Introduction to Neutrino Interaction Physics
NUFACT08 Summer School

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References

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1.1 Fermi theory of beta decay (1932)

- Existence of a point-like four fermion interaction (Fermi, 1932):

$$\text{n} \rightarrow p + e^- + \nu_e$$

- Lagrangian of the interaction:

$$L(x) = -\frac{G_F}{\sqrt{2}} \left[ \overline{\phi_p(x)} \gamma^\mu \phi_n(x) \right] \left[ \overline{\phi_e(x)} \gamma_\mu \phi_\nu(x) \right]$$

$$G_F = \text{Fermi coupling constant} = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}$$

- Gamow-Teller interaction when final spin different to initial nucleus:

$$L(x) = -\frac{G_F}{\sqrt{2}} \sum_i \left[ \overline{\phi_p(x)} \Gamma_i \phi_n(x) \right] \left[ \overline{\phi_e(x)} \Gamma_i \phi_\nu(x) \right] + h.c.$$  

Possible interactions:  
$$\Gamma_i = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu} = S, P, V, A, T$$
First neutrino cross-section calculation

- **Bethe-Peierls (1934):** calculation of first cross-section for inverse beta reaction
  \[ \bar{\nu}_e + p \rightarrow n + e^+ \quad \text{or} \quad \nu_e + n \rightarrow p + e^- \]  using Fermi theory

  \[ \sigma \approx 5 \times 10^{-44} \text{ cm}^2 \quad \text{for} \quad E(\bar{\nu}) = 2 \text{ MeV} \]
  Accurate to factor 2

  - **Conversion from natural units:**  \[ \hbar c = 197.3 \text{ MeV} \cdot \text{fm} \]
  - **Cross-section:** multiply by \( (\hbar c)^2 = 0.3894 \times 10^{-27} \text{ GeV}^2 \cdot \text{cm}^2 \)

- **Mean free path of antineutrino in water:**
  \[ \lambda = \frac{1}{n\sigma} \approx 1.5 \times 10^{21} \text{ cm} \approx 1600 \text{ light – years} \]

  \[ n = \frac{\text{num. free protons}}{\text{volume}} \approx 2 \frac{N_A}{A} \rho \]

  In water:
  \[ n = \frac{2 \times 6 \times 10^{23}}{18} = 6.7 \times 10^{22} \text{ cm}^{-3} \]

- **Probability of interaction:**
  \[ P = 1 - \exp \left( -\frac{L}{\lambda} \right) \approx \frac{L}{\lambda} = 6.7 \times 10^{-20} \text{ (m water)}^{-1} \]

  Need very intense source of antineutrinos to detect inverse beta reaction.
1.2 Neutrino discovery (1956)

- Reines and Cowan detect $\bar{v}_e + p \rightarrow n + e^+$ in 1953 (Hanford) (discovery confirmed 1956 in Savannah River):
  - Detection of two back-to-back $\gamma$s from prompt signal $e^+e^- \rightarrow \gamma\gamma$ at $t=0$.
  - Neutron thermalization: neutron capture in Cd, emission of late $\gamma$s $\langle t \rangle \sim 20$ ms

Publication Science 1956:
$\sigma = 6 \times 10^{-44}$ cm$^2 \pm 25\%$ (within 5% expected)
1956: parity violation discovery increases theory cross-section: $\sigma = (10 \pm 1.7) \times 10^{-44}$ cm$^2$
Reanalysis data in 1960:
$\sigma = (12+7-4) \times 10^{-44} \text{ cm}^2$

Nobel prize Reines 1995

4200 l scintillator

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1.3 Parity violation and V-A

- Parity violation in weak decays postulated by Lee & Yang in 1950
- Parity violation confirmed through forward-backward asymmetry of polarized $^{60}\text{Co}$ (Wu, 1957).

$^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$

More electrons emitted in direction opposite to $^{60}\text{Co}$ spins, implying maximal parity violation

- Helicity operator:

$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} \rightarrow \frac{\vec{\sigma} \cdot (-\vec{p})}{|\vec{p}|} = -H$$

Projects spin along direction of motion
1.3 Parity violation and V-A

- Goldhaber, Grodzins, Sunyar (1958) measure helicity of neutrino from K capture of $^{152}$Eu:

\[ ^{152}\text{Eu}(0^-) + e^- \rightarrow ^{152}\text{Sm}^*(1^-) + \nu_e \]
\[ ^{152}\text{Sm}^*(1^-) \rightarrow ^{152}\text{Sm}^*(0^+) + \gamma \]

\[ \rho_\gamma = -\rho_\nu \]
\[ \sigma_\gamma = -\sigma_\nu \]
\[ H_\gamma = H_\nu \]

Asymmetry of photon spectrum in magnetic field determines helicity of $\nu_e$:

\[ H(\nu_e) = -1 \Rightarrow H(\bar{\nu}_e) = +1 \]

Neutrinos are “left-handed” \( \downarrow \)  Antineutrinos are “right-handed” \( \uparrow \)
1.3 Parity violation and V-A

- Left and right handed projection operators:
  \[ \nu_L = P_L \nu = \frac{1}{2} (1 - \gamma_5) \nu \quad \nu_R = P_R \nu = \frac{1}{2} (1 + \gamma_5) \nu \]

- Chirality operator: \( \gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \)
  same as helicity operator for massless neutrinos (\( E=\rho \)).
  \[ \gamma_5 \nu_L = H \nu_L = -\nu_L \quad \gamma_5 \nu_R = H \nu_R = +\nu_R \]

- If only \( \nu_L \) interact and \( \nu_R \) do not interact, then \( \Gamma_i \) have to transform as: \( \bar{e} \Gamma_i \nu \rightarrow (P_L e) \Gamma_i (P_L \nu) = \bar{e} P_R \Gamma_i P_L \nu \)
  \[ V : P_R \gamma^\mu P_L = \frac{1}{2} \gamma^\mu (1 - \gamma_5) \quad A : P_R \gamma^\mu \gamma_5 P_L = -\frac{1}{2} \gamma^\mu (1 - \gamma_5) \]

- The only possible coupling is V-A, due to maximal parity violation in weak interactions (Feynman, Gell-Mann, 1958): 
  \[ L_{V-A} = -\frac{G_F}{\sqrt{2}} \left[ \bar{\phi}_p \gamma^\mu (1 - g_A \gamma_5) \phi_n \right] \left[ \bar{\phi}_e \gamma_\mu (1 - \gamma_5) \phi_\nu \right] \text{with} \ g_A = -1.2573 \pm 0.0028 \]
  (determined empirically)
1.4 Neutral currents

- Two types of weak interaction: charged current (CC) and neutral current (NC) from electroweak theory of Glashow, Weinberg, Salam.

- First example of NC observed in 1973, inside the Gargamelle bubble chamber filled with freon (CF$_3$Br): no muon!
1.5 Standard Model Neutrino Interactions

- Lagrangian for electroweak interactions:

\[
L_{\text{int}} = i \frac{g}{\sqrt{2}} \left[ j^{(+)}_\mu W^\mu + j^{(-)}_\mu W^{\mu +} \right] + ig \cos \theta_W j^{(3)}_\mu - g' \sin \theta_W j^{(Y/2)}_\mu \right] Z^\mu + \\
+ ig \sin \theta_W j^{(3)}_\mu + g' \cos \theta_W j^{(Y/2)}_\mu \right] A^\mu
\]

- 1\textsuperscript{st} term: charged current interactions (\(W^+, W^-\) exchange)
- 2\textsuperscript{nd} term: neutral current interactions (\(Z^0\) exchange)
- 3\textsuperscript{rd} term: electromagnetic interactions (photon exchange)

- Electron charge: \(e = g \sin \theta_W = g' \cos \theta_W\)

- 3\textsuperscript{rd} term: \(ej_{\mu}^{e.m.} = e(j^{(3)}_\mu + j^{(Y/2)}_\mu)\) (neutrinos only couple to \(W^\pm\) and \(Z^0\))
A) Neutrino electron interaction

\[ L_{\text{int}} = i \frac{g}{\sqrt{2}} \left[ j^{(+)}_\mu W^\mu + j^{(-)}_\mu W^\mu + j^{(z)}_\mu Z^\mu + ie_j^{e.m} \right] \]

- Where:
  \[ j^{(+)}_\mu = \overline{e}_{e,L} \gamma_\mu e_L = \frac{1}{2} \overline{e}_{e,L} \gamma_\mu (1 - \gamma_5) e \]
  \[ j^{(-)}_\mu = \overline{\nu}_{e,L} \gamma_\mu \nu_e = \frac{1}{2} \overline{e}_{e,L} \gamma_\mu (1 - \gamma_5) \nu_e \]
  \[ j^{(z)}_\mu = 2( j^{(3)}_\mu - \sin^2 \theta_W j^{e.m}_\mu ) = \]
  \[ = \overline{e}_{e,L} \gamma_\mu \nu_e - \overline{\nu}_{e,L} \gamma_\mu e_L + 2 \sin^2 \theta_W \overline{e} \gamma_\mu e = \]
  \[ = \frac{1}{2} \overline{e}_{e,L} \gamma_\mu (1 - \gamma_5) \nu_e - \frac{1}{2} \overline{\nu}_{e,L} \gamma_\mu (1 - \gamma_5) e + 2 \sin^2 \theta_W \overline{e} \gamma_\mu e \]

\[ \Rightarrow j^{(z)}_\mu = \frac{1}{2} \overline{e}_{e,L} \gamma_\mu (1 - \gamma_5) \nu_e + \overline{\nu}_{e,L} \gamma_\mu (g_V - g_A \gamma_5) e \]

- With:
  \[ g_V = -\frac{1}{2} + 2 \sin^2 \theta_W \quad g_A = -\frac{1}{2} \]
B) Quark weak interactions

\[ L_{\text{int}} = i \frac{g}{\sqrt{2}} \left[ j^{(+)}_\mu W^\mu + j^{(-)}_\mu W^{\mu +} \right] + i \frac{g}{2 \cos \theta_W} j^{(Z)}_\mu Z^\mu + ie j^{e,m}_\mu \]

Where:

\[ j^{(+)}_\mu = \frac{1}{2} \bar{u} \gamma_\mu (1 - \gamma_5) d \]

\[ j^{(-)}_\mu = \frac{1}{2} \bar{d} \gamma_\mu (1 - \gamma_5) u \]

\[ j^{(Z)}_\mu = \bar{u} \gamma_\mu (A_u - B_u \gamma_5) u + \bar{d} \gamma_\mu (A_d - B_d \gamma_5) d \]

With:

\[ A_u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad B_u = \frac{1}{2} \]

\[ A_d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \quad B_d = -\frac{1}{2} \]
After introducing Higgs field and spontaneous symmetry breaking:

\[ L_{\text{Higgs}} = -\left| D_\mu \phi \right|^2 - \mu^2 \left| \phi \right|^2 - \lambda \left| \phi \right|^4 \]

- **Masses:**
  \[ m_H = \sqrt{2\lambda} v \]
  \[ m_{W^\pm} = \frac{g v}{2} \left( \frac{m_{W^\pm}}{m_{Z^0}} \right)^2 = \frac{g^2}{g^2 + g'^2} = \cos^2 \theta_W \]
  \[ m_{Z^0} = \frac{\sqrt{g^2 + g'^2}}{2} v \]

- **Vacuum expectation value:**
  \[ v = \left( \sqrt{2G_F} \right)^{-1/2} \approx 246 \text{ GeV} \]

- **Effective Hamiltonian:**
  \[ H_{\text{eff}} = \frac{g^2}{4m_W^2} \left[ j^{(+)}(+) j^{(-)}(-) + h.c. \right] + \frac{g^2}{8m_Z^2 \cos^2 \theta_W} j^{(Z)}(+) j^{(Z)}(-) = \]
  \[ = \frac{G_F}{\sqrt{2}} \left[ 2 j^{(+)}(+) j^{(-)}(-) + h.c. + j^{(Z)}(+) j^{(Z)}(-) \right] \]
The vector boson masses are then predicted:

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2 \theta_W} = \frac{4\pi\alpha}{8m_W^2 \sin^2 \theta_W} \quad \alpha = 1/137.036
\]

- Masses:
  - \( m_W = 80.450 \pm 0.058 \text{ GeV} \)
  - \( m_Z = 91.1876 \pm 0.0021 \text{ GeV} \)
  - \( \sin^2 \theta_W = 0.22280 \pm 0.00035 \)

- Need radiative corrections:

\[
m_W = \frac{37.2805}{\sin \theta_W (1 - \Delta r)^{1/2}}
\]

with \( \Delta r \approx 0.03630 \pm 0.0011 \) for \( m_t = 172.7 \text{ GeV} \) \( m_H = 117 \text{ GeV} \)
yields: \( m_W = 80.51 \pm 0.11 \text{ GeV} \)
2. Neutrino Electron Scattering

2.1 Charged current
2.2 Neutral current
2.3 Interference charged and neutral current
2.4 Mass suppression
2.5 Number of neutrinos
2.1 Neutrino-electron CC scattering

- Only charged current: \( \nu_\mu + e^- \rightarrow \nu_e + \mu^- \) (Inverse Muon Decay)

\[ \begin{align*}
\nu_\mu & \rightarrow \mu^- \quad H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu \right) \left( \bar{e} \gamma^\mu (1 - \gamma^5) e \right) \\
\mu^- & \rightarrow W^- \\
e^- & \rightarrow \nu_e \\
W^- & \rightarrow \nu_\mu + e^- \\
\end{align*} \]

- Total spin \( J = 0 \)

\[ s = (p_{\nu_\mu} + p_e)^2 = 2m_e E_{\nu_\mu} \quad \text{(in LAB)} \]

\[ t = q^2 = -Q^2 = (p_{\nu_\mu} - p_\mu)^2 \quad \text{Inelasticity variable} \]

\[ y = \frac{p_e \cdot (p_{\nu_\mu} - p_\mu)}{p_\mu \cdot p_{\nu_\mu}} = \frac{E_{\nu_\mu} - E_\mu}{E_{\nu_\mu}} \quad \text{(in LAB)} \]

\[ \begin{align*}
\frac{d\sigma_{CC}(\nu_\mu e^-)}{dQ^2 dy} & = \frac{G_F^2 m_W^4}{\pi (Q^2 + m_W^2)^2} \Rightarrow \sigma_{CC}(\nu_\mu e^-) = \int_0^s G_F^2 m_W^4 \frac{dQ^2}{\pi (Q^2 + m_W^2)^2} \\
\end{align*} \]

- Total cross-section (ignoring mass terms): Measurement CHARM-II:

\[ \sigma_{CC}(\nu_\mu e^-) \approx \frac{G_F^2 s}{\pi} = \frac{2G_F^2 m_e E_{\nu_\mu}}{\pi} \quad \text{(in LAB)} \]

\[ \sigma(\nu_\mu e^-) = (1.651 \pm 0.093) \times 10^{-41} \left( \frac{E}{1 \text{GeV}} \right) \text{cm}^2 \]

- (cross-section proportional to energy!)

\[ \Rightarrow \sigma_{CC}(\nu_\mu e^-) = \frac{2G_F^2 m_e (\hbar c)^2 E_{\nu_\mu}}{\pi} = 1.72 \times 10^{-41} \left( \frac{E_{\nu_\mu}}{1 \text{GeV}} \right) \text{cm}^2 \quad \hbar c = 197.33 \text{ MeV} \cdot \text{fm} \]

\[ G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \]
2.2 Neutrino-electron NC scattering

- Only neutral current:
  \[ \nu^\mu + e^- \rightarrow \nu^\mu + e^- \]  
  Elastic scattering

Couples to e_L and e_R: J=0,1

Right handed current suppressed in backward direction:
\[ 1 - y = \frac{1 + \cos \theta^*}{2} \]

\[
\frac{d\sigma_{NC}(\nu^\mu e^-)}{dy} = \frac{G_F^2 s}{\pi} \frac{m_Z^4}{(Q_{\text{max}}^2 + m_Z^2)^2} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W (1 - y)^2 \right] 
\]

\[
\frac{d\sigma_{NC}(\bar{\nu}^\mu e^-)}{dy} = \frac{G_F^2 s}{\pi} \frac{m_Z^4}{(Q_{\text{max}}^2 + m_Z^2)^2} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 (1 - y)^2 + \sin^4 \theta_W \right] 
\]
2.2 Neutrino-electron NC scattering

- Only neutral current (total cross-section):

\[ \sigma_{NC}(\nu_\mu e^-) = \frac{G_F^2 s}{\pi} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right] = 0.16 \times 10^{-41} \left( \frac{E_\nu}{1 \text{GeV}} \right) \text{cm}^2 \]

\[ \sigma_{NC}(\bar{\nu}_\mu e^-) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( -\frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right] = 0.13 \times 10^{-41} \left( \frac{E_\nu}{1 \text{GeV}} \right) \text{cm}^2 \]

- Can obtain value of $\sin^2 \theta_W$ from neutrino electron elastic scattering (CHARM II):

\[ \sin^2 \theta_W = 0.2324 \pm 0.0058 \pm 0.0059 \]

\[ g_V = -0.035 \pm 0.017 \]

\[ g_A = -0.503 \pm 0.017 \]

\[ E_e \Theta^2 = 2m_e (1 - y) \]
2.3 Interference CC and NC

- Tree level Feynman diagrams: both neutral and charged currents

\[ \nu_e + e^- \rightarrow \nu_e + e^- \]

- Effective Hamiltonian:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e] [\bar{e} \gamma_\mu (1 - \gamma_5) \nu_e] + [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] [\bar{e} \gamma_\mu (g_V - g_A \gamma_5) e] \right\}
\]

\[
= \frac{G_F}{\sqrt{2}} \left\{ [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] [\bar{e} \gamma_\mu (1 + g_V - (1 + g_A) \gamma_5) e] \right\}
\]

(through a Fierz transformation)
2.3 Interference CC and NC

- Rearranging terms in charged and neutral current contributions for:

\[ \nu_e + e^- \rightarrow \nu_e + e^- \]

\[ g_L = \frac{1}{2} (1 + g_V + 1 + g_A) = -\frac{1}{2} + \sin^2 \theta_W + 1 = \frac{1}{2} + \sin^2 \theta_W \]

\[ g_R = \frac{1}{2} (1 + g_V - (1 + g_A)) = \sin^2 \theta_W \]

Then:

\[ \frac{d\sigma(\nu_e e^-)}{dy} = \frac{G_F^2 s}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W (1 - y)^2 \right] \]

\[ \Rightarrow \sigma(\nu_e e^-) = \frac{G_F^2 s}{\pi} \left[ \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} \sin^4 \theta_W \right] = 0.96 \times 10^{-41} \left( \frac{E_\nu}{1 \text{GeV}} \right) \text{cm}^2 \]

Also:

\[ \sigma(\bar{\nu}_e e^-) = \frac{G_F^2 s}{\pi} \left[ \frac{1}{3} \left( \frac{1}{2} + \sin^2 \theta_W \right)^2 + \sin^4 \theta_W \right] = 0.40 \times 10^{-41} \left( \frac{E_\nu}{1 \text{GeV}} \right) \text{cm}^2 \]

These cross-sections are a consequence of the interference of the charged and neutral current diagrams.
2.3 Interference CC and NC

- Neutrino pair production: \( e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e \)

Contribution from both W and Z graphs.

Then:

\[
\sigma(e^+ e^- \rightarrow \nu_e \bar{\nu}_e) = \frac{G_F^2 s}{12\pi} \left[ \left( \frac{1}{2} + 2\sin^2 \theta_W \right)^2 + \frac{1}{4} \right]
\]

- Only neutral current contribution to: \( e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu \)

\[
\sigma(e^+ e^- \rightarrow \nu_\mu \bar{\nu}_\mu) = \frac{G_F^2 s}{12\pi} \left[ \left( \frac{1}{2} - 2\sin^2 \theta_W \right)^2 + \frac{1}{4} \right]
\]

Caveat: below the Z pole!
Summary neutrino electron scattering processes:

<table>
<thead>
<tr>
<th>Process</th>
<th>Total cross-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{\mu} + e^- \rightarrow \mu^- + \nu_e$</td>
<td>$\frac{G_F^2 s}{\pi}$</td>
</tr>
<tr>
<td>$\nu_e + e^- \rightarrow \nu_e + e^-$</td>
<td>$\frac{G_F^2 s}{4\pi} \left( \frac{1}{3} (2\sin^2 \theta_w -1)^2 + \frac{4}{3} \sin^4 \theta_w \right)$</td>
</tr>
<tr>
<td>$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$</td>
<td>$\frac{G_F^2 s}{4\pi} \left( \frac{1}{3} (2\sin^2 \theta_w +1)^2 + 4\sin^4 \theta_w \right)$</td>
</tr>
<tr>
<td>$\nu_{\mu} + e^- \rightarrow \nu_{\mu} + e^-$</td>
<td>$\frac{G_F^2 s}{4\pi} \left( \frac{1}{3} (2\sin^2 \theta_w -1)^2 + \frac{4}{3} \sin^4 \theta_w \right)$</td>
</tr>
<tr>
<td>$\bar{\nu}<em>\mu + e^- \rightarrow \bar{\nu}</em>\mu + e^-$</td>
<td>$\frac{G_F^2 s}{4\pi} \left( \frac{1}{3} (2\sin^2 \theta_w -1)^2 + \frac{4}{3} \sin^4 \theta_w \right)$</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$</td>
<td>$\frac{G_F^2 s}{12\pi} \frac{1}{2} + 2\sin^2 \theta_w + 4\sin^4 \theta_w$</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$</td>
<td>$\frac{G_F^2 s}{12\pi} \frac{1}{2} - 2\sin^2 \theta_w + 4\sin^4 \theta_w$</td>
</tr>
</tbody>
</table>

$s = 2m_e E_{\nu}$ (in the LAB frame)
We have not taken into account the effect of initial and final state masses yet.

For example: $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$

Threshold: $s = m_e^2 + 2m_eE_\nu \geq m_\mu^2 \Rightarrow E_\nu \geq \frac{m_\mu^2 - m_e^2}{2m_e} \approx 11 \text{ GeV}$

Cross-section modification:

$$\sigma_{CC}(\nu_\mu e^-) = \frac{G_F^2}{\pi} \int_{Q_{min}^2}^{Q_{max}^2} \frac{m_W^4}{(Q^2 + m_W^2)^2} dQ^2 = \frac{G_F^2}{\pi} \frac{m_W^4}{(Q_{max}^2 + m_W^2)(Q_{min}^2 + m_W^2)} (Q_{max}^2 - Q_{min}^2) \approx \frac{G_F^2}{\pi} (Q_{max}^2 - Q_{min}^2) = \frac{G_F^2}{\pi} (s - m_\mu^2)$$

Therefore:

$$\sigma_{CC}(\nu_\mu e^-) = G_F^2 \frac{s}{\pi} \left[ 1 - \frac{m_\mu^2}{s} \right] = \sigma_{CC}^{massless}(\nu_\mu e^-) \left[ 1 - \frac{m_\mu^2}{s} \right]$$
2.5 Number of neutrinos

- Width of the Z-pole resonance: Breit-Wigner distribution

\[
\sigma(e^+ e^- \to f) = \frac{12 \pi (\hbar c)^2}{M_Z} \frac{s\Gamma_e \Gamma_f}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z}
\]

\[
\sigma_{\text{peak}}(e^+ e^- \to f) = \frac{12 \pi (\hbar c)^2}{M_Z} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} = \frac{12 \pi (\hbar c)^2}{M_Z} B(Z^0 \to e^+ e^-) B(Z^0 \to f \bar{f})
\]

\[\Gamma_Z = \Gamma_{\text{had}} + 3\Gamma_{l^+l^-} + N_{\nu} \Gamma_{\nu\bar{\nu}} = 2490 \text{ MeV} \quad \text{2 neutrinos}\]

\[\Gamma_{\text{had}} = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_b = 1741 \text{ MeV}\]

\[\Gamma_{l^+l^-} = 83.9 \text{ MeV}\]

\[\Gamma_{\nu\bar{\nu}} = 167.1 \text{ MeV}\]

\[\Rightarrow N_{\nu} = 2.9841 \pm 0.0083\]

- Only 3 neutrinos with mass less than the Z mass
3. Neutrino Nucleon Deep-Inelastic Scattering

3.1 Definition and variables
3.2 Charged current
3.3 Quark content of nucleons
3.4 Sum rules
3.5 Neutral current
3.6 A case study: $\sin^2\theta_W$ from neutrino interactions
3.7 Charm production in neutrino interactions
3.1 Definition and Variables

- Deep inelastic neutrino-nucleon scattering reactions have large $q^2$ 
  $\nu_l(p) + N \rightarrow l^-(p') + X$ 
  ($q^2 \gg m_N^2, E_\nu \gg m_N$):
- Quark-parton model valid due to asymptotic freedom of QCD, which makes quarks behave as free point-like particles.
- Infinite momentum frame: a parton takes a fraction $x (0<x<1)$, of momentum when struck by a neutrino. Final quark state:

$$ (xp_N + q)^2 = m_q^2 \Rightarrow x \approx -\frac{q^2}{2p_N \cdot q} \quad \text{if} \quad q^2 \gg m_q^2 $$

- Variables in DIS:
  \[ s = (p + p_N)^2 \approx 2M E_\nu = 2M E \]
  \[ Q^2 = -q^2 = -(p + p')^2 = 4EE' \sin^2 \frac{\theta}{2} \]
  \[ W^2 = E_x^2 - p_x^2 = -Q^2 + 2M \nu + M^2 \quad \text{Recoil mass} \]
  \[ \nu = \frac{q \cdot p_N}{M} = E - E' \]

Bjorken Variables 
(0<x<1, 0<y<1):

\[ x = \frac{-q^2}{2q \cdot p_N} = \frac{Q^2}{2M \nu} \]
\[ y = \frac{q \cdot p_N}{p \cdot p_N} = \frac{\nu}{E} = \frac{Q^2}{2M E x} \]
3.2 Charged current

- Neutrino proton CC scattering: \( \nu_\mu (p) + p \rightarrow \mu^- (p') + X \)
  \( u(x)dx \) = number of u-quarks in proton between \( x \) and \( x + dx \)
  \( u(x) = u_v (x) + u_s (x) \quad d(x) = d_v (x) + d_s (x) \)
  In the sea: \( u_s (x) = \bar{u} (x) \quad d_s (x) = \bar{d} (x) \)
  For proton (uud):
  \[ \int_0^1 u_v (x)dx = \int_0^1 [u(x) - \bar{u}(x)]dx = 2 \]
  \[ \int_0^1 d_v (x)dx = \int_0^1 [d(x) - \bar{d}(x)]dx = 1 \]

- Neutrino-quark/antineutrino-antiquark scattering:
  \( d\sigma_{CC} (\nu_\mu q) = \frac{d\sigma_{CC} (\bar{\nu}_\mu \bar{q})}{dy} = \frac{2G_F^2 m_q E}{\pi} \)
  with \( y = 1 - \frac{E}{E'} = \frac{1}{2} (1 - \cos \theta) \)

- Neutrino-antiquark/antineutrino-quark scattering:
  \( d\sigma_{CC} (\nu_\mu \bar{q}) = \frac{d\sigma_{CC} (\bar{\nu}_\mu q)}{dy} = \frac{2G_F^2 m_q E}{\pi} \)
  \( (1 - y)^2 \)

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3.2 Charged current

- **Scattering off proton:**
  \[
  \frac{d\sigma_{cc}(\nu_\mu p)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [d(x) + s(x)] + [\bar{u}(x) + \bar{c}(x)] (1 - y)^2 \right\}
  \]
  \[
  \frac{d\sigma_{cc}(\bar{\nu}_\mu p)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [u(x) + c(x)] (1 - y)^2 + [\bar{d}(x) + \bar{s}(x)] \right\}
  \]

- **Neutron (isospin symmetry):**
  \[
  \frac{d\sigma_{cc}(\nu_\mu n)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [u(x) + s(x)] + [\bar{d}(x) + \bar{c}(x)] (1 - y)^2 \right\}
  \]
  \[
  \frac{d\sigma_{cc}(\bar{\nu}_\mu n)}{dx dy} = \frac{G_F^2 ME}{\pi} 2x \left\{ [d(x) + c(x)] (1 - y)^2 + [\bar{u}(x) + \bar{s}(x)] \right\}
  \]
3.2 Charged current

- Structure functions:

\[ \frac{d^2 \sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 s}{2\pi} \left[ y^2 2xF_1(x,Q^2) + 2\left(1 - y - \frac{Mxy}{2E}\right)F_2(x,Q^2) \pm 2y\left(1 - \frac{y}{2}\right)xF_3(x,Q^2) \right] \]

\( F_i(x,Q^2) \) are the structure functions, which depend on the helicity structure of \( q-W \) interactions. For massless spin-1/2 partons, we have the Callan-Gross relationship*:

\( 2xF_1(x) = F_2(x) \)

\[ \frac{d^2 \sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 s}{2\pi} \left[ \left(1 - y\right)^2 + \left(1 - \frac{Mxy}{2E}\right)\right]F_2(x,Q^2) \pm 2y\left(1 - \frac{y}{2}\right)xF_3(x,Q^2) \]

Assuming massless target

* Deviations from the Callan-Gross relation are parameterised in terms of the “longitudinal” cross-section (i.e., gluon splitting \( g \rightarrow q\bar{q} \)):

\[ R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2(x)}{2xF_1(x)} \left(1 + \frac{4Mx^2}{Q^2}\right) \]
3.2 Charged current

Comparing the $y$ distribution of both cross-sections we can compare the parton distribution functions to the proton structure functions:

$$F_{2}^{\nu p}(x) = x[d(x) + \bar{u}(x) + s(x) + \bar{c}(x)]$$
$$xF_{3}^{\nu p}(x) = x[d(x) - \bar{u}(x) + s(x) - \bar{c}(x)]$$

$$F_{2}^{\nu p}(x) = x[u(x) + c(x) + \bar{d}(x) + \bar{s}(x)]$$
$$xF_{3}^{\nu p}(x) = x[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)]$$

Also, the neutron structure functions:

$$F_{2}^{\nu n}(x) = x[u(x) + \bar{d}(x) + s(x) + \bar{c}(x)]$$
$$xF_{3}^{\nu n}(x) = x[u(x) - \bar{d}(x) + s(x) - \bar{c}(x)]$$

$$F_{2}^{\nu n}(x) = x[d(x) + c(x) + \bar{u}(x) + \bar{s}(x)]$$
$$xF_{3}^{\nu n}(x) = x[d(x) + c(x) - \bar{u}(x) - \bar{s}(x)]$$
3.2 Charged current

- Scattering off isoscalar target (equal number neutrons and protons):

\[ q \equiv u + d + s + c \quad \bar{q} \equiv \bar{u} + \bar{d} + \bar{s} + \bar{c} \]

\[
F_{2}^{VN}(x) = x[q(x) + \bar{q}(x)]
\]

\[
xF_{3}^{VN}(x) = x[q(x) - \bar{q}(x) + 2(s(x) - c(x))] \]

\[
xF_{3}^{\bar{V}N}(x) = x[q(x) - \bar{q}(x) - 2(s(x) - c(x))] \]

\[
\frac{d\sigma_{CC}(\nu_{\mu}N)}{dx dy} = \frac{G_{F}^{2} 2ME}{2\pi} x\{q(x) + \bar{q}(x) (1 - y)^{2}\}
\]

\[
\frac{d\sigma_{CC}(\bar{\nu}_{\mu}N)}{dx dy} = \frac{G_{F}^{2} 2ME}{2\pi} x\{q(x)(1 - y)^{2} + \bar{q}(x)\}
\]

- Total cross-section:

\[
\sigma_{CC}(\nu_{\mu}N) = \frac{G_{F}^{2} s}{2\pi} \left[ \langle Q \rangle + \frac{1}{3} \langle \bar{Q} \rangle \right] = (0.677 \pm 0.014) \times 10^{-38} \text{ cm}^{2} / \text{GeV} \times E(\text{GeV})
\]

\[
\sigma_{CC}(\bar{\nu}_{\mu}N) = \frac{G_{F}^{2} s}{2\pi} \left[ \frac{1}{3} \langle Q \rangle + \langle \bar{Q} \rangle \right] = (0.334 \pm 0.008) \times 10^{-38} \text{ cm}^{2} / \text{GeV} \times E(\text{GeV})
\]
3.2 Charged current

- Structure functions:

Scaling violations
3.3 Quark content of nucleons

- Quark content of nucleons from CC cross-sections
- Define: \[ U = \int_0^1 xu(x)dx \text{ , etc.} \]
- Experimental values from y distribution of cross-sections yields:
  
  Since \[ r = \frac{\sigma_{CC}(\overline{\nu}N)}{\sigma_{CC}(\nu N)} = 0.493 \pm 0.016 \text{ (measured)} \]

  then: \[ \frac{\overline{Q}}{Q} = \frac{3r - 1}{3 - r} \approx 0.191 \quad \Rightarrow \quad Q = 0.405 \quad \text{and} \quad \overline{Q} = 0.078 \]

  therefore: \[ Q_V = Q - \overline{Q} \approx 0.33 \quad \frac{\overline{Q}}{Q + \overline{Q}} = 0.16 \pm 0.03 \]

  \[ \int_0^1 F_2^{\nu N}(x)dx = Q + \overline{Q} \approx 0.48 \]

- Quarks and antiquarks carry 48% of proton momentum, valence quarks only 33% and sea quarks only 7.8% (u and d sea quarks carry 6%, s quarks carry 1.3% and c quarks 0.5%).
3.3 Quark content of nucleons

Parton distribution functions as a function of x, fitted from structure functions:

\[ u(x)dx = \text{number of u-quarks in proton between } x \text{ and } x + dx \]
\[ u(x) = u_V(x) + u_S(x) \quad d(x) = d_V(x) + d_S(x) \]

In the sea:
\[ d_S(x) = \bar{d}(x) \quad u_S(x) = \bar{u}(x) \]

For proton (uud):
\[ \int_0^1 u_V(x)dx = \int_0^1 [u(x) - \bar{u}(x)]dx = 2 \]
\[ \int_0^1 d_V(x)dx = \int_0^1 [d(x) - \bar{d}(x)]dx = 1 \]
3.4 Sum rules

- Sum rules:
  - Gross-Llewellyn Smith: 
    \[ S_{GLS} = \frac{1}{2} \int_0^1 (F_3^\nu (x) + F_3^{\bar{\nu}} (x)) \, dx \]
    \[ S_{GLS} = \int_0^1 (q(x) - \bar{q}(x)) \, dx = 3 \left[ 1 - \frac{\alpha_s}{\pi} - a \left( \frac{\alpha_s}{\pi} \right)^2 - b \left( \frac{\alpha_s}{\pi} \right)^3 \right] = 2.64 \pm 0.06 \]
  - Adler:
    \[ S_A = \frac{1}{2} \int_0^1 \frac{1}{x} (F_2^{\nu n} (x) + F_2^{\nu p} (x)) \, dx = \int_0^1 (u_\nu (x) - d_\nu (x)) \, dx = 1 \]
  - Gottfried:
    \[ S_G = \frac{1}{2} \int_0^1 \frac{1}{x} (F_2^{\mu n} (x) + F_2^{\mu p} (x)) \, dx = \frac{1}{3} \int_0^1 (u(x) + \bar{u}(x) - d(x) - \bar{d}(x)) \, dx = \frac{1}{3} \]
    \[ S_G = 0.235 \pm 0.026 \quad \text{Maybe isospin asymmetry:} \quad \bar{u}(x) \neq \bar{d}(x) \]
  - Bjorken:
    \[ S_B = \int_0^1 (F_1^{\nu p} (x) + F_1^{\nu n} (x)) \, dx = 1 - \frac{2\alpha_s(Q^2)}{3\pi} \]
3.5 Neutral current

- Neutral currents:
  \[ d\sigma_{NC}(\nu_\mu q) = \frac{d\sigma_{NC}(\bar{\nu}_\mu \bar{q})}{dy} = \frac{G_F^2 m_q E_\nu}{2\pi} \left\{ (g_V + g_A)^2 + (g_V - g_A)^2 (1 - y)^2 + \frac{m_q}{E_\nu} (g_A^2 - g_V^2) y \right\} \]
  \[ d\sigma_{NC}(\bar{\nu}_\mu q) = \frac{d\sigma_{NC}(\nu_\mu \bar{q})}{dy} = \frac{G_F^2 m_q E_\nu}{2\pi} \left\{ (g_V - g_A)^2 + (g_V + g_A)^2 (1 - y)^2 + \frac{m_q}{E_\nu} (g_A^2 - g_V^2) y \right\} \]

- Coupling constants:
  \[ g_V = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad g_A = \frac{1}{2} \quad \text{for } q=u,c \]
  \[ g'_V = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \quad g'_A = -\frac{1}{2} \quad \text{for } q=d,s \]
  \[ g_L = \frac{1}{2} \left( g_V + g_A \right) \]
  \[ g_R = \frac{1}{2} \left( g_V - g_A \right) \]
Neutral currents off nucleons (neglecting c and s quark contributions):

\[
\nu_\mu + N \rightarrow \nu_\mu + X
\]

\[
\frac{d\sigma_{NC}(\nu_\mu N)}{dxdy} = \frac{G_F^2 ME}{\pi} x \left\{ (g_L^2 + g'_L^2) [q + \bar{q}(1-y)^2] + (g_R^2 + g'_R^2) [\bar{q} + q(1-y)^2] \right\}
\]

\[
\frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{dxdy} = \frac{G_F^2 ME}{\pi} x \left\{ (g_R^2 + g'_R^2) [q + \bar{q}(1-y)^2] + (g_L^2 + g'_L^2) [\bar{q} + q(1-y)^2] \right\}
\]

Defining:

\[
R_v \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} \quad R_{\bar{v}} \equiv \frac{\sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\bar{\nu} N)} \quad r \equiv \frac{\sigma_{CC}(\bar{\nu} N)}{\sigma_{CC}(\nu N)}
\]

yields:

\[
g_L^2 + g'_L^2 = \frac{R_v - r^2 R_{\bar{v}}}{1 - r^2}
\]

\[
g_R^2 + g'_R^2 = \frac{r(R_{\bar{v}} - R_v)}{1 - r^2}
\]

\[
R_v = \left( g_L^2 + g'_L^2 \right) + r \left( g_R^2 + g'_R^2 \right) = \frac{1}{2} - \sin^2 \theta_W + (1 + r) \frac{5}{9} \sin^4 \theta_W
\]

\[
R_{\bar{v}} = \left( g_L^2 + g'_L^2 \right) + \frac{1}{r} \left( g_R^2 + g'_R^2 \right) = \frac{1}{2} - \sin^2 \theta_W + \left( 1 + \frac{1}{r} \right) \frac{5}{9} \sin^4 \theta_W
\]

(Llewelyn-Smith relationships)

3.5 Neutral current
3.5 Neutral currents

- More relationships from the combination of neutrino and antineutrino tagged interactions:

\[ R^+ = \frac{d\sigma_{NC}(\nu_\mu N)}{d\sigma_{CC}(\nu_\mu N)} + \frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{d\sigma_{CC}(\bar{\nu}_\mu N)} = \frac{1}{2} - \sin^2 \theta_W + \frac{10}{9} \sin^4 \theta_W \]

\[ R^- = \frac{d\sigma_{NC}(\nu_\mu N)}{d\sigma_{CC}(\nu_\mu N)} - \frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{d\sigma_{CC}(\bar{\nu}_\mu N)} = \frac{R_v - rR_v}{1 - r} = \frac{1}{2} - \sin^2 \theta_W \]  

(Paschos-Wolfenstein relationship)

- Paschos-Wolfenstein relation removes the effects of sea quark differences (especially at low x) since the neutrino and antineutrino cross-sections are equal. It would also remove error from c quark.

- All of these relationships can be used in neutrino experiments to test the electroweak theory and measure \( \sin^2 \theta_W \)
3.6 $\sin^2\theta_W$

- Llewellyn-Smith relationship used to measure $\sin^2\theta_W$ by performing ratios of charged current to neutral current of neutrino nucleon scattering.

$$R_\nu = \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} = \frac{1}{2} - \sin^2 \theta_W + (1 + r) \frac{5}{9} \sin^4 \theta_W$$

$$R_{\bar{\nu}} = \frac{\sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\bar{\nu} N)} = \frac{1}{2} - \sin^2 \theta_W + \left(1 + \frac{1}{r}\right) \frac{5}{9} \sin^4 \theta_W$$

- CHARM, CDHS and CCFR and NuTeV are all large sampling calorimeters that can measure large statistics CC and NC data:

CCFR/NuTeV
3.6 $\sin^2 \theta_W$

- The ratio of NC to CC data from an average of different experiments (CDHS, CHARM, CCFR, NUTEV) gives a value of $\sin^2 \theta_W$.
- This on-shell value relates to the $W$ and $Z$ boson masses:
  \[ \sin^2 \theta_{W_{\text{on-shell}}} = 1 - \frac{M_W^2}{M_Z^2} \]
  
  - For example, the CDHS experiment at CERN obtained:
    \[ R_\nu = 0.3072 \pm 0.0033 \quad R_{\bar{\nu}} = 0.382 \pm 0.016 \]
    \[ \Rightarrow \sin^2 \theta_W = 0.233 \pm 0.003 \pm 0.005 \]

- The world average value is:
  \[ \text{World average} : \sin^2 \theta_W = 0.2227 \pm 0.00037 \]
  
  - Example of data from the CHARM experiment.
**3.6 \sin^2 \theta_W**

- NuTeV experiment at Fermilab uses Paschos-Wolfenstein relationship and obtains reduced systematic errors but their result is >3σ away from world average:

\[ NUTEV : \ R_v = 0.3916 \pm 0.0013 \quad R_{\bar{v}} = 0.4050 \pm 0.0027 \]

\[ \Rightarrow \ \sin^2 \theta_W = 0.22773 \pm 0.00135 \pm 0.00095 \]

*World average:*

\[ \sin^2 \theta_W = 0.2227 \pm 0.00037 \]

- Charged current events had a muon (\(\mu^-\) from neutrinos and \(\mu^+\) from antineutrinos) and neutral current events were “short” events.

- Sign-selected neutrino beam, tags neutrino and antineutrino interactions (selected by decay of \(\pi^+\) and \(\pi^-\)).

- Allows use of Paschos-Wolfenstein formula to reduce systematics.
3.7 Charm production

- Production of charm can be carried out from deep inelastic neutrino scattering from d or s quarks:

\[
(q + \xi p)^2 = q^2 + 2\xi p \cdot q + \xi^2 M^2 = p'^2 = m_c^2
\]

Therefore:

\[
\xi \approx \frac{-q^2 + m_c^2}{2p \cdot q} = \frac{Q^2 + m_c^2}{2M_N} = \frac{Q^2 + m_c^2}{2M_N} = \frac{Q^2 + m_c^2}{Q^2 / x} = x \left(1 + \frac{m_c^2}{Q^2}\right)
\]

- Slow rescaling model (LO): effect of a heavy quark threshold
- Replace:

\[
x = \frac{Q^2}{2M_N} \rightarrow \xi = x \left(1 + \frac{m_c^2}{Q^2}\right)
\]

- Cross-section:

\[
\frac{d^3 \sigma^\nu}{d\xi dy dz} = \frac{G_F^2 M_N \xi}{\pi} \left\{ \left[ u(\xi, Q^2) + d(\xi, Q^2) \right] |V_{cd}|^2 + 2s(\xi, Q^2) |V_{cs}|^2 \right\} \left(1 - y + \frac{xy}{\xi}\right) D(z)
\]

- Fragmentation of charm quark into hadrons:

\[
D(z) \propto \frac{1}{z} \left( \frac{1}{1 - \frac{\xi p}{1 - z}} \right)^{-2}
\]

(Petersen function, but there are others)
3.7 Charm production

- Production of opposite sign dimuon events is signal of charm production because of semileptonic decay of charm:

  \[
  \nu_\mu + \left( \frac{d}{s} \right) \rightarrow \mu^- + c + X \\
  \\leftrightarrow \mu^+ + \nu_\mu + X'
  \]

  \[
  \bar{\nu}_\mu + \left( \frac{\bar{d}}{\bar{s}} \right) \rightarrow \mu^+ + \bar{c} + X \\
  \\leftrightarrow \mu^- + \bar{\nu}_\mu + X'
  \]

- Charm production can probe strange sea, measure charm mass and \( V_{cd} \)

- High statistics opposite sign dimuon samples were acquired by CDHS, CCFR, NOMAD, CHORUS, NUTEV
### 3.7 Charm production

- Some results from opposite sign dimuons:
  - Cross-section: between 0.2%-1% depending on energy
  - Measurement charm mass (average): \( \langle m_c^{LO} \rangle = 1.43 \pm 0.10 \)
  - Strange sea asymmetry
  - Measurement \( V_{cd} \) (average):
    \[
    V_{cd}^{LO} = 0.232 \pm 0.010
    \]
    \[
    V_{cd}^{NLO} = 0.246 \pm 0.016
    \]