

# Introduction to Neutrino Interaction Physics

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# References

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2. C.H. Kim & A. Pevsner “Neutrinos in Physics and Astrophysics”, Harwood Academic Publishers, 1993.
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5. H.V. Klapdor-Kleingrothaus & K. Zuber, “Particle Astrophysics”, Institute of Physics Publishing, 1997.
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# 1. History and Introduction

1.1 Fermi Theory

1.2 Neutrino discovery

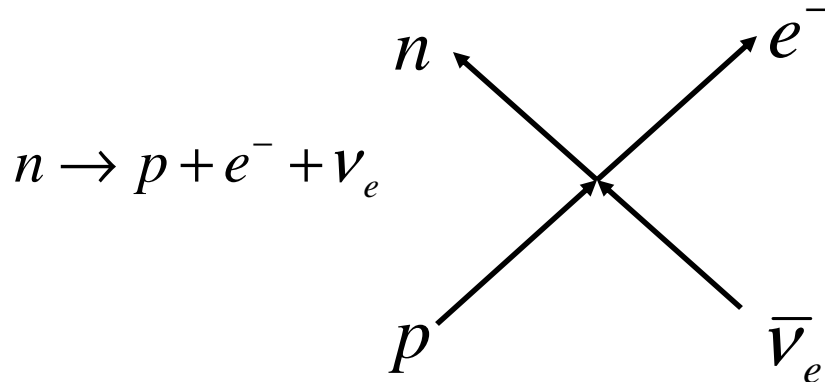
1.3 Parity violation and V-A theory

1.4 Neutral currents

1.5 Standard model neutrino interactions

# 1.1 Fermi theory of beta decay (1932)

- Existence of a point-like four fermion interaction (Fermi, 1932):



- Lagrangian of the interaction:

$$L(x) = -\frac{G_F}{\sqrt{2}} [\bar{\phi}_p(x) \gamma^\mu \phi_n(x)] [\bar{\phi}_e(x) \gamma_\mu \phi_\nu(x)]$$

$G_F$  = Fermi coupling constant =  $(1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}$

- Gamow-Teller interaction when final spin different to initial nucleus:

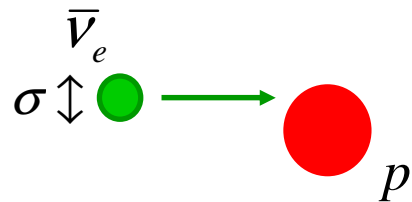
$$L(x) = -\frac{G_F}{\sqrt{2}} \sum_i [\bar{\phi}_p(x) \Gamma_i \phi_n(x)] [\bar{\phi}_e(x) \Gamma_i \phi_\nu(x)] + h.c.$$

Possible interactions:  $\Gamma_i = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu} = S, P, V, A, T$

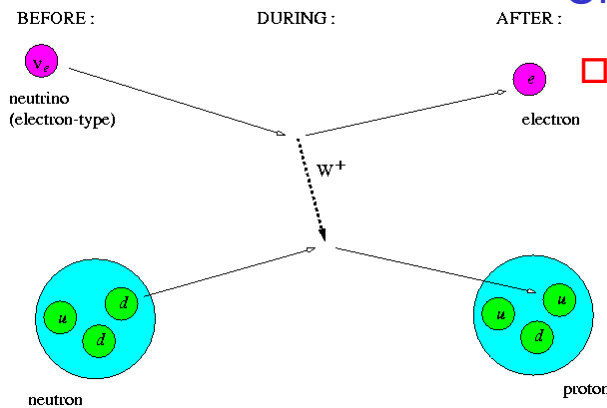
# First neutrino cross-section calculation

- **Bethe-Peierls (1934):** calculation of first cross-section for inverse beta reaction  $\bar{\nu}_e + p \rightarrow n + e^+$  or  $\nu_e + n \rightarrow p + e^-$  using Fermi theory

$\sigma \approx 5 \times 10^{-44} \text{ cm}^2$  for  $E(\bar{\nu}) = 2 \text{ MeV}$  **Accurate to factor 2**  
 Conversion from natural units:  $\hbar c = 197.3 \text{ MeV} \cdot \text{fm}$



Cross-section: multiply by  $(\hbar c)^2 = 0.3894 \times 10^{-27} \text{ GeV}^2 \cdot \text{cm}^2$



- **Mean free path of antineutrino in water:**

$$\lambda = \frac{1}{n\sigma} \approx 1.5 \times 10^{21} \text{ cm} \approx 1600 \text{ light-years}$$

$$n = \frac{\text{num. free protons}}{\text{volume}} \approx 2 \frac{N_A}{A} \rho$$

In water:  $n = \frac{2 \times 6 \times 10^{23}}{18} = 6.7 \times 10^{22} \text{ cm}^{-3}$

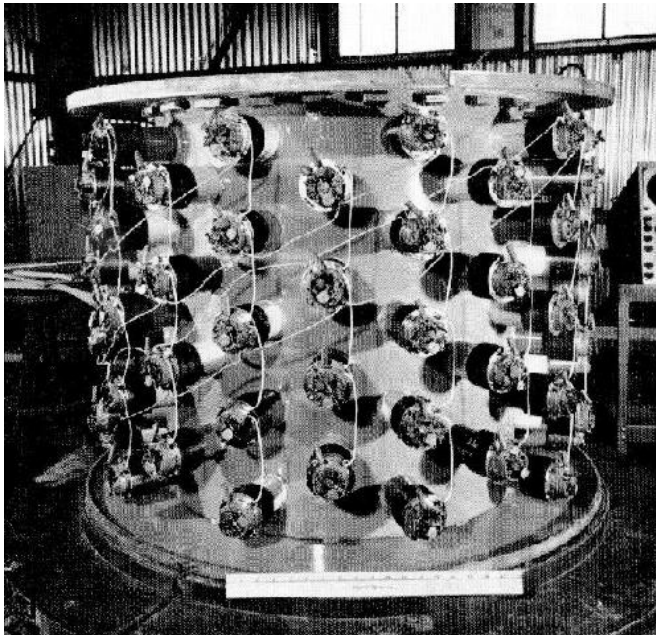
- **Probability of interaction:**

$$P = 1 - \exp\left(-\frac{L}{\lambda}\right) \approx \frac{L}{\lambda} = 6.7 \times 10^{-20} (\text{m water})^{-1}$$

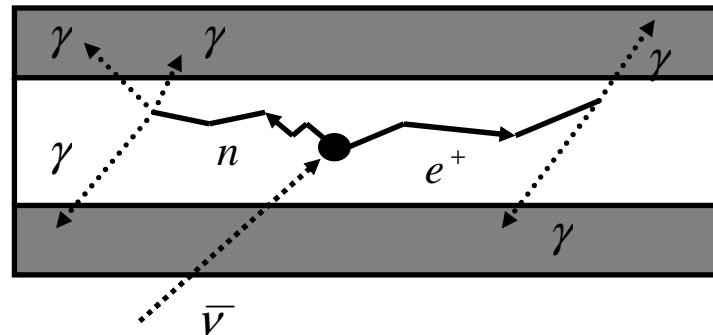
Need very intense source of antineutrinos to detect inverse beta reaction.

# 1.2 Neutrino discovery (1956)

- Reines and Cowan detect  $\bar{\nu}_e + p \rightarrow n + e^+$  in 1953 (Hanford) (discovery confirmed 1956 in Savannah River):
  - Detection of two back-to-back  $\gamma$ s from prompt signal  $e^+e^- \rightarrow \gamma\gamma$  at  $t=0$ .
  - Neutron thermalization: neutron capture in Cd, emission of late  $\gamma$ s  $\langle t \rangle \sim 20$  ms



4200 I scintillator



Scintillator  
 H<sub>2</sub>O + CdCl<sub>2</sub>  
 Scintillator

**Publication Science 1956:**

$\sigma = 6 \times 10^{-44} \text{ cm}^2 \pm 25\%$  (within 5% expected)

**1956: parity violation discovery increases theory cross-section:  $\sigma = (10 \pm 1.7) \times 10^{-44} \text{ cm}^2$**

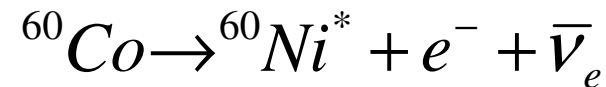
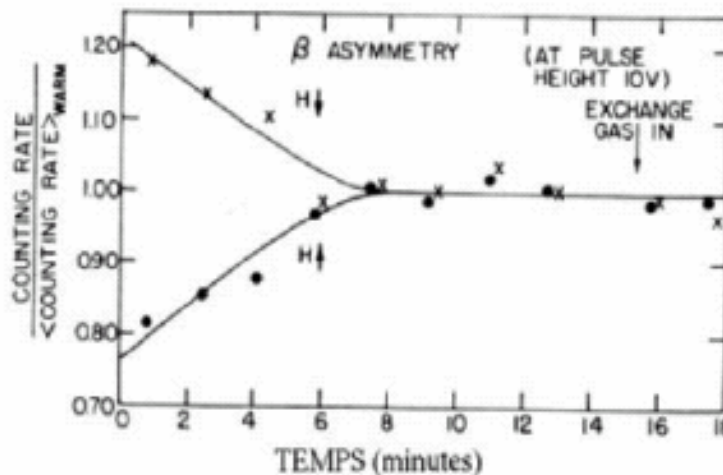
**Reanalysis data in 1960:**

$\sigma = (12 + 7 - 4) \times 10^{-44} \text{ cm}^2$

**Nobel prize Reines 1995**

# 1.3 Parity violation and V-A

- Parity violation in weak decays postulated by Lee & Yang in 1950
- Parity violation confirmed through forward-backward asymmetry of polarized  $^{60}\text{Co}$  (Wu, 1957).



More electrons emitted in direction opposite to  $^{60}\text{Co}$  spins, implying maximal parity violation

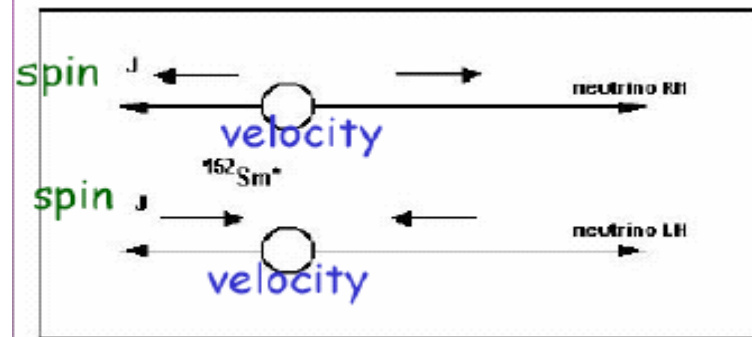
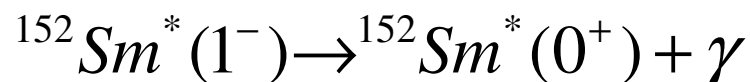
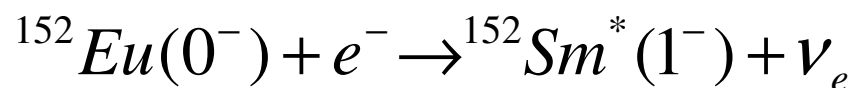
- Helicity operator: 
$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} \xrightarrow{P} \frac{\vec{\sigma} \cdot (-\vec{p})}{|\vec{p}|} = -H$$

Projects spin along direction of motion



# 1.3 Parity violation and V-A

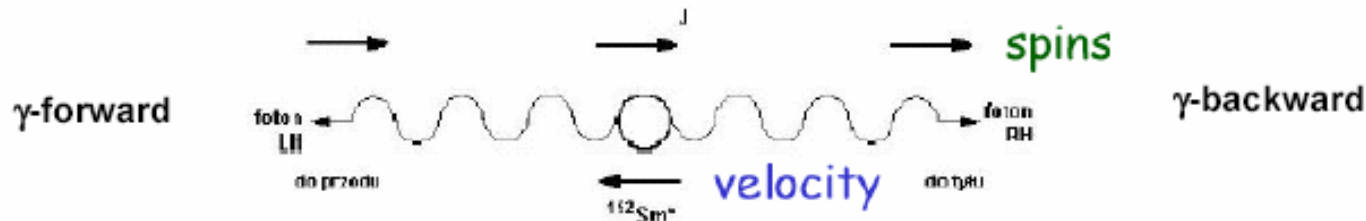
- Goldhaber, Grodzins, Sunyar (1958) measure helicity of neutrino from K capture of  $^{152}\text{Eu}$ :



$$\rho_\gamma = -\rho_\nu$$

$$\sigma_\gamma = -\sigma_\nu$$

$$H_\gamma = H_\nu$$



Asymmetry of photon spectrum in magnetic field determines helicity of  $\nu_e$ :

$$H(\nu_e) = -1 \Rightarrow H(\bar{\nu}_e) = +1$$

Neutrinos are “left-handed”



Antineutrinos are “right-handed”



# 1.3 Parity violation and V-A

- Left and right handed projection operators:

$$\nu_L = P_L \nu = \frac{1}{2}(1 - \gamma_5)\nu \quad \nu_R = P_R \nu = \frac{1}{2}(1 + \gamma_5)\nu$$

- Chirality operator:  $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

same as helicity operator for massless neutrinos ( $E=p$ ).

$$\gamma_5 \nu_L = H \nu_L = -\nu_L \quad \gamma_5 \nu_R = H \nu_R = +\nu_R$$

- If only  $\nu_L$  interact and  $\nu_R$  do not interact, then  $\Gamma_i$  have to transform as:  $\bar{e}\Gamma_i\nu \rightarrow (\overline{P_L e})\Gamma_i(P_L\nu) = \bar{e}P_R\Gamma_iP_L\nu$

$$V: P_R\gamma^\mu P_L = \frac{1}{2}\gamma^\mu(1 - \gamma_5) \quad A: P_R\gamma^\mu\gamma_5 P_L = -\frac{1}{2}\gamma^\mu(1 - \gamma_5)$$

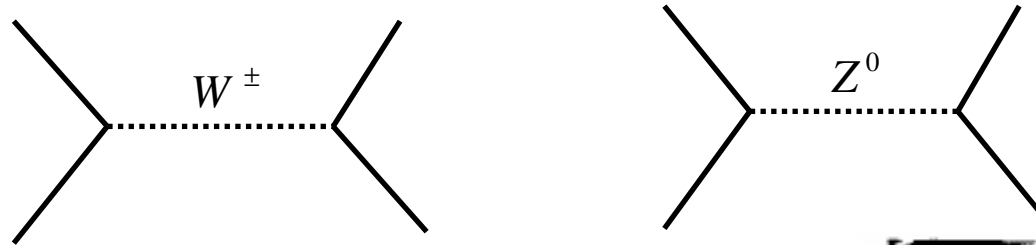
- The only possible coupling is V-A, due to maximal parity violation in weak interactions (Feynman, Gell-Mann, 1958):

$$L_{V-A} = -\frac{G_F}{\sqrt{2}} \left[ \bar{\phi}_p \gamma^\mu (1 - g_A \gamma_5) \phi_n \right] \left[ \bar{\phi}_e \gamma_\mu (1 - \gamma_5) \phi_\nu \right] \text{ with } g_A = -1.2573 \pm 0.0028$$

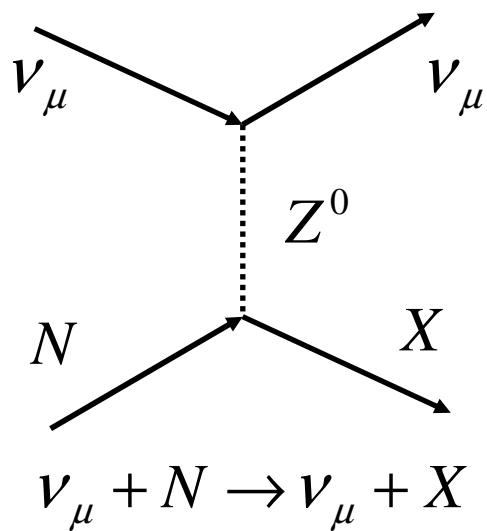
(determined empirically)

# 1.4 Neutral currents

- Two types of weak interaction: charged current (CC) and neutral current (NC) from electroweak theory of Glashow, Weinberg, Salam.



- First example of NC observed in 1973, inside the Gargamelle bubble chamber filled with freon ( $CF_3Br$ ): no muon!



# 1.5 Standard Model Neutrino Interactions

- Lagrangian for electroweak interactions:

$$L_{\text{int}} = i \frac{g}{\sqrt{2}} [j_{\mu}^{(+)} W^{\mu} + j_{\mu}^{(-)} W^{\mu+}] + i [g \cos \theta_W j_{\mu}^{(3)} - g' \sin \theta_W j_{\mu}^{(Y/2)}] Z^{\mu} + \\ + i [g \sin \theta_W j_{\mu}^{(3)} + g' \cos \theta_W j_{\mu}^{(Y/2)}] A^{\mu}$$

- 1<sup>st</sup> term: charged current interactions ( $W^+$ ,  $W^-$  exchange)
- 2<sup>nd</sup> term: neutral current interactions ( $Z^0$  exchange)
- 3<sup>rd</sup> term: electromagnetic interactions (photon exchange)

- Electron charge:  $e = g \sin \theta_W = g' \cos \theta_W$

- 3<sup>rd</sup> term:  $e j_{\mu}^{e.m.} = e (j_{\mu}^{(3)} + j_{\mu}^{(Y/2)})$   
(neutrinos only couple to  $W^{\pm}$  and  $Z^0$ )

# S.M. interactions (cont)

- A) Neutrino electron interaction

$$L_{\text{int}} = i \frac{g}{\sqrt{2}} [j_{\mu}^{(+)} W^{\mu} + j_{\mu}^{(-)} W^{\mu+}] + i \frac{g}{2 \cos \theta_W} j_{\mu}^{(Z)} Z^{\mu} + i e j_{\mu}^{e.m.}$$

- Where:  $j_{\mu}^{(+)} = \bar{\nu}_{e,L} \gamma_{\mu} e_L = \frac{1}{2} \bar{\nu}_e \gamma_{\mu} (1 - \gamma_5) e$

$$j_{\mu}^{(-)} = \bar{e}_L \gamma_{\mu} \nu_{e,L} = \frac{1}{2} \bar{e} \gamma_{\mu} (1 - \gamma_5) \nu_e$$

$$j_{\mu}^{(Z)} = 2(j_{\mu}^{(3)} - \sin^2 \theta_W j_{\mu}^{e.m.}) =$$

$$= \bar{\nu}_{e,L} \gamma_{\mu} \nu_{e,L} - \bar{e}_L \gamma_{\mu} e_L + 2 \sin^2 \theta_W \bar{e} \gamma_{\mu} e =$$

$$= \frac{1}{2} \bar{\nu}_e \gamma_{\mu} (1 - \gamma_5) \nu_e - \frac{1}{2} \bar{e} \gamma_{\mu} (1 - \gamma_5) e + 2 \sin^2 \theta_W \bar{e} \gamma_{\mu} e$$

$$\Rightarrow j_{\mu}^{(Z)} = \frac{1}{2} \bar{\nu}_e \gamma_{\mu} (1 - \gamma_5) \nu_e + \bar{e} \gamma_{\mu} (g_V - g_A \gamma_5) e$$

- With:  $g_V = -\frac{1}{2} + 2 \sin^2 \theta_W$        $g_A = -\frac{1}{2}$

# S.M. interactions (cont)

- B) Quark weak interactions

$$L_{\text{int}} = i \frac{g}{\sqrt{2}} [j_{\mu}^{(+)} W^{\mu} + j_{\mu}^{(-)} W^{\mu+}] + i \frac{g}{2 \cos \theta_W} j_{\mu}^{(Z)} Z^{\mu} + i e j_{\mu}^{e.m}$$

- Where: 
$$j_{\mu}^{(+)} = \frac{1}{2} \bar{u} \gamma_{\mu} (1 - \gamma_5) d$$

$$j_{\mu}^{(-)} = \frac{1}{2} \bar{d} \gamma_{\mu} (1 - \gamma_5) u$$

$$j_{\mu}^{(Z)} = \bar{u} \gamma_{\mu} (A_u - B_u \gamma_5) u + \bar{d} \gamma_{\mu} (A_d - B_d \gamma_5) d$$

- With:

$$A_u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad B_u = \frac{1}{2}$$

$$A_d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \quad B_d = -\frac{1}{2}$$

# S.M. interactions (cont)

- After introducing Higgs field and spontaneous symmetry breaking:

$$L_{Higgs} = -|D_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4$$

- Masses:

$$m_H = \sqrt{2\lambda} v$$

$$m_{W^\pm} = \frac{gv}{2} \quad \left( \frac{m_{W^\pm}}{m_{Z^0}} \right)^2 = \frac{g^2}{g^2 + g'^2} = \cos^2 \theta_W$$

$$m_{Z^0} = \frac{\sqrt{g^2 + g'^2}}{2} v$$

- Vacuum expectation value:  $v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$

- Effective Hamiltonian:

$$\begin{aligned} H_{eff} &= \frac{g^2}{4m_W^2} [j^{(+)\mu} j_\mu^{(-)} + h.c.] + \frac{g^2}{8m_Z^2 \cos^2 \theta_W} j^{(Z)\mu} j_\mu^{(Z)} = \\ &= \frac{G_F}{\sqrt{2}} [2j^{(+)\mu} j_\mu^{(-)} + h.c. + j^{(Z)\mu} j_\mu^{(Z)}] \end{aligned}$$

# S.M. interactions (cont)

- The vector boson masses are then predicted:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2 \theta_W} = \frac{4\pi\alpha}{8m_W^2 \sin^2 \theta_W} \quad \alpha = 1/137.036$$

- Masses:

$$m_W^2 = \left( \frac{37.2805}{\sin \theta_W} \right)^2$$

$$m_W = 80.450 \pm 0.058 \text{ GeV}$$

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

$$\sin^2 \theta_W = 0.22280 \pm 0.00035$$

- Need radiative corrections:

$$m_W = \frac{37.2805}{\sin \theta_W (1 - \Delta r)^{1/2}}$$

with  $\Delta r \approx 0.03630 \pm 0.0011$  for  $m_t = 172.7 \text{ GeV}$   $m_H = 117 \text{ GeV}$

yields:  $m_W = 80.51 \pm 0.11 \text{ GeV}$



## 2. Neutrino Electron Scattering

2.1 Charged current

2.2 Neutral current

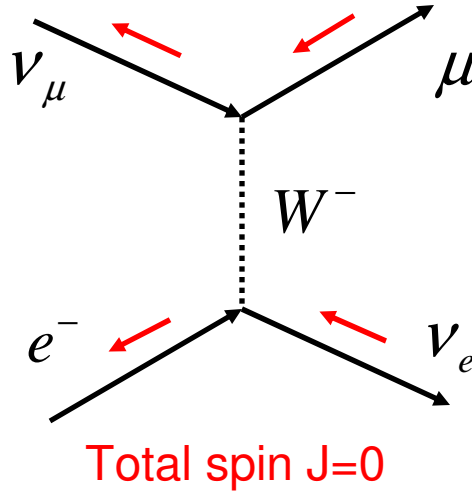
2.3 Interference charged and neutral current

2.4 Mass suppression

2.5 Number of neutrinos

# 2.1 Neutrino-electron CC scattering

□ Only charged current:  $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$  (Inverse Muon Decay)



$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \mu] [\bar{e} \gamma_\mu (1 - \gamma_5) \nu_e]$$

$$s = (p_{\nu_\mu} + p_e)^2 = 2m_e E_{\nu_\mu} \text{ (in LAB)}$$

$$t = q^2 = -Q^2 = (p_{\nu_\mu} - p_\mu)^2 \quad \text{Inelasticity variable}$$

$$y = \frac{p_e \cdot (p_{\nu_\mu} - p_\mu)}{p_e \cdot p_{\nu_\mu}} = \frac{E_{\nu_\mu} - E_\mu}{E_{\nu_\mu}} \text{ (in LAB)} \quad (0 < y < 1)$$

$$\frac{d\sigma_{CC}(\nu_\mu e^-)}{dQ^2 dy} = \frac{G_F^2}{\pi} \frac{m_W^4}{(Q^2 + m_W^2)^2} \Rightarrow \sigma_{CC}(\nu_\mu e^-) = \int_0^s \frac{G_F^2}{\pi} \frac{m_W^4}{(Q^2 + m_W^2)^2} dQ^2$$

Total cross-section (ignoring mass terms):

Measurement CHARM-II:

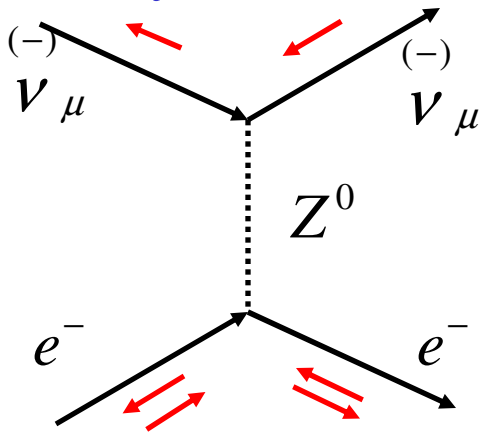
$$\sigma_{CC}(\nu_\mu e^-) \approx \frac{G_F^2 s}{\pi} = \frac{2G_F^2 m_e}{\pi} E_{\nu_\mu} \text{ (in LAB)} \quad \sigma(\nu_\mu e^-) = (1.651 \pm 0.093) \times 10^{-41} \left( \frac{E}{1 \text{ GeV}} \right) \text{ cm}^2$$

(cross-section proportional to energy!)

$$\Rightarrow \sigma_{CC}(\nu_\mu e^-) = \frac{2G_F^2 m_e (\hbar c)^2 E_{\nu_\mu}}{\pi} = 1.72 \times 10^{-41} \left( \frac{E_{\nu_\mu}}{1 \text{ GeV}} \right) \text{ cm}^2 \quad \begin{aligned} \hbar c &= 197.33 \text{ MeV} \cdot \text{fm} \\ G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2} \end{aligned}$$

## 2.2 Neutrino-electron NC scattering

□ Only neutral current:



$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$  Elastic scattering

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\mu (1 - \gamma_5) \nu_\mu] [\bar{e} \gamma_\mu (g_V - g_A \gamma_5) e]$$

$$\bar{e} \gamma_\mu (g_V - g_A \gamma_5) e = g_L \bar{e} \gamma_\mu (1 - \gamma_5) e + g_R \bar{e} \gamma_\mu (1 + \gamma_5) e$$

$$g_V = -\frac{1}{2} + 2 \sin^2 \theta_W \quad g_L = \frac{1}{2} (g_V + g_A) = -\frac{1}{2} + \sin^2 \theta_W$$

$$g_R = \frac{1}{2} (g_V - g_A) = \sin^2 \theta_W$$

Couples to  $e_L$  and  $e_R$ :  $J=0,1$   $g_A = -\frac{1}{2}$

Right handed current suppressed in backward direction:

$$1 - y = \frac{1 + \cos \theta^*}{2}$$

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi} \frac{m_Z^4}{(Q_{\text{max}}^2 + m_Z^2)^2} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W (1 - y)^2 \right]$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi} \frac{m_Z^4}{(Q_{\text{max}}^2 + m_Z^2)^2} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 (1 - y)^2 + \sin^4 \theta_W \right]$$

## 2.2 Neutrino-electron NC scattering

- Only neutral current (total cross-section):  $\overset{(-)}{\nu}_\mu + e^- \rightarrow \overset{(-)}{\nu}_\mu + e^-$

$$\sigma_{NC}(\nu_\mu e^-) = \frac{G_F^2 s}{\pi} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right] = 0.16 \times 10^{-41} \left( \frac{E_\nu}{1 \text{ GeV}} \right) \text{ cm}^2$$

$$\sigma_{NC}(\bar{\nu}_\mu e^-) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right] = 0.13 \times 10^{-41} \left( \frac{E_\nu}{1 \text{ GeV}} \right) \text{ cm}^2$$

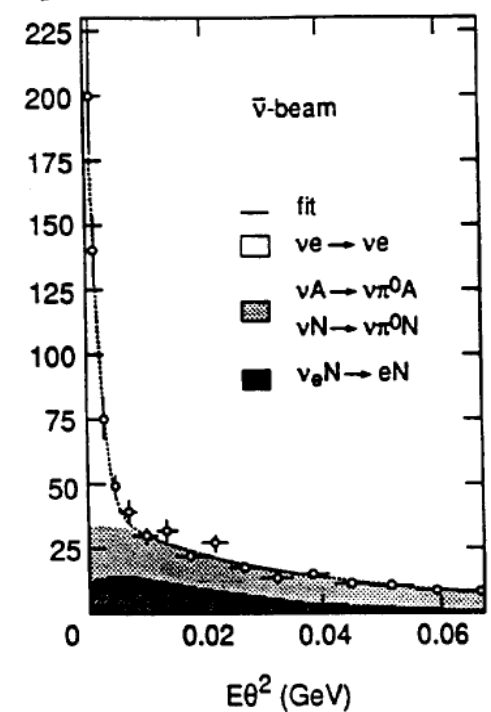
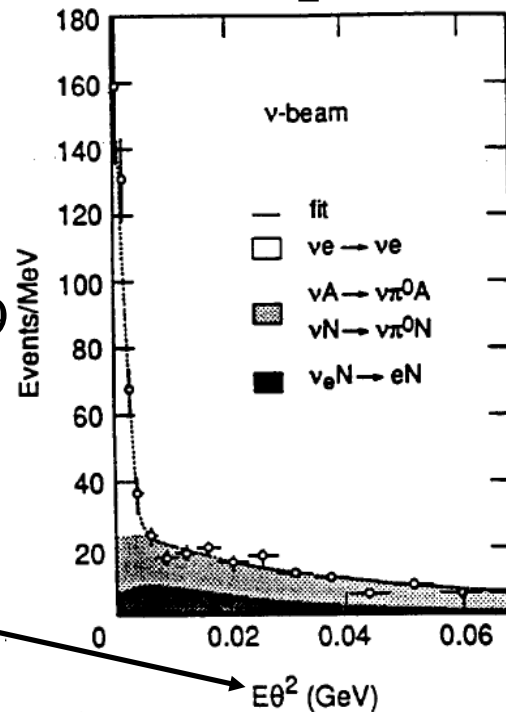
- Can obtain value of  $\sin^2 \theta_W$  from neutrino electron elastic scattering (CHARM II):

$$\sin^2 \theta_W = 0.2324 \pm 0.0058 \pm 0.0059$$

$$g_V = -0.035 \pm 0.017$$

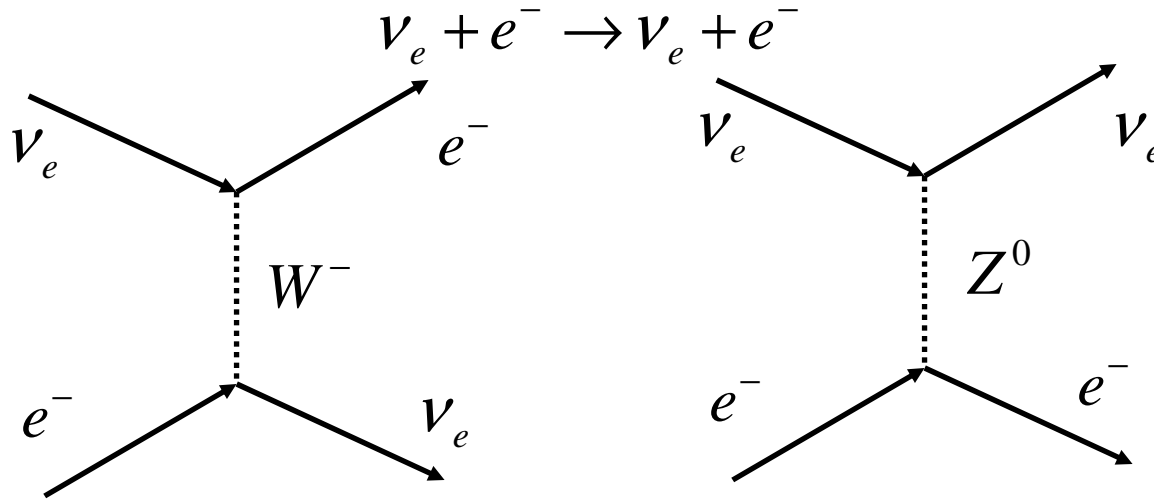
$$g_A = -0.503 \pm 0.017$$

$$E_e \Theta^2 = 2m_e(1-y)$$



## 2.3 Interference CC and NC

- Tree level Feynman diagrams: both neutral and charged currents



- Effective Hamiltonian:

$$\begin{aligned}
 H_{eff} &= \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e] [\bar{e} \gamma_\mu (1 - \gamma_5) \nu_e] + [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] [\bar{e} \gamma_\mu (g_V - g_A \gamma_5) e] \right\} \\
 &= \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] [\bar{e} \gamma_\mu (1 + g_V - (1 + g_A) \gamma_5) e] \right\}
 \end{aligned}$$

(through a Fierz transformation)

## 2.3 Interference CC and NC

- Rearranging terms in charged and neutral current contributions for:

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

$$g_L = \frac{1}{2}(1 + g_V + 1 + g_A) = -\frac{1}{2} + \sin^2 \theta_W + 1 = \frac{1}{2} + \sin^2 \theta_W$$

$$g_R = \frac{1}{2}(1 + g_V - (1 + g_A)) = \sin^2 \theta_W$$

Then: 
$$\frac{d\sigma(\nu_e e^-)}{dy} = \frac{G_F^2 s}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W (1-y)^2 \right]$$

$$\Rightarrow \sigma(\nu_e e^-) = \frac{G_F^2 s}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right] = 0.96 \times 10^{-41} \left( \frac{E_\nu}{1 \text{ GeV}} \right) \text{ cm}^2$$

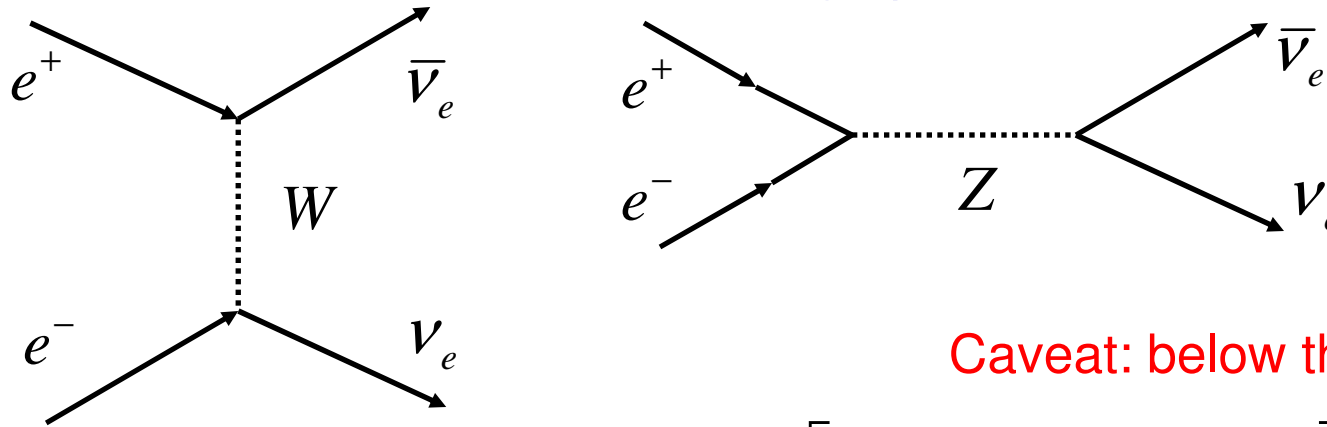
$$\text{Also : } \sigma(\bar{\nu}_e e^-) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right] = 0.40 \times 10^{-41} \left( \frac{E_\nu}{1 \text{ GeV}} \right) \text{ cm}^2$$

These cross-sections are a consequence of the interference of the charged and neutral current diagrams.

## 2.3 Interference CC and NC

- Neutrino pair production:  $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$

Contribution from both W and Z graphs.



Caveat: below the Z pole!

Then:

$$\sigma(e^+e^- \rightarrow \nu_e \bar{\nu}_e) = \frac{G_F^2 s}{12\pi} \left[ \left( \frac{1}{2} + 2 \sin^2 \theta_W \right)^2 + \frac{1}{4} \right]$$

- Only neutral current contribution to:  $e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$

$$\sigma(e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu) = \frac{G_F^2 s}{12\pi} \left[ \left( \frac{1}{2} - 2 \sin^2 \theta_W \right)^2 + \frac{1}{4} \right]$$

# Neutrino-electron scattering summary

- Summary neutrino electron scattering processes:

Process	Total cross-section
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	$\frac{G_F^2 s}{\pi}$
$\nu_e + e^- \rightarrow \nu_e + e^-$	$\frac{G_F^2 s}{4\pi} \left[ (2\sin^2 \theta_w - 1)^2 + \frac{4}{3} \sin^4 \theta_w \right]$
$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	$\frac{G_F^2 s}{4\pi} \left[ \frac{1}{3} (2\sin^2 \theta_w + 1)^2 + 4\sin^4 \theta_w \right]$
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	$\frac{G_F^2 s}{4\pi} \left[ (2\sin^2 \theta_w - 1)^2 + \frac{4}{3} \sin^4 \theta_w \right]$
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$	$\frac{G_F^2 s}{4\pi} \left[ \frac{1}{3} (2\sin^2 \theta_w - 1)^2 + 4\sin^4 \theta_w \right]$
$e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$	$\frac{G_F^2 s}{12\pi} \left[ \frac{1}{2} + 2\sin^2 \theta_w + 4\sin^4 \theta_w \right]$
$e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$	$\frac{G_F^2 s}{12\pi} \left[ \frac{1}{2} - 2\sin^2 \theta_w + 4\sin^4 \theta_w \right]$

$$s = 2m_e E_\nu \text{ (in the LAB frame)}$$



## 2.4 Mass suppression

- We have not taken into account the effect of initial and final state masses yet
- For example:  $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$

Threshold  $s = m_e^2 + 2m_e E_\nu \geq m_\mu^2 \Rightarrow E_\nu \geq \frac{m_\mu^2 - m_e^2}{2m_e} \approx 11 \text{ GeV}$

- Cross-section modification:

$$\begin{aligned} \sigma_{CC}(\nu_\mu e^-) &= \int_{Q_{\min}^2}^{Q_{\max}^2} \frac{G_F^2}{\pi} \frac{m_W^4}{(Q^2 + m_W^2)^2} dQ^2 = \frac{G_F^2}{\pi} \frac{m_W^4}{(Q_{\max}^2 + m_W^2)(Q_{\min}^2 + m_W^2)} (Q_{\max}^2 - Q_{\min}^2) \approx \\ &\approx \frac{G_F^2}{\pi} (Q_{\max}^2 - Q_{\min}^2) = \frac{G_F^2}{\pi} (s - m_\mu^2) \end{aligned}$$

- Therefore:

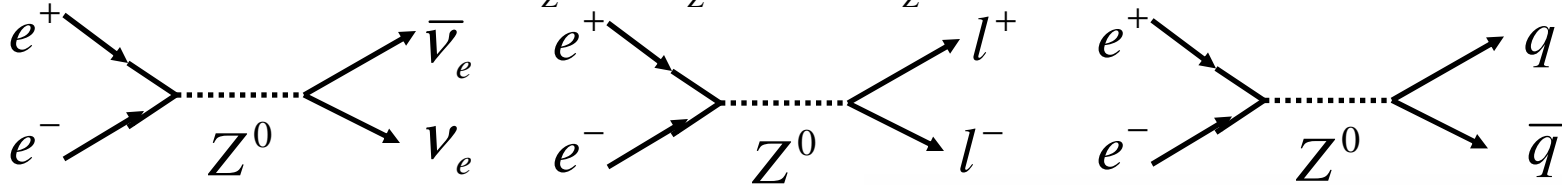
$$\sigma_{CC}(\nu_\mu e^-) = \frac{G_F^2 s}{\pi} \left( 1 - \frac{m_\mu^2}{s} \right) = \sigma_{CC}^{\text{massless}}(\nu_\mu e^-) \left( 1 - \frac{m_\mu^2}{s} \right)$$

# 2.5 Number of neutrinos

- Width of the Z-pole resonance: Breit-Wigner distribution

$$\sigma(e^+e^- \rightarrow f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{s\Gamma_e\Gamma_f}{(s - M_Z^2)^2 + s^2\Gamma_Z^2 / M_Z}$$

$$\sigma_{peak}(e^+e^- \rightarrow f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{\Gamma_e\Gamma_f}{\Gamma_Z^2} = \frac{12\pi(\hbar c)^2}{M_Z} B(Z^0 \rightarrow e^+e^-)B(Z^0 \rightarrow f\bar{f})$$



$$\Gamma_Z = \Gamma_{had} + 3\Gamma_{l^+l^-} + N_\nu\Gamma_{\nu\bar{\nu}} = 2490\text{MeV} \quad \text{2 neutrinos}$$

$\sigma(Z \rightarrow \text{hadrons})$

$$\Gamma_{had} = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_b = 1741\text{MeV}$$

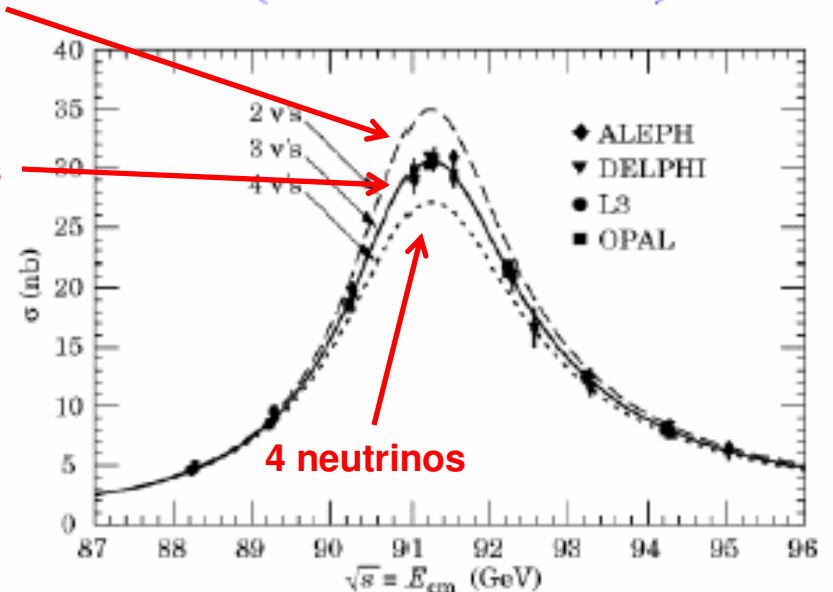
$$\Gamma_{l^+l^-} = 83.9\text{MeV}$$

$$\Gamma_{\nu\bar{\nu}} = 167.1\text{MeV}$$

$$\Rightarrow N_\nu = 2.9841 \pm 0.0083$$

- Only 3 neutrinos with mass less than the Z mass

3 neutrinos



Neutrino Interac  
NUFACT08 Sun

- # 3. Neutrino Nucleon Deep-Inelastic Scattering
- 3.1 Definition and variables
  - 3.2 Charged current
  - 3.3 Quark content of nucleons
  - 3.4 Sum rules
  - 3.5 Neutral current
  - 3.6 A case study:  $\sin^2\theta_W$  from neutrino interactions
  - 3.7 Charm production in neutrino interactions

# 3.1 Definition and Variables

- Deep inelastic neutrino-nucleon scattering reactions have large  $q^2$   $\nu_l(p) + N \rightarrow l^-(p') + X$  ( $q^2 \gg m_N^2, E_\nu \gg m_N$ ):
- Quark-parton model valid due to asymptotic freedom of QCD, which makes quarks behave as free point-like particles.
- Infinite momentum frame: a parton takes a fraction  $x$  ( $0 < x < 1$ ), of momentum when struck by a neutrino. Final quark state:

$$(xp_N + q)^2 = m_q^2 \Rightarrow x \approx -\frac{q^2}{2p_N \cdot q} \quad \text{if } q^2 \gg m_q^2$$

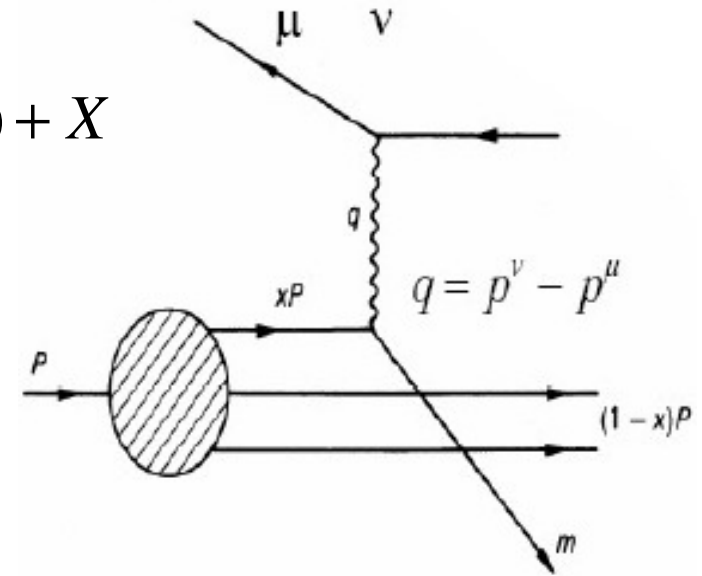
- Variables in DIS:

$$s = (p + p_N)^2 \approx 2ME_\nu = 2ME$$

$$Q^2 = -q^2 = -(p + p')^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$W^2 = E_X^2 - p_X^2 = -Q^2 + 2M\nu + M^2 \quad \text{Recoil mass}$$

$$\nu = \frac{q \cdot p_N}{M} = E - E'$$



Bjorken Variables

( $0 < x < 1, 0 < y < 1$ ):

$$x = \frac{-q^2}{2q \cdot p_N} = \frac{Q^2}{2M\nu}$$

$$y = \frac{q \cdot p_N}{p \cdot p_N} = \frac{\nu}{E} = \frac{Q^2}{2MEx}$$

## 3.2 Charged current

- Neutrino proton CC scattering:  $\nu_\mu(p) + p \rightarrow \mu^-(p') + X$

$u(x)dx$  = number of u-quarks in proton between  $x$  and  $x+dx$

$$u(x) = u_V(x) + u_S(x) \quad d(x) = d_V(x) + d_S(x)$$

In the sea:  $u_S(x) = \bar{u}(x) \quad d_S(x) = \bar{d}(x)$

For proton (uud):

$$\int_0^1 u_V(x) dx = \int_0^1 [u(x) - \bar{u}(x)] dx = 2$$

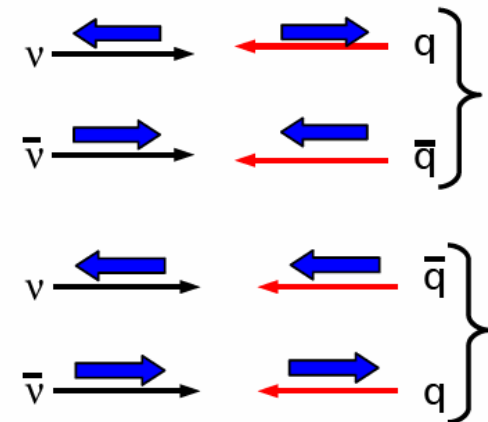
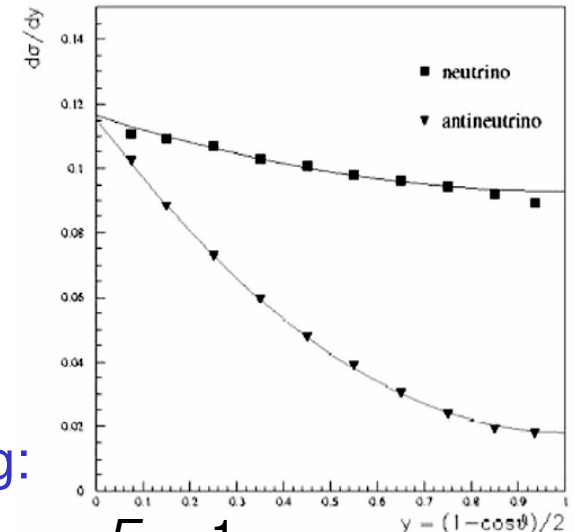
$$\int_0^1 d_V(x) dx = \int_0^1 [d(x) - \bar{d}(x)] dx = 1$$

- Neutrino-quark/antineutrino-antiquark scattering:

$$\frac{d\sigma_{CC}(\nu_\mu q)}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu \bar{q})}{dy} = \frac{2G_F^2 m_q E}{\pi} \quad \text{with } y = 1 - \frac{E}{E'} = \frac{1}{2}(1 - \cos\theta)$$

- Neutrino-antiquark/antineutrino-quark scattering:

$$\frac{d\sigma_{CC}(\nu_\mu \bar{q})}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu q)}{dy} = \frac{2G_F^2 m_q E}{\pi} (1-y)^2$$



## 3.2 Charged current

□ Scattering off proton:  $proton = uud + (u\bar{u}) + (d\bar{d}) + (s\bar{s}) + (c\bar{c})$

$$\frac{d\sigma_{CC}(v_\mu p)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [d(x) + s(x)] + [\bar{u}(x) + \bar{c}(x)] (1-y)^2 \right\}$$

$$\frac{d\sigma_{CC}(\bar{v}_\mu p)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [u(x) + c(x)] (1-y)^2 + [\bar{d}(x) + \bar{s}(x)] \right\}$$

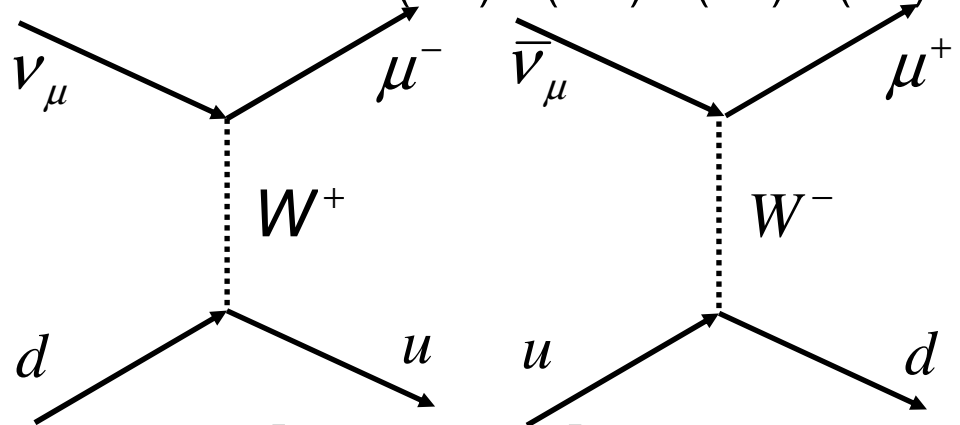
□ Neutron (isospin symmetry):  $neutron = uud + (u\bar{u}) + (d\bar{d}) + (s\bar{s}) + (c\bar{c})$

$$u_n(x) = d_p(x) \equiv d(x)$$

$$d_n(x) = u_p(x) \equiv u(x)$$

$$s_n(x) = s_p(x) \equiv s(x)$$

$$c_n(x) = c_p(x) \equiv c(x)$$



$$\frac{d\sigma_{CC}(v_\mu n)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [u(x) + s(x)] + [\bar{d}(x) + \bar{c}(x)] (1-y)^2 \right\}$$

$$\frac{d\sigma_{CC}(\bar{v}_\mu n)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [d(x) + c(x)] (1-y)^2 + [\bar{u}(x) + \bar{s}(x)] \right\}$$

## 3.2 Charged current

□ Structure functions:

$$\frac{d^2\sigma^{v,\bar{v}}}{dxdy} = \frac{G_F^2 s}{2\pi} \left[ y^2 2xF_1(x, Q^2) + 2 \left( 1 - y - \frac{Mxy}{2E} \right) F_2(x, Q^2) \pm 2y \left( 1 - \frac{y}{2} \right) xF_3(x, Q^2) \right]$$

$F_i(x, Q^2)$  are the structure functions, which depend on the helicity structure of  $q$ - $W$  interactions. For massless spin-1/2 partons, we have the **Callan-Gross** relationship\*:  $2xF_1(x) = F_2(x)$

$$\begin{aligned} \frac{d^2\sigma^{v,\bar{v}}}{dxdy} &= \frac{G_F^2 s}{2\pi} \left[ \left( (1-y)^2 + \left( 1 - \frac{Mxy}{2E} \right) \right) F_2(x, Q^2) \pm 2y \left( 1 - \frac{y}{2} \right) xF_3(x, Q^2) \right] = \\ &= \frac{G_F^2 s}{2\pi} \left[ \left( 1 + (1-y)^2 \right) F_2(x, Q^2) \pm \left( 1 - (1-y)^2 \right) xF_3(x, Q^2) \right] \end{aligned}$$

Assuming massless target

\* Deviations from the Callan-Gross relation are parameterised in terms of the “longitudinal” cross-section (ie.gluon splitting  $g \rightarrow qq$ ):

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2(x)}{2xF_1(x)} \left( 1 + \frac{4Mx^2}{Q^2} \right)$$

## 3.2 Charged current

- Comparing the  $y$  distribution of both cross-sections we can compare the parton distribution functions to the proton structure functions:

$$F_2^{vp}(x) = x[d(x) + \bar{u}(x) + s(x) + \bar{c}(x)]$$
$$xF_3^{vp}(x) = x[d(x) - \bar{u}(x) + s(x) - \bar{c}(x)]$$
$$F_2^{\bar{v}p}(x) = x[u(x) + c(x) + \bar{d}(x) + \bar{s}(x)]$$
$$xF_3^{\bar{v}p}(x) = x[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)]$$

- Also, the neutron structure functions:

$$F_2^{vn}(x) = x[u(x) + \bar{d}(x) + s(x) + \bar{c}(x)]$$
$$xF_3^{vn}(x) = x[u(x) - \bar{d}(x) + s(x) - \bar{c}(x)]$$
$$F_2^{\bar{v}n}(x) = x[d(x) + c(x) + \bar{u}(x) + \bar{s}(x)]$$
$$xF_3^{\bar{v}n}(x) = x[d(x) + c(x) - \bar{u}(x) - \bar{s}(x)]$$



## 3.2 Charged current

- Scattering off isoscalar target (equal number neutrons and protons):

$$q \equiv u + d + s + c \qquad \bar{q} \equiv \bar{u} + \bar{d} + \bar{s} + \bar{c}$$

$$F_2^{vN}(x) = x[q(x) + \bar{q}(x)]$$

$$xF_3^{vN}(x) = x[q(x) - \bar{q}(x) + 2(s(x) - c(x))]$$

$$xF_3^{\bar{v}N}(x) = x[q(x) - \bar{q}(x) - 2(s(x) - c(x))]$$

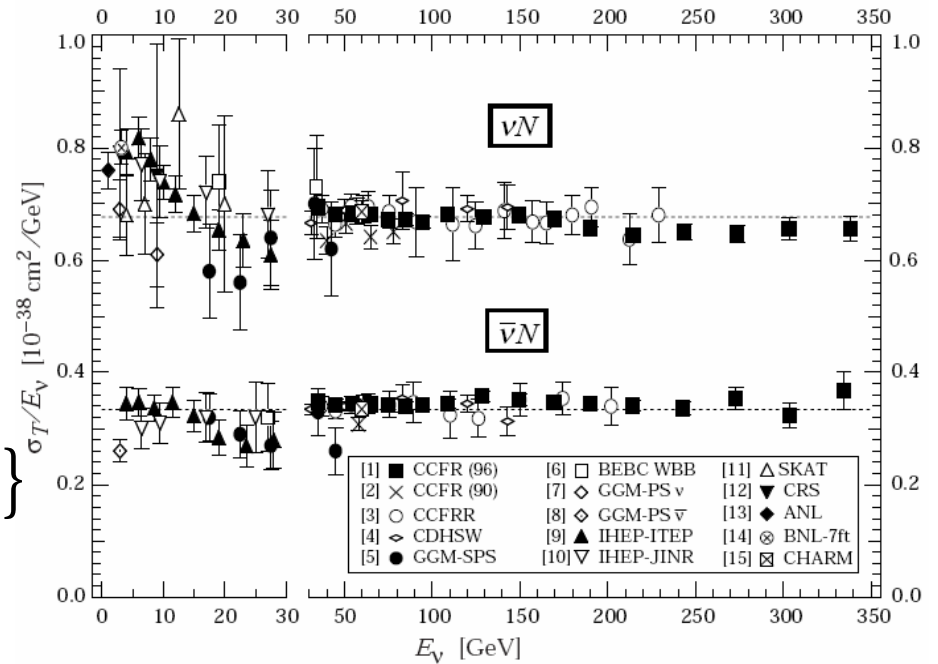
$$\frac{d\sigma_{CC}(v_\mu N)}{dx dy} = \frac{G_F^2 2ME}{2\pi} x \{ q(x) + \bar{q}(x) (1-y)^2 \}$$

$$\frac{d\sigma_{CC}(\bar{v}_\mu N)}{dx dy} = \frac{G_F^2 2ME}{2\pi} x \{ q(x)(1-y)^2 + \bar{q}(x) \}$$

- Total cross-section:

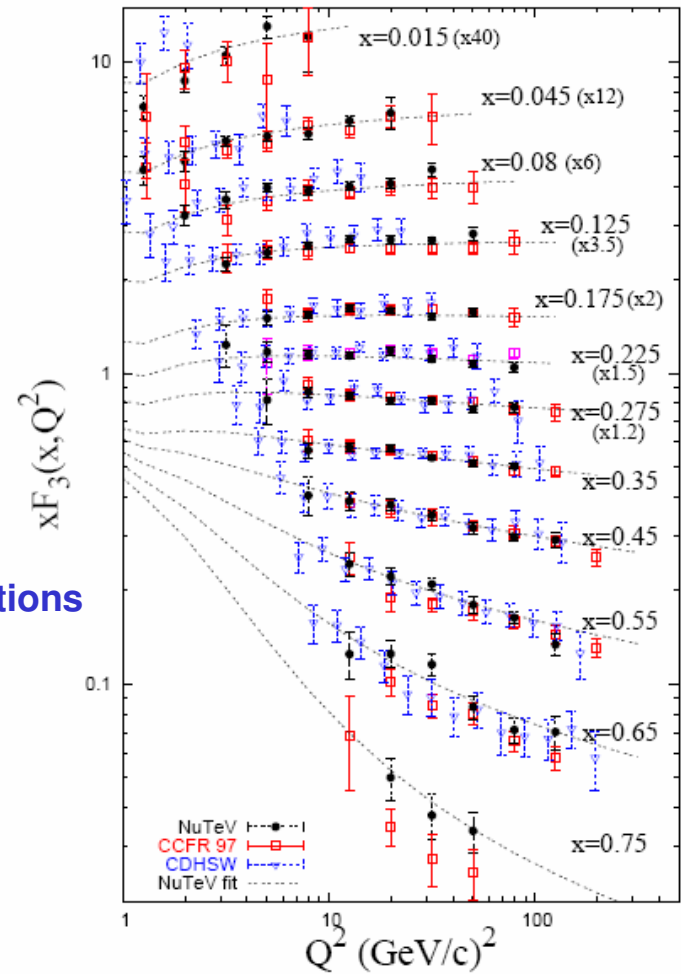
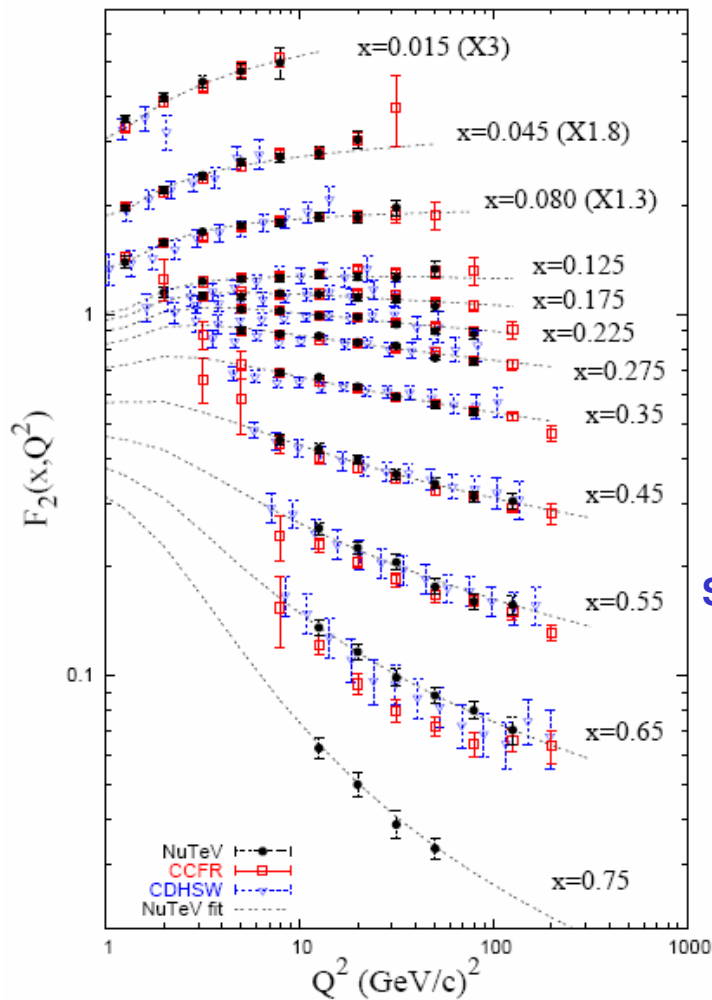
$$\sigma_{CC}(v_\mu N) = \frac{G_F^2 s}{2\pi} \left[ \langle Q \rangle + \frac{1}{3} \langle \bar{Q} \rangle \right] = (0.677 \pm 0.014) \times 10^{-38} \text{ cm}^2 / \text{GeV} \times E(\text{GeV})$$

$$\sigma_{CC}(\bar{v}_\mu N) = \frac{G_F^2 s}{2\pi} \left[ \frac{1}{3} \langle Q \rangle + \langle \bar{Q} \rangle \right] = (0.334 \pm 0.008) \times 10^{-38} \text{ cm}^2 / \text{GeV} \times E(\text{GeV})$$



# 3.2 Charged current

□ Structure functions:



Scaling violations

## 3.3 Quark content of nucleons

- Quark content of nucleons from CC cross-sections

- Define: 
$$U = \int_0^1 xu(x)dx, \text{ etc.}$$

- Experimental values from  $y$  distribution of cross-sections yields:

Since 
$$r \equiv \frac{\sigma_{CC}(\bar{\nu}N)}{\sigma_{CC}(\nu N)} = 0.493 \pm 0.016 \text{ (measured)}$$

then: 
$$\frac{\bar{Q}}{Q} = \frac{3r - 1}{3 - r} \approx 0.191 \quad \Rightarrow \quad Q = 0.405 \quad \text{and} \quad \bar{Q} = 0.078$$

therefore: 
$$Q_V = Q - \bar{Q} \approx 0.33 \quad \frac{\bar{Q}}{Q + \bar{Q}} = 0.16 \pm 0.03$$

$$\int_0^1 F_2^{\nu N}(x) dx = Q + \bar{Q} \approx 0.48$$

- Quarks and antiquarks carry 48% of proton momentum, valence quarks only 33% and sea quarks only 7.8% (u and d sea quarks carry 6%, s quarks carry 1.3% and c quarks 0.5%).

## 3.3 Quark content of nucleons

- Parton distribution functions as a function of  $x$ , fitted from structure functions:

$u(x)dx$  = number of u-quarks in proton between  $x$  and  $x+dx$

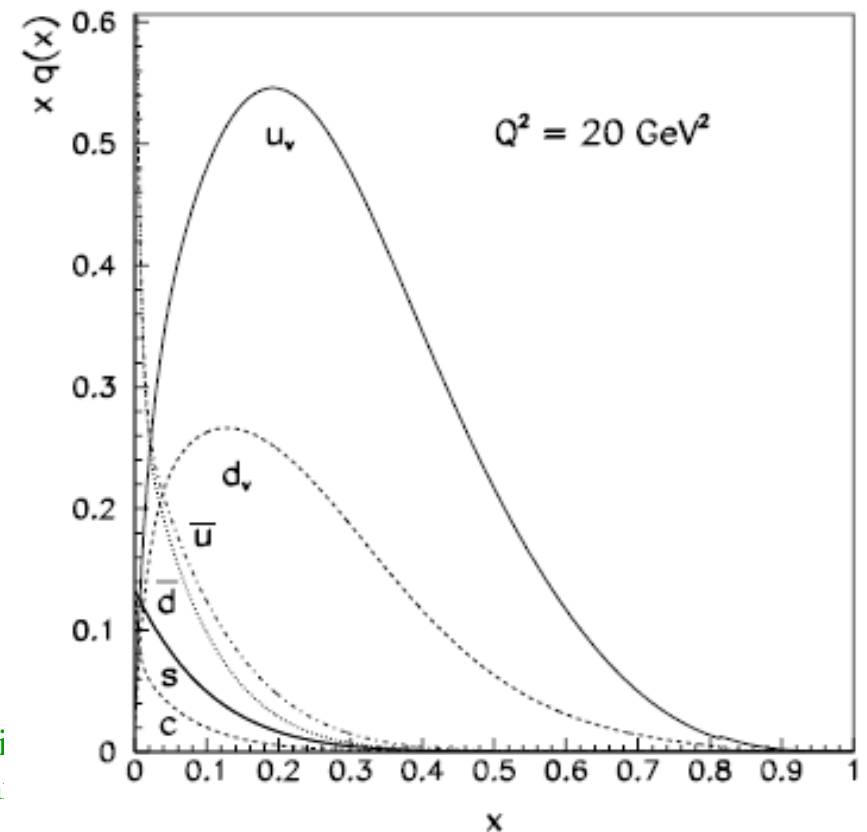
$$u(x) = u_V(x) + u_S(x) \quad d(x) = d_V(x) + d_S(x)$$

In the sea:  $d_S(x) = \bar{d}(x) \quad u_S(x) = \bar{u}(x)$

For proton (uud):

$$\int_0^1 u_V(x) dx = \int_0^1 [u(x) - \bar{u}(x)] dx = 2$$

$$\int_0^1 d_V(x) dx = \int_0^1 [d(x) - \bar{d}(x)] dx = 1$$



## 3.4 Sum rules

□ Sum rules:

– Gross-Llewellyn Smith: 
$$S_{GLS} = \frac{1}{2} \int_0^1 (F_3^{\nu} (x) + F_3^{\bar{\nu}} (x)) dx$$

$$S_{GLS} = \int_0^1 (q(x) - \bar{q}(x)) dx = 3 \left[ 1 - \frac{\alpha_s}{\pi} - a \left( \frac{\alpha_s}{\pi} \right)^2 - b \left( \frac{\alpha_s}{\pi} \right)^3 \right] = 2.64 \pm 0.06$$

– Adler:

$$S_A = \frac{1}{2} \int_0^1 \frac{1}{x} (F_2^{\nu n} (x) + F_2^{\nu p} (x)) dx = \int_0^1 (u_V (x) - d_V (x)) dx = 1$$

– Gottfried:

$$S_G = \frac{1}{2} \int_0^1 \frac{1}{x} (F_2^{\mu n} (x) + F_2^{\mu p} (x)) dx = \frac{1}{3} \int_0^1 (u(x) + \bar{u}(x) - d(x) - \bar{d}(x)) dx = \frac{1}{3}$$

$$S_G = 0.235 \pm 0.026 \quad \text{Maybe isospin asymmetry: } \bar{u}(x) \neq \bar{d}(x)$$

– Bjorken:

$$S_B = \int_0^1 (F_1^{\bar{\nu} p} (x) + F_1^{\nu p} (x)) dx = 1 - \frac{2\alpha_s(Q^2)}{3\pi}$$

## 3.5 Neutral current

□ Neutral currents:  $\bar{\nu}_\mu + p \rightarrow \bar{\nu}_\mu + X$

$$\frac{d\sigma_{NC}(\nu_\mu q)}{dy} = \frac{d\sigma_{NC}(\bar{\nu}_\mu \bar{q})}{dy} =$$

$$\frac{G_F^2 m_q E_\nu}{2\pi} \left\{ (g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 + \frac{m_q}{E_\nu} (g_A^2 - g_V^2) y \right\}$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu q)}{dy} = \frac{d\sigma_{NC}(\nu_\mu \bar{q})}{dy} =$$

$$\frac{G_F^2 m_q E_\nu}{2\pi} \left\{ (g_V - g_A)^2 + (g_V + g_A)^2 (1-y)^2 + \frac{m_q}{E_\nu} (g_A^2 - g_V^2) y \right\}$$

□ Coupling constants:

$$g_V = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W$$

$$g_A = \frac{1}{2} \quad \text{for } q=u,c$$

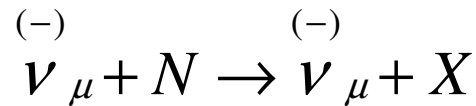
$$g'_V = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$$

$$g'_A = -\frac{1}{2} \quad \text{for } q=d,s$$

$$\begin{cases} g_L = \frac{1}{2} (g_V + g_A) \\ g_R = \frac{1}{2} (g_V - g_A) \end{cases}$$

## 3.5 Neutral current

- Neutral currents off nucleons (neglecting c and s quark contributions):



$$\frac{d\sigma_{NC}(\nu_\mu N)}{dx dy} = \frac{G_F^2 ME}{\pi} x \left\{ (g_L^2 + g_L'^2) [q + \bar{q}(1-y)^2] + (g_R^2 + g_R'^2) [\bar{q} + q(1-y)^2] \right\}$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{dx dy} = \frac{G_F^2 ME}{\pi} x \left\{ (g_R^2 + g_R'^2) [q + \bar{q}(1-y)^2] + (g_L^2 + g_L'^2) [\bar{q} + q(1-y)^2] \right\}$$

- Defining:  $R_\nu \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)}$        $R_{\bar{\nu}} \equiv \frac{\sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\bar{\nu} N)}$        $r \equiv \frac{\sigma_{CC}(\bar{\nu} N)}{\sigma_{CC}(\nu N)}$

yields:  $g_L^2 + g_L'^2 = \frac{R_\nu - r^2 R_{\bar{\nu}}}{1 - r^2}$        $g_R^2 + g_R'^2 = \frac{r(R_{\bar{\nu}} - R_\nu)}{1 - r^2}$

$$R_\nu = (g_L^2 + g_L'^2) + r(g_R^2 + g_R'^2) = \frac{1}{2} - \sin^2 \theta_W + (1+r) \frac{5}{9} \sin^4 \theta_W$$

$$R_{\bar{\nu}} = (g_L^2 + g_L'^2) + \frac{1}{r}(g_R^2 + g_R'^2) = \frac{1}{2} - \sin^2 \theta_W + \left(1 + \frac{1}{r}\right) \frac{5}{9} \sin^4 \theta_W$$

**(Llewellyn-Smith relationships)**

## 3.5 Neutral currents

- More relationships from the combination of neutrino and antineutrino tagged interactions:

$$R^+ = \frac{\frac{d\sigma_{NC}(v_\mu N)}{dy} + \frac{d\sigma_{NC}(\bar{v}_\mu N)}{dy}}{\frac{d\sigma_{CC}(v_\mu N)}{dy} + \frac{d\sigma_{CC}(\bar{v}_\mu N)}{dy}} = \frac{1}{2} - \sin^2 \theta_W + \frac{10}{9} \sin^4 \theta_W$$

$$R^- = \frac{\frac{d\sigma_{NC}(v_\mu N)}{dy} - \frac{d\sigma_{NC}(\bar{v}_\mu N)}{dy}}{\frac{d\sigma_{CC}(v_\mu N)}{dy} - \frac{d\sigma_{CC}(\bar{v}_\mu N)}{dy}} = \frac{R_\nu - rR_{\bar{\nu}}}{1-r} = \frac{1}{2} - \sin^2 \theta_W$$

**(Paschos-Wolfenstein relationship)**

- Paschos-Wolfenstein relation removes the effects of sea quark differences (especially at low x) since the neutrino and antineutrino cross-sections are equal. It would also remove error from c quark
- All of these relationships can be used in neutrino experiments to test the electroweak theory and measure  $\sin^2 \theta_W$



## 3.6 $\sin^2\theta_W$

- Llewellyn-Smith relationship used to measure  $\sin^2\theta_W$  by performing ratios of charged current to neutral current of neutrino nucleon scattering.

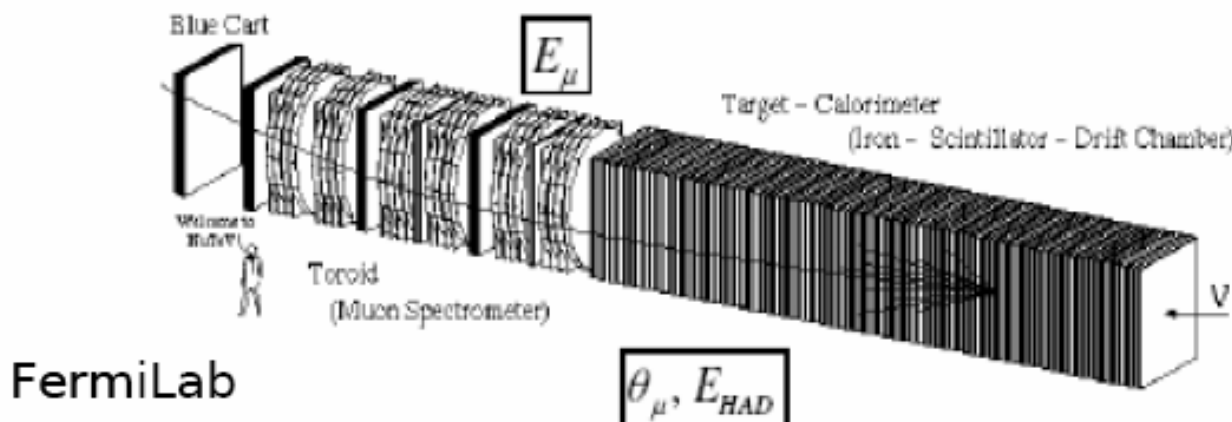
$$R_\nu = \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} = \frac{1}{2} - \sin^2\theta_W + (1+r)\frac{5}{9}\sin^4\theta_W$$

(Llewellyn-Smith relationships)

$$R_{\bar{\nu}} = \frac{\sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\bar{\nu} N)} = \frac{1}{2} - \sin^2\theta_W + \left(1 + \frac{1}{r}\right)\frac{5}{9}\sin^4\theta_W$$

- CHARM, CDHS and CCFR and NuTeV are all large sampling calorimeters that can measure large statistics CC and NC data:

CCFR/NuTeV



## 3.6 $\sin^2\theta_W$

- The ratio of NC to CC data from an average of different experiments (CDHS, CHARM, CCFR, NUTEV) gives a value of  $\sin^2\theta_W$
- This on-shell value relates to the W and Z boson masses:

$$\sin^2 \theta_W^{on-shell} = 1 - \frac{M_W^2}{M_Z^2}$$

- For example, the CDHS experiment at CERN obtained:

$$R_\nu = 0.3072 \pm 0.0033 \quad R_{\bar{\nu}} = 0.382 \pm 0.016$$

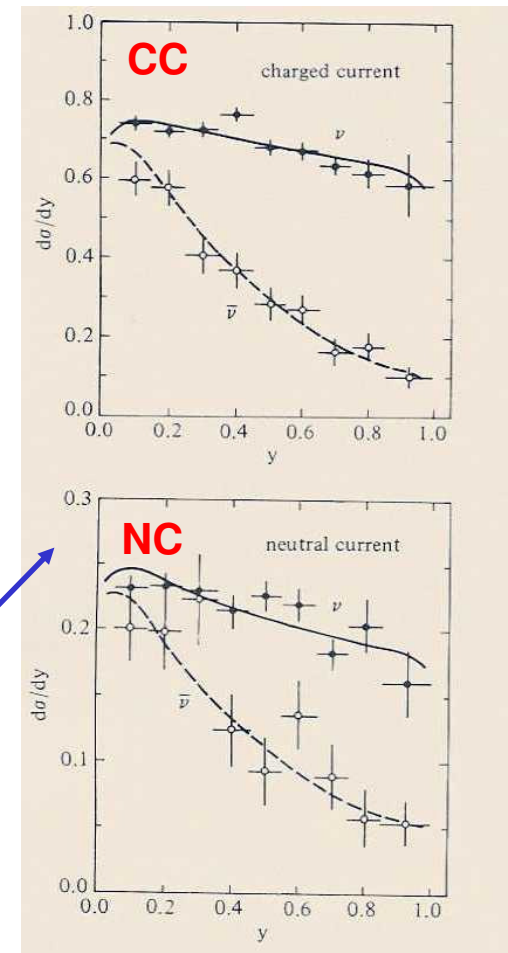
$$\Rightarrow \sin^2 \theta_W = 0.233 \pm 0.003 \pm 0.005$$

- The world average value is:

$$\text{World average : } \sin^2 \theta_W = 0.2227 \pm 0.00037$$

- Example of data from the CHARM experiment

**CHARM data**



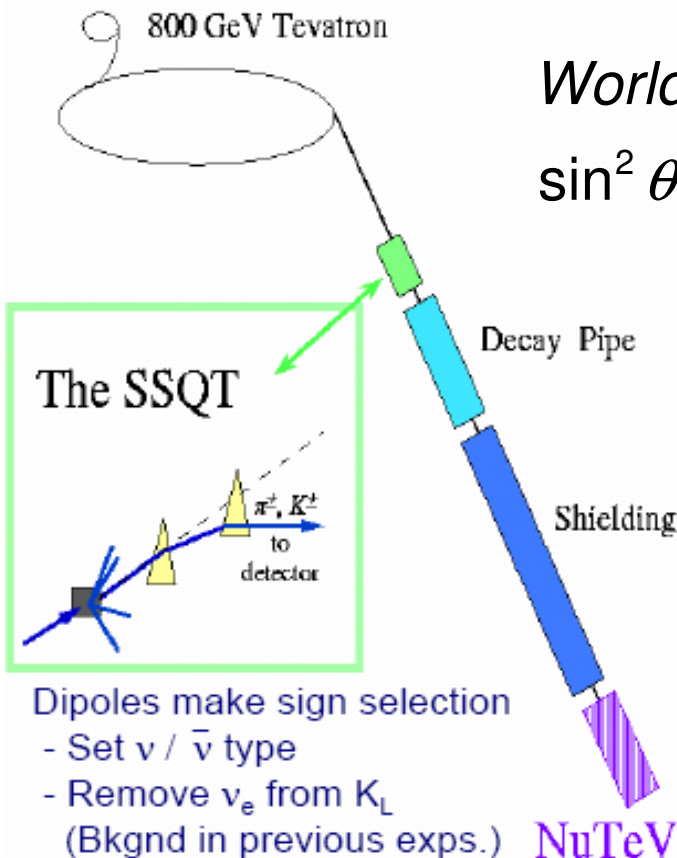
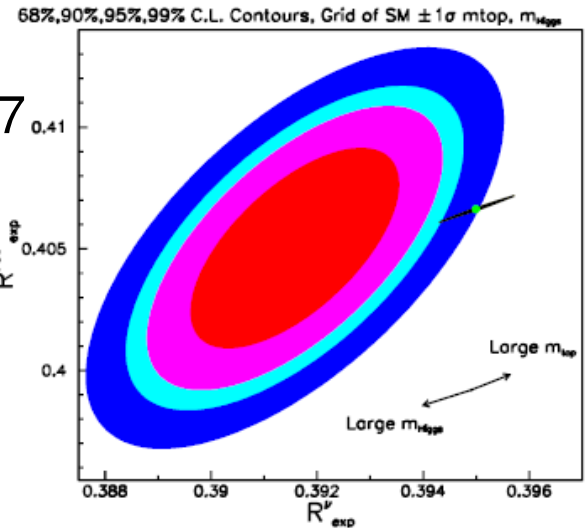
# 3.6 $\sin^2\theta_W$

- NuTeV experiment at Fermilab uses Paschos-Wolfenstein relationship and obtains reduced systematic errors but their result is  $>3\sigma$  away from world average:

NUTEV :  $R_\nu = 0.3916 \pm 0.0013$      $R_{\bar{\nu}} = 0.4050 \pm 0.0027$   
 $\Rightarrow \sin^2 \theta_W = 0.22773 \pm 0.00135 \pm 0.00095$

World average :

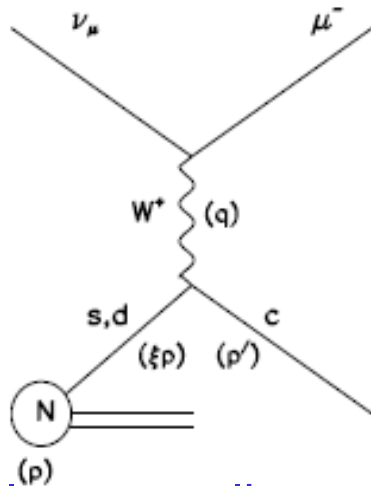
$\sin^2 \theta_W = 0.2227 \pm 0.00037$



- Charged current events had a muon ( $\mu^-$  from neutrinos and  $\mu^+$  from antineutrinos) and neutral current events were “short” events.
- Sign-selected neutrino beam, tags neutrino and antineutrino interactions (selected by decay of  $\pi^+$  and  $\pi^-$ ).
- Allows use of Paschos-Wolfenstein formula to reduce systematics.

## 3.7 Charm production

- Production of charm can be carried out from deep inelastic neutrino scattering from d or s quarks:



$$(q + \xi p)^2 = q^2 + 2\xi p \cdot q + \xi^2 M^2 = p'^2 = m_c^2$$

Therefore:

$$\xi \approx \frac{-q^2 + m_c^2}{2p \cdot q} = \frac{Q^2 + m_c^2}{2M\nu} = \frac{Q^2 + m_c^2}{2M\nu} = \frac{Q^2 + m_c^2}{Q^2/x} = x \left( 1 + \frac{m_c^2}{Q^2} \right)$$

- Slow rescaling model (LO): effect of a heavy quark threshold

- Replace:  $x = \frac{Q^2}{2M\nu} \rightarrow \xi = x \left( 1 + \frac{m_c^2}{Q^2} \right)$

- Cross-section:

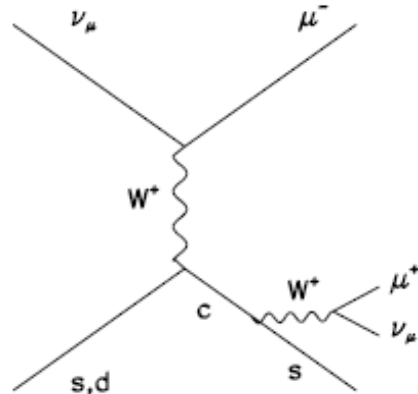
$$\frac{d^3\sigma^{\nu}}{d\xi dy dz} = \frac{G_F^2 M E \xi}{\pi} \left\{ [u(\xi, Q^2) + d(\xi, Q^2)] |V_{cd}|^2 + 2s(\xi, Q^2) |V_{cs}|^2 \right\} \left( 1 - y + \frac{xy}{\xi} \right) D(z)$$

- Fragmentation of charm quark into hadrons:  $D(z) \propto \frac{1}{z} \left( 1 - \frac{1}{z} - \frac{\epsilon p}{1-z} \right)^{-2}$

(Petersen function, but there are others)

# 3.7 Charm production

- Production of **opposite sign dimuon** events is signal of charm production because of semileptonic decay of charm:



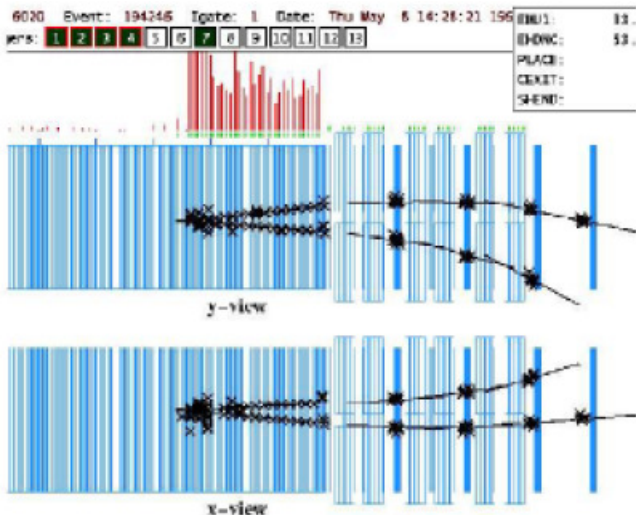
$$\nu_\mu + \begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \mu^- + c + X$$

$$\Leftrightarrow \mu^+ + \nu_\mu + X'$$

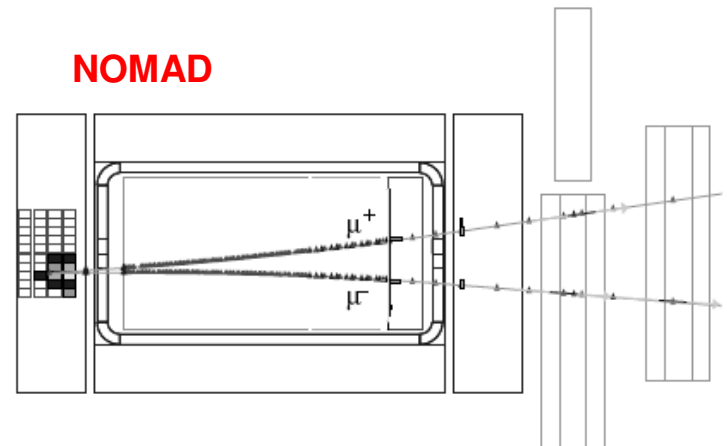
$$\bar{\nu}_\mu + \begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix} \rightarrow \mu^+ + \bar{c} + X$$

$$\Leftrightarrow \mu^- + \bar{\nu}_\mu + X'$$

- Charm production can probe strange sea, measure charm mass and  $V_{cd}$



CCFR/NUTEV



NOMAD

- High statistics opposite sign dimuon samples were acquired by CDHS, CCFR, NOMAD, CHORUS, NUTEV

# 3.7 Charm production

□ Some results from **opposite sign dimuons**:

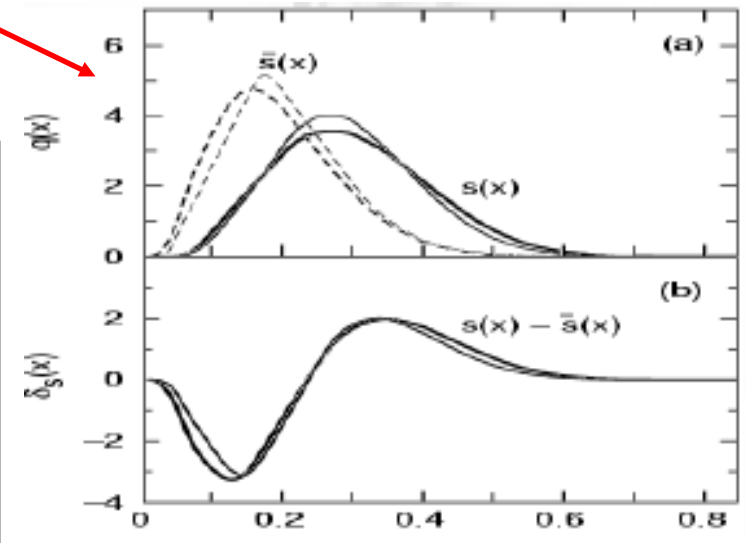
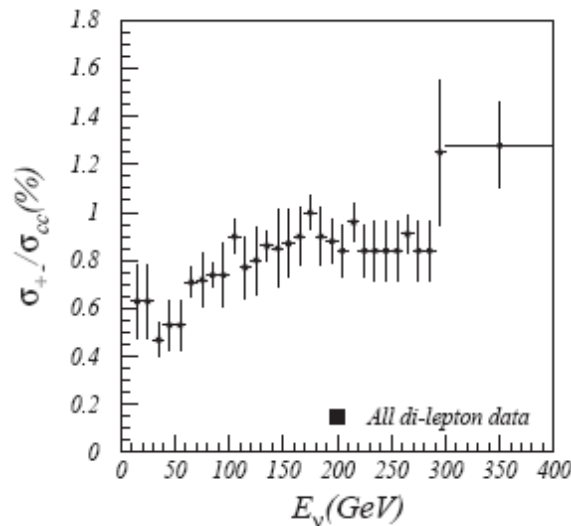
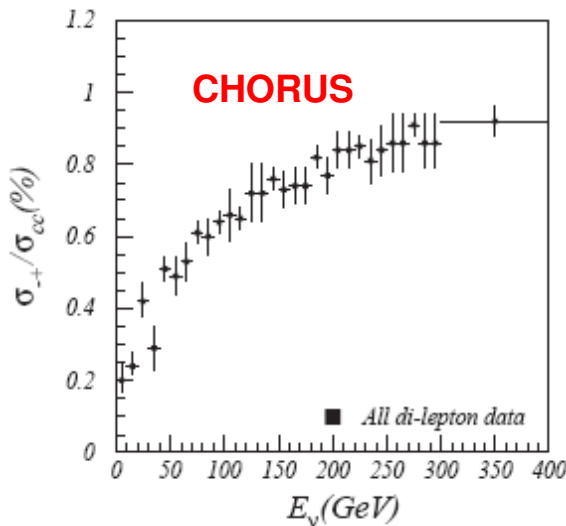
- Cross-section: between 0.2%-1% depending on energy
- Measurement charm mass (average):  $\langle m_c^{LO} \rangle = 1.43 \pm 0.10$
- Strange sea asymmetry
- Measurement  $V_{cd}$  (average):

$$V_{cd}^{LO} = 0.232 \pm 0.010$$

$$V_{cd}^{NLO} = 0.246 \pm 0.016$$

$$m_c^{NLO} = 1.70 \pm 0.019 \text{ (NUTEV)}$$

$$m_c^{NLO} = 1.58 \pm 0.09_{-0.09}^{+0.04} \text{ (NOMAD)}$$



Review: G di Lellis et al, Phys. Rep. 399, 2004, 227.