Accelerator Physics
Transverse motion

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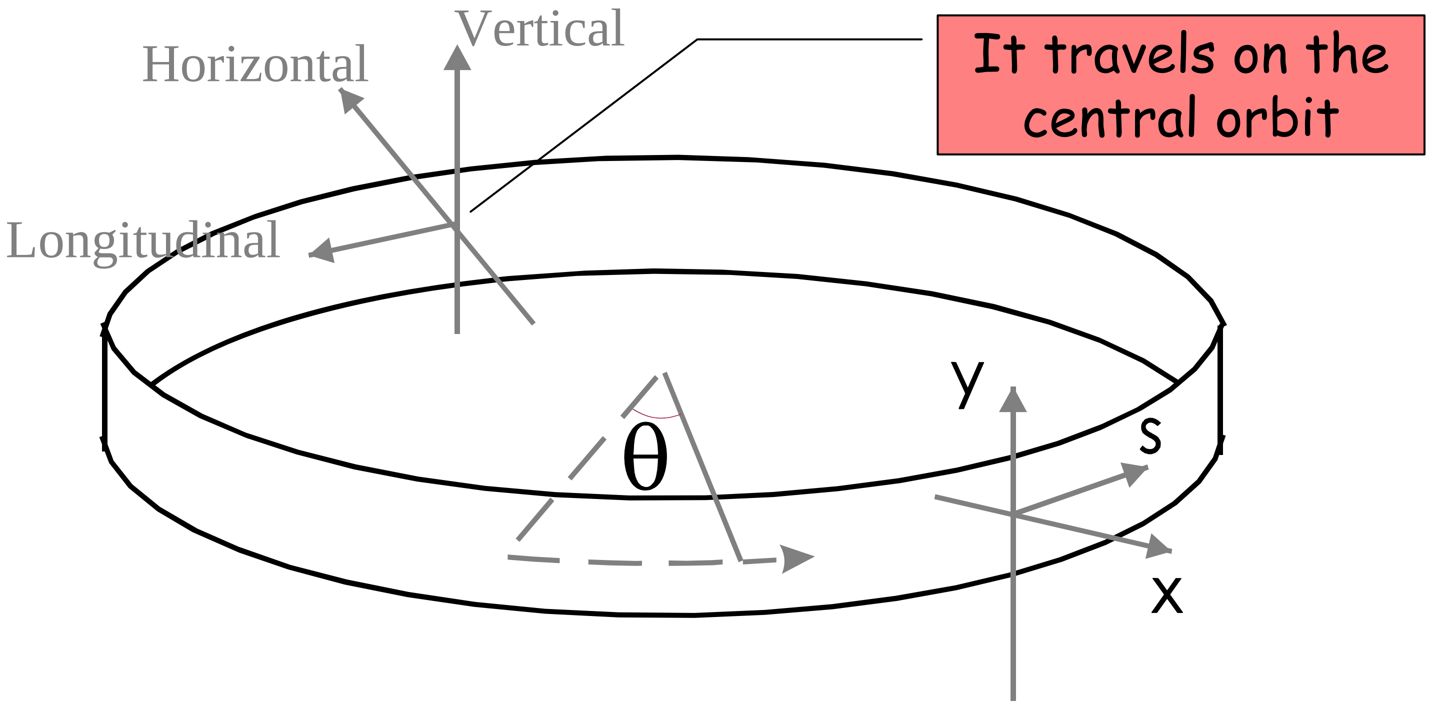
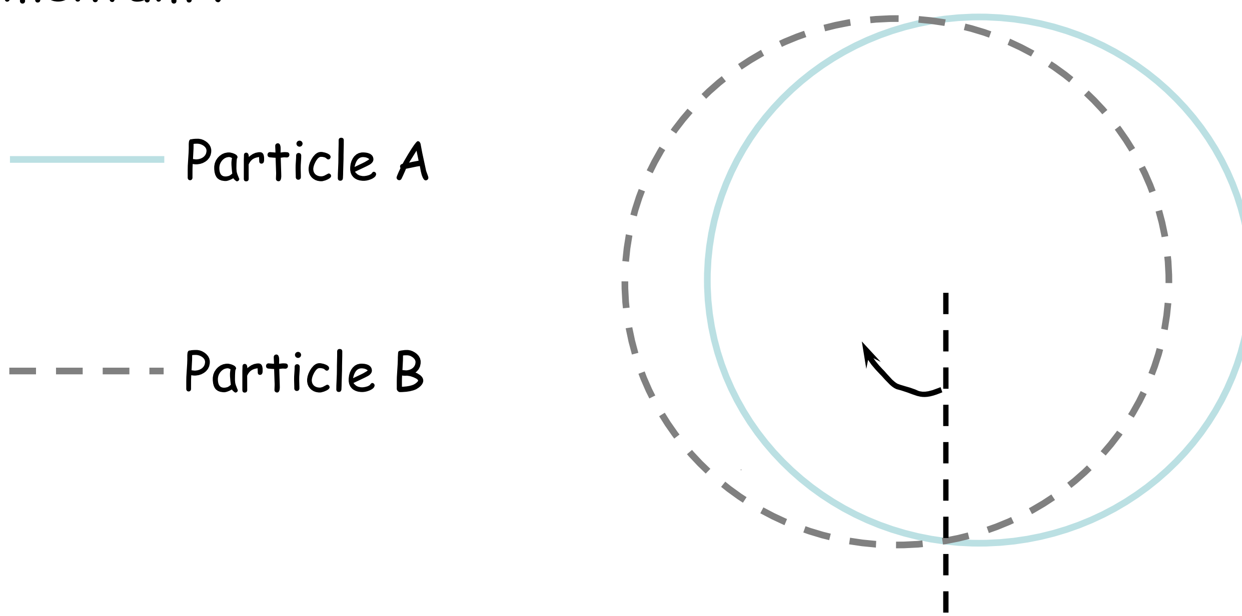
Accelerator co-ordinates



Rotating Cartesian Co-ordinate System

Two particles in a dipole field

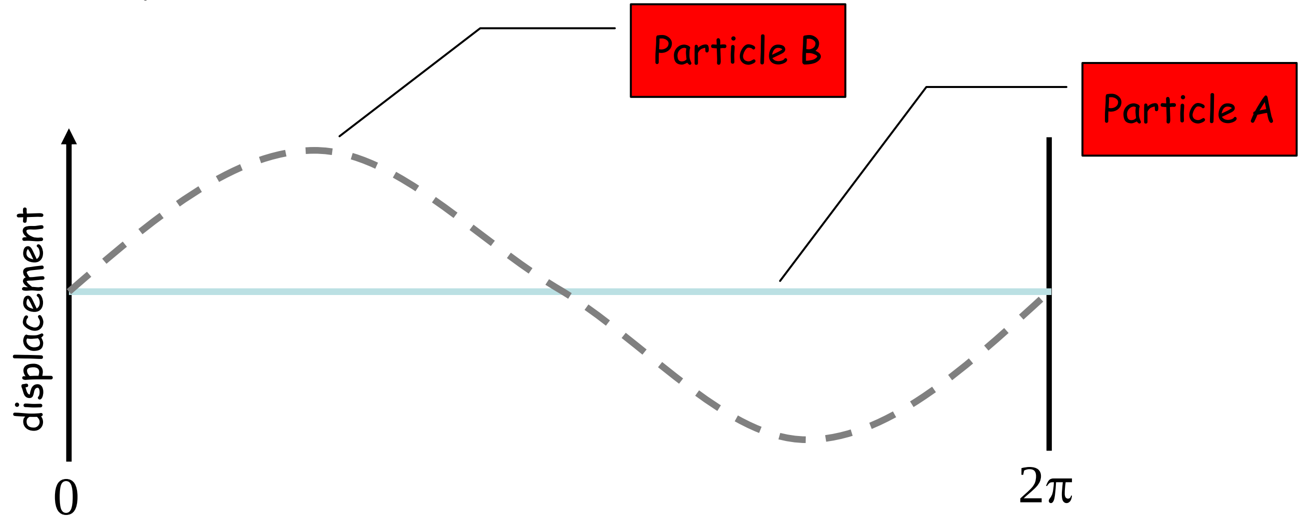
- What happens with two particles that travel in a dipole field with different initial angles, but with equal initial position and equal momentum ?



- Assume that $B\rho$ is the same for both particles.
- Lets unfold these circles……

The 2 trajectories unfolded

- ✓ The horizontal displacement of particle B with respect to particle A.



- ✓ Particle B oscillates around particle A.
- ✓ This type of oscillation forms the basis of all transverse motion in an accelerator.
- ✓ It is called **'Betatron Oscillation'**

Dipole magnet

- A dipole with a uniform dipolar field deviates a particle by an angle θ.
- The deviation angle θ depends on the length L and the magnetic field B.
- The angle θ can be calculated:

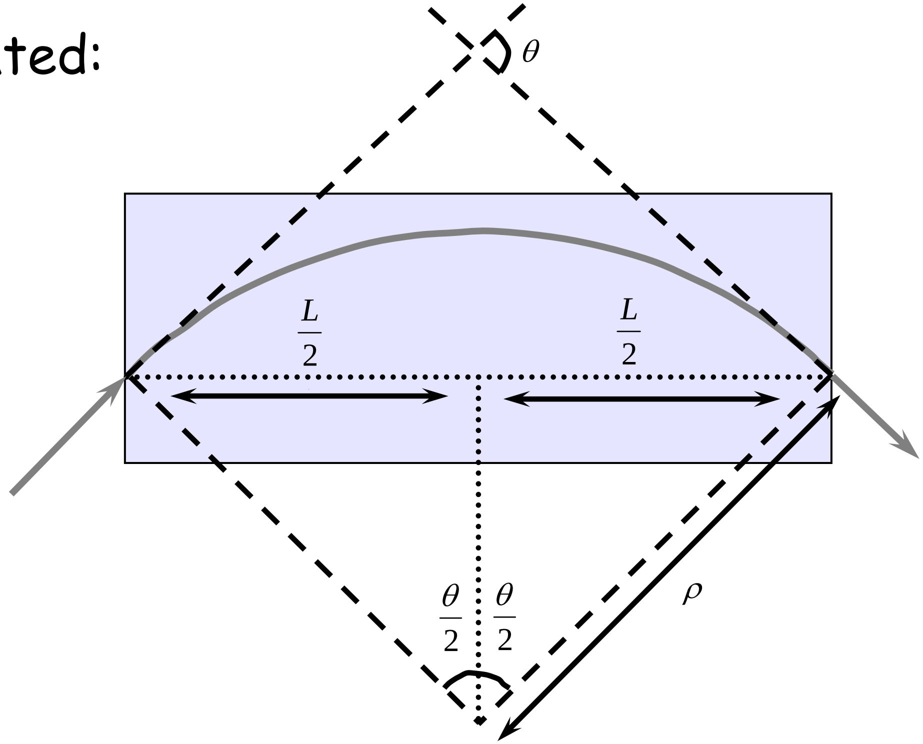
$$\sin\left(\frac{\theta}{2}\right) = \frac{L}{2\rho} = \frac{1}{2}\frac{LB}{(B\rho)}$$

- If θ is small:

$$\sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2}$$

- So we can write:

$$\theta = \frac{LB}{(B\rho)}$$



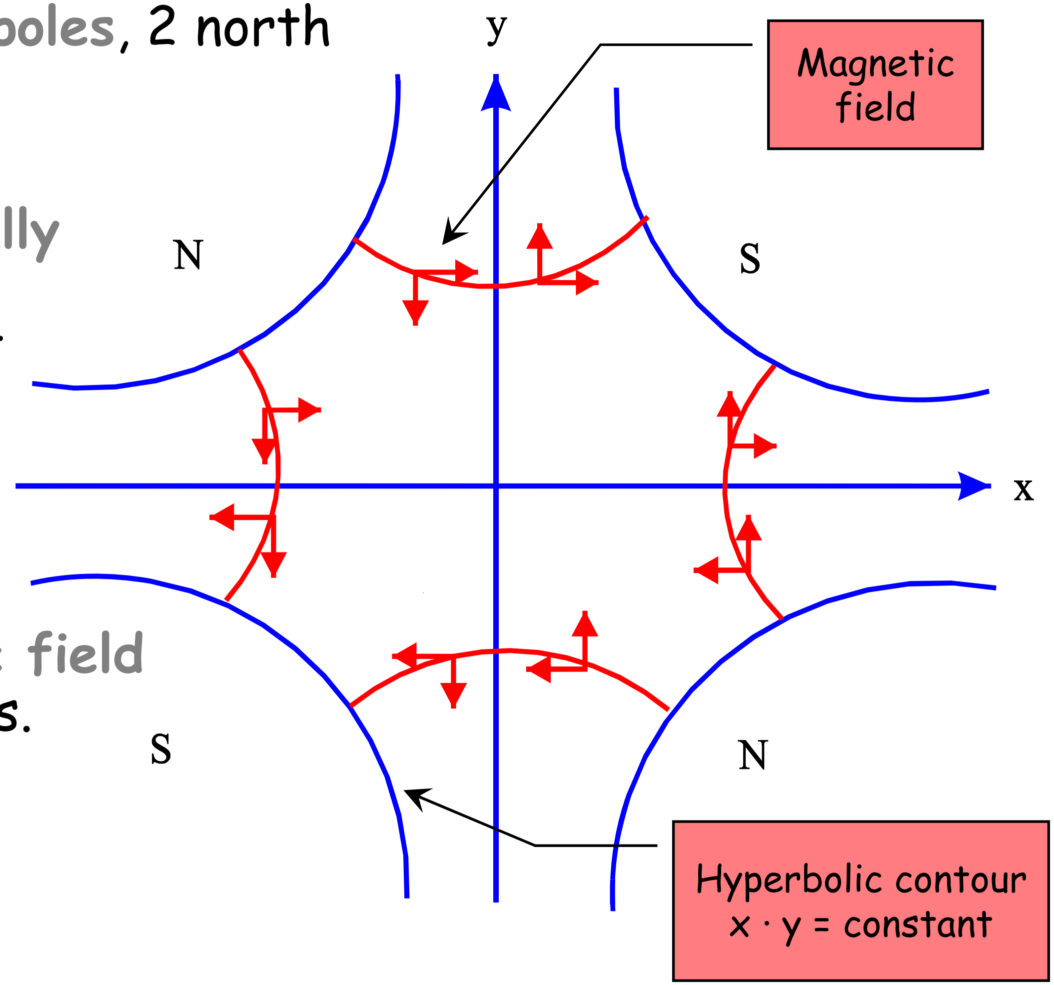
'Stable' or 'unstable' motion ?

- ✓ Since the horizontal trajectories close we can say that the horizontal motion in our simplified accelerator with only a horizontal dipole field is **'stable'**
- ✓ What can we say about the vertical motion in the same simplified accelerator ? Is it **'stable'** or **'unstable'** and why ?
- ✓ What can we do to make this motion stable ?
- ✓ We need some element that 'focuses' the particles back to the reference trajectory.
- ✓ This extra focusing can be done using:

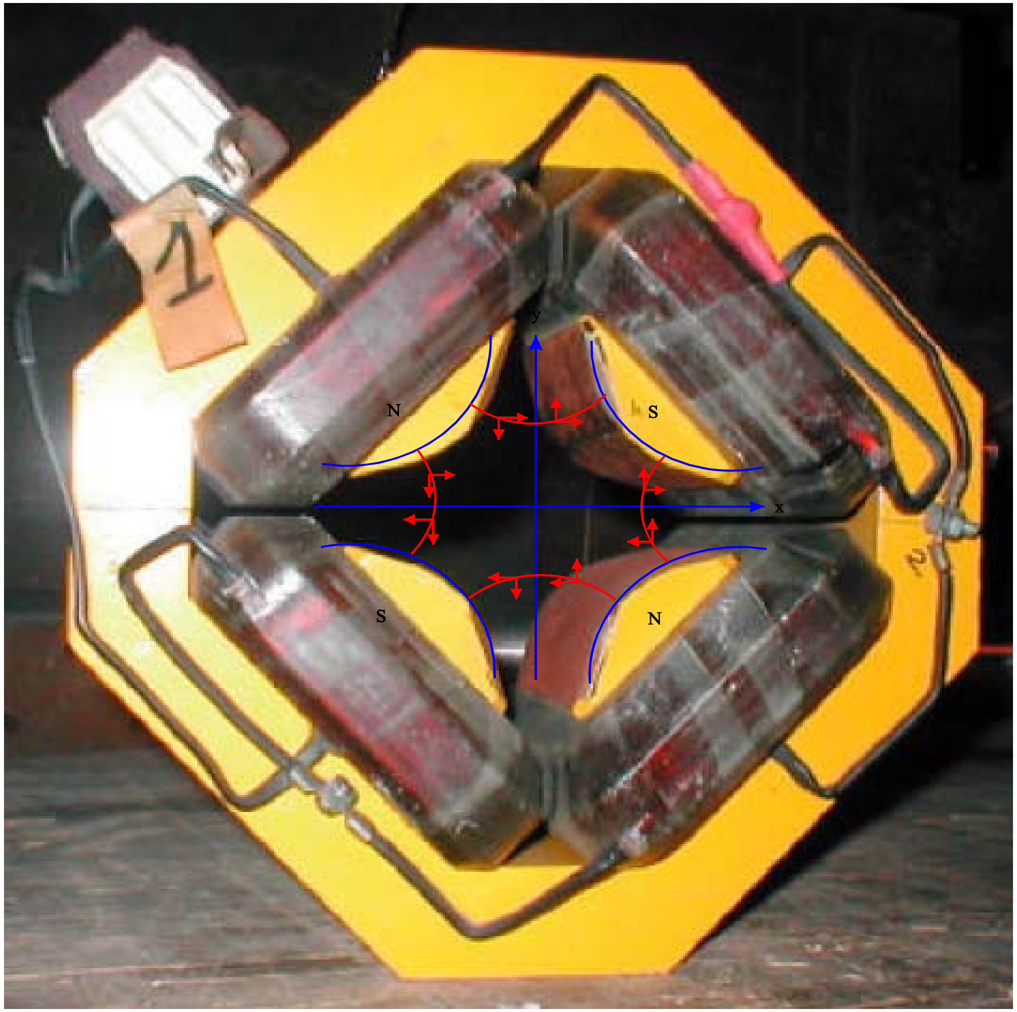
Quadrupole magnets

Quadrupole Magnet

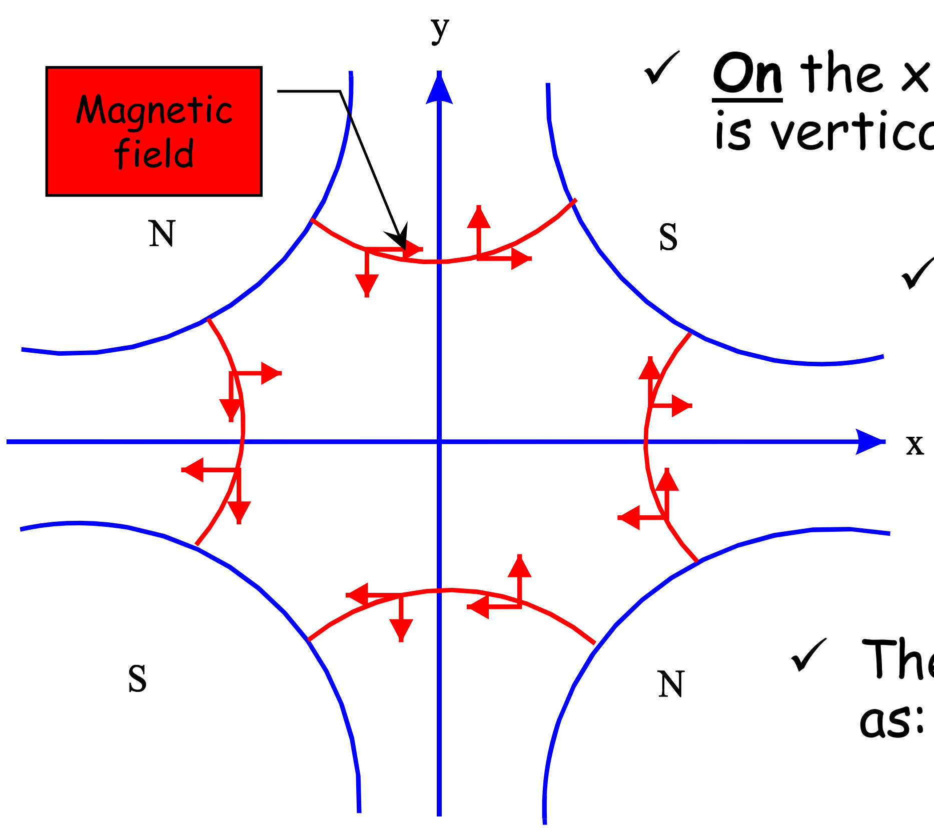
- ✓ A Quadrupole has 4 poles, 2 north and 2 south
- ✓ They are symmetrically arranged around the centre of the magnet
- ✓ There is no magnetic field along the central axis.



Resistive Quadrupole magnet

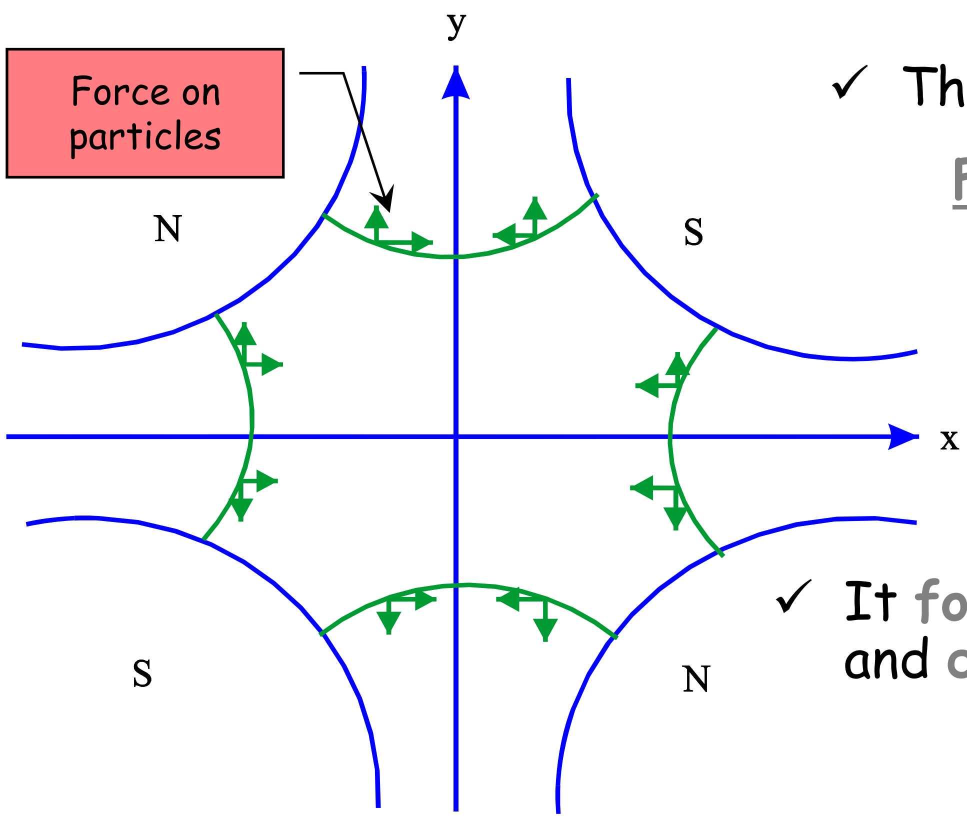


Quadrupole fields



- ✓ **<u>On</u>** the x-axis (horizontal) the field is vertical and given by: $B_y \propto x$
- ✓ **<u>On</u>** the y-axis (vertical) the field is horizontal and given by: $B_x \propto y$
- ✓ The field <u>gradient</u>, <u>K</u> is defined as: $\frac{d(By)}{dx} \quad (Tm^{-1})$
- ✓ The <u>'normalised gradient'</u>, <u>k</u> is defined as: $\frac{K}{(B\rho)}(m^{-2})$

Types of quadrupoles



✓ This is a:

Focusing Quadrupole (QF)

✓ It focuses the beam horizontally and defocuses the beam vertically.

✓ Rotating this magnet by 90° will give a:

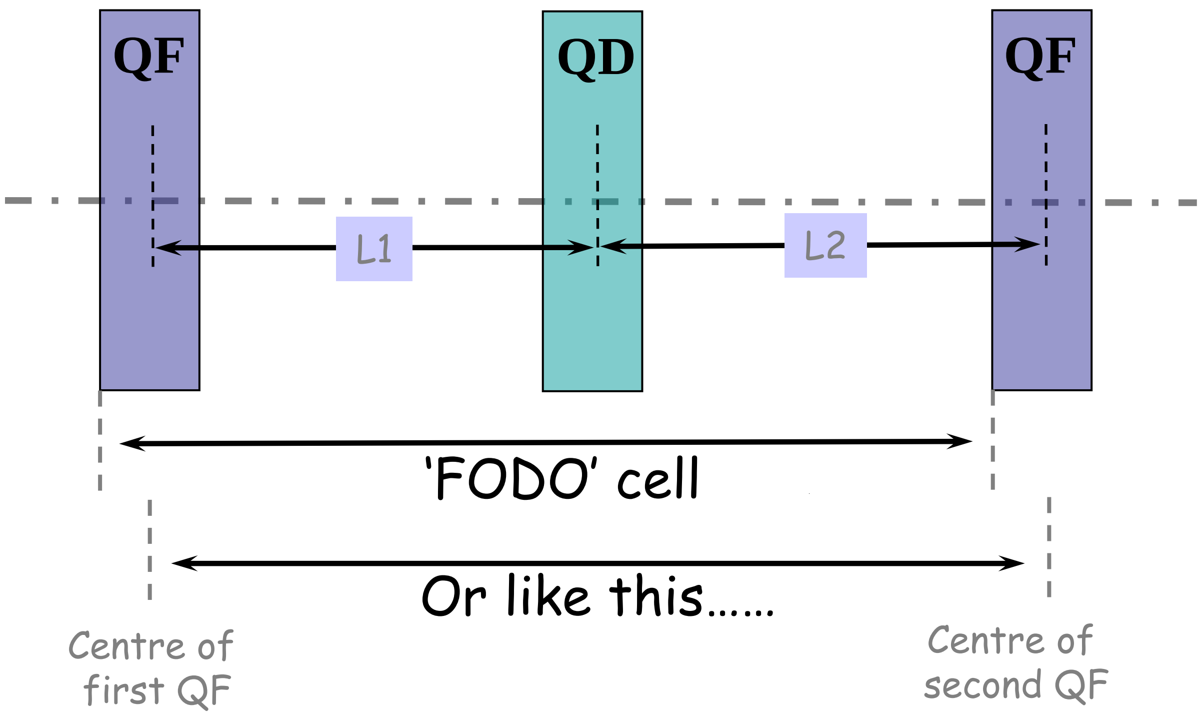
Defocusing Quadrupole (QD)

Focusing and Stable motion

- Using a combination of focusing (QF) and defocusing (QD) quadrupoles solves our problem of 'unstable' vertical motion.
- It will keep the beams focused in **both planes** when the position in the accelerator, type and strength of the quadrupoles are well chosen.
- By now our accelerator is composed of:
  - **Dipoles**, constrain the beam to some closed path (orbit).
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions.
- A combination of focusing and defocusing sections that is very often used is the so called: FODO lattice.
- This is a configuration of magnets where focusing and defocusing magnets alternate and are separated by non-focusing drift spaces.

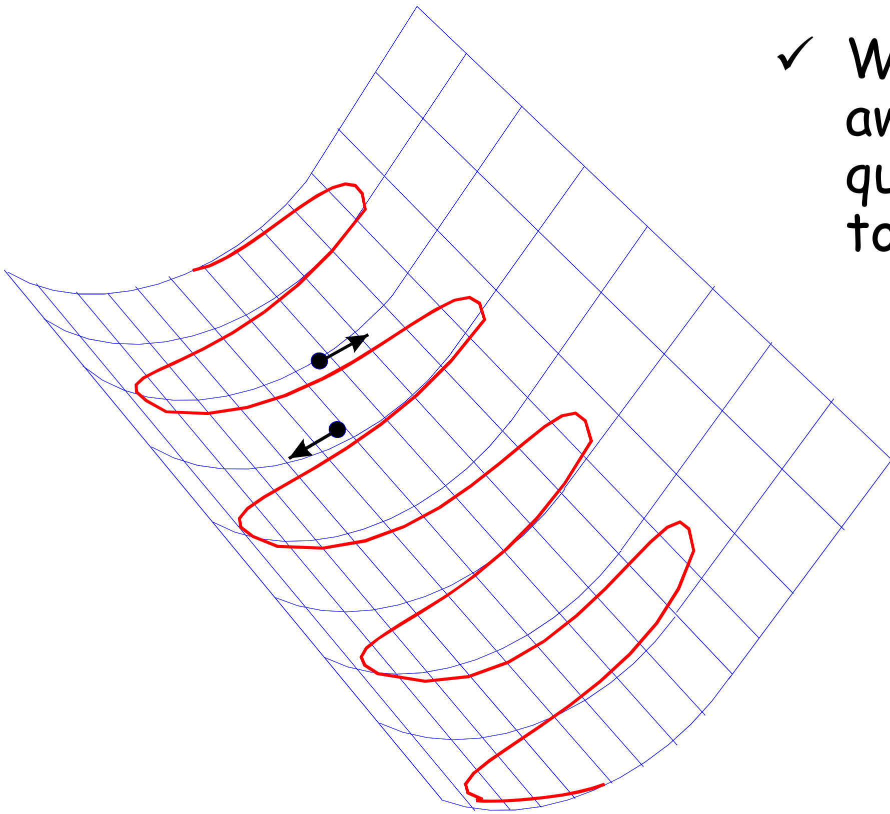
FODO cell

✓ The 'FODO' cell is defined as follows:



The mechanical equivalent

- ✓ The gutter below illustrates how the particles in our accelerator behave due to the quadrupolar fields.

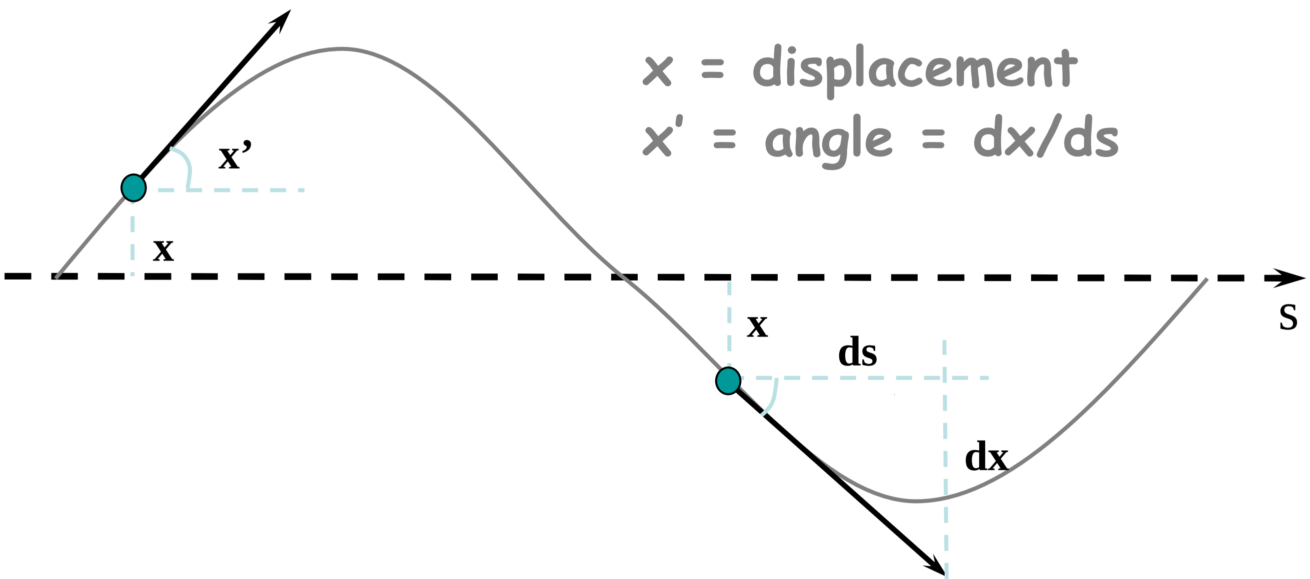


- ✓ Whenever a particle beam diverges away from the central orbit the quadrupoles focus them back towards the central orbit.

- ✓ **How can we represent the focusing gradient of a quadrupole in this mechanical equivalent ?**

The particle characterized

- A particle during its transverse motion in our accelerator is characterized by:
  - Position or displacement from the central orbit.
  - Angle with respect to the central orbit.



x = displacement

x' = angle = dx/ds

- This is a motion with a constant restoring coefficient, similar to the pendulum

Hill's equation

These transverse oscillations are called **<u>Betatron Oscillations</u>**, and they exist in both horizontal and vertical planes.
The number of such oscillations/turn is $\mathbf{Q_x}$ or $\mathbf{Q_y}$. (**<u>Betatron Tune</u>**)

<u>(Hill's Equation)</u> describes this motion

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

If the restoring coefficient (**K**) is constant in "**s**" then this is just SHM

Remember "**s**" is just longitudinal displacement around the ring

Hill's equation (2)

- ✓ In a real accelerator K varies strongly with 's'.
- ✓ Therefore we need to solve Hill's equation for K varying as a function of 's'

$$\boxed{\frac{d^2x}{ds^2} + K(s)x = 0}$$

- ✓ The phase advance and the amplitude modulation of the oscillation are determined by the variation of K around (along) the machine.

- ✓ The overall oscillation amplitude will depend on the initial conditions.

Solution of Hill's equation (1)

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- ✓ 2nd order differential equation.
- ✓ Guess a solution:

$$x = \sqrt{\varepsilon.\beta(s)}\cos(\phi(s) + \phi_0)$$

- ✓ ε and ϕ_0 are constants, which depend on the <u>initial conditions</u>.
- ✓ $\beta(s)$ = the <u>amplitude modulation</u> due to the changing focusing strength.
- ✓ $\phi(s)$ = the <u>phase advance</u>, which also depends on focusing strength.

Benasque

Solution of Hill's equation (2)

CERN

- ✓ Define some parameters

$$\alpha = -\beta'/2$$

$$\beta = \omega^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

- ✓ …and let $\phi = (\phi(s) + \phi_0)$

Remember ϕ is still a function of s

$$x = \sqrt{\varepsilon}.\omega(s)\cos\phi$$

- ✓ In order to solve Hill's equation we differentiate our guess, which results in:

$$x' = \sqrt{\varepsilon}\frac{d\omega}{ds}\cos\phi - \sqrt{\varepsilon}\omega\phi'\sin\phi$$

- ✓ ……and differentiating a second time gives:

$$x'' = \sqrt{\varepsilon}\omega''\cos\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega\phi''\sin\phi - \sqrt{\varepsilon}\omega\phi'^2\cos\phi$$

- ✓ Now we need to substitute these results in the original equation.

Solution of Hill's equation (3)

- ✓ So we need to substitute $x = \sqrt{\varepsilon.\beta(s)}\cos\left(\phi(s)+\phi_0\right)$

  and its second derivative

$$x'' = \sqrt{\varepsilon}\omega''\cos\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega\phi''\sin\phi - \sqrt{\varepsilon}\omega\phi'^2\cos\phi$$

  into our initial differential equation $\dfrac{d^2x}{ds^2} + K(s)x = 0$

- ✓ This gives:

$$\sqrt{\varepsilon}\omega''\cos\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega\phi''\sin\phi - \sqrt{\varepsilon}\omega\phi'^2\cos\phi$$
$$+K(s)\sqrt{\varepsilon}\omega\cos\phi = 0$$

Sine and Cosine are orthogonal and will never be 0 at the same time

Solution of Hill's equation (4)

$$\sqrt{\varepsilon}\omega''\cos\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega'\phi'\sin\phi - \sqrt{\varepsilon}\omega\phi''\sin\phi - \sqrt{\varepsilon}\omega\phi'^2\cos\phi$$
$$+K(s)\sqrt{\varepsilon}\omega\cos\phi = 0$$

- ✓ Using the 'Sin' terms $\longrightarrow$ $2\omega'\phi' + \omega\phi'' = 0$ $\longrightarrow$ $2\omega\omega'\phi' + \omega^2\phi'' = 0$
- ✓ We defined $\beta = \omega^2$, which after differentiating gives $\beta' = 2\omega\omega'$
- ✓ Combining $2\omega\omega'\phi' + \omega^2\phi'' = 0$ and $\beta' = 2\omega\omega'$ gives:

$$\frac{d\beta}{ds} = \frac{d\beta}{d\omega}\frac{d\omega}{ds}$$

$$\beta'\phi' + \beta\phi'' = (\beta\phi')' = 0$$

- ✓ Which is the case as: $\beta\phi' = const. = 1$ since $\phi' = \frac{d\phi}{ds} = \frac{1}{\beta}$
- ✓ **So our guess seems to be correct**

Solution of Hill's equation (5)

- ✓ Since our solution was correct we have the following for x:

$$x = \sqrt{\varepsilon.\beta}\cos\phi$$

$$\frac{d\omega}{ds} = \frac{\beta'}{2\omega} = -\frac{\alpha}{\sqrt{\beta}}$$

- ✓ For x' we have now: $x' = \sqrt{\varepsilon}\frac{d\omega}{ds}\cos\phi - \sqrt{\varepsilon}\omega\phi'\sin\phi$

$$\omega = \sqrt{\beta}$$

- ✓ Thus the expression for x' finally becomes:

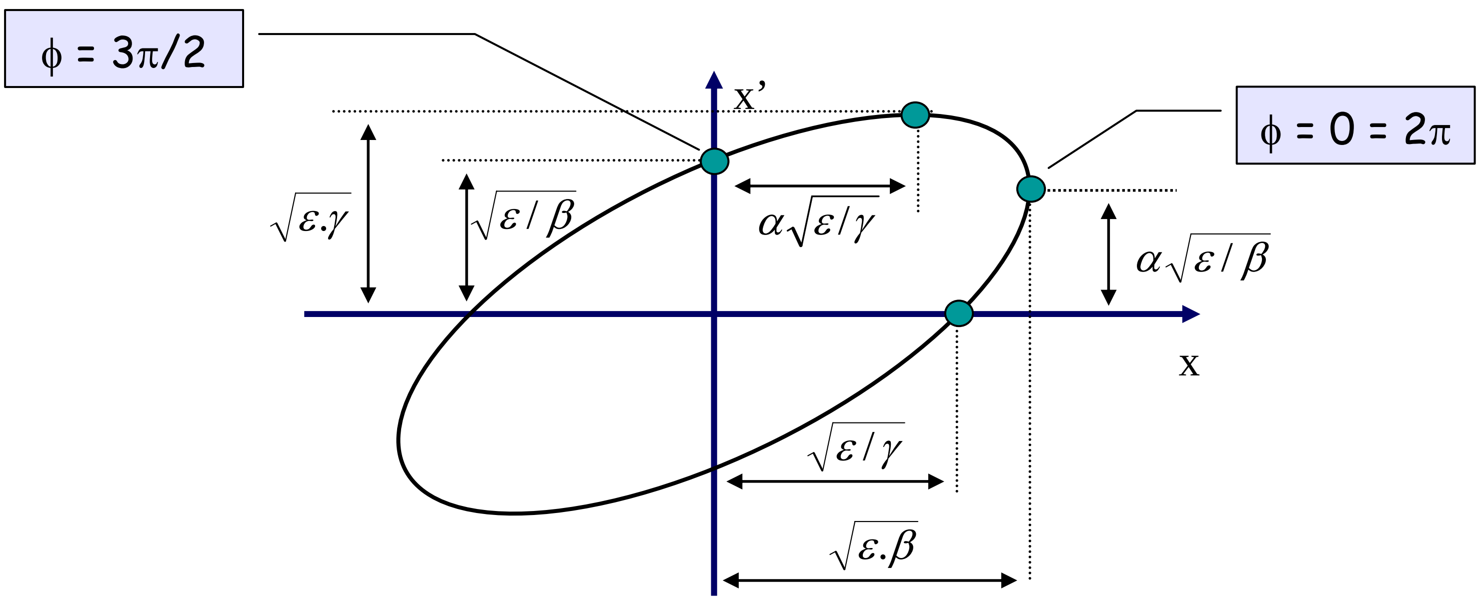
$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos\phi - \sqrt{\varepsilon/\beta}\sin\phi$$

Phase Space Ellipse

- ✓ So now we have an expression for x and x'

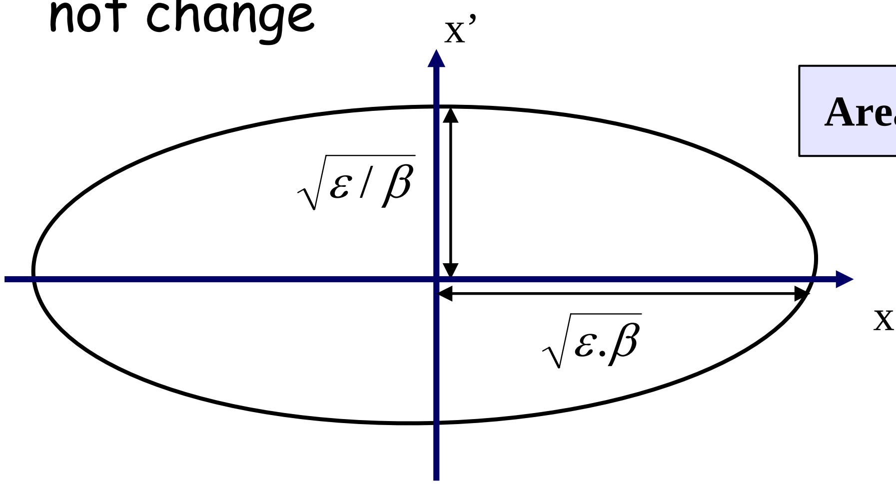
$$x = \sqrt{\varepsilon.\beta}\cos\phi$$ and $$x' = -\alpha\sqrt{\varepsilon/\beta}\cos\phi - \sqrt{\varepsilon/\beta}\sin\phi$$

- ✓ If we plot x' versus x we get an ellipse, which is called the phase space ellipse.

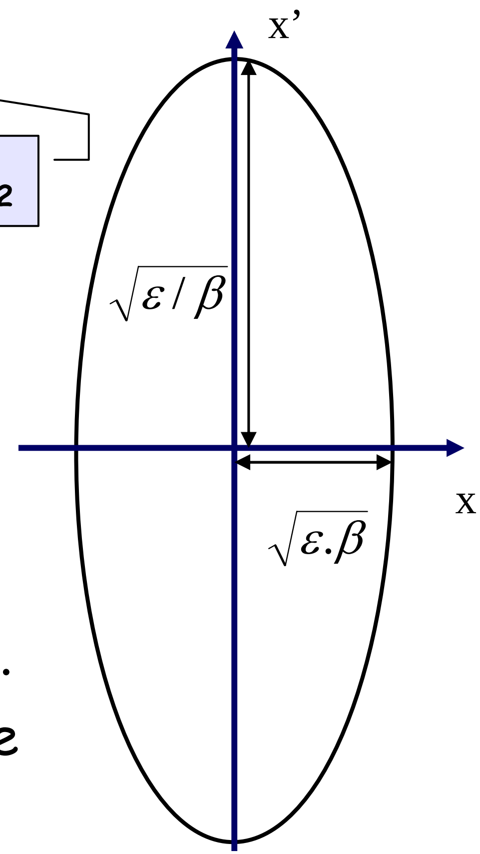


Phase Space Ellipse (2)

- As we move around the machine the shape of the ellipse will change as β changes under the influence of the quadrupoles
- However the area of the ellipse (πε) does not change

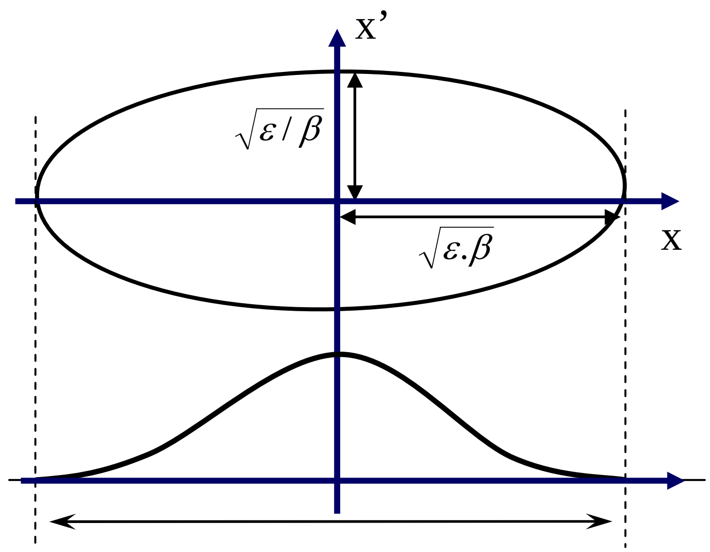


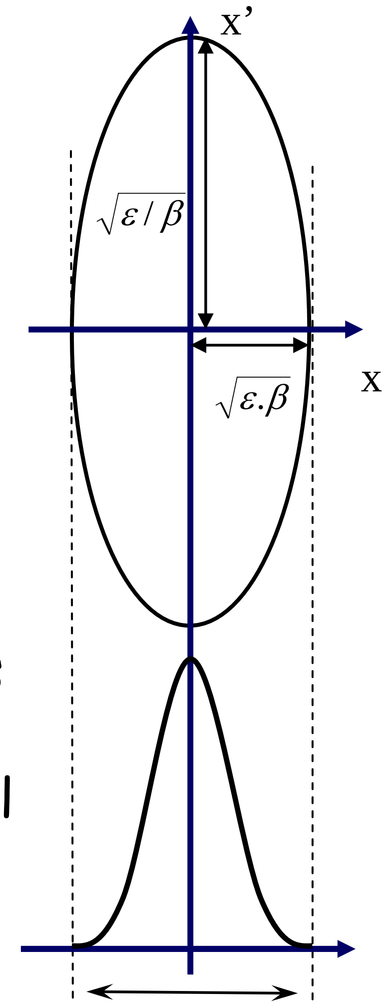
Area = π · r_1 · r_2



- ε is called the transverse emittance and is determined by the initial beam conditions.
- The units are meter · radians, but in practice we use more often mm · mrad.

Phase Space Ellipse (3)

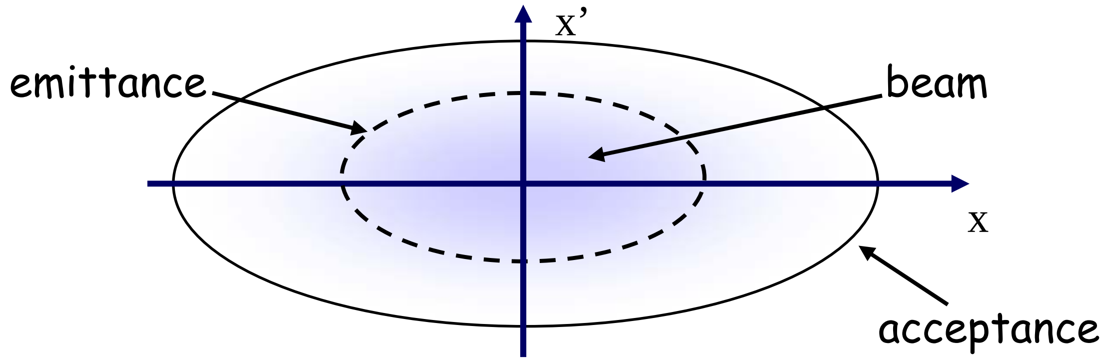




- ✓ The projection of the ellipse on the x-axis gives the Physical transverse beam size.
- ✓ The variation of $\beta(s)$ around the machine will tell us how the transverse beam size will vary.

Emittance & Acceptance

- To be rigorous we should define the emittance slightly differently.
  - Observe all the particles at a single position on one turn and measure both their position and angle.
  - This will give a large number of points in our phase space plot, each point representing a particle with its co-ordinates x, x'.



- The emittance is the area of the ellipse, which contains a defined percentage, of the particles.
- The acceptance is the maximum area of the ellipse, which the emittance can attain without losing particles.

Matrix Formalism

- ✓ Lets represent the particles transverse position and angle by a column matrix.

$$\begin{pmatrix} x \\ x' \end{pmatrix}$$

- ✓ As the particle moves around the machine the values for x and x' will vary under influence of the dipoles, quadrupoles and drift spaces.
- ✓ These modifications due to the different types of magnets can be expressed by a **Transport Matrix M**
- ✓ If we know x_1 and x_1' at some point s_1 then we can calculate its position and angle after the next magnet at position S_2 using:

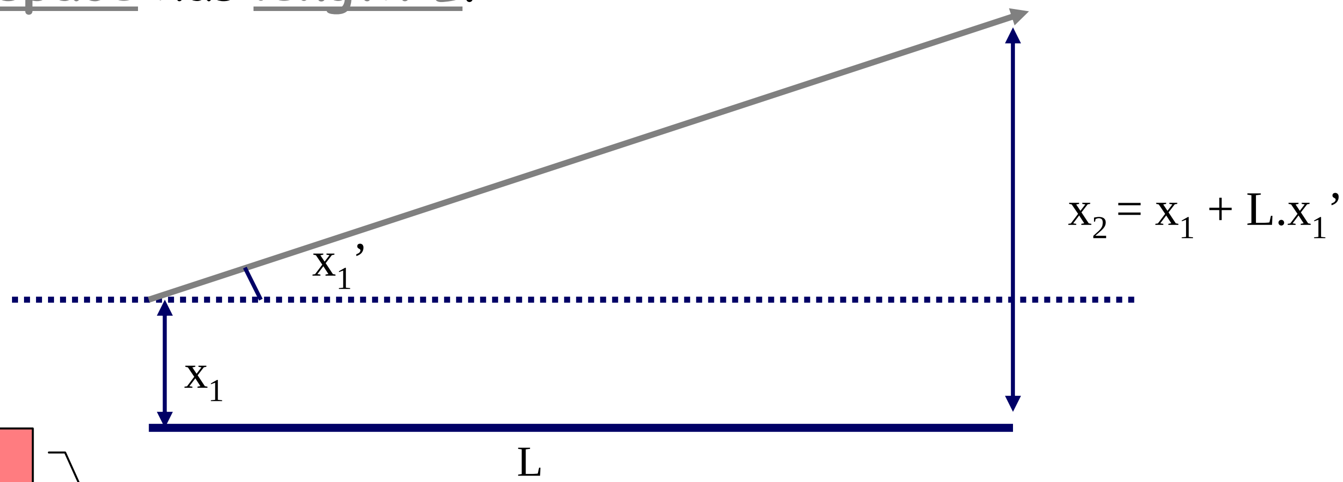
$$\begin{pmatrix} x(s_2) \\ x(s_2)' \end{pmatrix} = M \begin{pmatrix} x(s_1) \\ x(s_1)' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x(s_1) \\ x(s_1)' \end{pmatrix}$$

How to apply the formalism

- ✓ If we want to know how a particle behaves in our machine as it moves around using the matrix formalism, we need to:
  - ✓ Split our machine into separate element as dipoles, focusing and defocusing quadrupoles, and drift spaces.
  - ✓ Find the matrices for all of these components
  - ✓ Multiply them all together
  - ✓ Calculate what happens to an individual particle as it makes one or more turns around the machine

Matrix for a drift space

- ✓ A drift space contains no magnetic field.
- ✓ A drift space has length L.

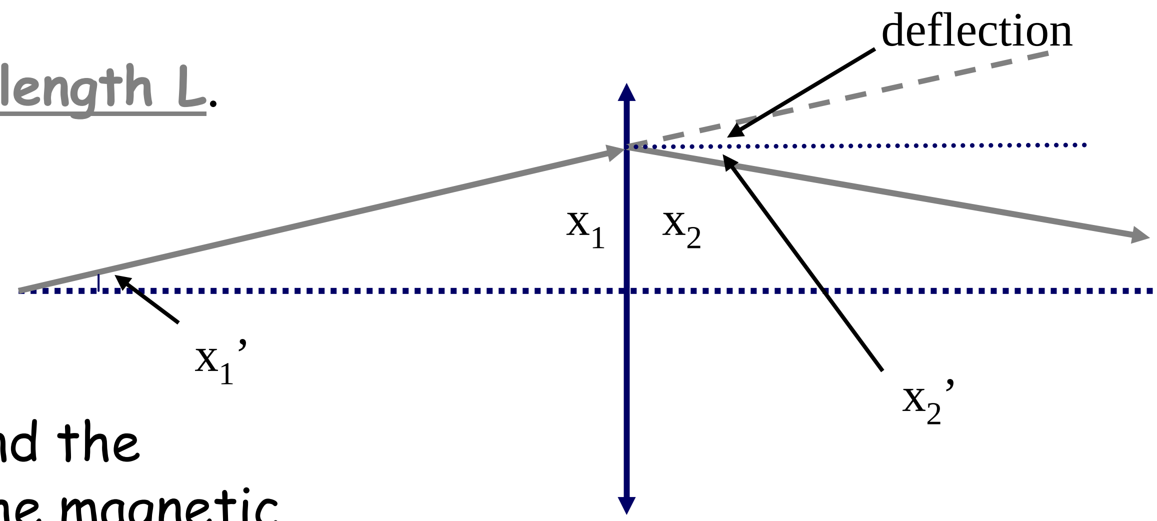


x_1' small

$$\left.\begin{aligned} x_2 &= x_1 + L x_1' \\ x_2' &= 0 + x_1' \end{aligned}\right\} \longrightarrow \boxed{\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}}$$

Matrix for a quadrupole

✓ A quadrupole of length L.



Remember $B_y \propto x$ and the deflection due to the magnetic field is: $\dfrac{LB_y}{(B\rho)} = -\dfrac{LK}{(B\rho)} \cdot x$

Provided L is small

$$\left.\begin{aligned} x_2 &= x_1 + 0 \\ x_2' &= -\frac{LK}{(B\rho)} x_1 + x_1' \end{aligned}\right\} \longrightarrow \begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{LK}{(B\rho)} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

Matrix for a quadrupole (2)

- ✓ We found :

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{LK}{(B\rho)} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

- ✓ Define the focal length of the quadrupole as f= $\frac{(B\rho)}{KL}$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

A quick recap......

- ✓ We solved Hill's equation, which led us to the definition of transverse emittance and allowed us to describe particle motion in phase space in terms of β, α, etc...
- ✓ We constructed the Transport Matrices corresponding to drift spaces and quadrupoles.
- ✓ **Now we must combine these matrices with the solution of Hill's equation to evaluate β, α, etc...**

Matrices & Hill's equation

- We can multiply the matrices of our drift spaces and quadrupoles together to form a transport matrix that describes a larger section of our accelerator.
- These matrices will move our particle from one point $(x(s_1),x'(s_1))$ on our phase space plot to another $(x(s_2),x'(s_2))$, as shown in the matrix equation below.

$$\begin{pmatrix} x(s_2) \\ x'(s_2) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x(s_1) \\ x'(s_1) \end{pmatrix}$$

- The elements of this matrix are fixed by the elements through which the particles pass from point s_1 to point s_2.
- However, we can also express (x,x') as solutions of Hill's equation.

$$x = \sqrt{\varepsilon.\beta}\cos\phi \quad \text{and} \quad x' = -\alpha\sqrt{\varepsilon/\beta}\cos\phi - \sqrt{\varepsilon/\beta}\sin\phi$$

Matrices & Hill's equation (2)

$$x = \sqrt{\varepsilon.\beta}\cos(\mu+\phi)$$

$$x = \sqrt{\varepsilon.\beta}\cos\phi$$

$$\begin{pmatrix} x(s_2) \\ x'(s_2) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x(s_1) \\ x'(s_1) \end{pmatrix}$$

$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos(\mu+\phi) - \sqrt{\varepsilon/\beta}\sin(\mu+\phi)$$

$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos\phi - \sqrt{\varepsilon/\beta}\sin\phi$$

- ✓ Assume that our transport matrix describes a complete turn around the machine.
- ✓ Therefore : $\beta(s_2) = \beta(s_1)$
- ✓ Let μ be the change in betatron phase over one complete turn.
- ✓ Then we get for $x(s_2)$:

$$x(s_2) = \sqrt{\varepsilon.\beta}\cos(\mu+\phi) = a\sqrt{\varepsilon.\beta}\cos\phi - b\alpha\sqrt{\varepsilon/\beta}\cos\phi - b\sqrt{\varepsilon/\beta}\sin\phi$$

Matrices & Hill's equation (3)

- So, for the position x at s2 we have...

$$\sqrt{\varepsilon.\beta}\,\underbrace{\cos(\mu+\phi)}_{\cos\phi\cos\mu-\sin\phi\sin\mu} = a\sqrt{\varepsilon.\beta}\cos\phi - b\alpha\sqrt{\varepsilon/\beta}\cos\phi - b\sqrt{\varepsilon/\beta}\sin\phi$$

- Equating the 'sin' terms gives: $-\sqrt{\varepsilon.\beta}\sin\mu\sin\phi = -b\sqrt{\varepsilon/\beta}\sin\phi$
- Which leads to: $b = \beta\sin\mu$
- Equating the 'cos' terms gives:

$$\sqrt{\varepsilon.\beta}\cos\mu\cos\phi = a\sqrt{\varepsilon.\beta}\cos\phi - \alpha\sqrt{\varepsilon.\beta}\sin\mu\cos\phi$$

- Which leads to: $a = \cos u + \alpha\sin\mu$
- We can repeat this for c and d.

Matrices & Twiss parameters

✓ Remember previously we defined:

$$\alpha = \frac{-\beta'}{2} = -\omega\omega'$$

$$\beta = \omega^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

✓ These are called <u>TWISS parameters</u>

✓ Remember also that m is the total betatron phase advance over one complete turn is.

$$Q = \frac{\mu}{2\pi}$$

Number of betatron oscillations per turn

✓ Our transport matrix becomes now:

$$\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Lattice parameters

$$\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- ✓ This matrix describes one complete turn around our machine and will vary depending on the starting point (s).
- ✓ If we start at any point and multiply all of the matrices representing each element all around the machine we can calculate α, β, γ and μ for that specific point, which then will give us $\beta(s)$ and Q
- ✓ If we repeat this many times for many different initial positions (s) we can calculate our Lattice Parameters for all points around the machine.

Lattice calculations and codes

- ✓ Obviously m (or Q) is not dependent on the initial position 's', but we can calculate the change in betatron phase, dm, from one element to the next.
- ✓ Computer codes like "MAD" or "Transport" vary lengths, positions and strengths of the individual elements to obtain the desired beam dimensions or envelope '$\beta(s)$' and the desired 'Q'.
- ✓ Often a machine is made of many individual and identical sections (FODO cells). In that case we only calculate a single cell and not the whole machine, as the the functions β (s) and dμ will repeat themselves for each identical section.
- ✓ The insertion section have to be calculated separately.

The $\beta(s)$ and Q relation.

- ✓ $Q = \frac{\mu}{2\pi}$, where μ = Δφ over a complete turn
- ✓ But we also know: $\frac{d\phi(s)}{ds} = \frac{1}{\beta(s)}$
- ✓ This leads to: $Q = \frac{1}{2\pi}\int_{o}^{s} \frac{ds}{\beta(s)}$

Over one complete turn

- ✓ Increasing the focusing strength decreases the size of the beam envelope (β) and increases Q and vice versa.

Tune corrections

- ✓ What happens if we change the focusing strength slightly?
- ✓ The Twiss matrix for our 'FODO' cell is given by:

$$\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- ✓ Add a small QF quadrupole, with strength dK and length ds.
- ✓ This will modify the 'FODO' lattice, and add a horizontal focusing term:

$$\begin{pmatrix} 1 & 0 \\ -dkds & 1 \end{pmatrix} \qquad dk = \frac{dK}{(B\rho)} \qquad f = \frac{(B\rho)}{dKds}$$

- ✓ The new Twiss matrix representing the modified lattice is:

$$\begin{pmatrix} 1 & 0 \\ -dkds & 1 \end{pmatrix}\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Tune corrections (2)

- This gives $\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -dkds(\cos\mu + \sin\mu) - \gamma\sin\mu & -dkds\beta\sin\mu + \cos\mu - \alpha\sin\mu \end{pmatrix}$

- This extra quadrupole will modify the phase advance μ for the FODO cell.

New phase advance — $\mu_1 = \mu + d\mu$ — Change in phase advance

- If dμ is small then we can ignore changes in β

- So the new Twiss matrix is just:

$$\begin{pmatrix} \cos\mu_1 + \alpha\sin\mu_1 & \beta\sin\mu_1 \\ -\gamma\sin\mu_1 & \cos\mu_1 - \alpha\sin\mu_1 \end{pmatrix}$$

Tune corrections (3)

- These two matrices represent the same FODO cell therefore:

$$\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -dkds(\cos\mu + \sin\mu) - \gamma\sin\mu & -dkds\,\beta\sin\mu + \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- Which equals:

$$\begin{pmatrix} \cos\mu_1 + \alpha\sin\mu_1 & \beta\sin\mu_1 \\ -\gamma\sin\mu_1 & \cos\mu_1 - \alpha\sin\mu_1 \end{pmatrix}$$

- Combining and compare the first and the fourth terms of these two matrices gives:

$$2\cos\mu_1 = 2\cos\mu - dk\,ds\,\beta\sin\mu$$

Only valid for change in $\delta\ell$

$<<$

Tune corrections (4)

$$2\cos\mu_1 = 2\cos\mu - dk\, ds\, \beta \sin\mu$$

Remember $\mu_1 = \mu + d\mu$ and $d\mu$ is small

$$2\cos\mu - 2\sin\mu\, d\mu$$

$$2\sin\mu\, d\mu = dk\, ds\, \beta \sin\mu$$

$$d\mu = \frac{1}{2} dk\, ds\, \beta$$

,but: $dQ = d\mu/2\pi$

In the **horizontal plane** this is a **QF**

$$dQh = +\frac{1}{4\pi} dk.ds.\beta h$$

If we follow the same reasoning for both transverse planes for both **QF** and **QD** quadrupoles

QD

$$dQv = +\frac{1}{4\pi}\beta v.dk_D.ds_D - \frac{1}{4\pi}\beta v.dk_F.ds_F$$

$$dQh = -\frac{1}{4\pi}\beta h.dk_D.ds_D + \frac{1}{4\pi}\beta h.dk_F.ds_F$$

QF

Tune corrections (5)

Let $\mathbf{dk_F = dk}$ for **QF** and $\mathbf{dk_D = dk}$ for **QD**

$\beta_{\mathbf{hF}}, \beta_{\mathbf{vF}} = \beta$ at **QF** and $\beta_{\mathbf{hD}}, \beta_{\mathbf{vD}} = \beta$ at **QD**

Then:

$$\boxed{\begin{pmatrix} dQv \\ dQh \end{pmatrix} = \underline{\begin{pmatrix} \frac{1}{4\pi}\beta_{vD} & \frac{-1}{4\pi}\beta_{vF} \\ \frac{-1}{4\pi}\beta_{hD} & \frac{1}{4\pi}\beta_{hF} \end{pmatrix}} \begin{pmatrix} dk_D ds \\ dk_F ds \end{pmatrix}}$$

This matrix relates the change in the tune to the change in strength of the quadrupoles.
We can invert this matrix to calculate change in quadrupole field needed for a given change in tune