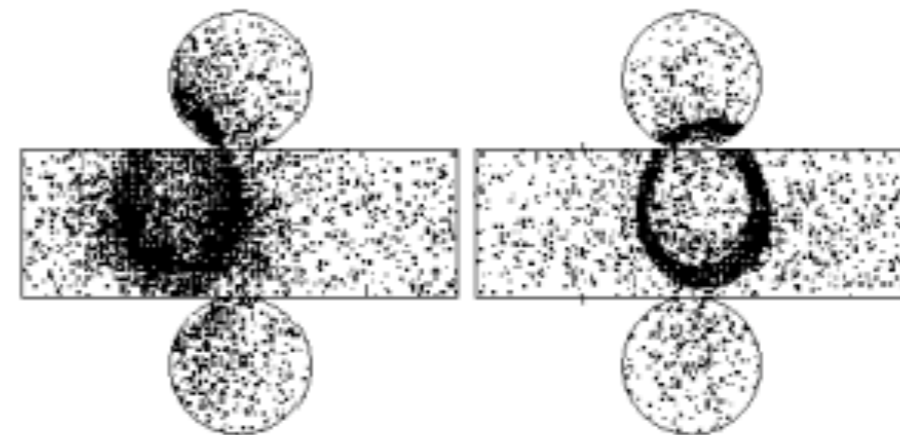
ν -detection via CC int.:

ν_e -tag: electron

ν_μ -tag: muon

e-like

μ -like



Contained events:

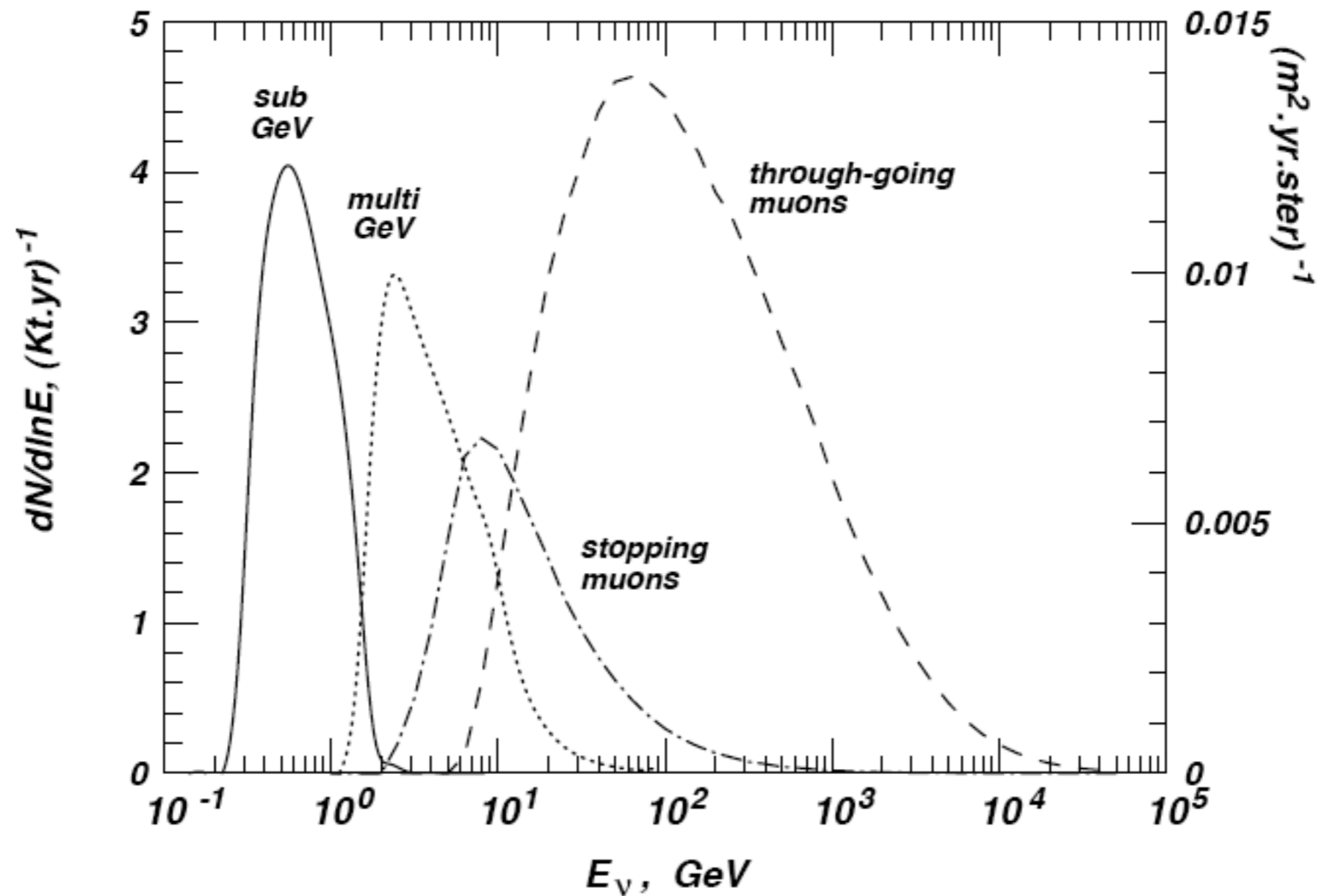
- **CONTAINED EVENTS**: when the interaction point is inside the detector.
 - If all the particles produced in the interaction deposit their energy inside the detector the event is called **FULLY CONTAINED** (FC). Flavour, kinetic energy and direction of the neutrino are known. FC EVENTS are subdivided by Super Kamiokande in **SUB-GeV** events (when $E_\nu < 1.2$ GeV) or **MULTI-GeV** (if $E_\nu > 1.2$ GeV).
 - When some of the particles produced in the interaction escape from the detector (typically a μ) the event is called a **PARTIALLY CONTAINED** (PC) event. Generally PC events are produced by relatively high energy neutrinos and/or events produced nearby the border of the fiducial volume.

Non-Contained events:

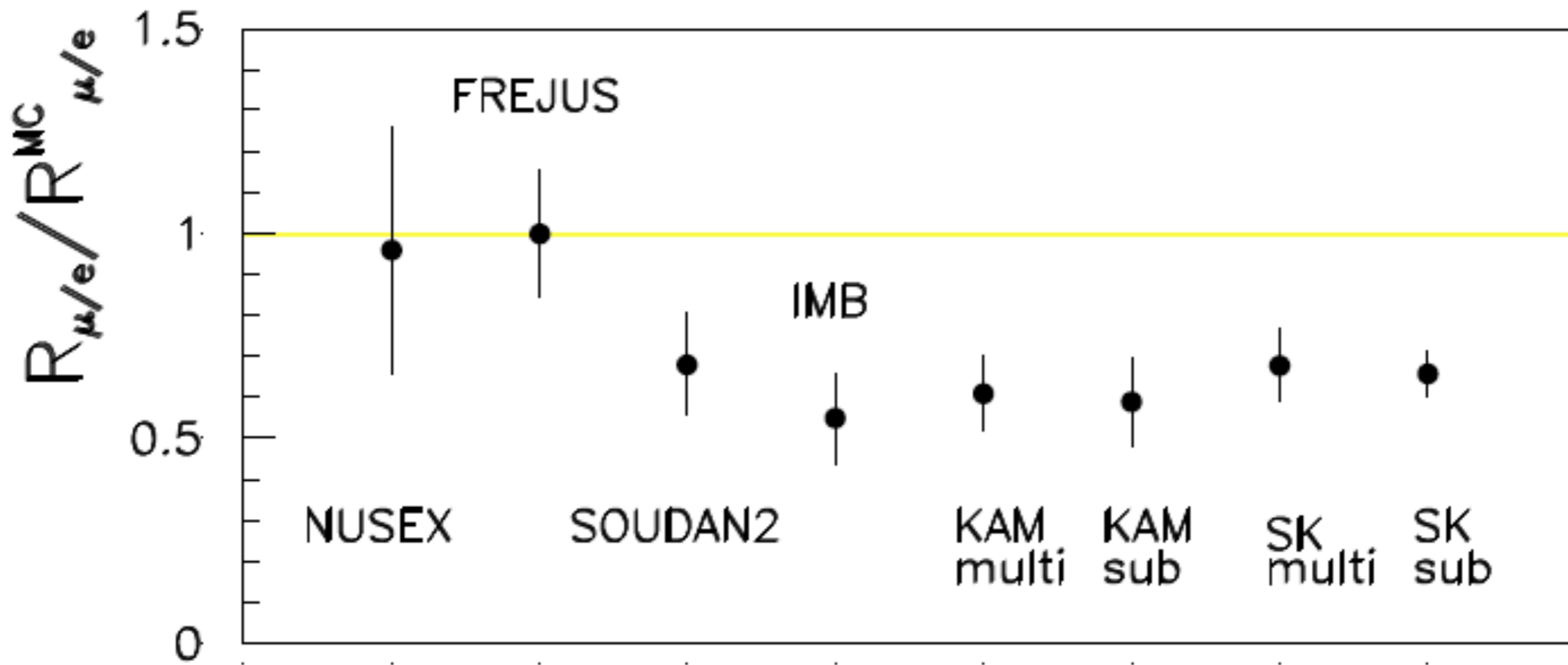
- **NOT CONTAINED EVENTS:** when the interaction point is outside the the detector (rocks surrounding the detector). If the incoming neutrino is sufficiently energetic then the produced μ^\pm can enter in the detector (e^\pm are absorbed in the rock). These events are classified as:
 - **STOPPING MUONS** if the μ produced outside the detector stops inside the detector. This arise for typical neutrino energies ~ 10 GeV. Then the energy of the μ is well known.
 - **THROUGH GOING MUONS** if the μ produced outside the detector crosses all the detector and exits. This arises for typical neutrino energies ~ 100 GeV.

As down-going muons produced by ν -interactions in the rocks outside the detector cannot be distinguished by COSMIC RAYS muons only **UP-GOING NOT CONTAINED** events are considered (a μ with GeV energy cannot cross all the earth !!)

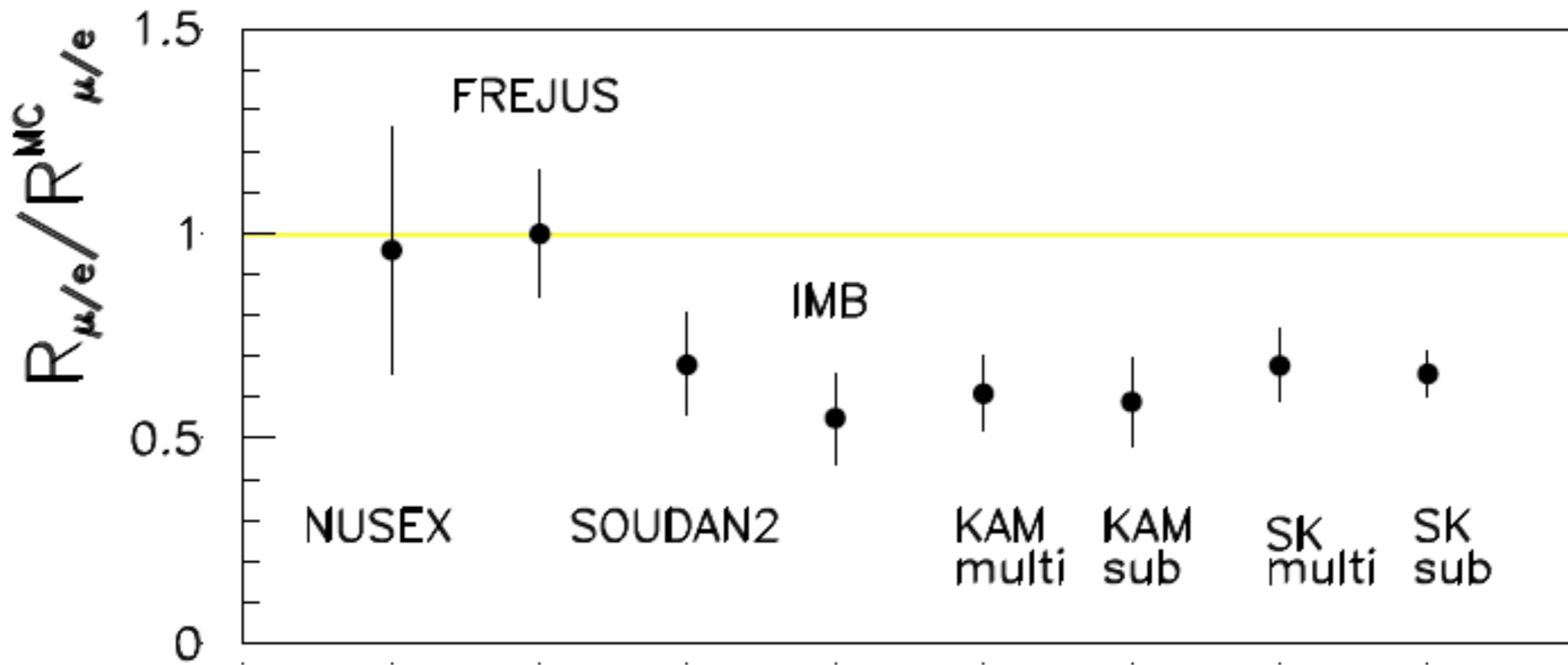
Atmospheric neutrino spectra in event classes



The atmospheric neutrino problem



The atmospheric neutrino problem



$$R = \frac{\left(\frac{\nu_{\mu}}{\nu_e}\right)_{exp}}{\left(\frac{\nu_{\mu}}{\nu_e}\right)_{the}} \approx 0.6$$

HALF of the ν_{μ} FLUX or DOUBLE of the ν_e FLUX

SuperK data

SK gives the most precise determination of the flux double ratio:

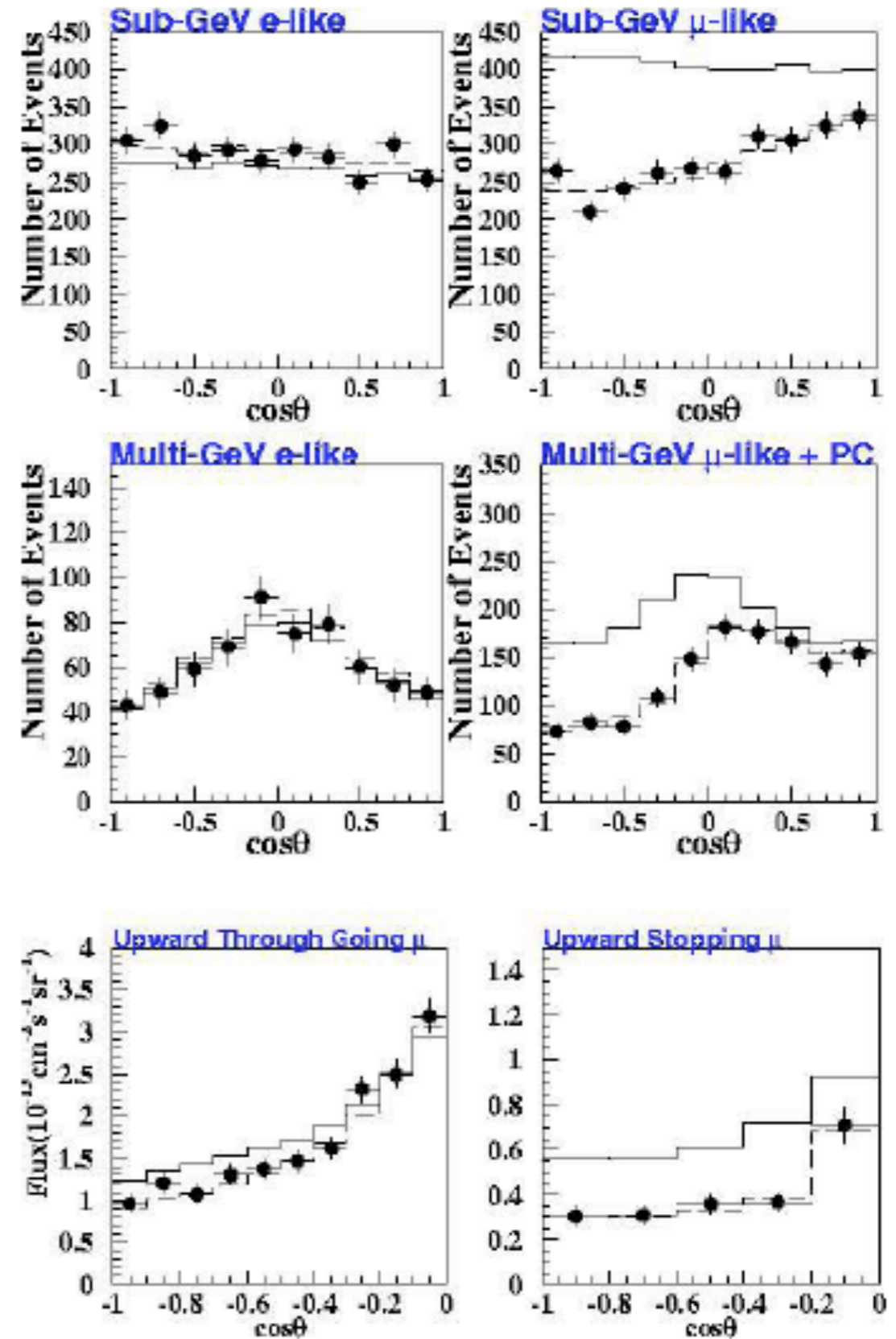
$$\left\{ \begin{array}{l} R = 0.638 \pm 0.016 \pm 0.050 \quad \text{SUB-GeV events} \\ R = 0.658 \pm 0.030 \pm 0.078 \quad \text{MULTI-GeV events} \end{array} \right.$$

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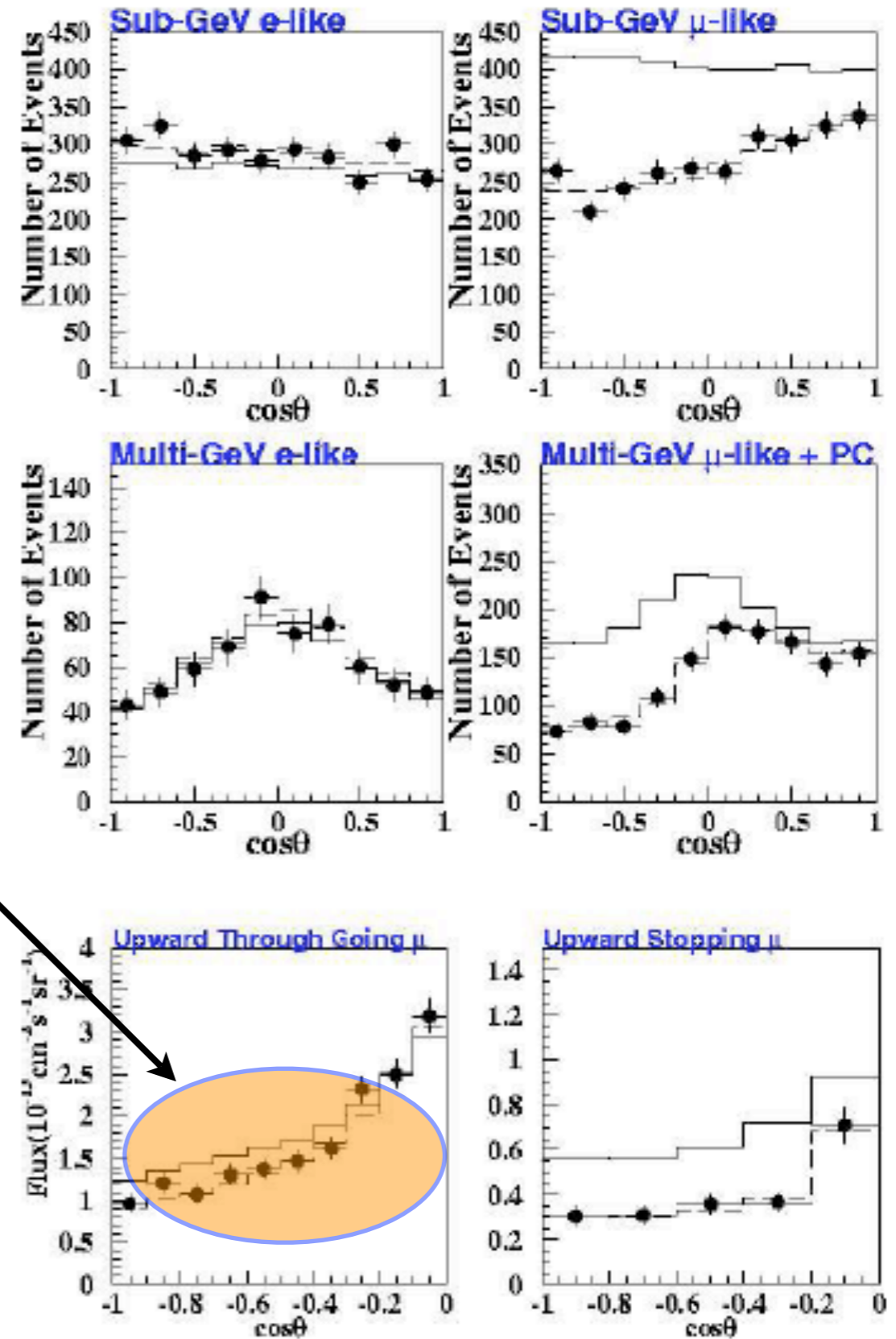
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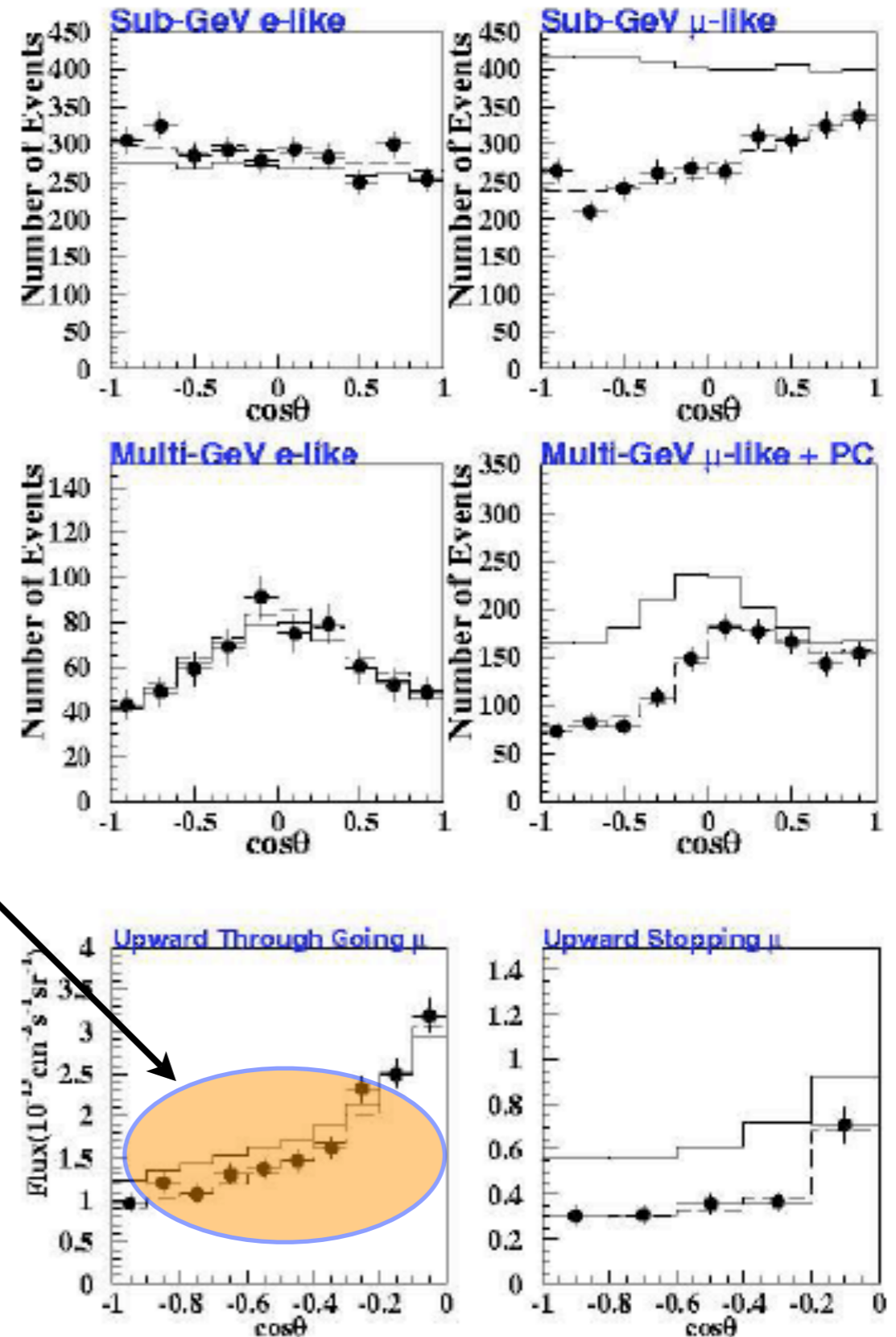
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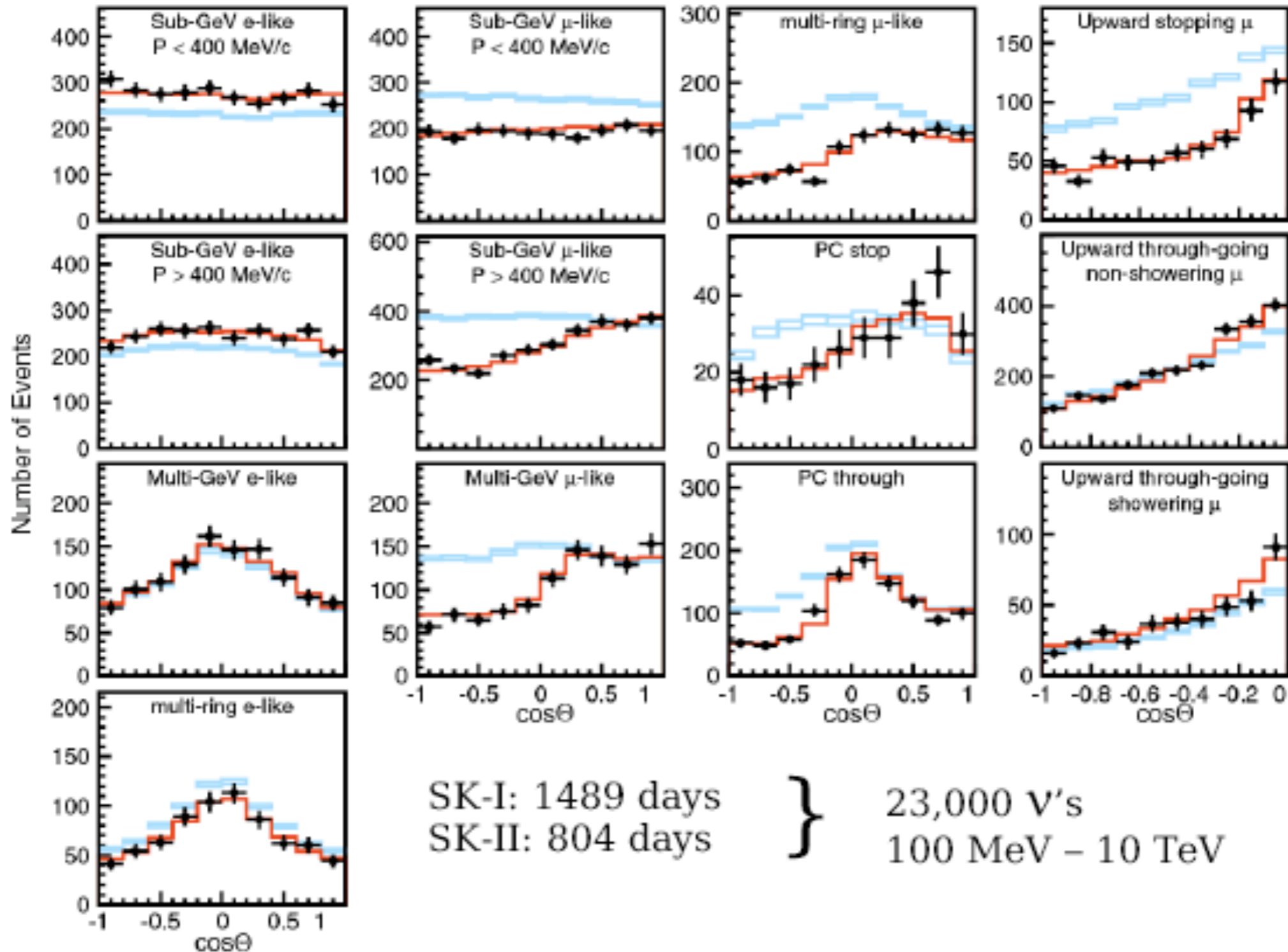
However...

Muon neutrino fluxes disagree strongly:

- 1) at all angles for Sub-GeV
- 2) mainly for up-going events for Multi-GeV



More recent SuperK data



- The μ -like flux is SUPPRESSED both in the SUB and MULTI-GeV channel. Suppression is also evident in NOT CONTAINED events. The suppression has a clear DEPENDENCE from the zenith angle (larger for $\cos\theta = -1$ and smaller for $\cos\theta = 1$):

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- The high energy part of the μ -like flux represented by the UPWARD THROUGH GOING muons seems **LESS SUPPRESSED** (smaller L/E).
- The angular dependence of ν_μ can be described by an **UP-DOWN ASYMMETRY**:

$$A_\mu = \frac{U - D}{U + D} = -0.316 \pm 0.042 \pm 0.005$$

NO ASYMMETRY is **EXCLUDED** at more than 6σ .



$\nu_\mu \rightarrow \nu_x$ ($x \neq e$) NEUTRINO OSCILLATIONS
BEST EXPLANATION for the **RATIO SUPPRESSION**
 and
ZENITH ANGLE DEPENDENCE

Accelerator neutrino experiments

Conventional neutrino beams
are produced through π and K decay

$$p + \text{target} \rightarrow \pi^\pm + X$$

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

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$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

Main beam: ν_μ

Components of $\nu_e, \bar{\nu}_e, \bar{\nu}_\mu$

Signals: advantages and limitations

- **MULTIPLICITY of CHANNELS:**

Accelerator neutrino fluxes can be used to search both for ν_μ disappearance or for ν_e , ν_τ appearance experiments.

- The main limitation for the ν_e appearance channel is represented by the usual not complete knowledge of K meson production (K/ π ratio) in proton interaction on target. K_{e3} decays represent the **MAIN BACKGROUND** to ν_e appearance experiments. Even if the ν_e flux contamination can be reduced to few %, it is the enough to reduce the sensitivity in the ν_e APPEARANCE channel;
- The main limitation for the ν_τ appearance channel is represented by the high energy τ threshold production (around 2 GeV). Low energetic τ s are also difficult to distinguish from Cosmic Rays background due to their hadronic decay;

Using **accelerator neutrino beams**, it is possible to test the SK results for atmospheric neutrinos using a neutrino source that is **better controlled** (as it was the case for KamLAND for solar neutrinos).

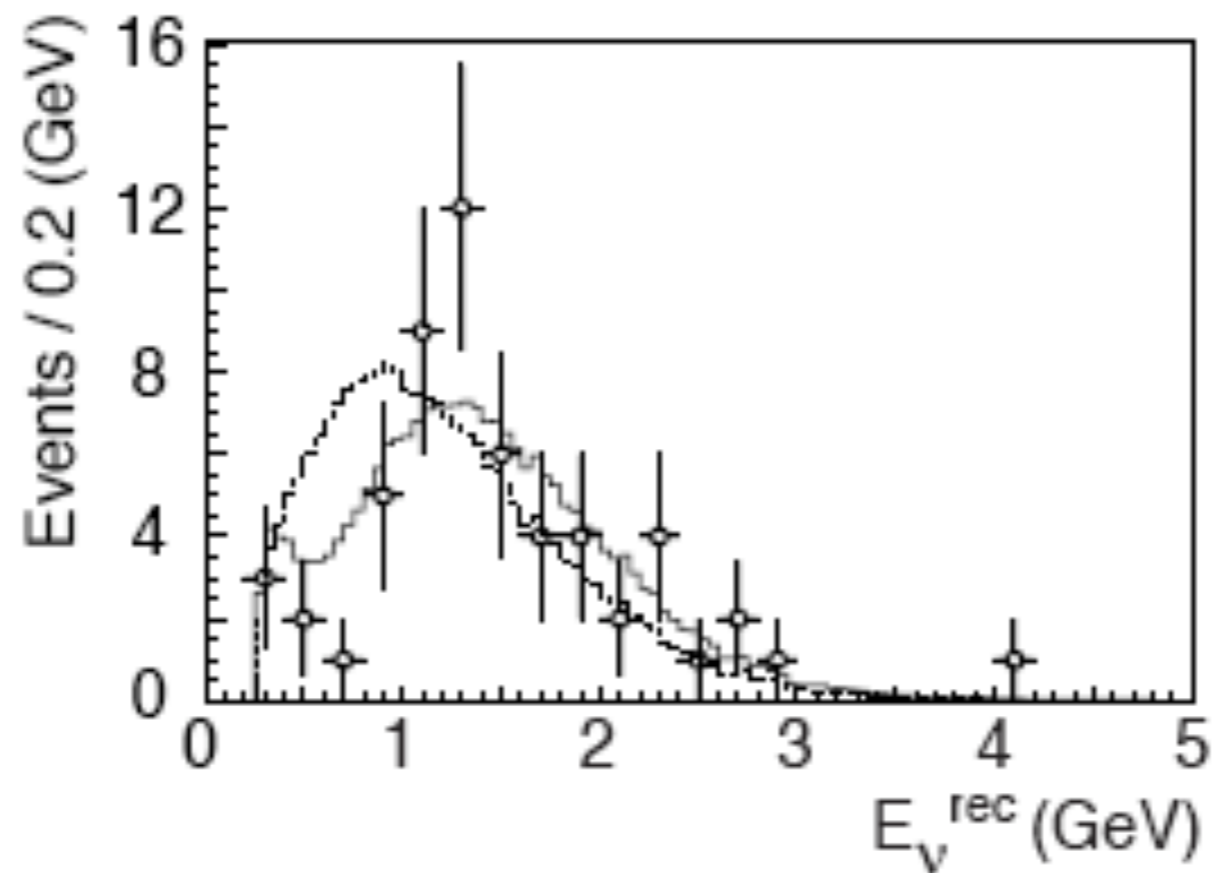
The typical baseline is $O(10^2)$ Km,
the typical neutrino energy $E \geq 1$ GeV

Three experiments:

- 1) K2K (finished)
- 2) MINOS (still taking data)
- 3) CNGS (no results yet)

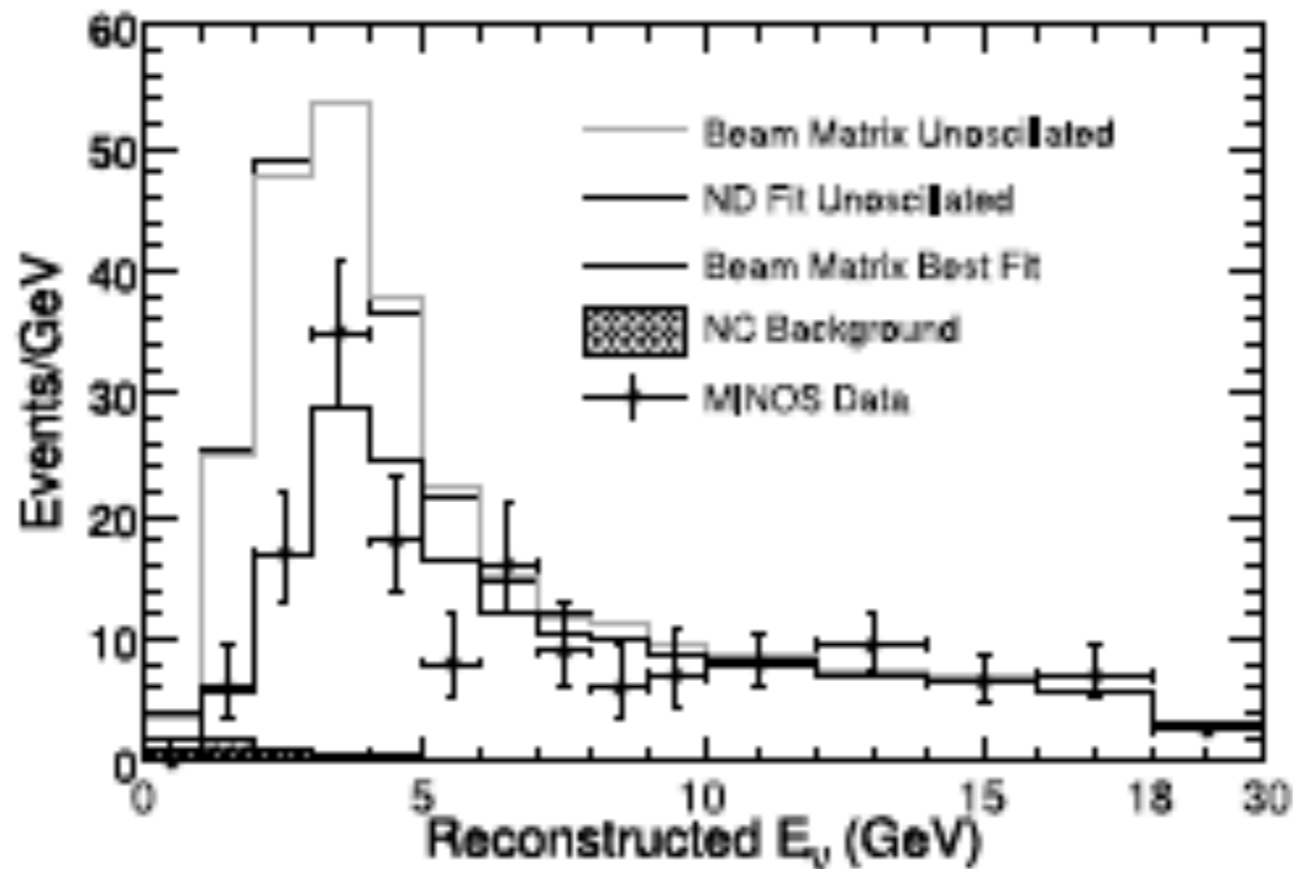
I) K2K: KEK to Kamioka; $L = 235$ Km, $\langle E \rangle = 1.4$ GeV

107 observed events
151⁺¹²₋₁₀ expected



2) MINOS: FermiLab to Soudan; $L = 730$ Km, $\langle E \rangle = 4$ GeV

122 observed events below 10 GeV
 239 ± 11 expected



FIT TO “ATMO” DATA

- **All experiments** testing the atmospheric neutrino flux observe a ratio ν_{μ}/ν_e in disagreement with the expectations
- **K2K and MINOS** have observed a similar deficit for ν_{μ} neutrinos produced in (conventional) accelerator beams

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Knowing that the deficit mainly comes from neutrinos that:

- Travel a distance $L \approx 10^2 - 10^4$ km (UP-GOING NEUTRINOS)
- Have a energy $E_{\nu} \approx 1$ GeV (SUB and MULTI GeV channels)

One obtains the following estimation for the MASS DIFFERENCE:

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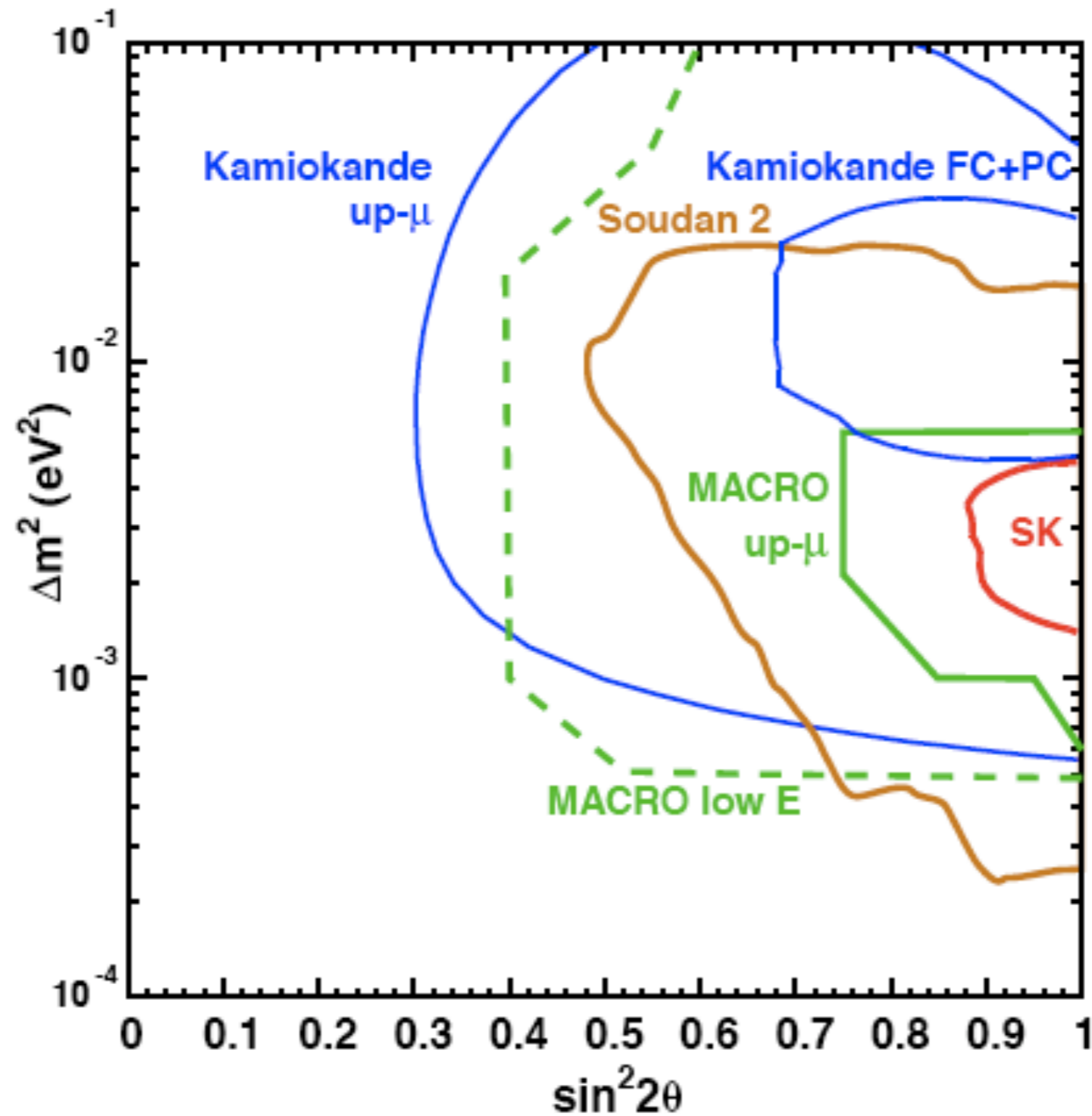
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Moreover assuming that down-going ν_μ don't oscillate, while all the up-going ν_μ do, one obtains the following lower limit to the mixing angle

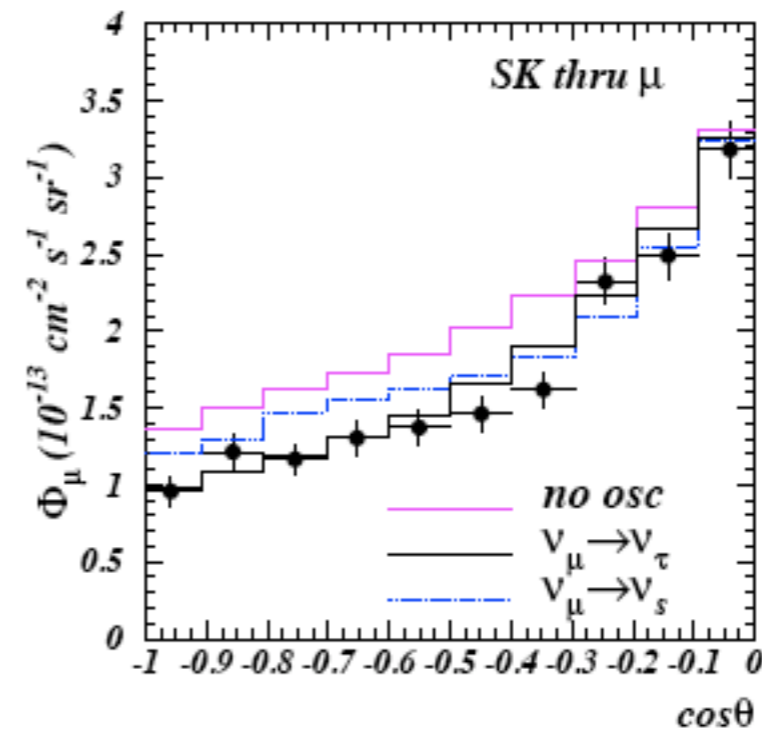
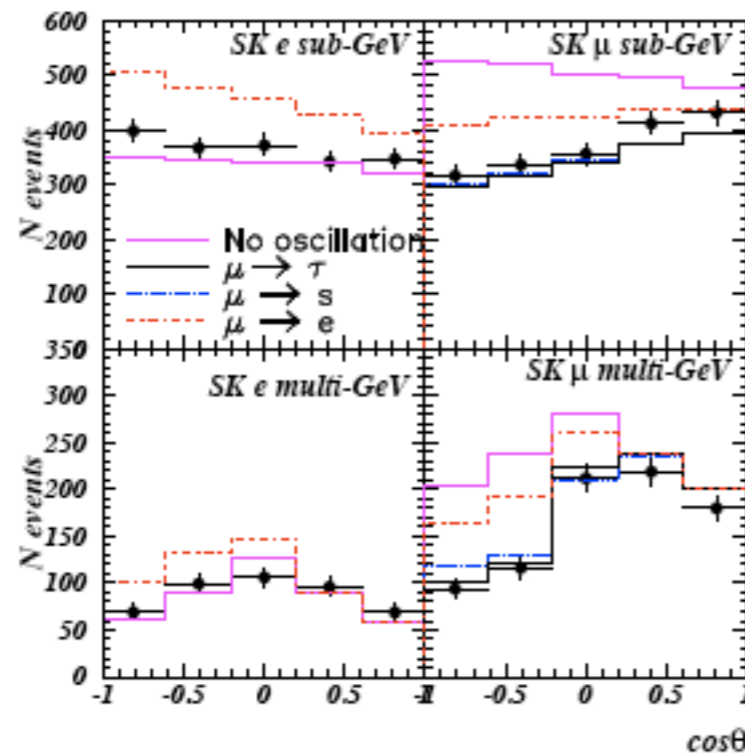
$$|\mathcal{A}_\mu| \sim \frac{\sin^2 2\theta}{4 - \sin^2 2\theta} > 0.27 \quad \rightarrow \quad \star \sin^2 2\theta > 0.85 \star$$

Only one possible Δm^2_{23}



Three possible two-families oscillations

- $\nu_\mu \rightarrow \nu_e$ oscillation is excluded by SK and reactor experiments:
 - SK does not measure any excess of ν_e total flux or any zenith angle dependence
 - Reactor experiments (CHOOZ - see later) exclude ν_e disappearance in the ATM mass difference range
- $\nu_\mu \rightarrow \nu_s$ oscillation is disfavored by SK data. ν_s behavior differs from ν_τ due to matter effects in crossing the earth

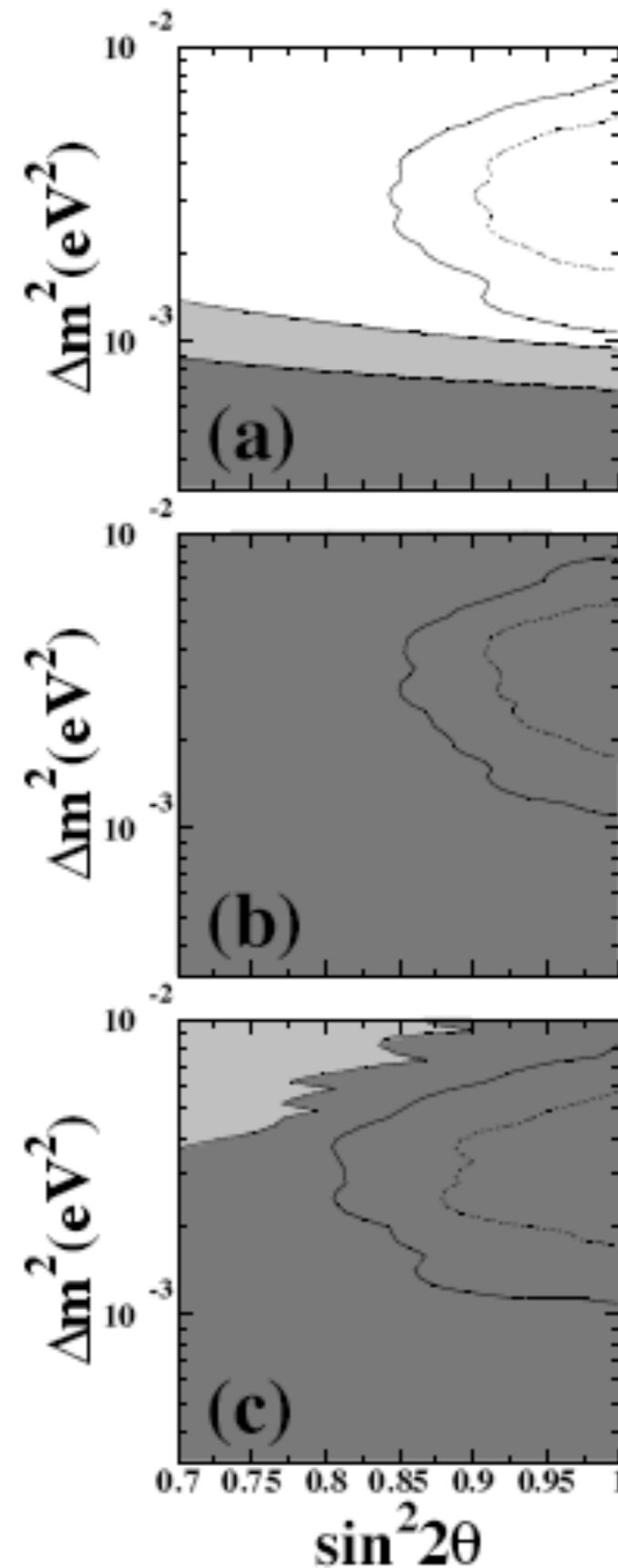


Excluded regions for:

(a) $\nu_\mu \rightarrow \nu_\tau$

(b) $\nu_\mu \rightarrow \nu_s$ ($\Delta m^2_{23} > 0$)

(c) $\nu_\mu \rightarrow \nu_s$ ($\Delta m^2_{23} < 0$)



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- $\nu_\mu \rightarrow \nu_\tau$ oscillation gives the best fit to data

$$\begin{cases} \Delta m_{\text{ATM}}^2 = 2.5 \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta \geq 0.9 \end{cases}$$

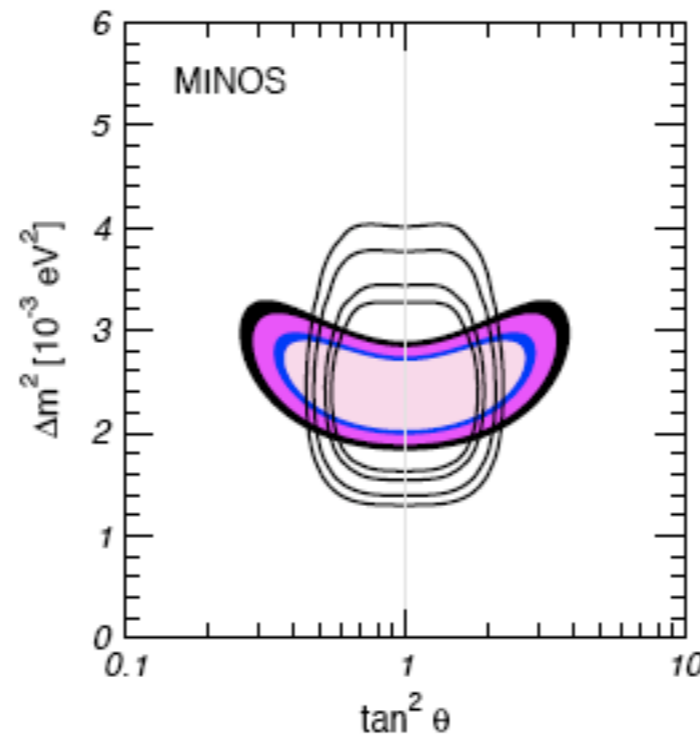
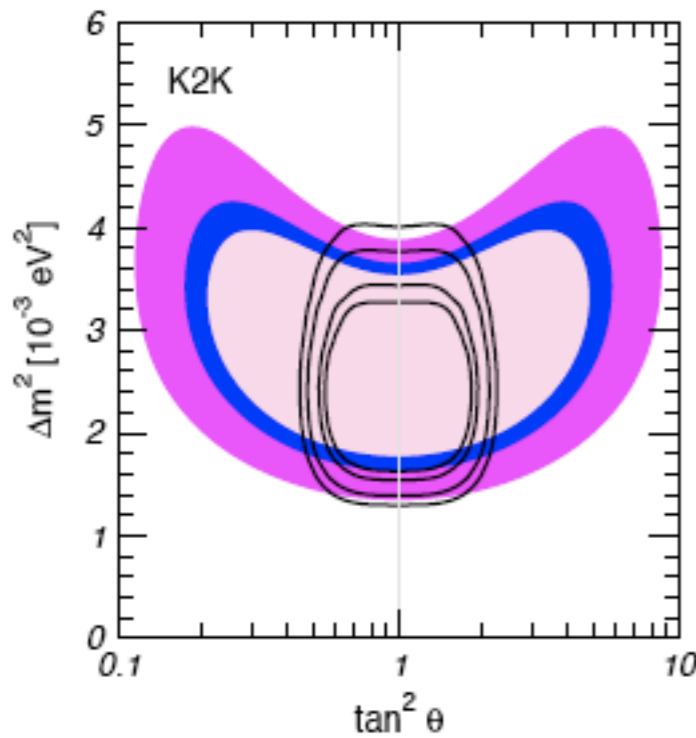
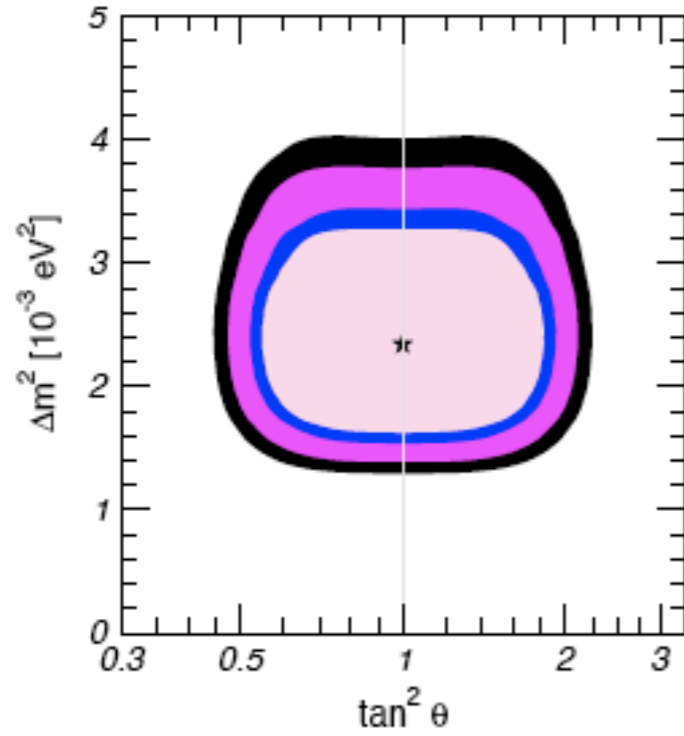
Three possible two-families oscillations

- $\nu_\mu \rightarrow \nu_e$ oscillation is excluded by SK and reactor experiments:
 - SK does not measure any excess of ν_e total flux or any zenith angle dependence
 - Reactor experiments (CHOOZ - see later) exclude ν_e disappearance in the ATM mass difference range
- $\nu_\mu \rightarrow \nu_s$ oscillation is disfavored by SK data. ν_s behavior differs from ν_τ due to matter effects in crossing the earth
- $\nu_\mu \rightarrow \nu_\tau$ oscillation gives the best fit to data

$$\begin{cases} \Delta m_{\text{ATM}}^2 = 2.5 \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta \geq 0.9 \end{cases}$$

$\nu_\mu \rightarrow \nu_\tau$ OSCILLATION EXPLAINS NEUTRINO ATMOSPHERIC PROBLEM

Global analysis of atmospheric data



K2K and MINOS

CONCLUSIONS

- The atmospheric neutrino flux is better understood than the Sun's: it took only 20 years to be convinced that particle physics must be changed.
- The most recent data give:

$$\Delta m_{31}^2 = \begin{cases} -2.37 \pm 0.15 \begin{pmatrix} +0.43 \\ -0.46 \end{pmatrix} \times 10^{-3} \text{ eV}^2 & \text{(inverted hierarchy),} \\ +2.46 \pm 0.15 \begin{pmatrix} +0.47 \\ -0.42 \end{pmatrix} \times 10^{-3} \text{ eV}^2 & \text{(normal hierarchy),} \end{cases}$$

$$\theta_{23} = 42.3 \begin{matrix} +5.1 \\ -3.3 \end{matrix} \begin{pmatrix} +11.3 \\ -7.7 \end{pmatrix},$$

Gonzalez-García and Maltoni, 2008